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# Teaching guide

for Seeing through arithmetic 6

This is the Teaching Guide for *Seeing Through Arithmetic 6*, the sixth book of The Basic Mathematics Program, which is a unit of THE CURRICULUM FOUNDATION SERIES ®, published by Scott, Foresman and Company.

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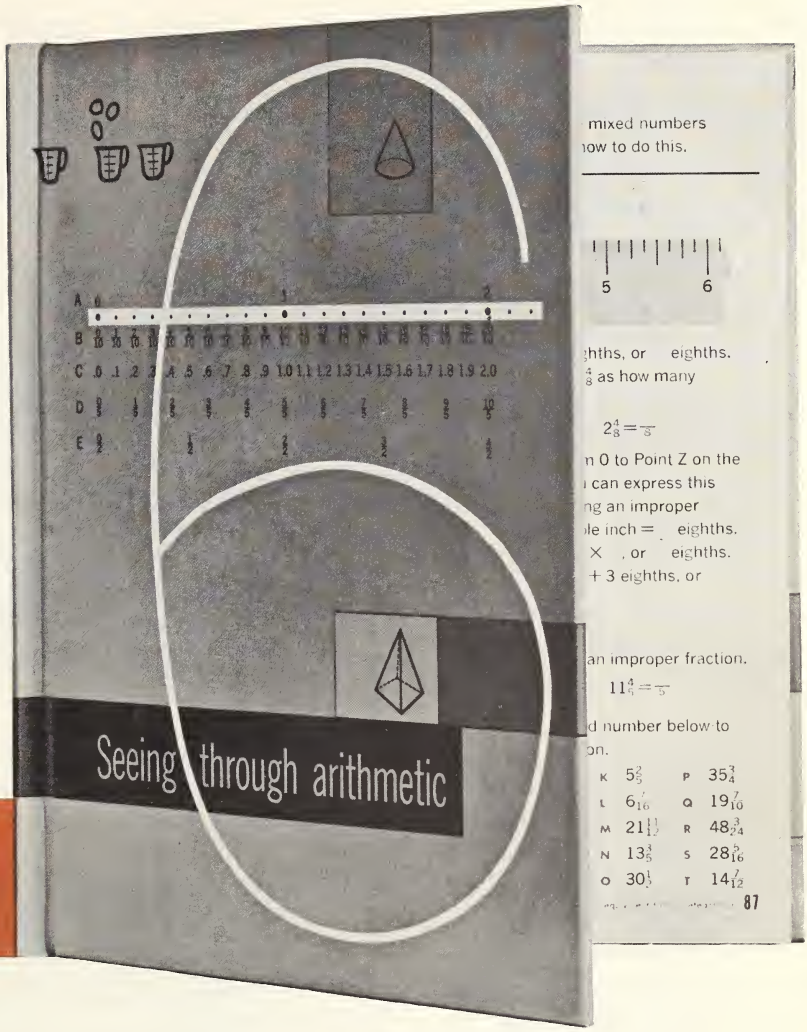
This book was prepared by the editorial staff under the direction of George E. Russell, Directing Editor of The Basic Mathematics Program.

This book was designed by William Nicoll and illustrated by Janet LaSalle.

# Teachin




guide

# Scott, Foresman and Company

Chicago, Atlanta, Dallas, Palo Alto, Fair Lawn, N.J.

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# Getting acquainted with this book

your guide to an up-to-date point of view about arithmetic and methods of teaching it

In this *Teaching Guide* you will find several different kinds of help in planning your work.

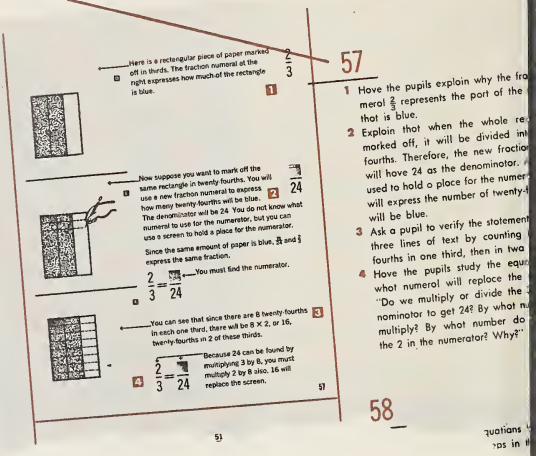
The various kinds of help are so organized as to meet the varied needs of teachers. For some, the brief notes keyed to the reproduced pages from the pupils' book may be sufficient. Others may want to use the Expanded Notes and activity suggestions. You can decide for yourself how much use to make of these various materials in the *Teaching Guide*.

A section called

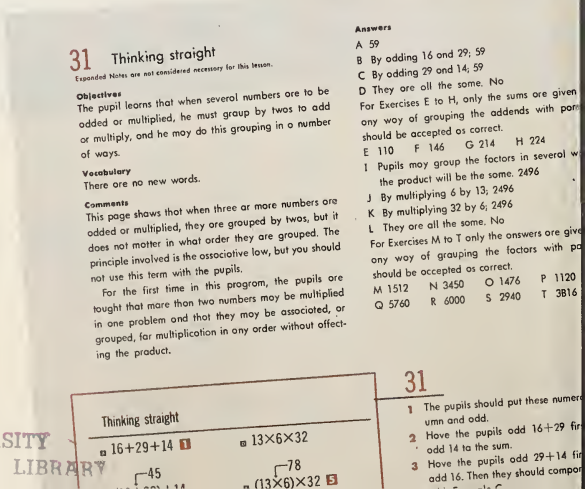
## Lesson briefs

on pages 21-269

provides a quick overview of how to teach each block of work. The teaching suggestions for each page are keyed to a reproduction of the page from the pupils' book.


Here is a rectangular piece of paper marked off in thirds. The fraction numeral at the right expresses how much of the rectangle is blue.
Now suppose you want to mark off the same rectangle in twenty-fourths. You will use a new fraction numeral to express how many twenty-fourths will be blue. The denominator will be 24. You do not know what numeral to use for the numerator, but you can use a screen to hold a place for the numerator.
Since the same amount of paper is blue, $\frac{2}{3}$ and $\frac{\square}{24}$ express the same fraction.
$\frac{2}{3} = \frac{\square}{24}$ You must find the numerator.
You can see that since there are 8 twenty-fourths in each one third, there will be 8 × 2, or 16, twenty-fourths in 2 of these thirds.
$\frac{2}{3} = \frac{16}{24}$ Because 24 can be found by multiplying 3 by 8, you must multiply 2 by 8 also. 16 will replace the screen.
57
1 Have the pupils explain why the fra
merol $\frac{2}{3}$ represents the part of the
that is blue.
2 Explain that when the whole re
marked off, it will be divided int
fourths. Therefore, the new fractio
will have 24 as the denominator.
used to hold a place for the numer
will express the number of twenty-f
will be blue.
3 Ask a pupil to verify the statement
three lines of text by counting
fourths in one third, then in two
4 Have the pupils study the equi
what numeral will replace the
"Do we multiply or divide the
nominator to get 24? By what nu
multiply? By what number do
the 2 in the numerator? Why?"
58
53

Also included are objectives, a list of new vocabulary, brief general comments on the block of work, and an answer key for all problems and exercises.


31 Thinking straight
Expanded Notes are not considered necessary for this lesson.
Objectives
The pupil learns that when several numbers are to be added or multiplied, he must group by twos to add or multiply, and he may do this grouping in a number of ways.
Vocabulary
There are no new words.
Comments
This page shows that when three or more numbers are added or multiplied, they are grouped by twos, but it does not matter in what order they are grouped. The principle involved is the associative law, but you should not use this term with the pupils.
For the first time in this program, the pupils are taught that more than two numbers may be multiplied in one problem and that they may be associated, or grouped, for multiplication in any order without affecting the product.
Answers
A 59
B By adding 16 and 29; 59
C By adding 29 and 14; 59
D They are all the same. No
For Exercises E to H, only the sums are given
any way of grouping the addends with pos
should be accepted as correct.
E 110 F 146 G 214 H 224
I Pupils may group the factors in several w
the product will be the same. 2496
J By multiplying 6 by 13; 2496
K By multiplying 32 by 6; 2496
L They are all the same. No
For Exercises M to T only the answers are give
any way of grouping the factors with pa
should be accepted as correct.
M 1512 N 3450 O 1476 P 1120
Q 5760 R 6000 S 2940 T 3816
Thinking straight
16+29+14 1
13×6×32
45
78
(13×6)×32 5
31
1 The pupils should put these numer
umn and add.
2 Have the pupils add 16+29 fir
add 14 to the sum.
3 Have the pupils add 29+14 fir
add 16. Then they should compar

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A section called

## Expanded notes

on pages 271-369

gives detailed suggestions for teaching each lesson. The Expanded Notes for each block of work are organized to help provide for individual differences. First there are teaching suggestions for the class as a whole, then special suggestions for able pupils, and finally, suggestions for the slow learners. Sections called "Charting the Course" are interspersed throughout the Expanded Notes to introduce a block of work or related blocks of work. They explain the fundamental mathematical ideas that are to be taught, and why the ideas are introduced at that point. They may be read all at one time or individually as the work with the book progresses. You should, however, have these general principles in mind as you examine the more detailed suggestions that are provided for teaching each lesson. You will be in the best possible position to plan your lessons if you study both these Expanded Notes and the section entitled Lesson Briefs.

A section called

## Activities

on pages 371-399

provides games and activities useful in arithmetic. References to them are made in the Expanded Notes, but they can be used at other times.

A section called

## Reference materials

on pages 401-424

provides you with a useful bibliography, a table of mathematical content, a description of the problem types, a grade-placement chart for the program, a suggested time schedule, alphabetical word list, and an index.

You will find it profitable
to read the brief overview
of the major features of the
*Seeing Through Arithmetic*
program

described in the section

## Guiding principles

that begins on the next page.

Explained there
are the features of this program,
all of which
have been designed to help you
and your pupils discover
that seeing through arithmetic
can be an interesting
and satisfying
experience.

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# Guiding principles

Arithmetic is a fundamental subject. Pupils should learn to use it with ease,

with confidence,

and with enjoyment.

The best way to reach this goal

is to help pupils gain insight into arithmetic.

The objective: **insight**

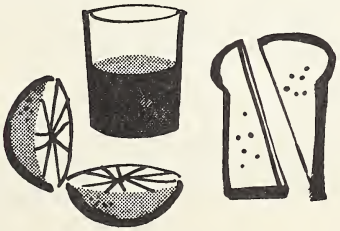
"Insight" is a term that has come into prominence in modern psychology. Insight depends upon the recognition of relationships. The number system, for example, involves many relationships. When pupils have gained insight as to these relationships, a great many facts that would otherwise remain isolated are recognized as elements of a few general principles. The processes of arithmetic become understandable, and not just a set of arbitrary rules. It is then possible for pupils to gain skill in the computational processes with less reliance upon sheer memory and with more satisfaction.

*Seeing Through Arithmetic 6* provides for the development of insight and skill in many ways. Among these are:

1. Fundamental mathematical ideas, and in particular those underlying fractions, rates, and comparisons, are clarified as they have never been in the past.
2. Pupils are taught to use powerful tools of analysis and communication (such as the equation) to help them in problem solving.
3. The learning of arithmetic is simplified, and the time required to develop understanding is shortened, by the way the work is organized.
4. New and improved methods of presentation are systematically used to make the learning of arithmetic easier and more enjoyable than it has been in the past.

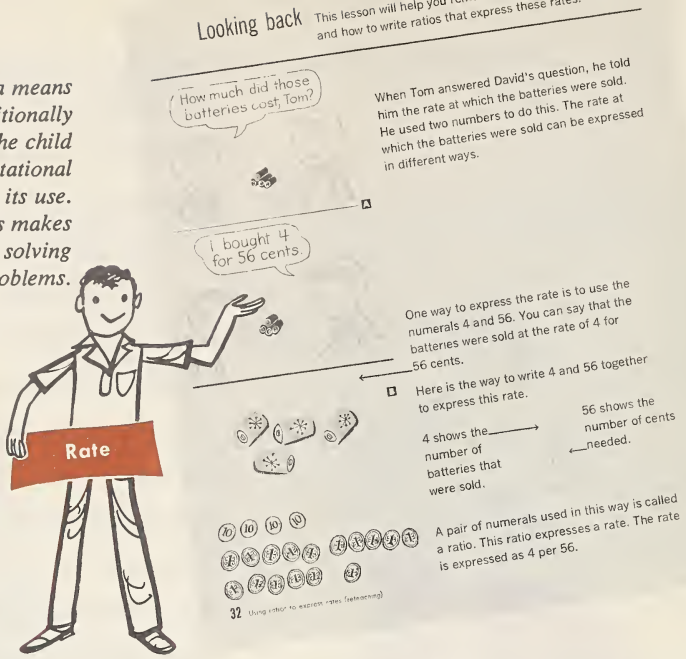
These features and the basis for them are explained in the following sections.

# 1 Developing fundamental mathematical ideas



A modern arithmetic program must emphasize fundamental ideas. These ideas are mathematical concepts and principles. For example, the concept of *fraction* is obviously fundamental, and so also are the principles that underlie computation with fractions. Although these ideas have innumerable practical applications and their origins are rooted in concrete situations, in learning arithmetic the mathematical concepts themselves must be in the focus of attention. The fundamental ideas should stand out and not be obscured by an approach which prematurely dwells upon the rules and the details of computational processes.

*The ratio symbol as a means of expressing rate has traditionally been overlooked. The child has been taught computational procedures to get around its use. The use of equations of ratios makes clear thinking possible in solving rate problems.*


Looking back
How much did those batteries cost, Tom?
When Tom answered David's question, he told him the rate at which the batteries were sold. He used two numbers to do this. The rate at which the batteries were sold can be expressed in different ways.
I bought 4 for 56 cents.
One way to express the rate is to use the numerals 4 and 56. You can say that the batteries were sold at the rate of 4 for 56 cents.
Here is the way to write 4 and 56 together to express this rate.
4 shows the number of batteries that were sold.
56 shows the number of cents needed.
A pair of numerals used in this way is called a ratio. This ratio expresses a rate. The rate is expressed as 4 per 56.
Rate
32

In *Seeing Through Arithmetic 6* and the other books of this program, the emphasis is upon helping pupils learn basic mathematical ideas. Concrete situations represented by pictures are used to introduce and develop mathematical concepts and principles, and not merely as decorative illustrations.

This emphasis upon fundamental mathematical concepts is in keeping with current recommendations for improvement in the mathematics curriculum. One reason the teaching of mathematical ideas is important is that in the long run it leads to better achievement. When pupils do not understand arithmetic, and attempt to learn it by methods that rely mainly upon drill, the burden upon memory becomes unbearable. Much forgetting occurs, and large amounts of instructional time must be devoted to reteaching and maintenance of skills. Dislike of arithmetic commonly is associated with such situations.

When, however, pupils *understand* the basic concepts, reason can come to the aid of memory. What is done in arithmetic "makes sense" and is not a blind following of rules. Satisfaction replaces excessive frustration, and liking arithmetic often replaces hating it. The ability to remember and use arithmetic is enhanced rather than blocked.

*Seeing Through Arithmetic 6* contains many examples of teaching that emphasize understanding and insight. Because fractions are such an important part of the work at this stage, the next few paragraphs will discuss in some detail the teaching of fractions and of certain related concepts.

The teacher (but not necessarily the pupils) should recognize the distinction between a fraction numeral (a symbol for a fraction) and the idea it stands for (the fraction itself). For example, suppose a pupil has partitioned a bar of candy into 4 equal parts and eaten one of them, so that only 3 parts are left. He can describe how much of his candy remains by saying "three fourths" or by writing the *fraction numeral* $\frac{3}{4}$. There are, however, many other fraction numerals (for example, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{75}{100}$) that may be used to describe the same amount or part of the original bar. Thus $\frac{3}{4}$ and $\frac{6}{8}$ are not to be thought of as two different fractions. Instead, they are two different fraction numerals which represent or name the same fraction.

Moreover, the same symbols may be used to express other situations that are quite different. For example, suppose 3 pieces of a certain kind of candy cost 4 cents. Then 6 pieces will cost 8 cents, 9 pieces

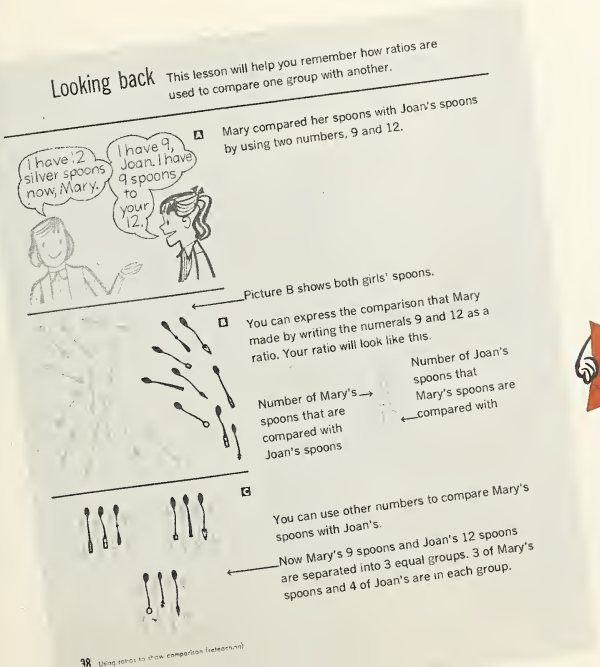
will cost 12 cents, and so on. This situation involves the concept of *rate*, and in this example the rate is "3 pieces for 4 cents" or, more briefly, "3 for 4." This idea is simple and familiar to children.

In this situation, however, the 3 pieces of candy are clearly not a part of, or "fraction" of, the 4 cents. Rate situations like this one are expressed by using *ratios*. For example, the ratio "3 to 4" can be written in the form 3/4. Since fraction numerals can also be written in this way, it takes some clear thinking to see why this form is justified. Eventually, when pupils have developed a clear understanding of *both* fractions and rates (and also comparisons), they may use fraction numerals as terms of a ratio. For example, the ratio ¾ over 1 expresses the same rate as does 3 over 4. Understanding of such expressions rarely develops spontaneously in pupils. It must be carefully developed over a period of time by activities specifically planned for this purpose.

There is another type of everyday situation which is described by using ratios. This situation occurs when two quantities are compared. For example, suppose one girl has a collection of 6 dolls and her friend has 8 dolls. The ratio "6 to 8," or the equal ratio "3 to 4," or the symbol 3/4 can all be used to express a *comparison* of one of the sets of dolls to the other. This situation of the dolls involves neither the fraction idea nor the rate idea.

A careful analysis of situations of these types shows that although they are quite different, when they are expressed mathematically there are some basic similarities in the way the symbols are treated. *There are also some basic differences.* In the past these situations have not been adequately explained to pupils, and the result has been not only lack of understanding but actual confusion and the abandonment of efforts to understand.

In *Seeing Through Arithmetic 5* and earlier books of the program, a beginning was made toward the development of those basic understandings. In *Seeing Through Arithmetic 6,* these ideas are retaught and extended. They are not difficult for pupils who meet them initially under proper guidance and who learn through carefully planned activities. To older pupils and adults the ideas sometimes seem more difficult than they really are, in part because some of these persons have more or less successfully developed formal ways of dealing with the situations without understanding them. In *Seeing Through Arithmetic 6* a foundation is laid for the development of broader


Looking back
This lesson will help you remember how ratios are used to compare one group with another.
I have 12 silver spoons now, Mary.
I have 9, Joan. I have 9 spoons to your 12.
Mary compared her spoons with Joan's spoons by using two numbers, 9 and 12.
Picture B shows both girls' spoons.
You can express the comparison that Mary made by writing the numerals 9 and 12 as a ratio. Your ratio will look like this.
Number of Mary's spoons that are compared with Joan's spoons
Number of Joan's spoons that Mary's spoons are compared with
You can use other numbers to compare Mary's spoons with Joan's.
Now Mary's 9 spoons and Joan's 12 spoons are separated into 3 equal groups. 3 of Mary's spoons and 4 of Joan's are in each group.

*The ratio symbol is also necessary in solving problems that involve ideas of comparison. Problems usually solved by meaningless computational short cuts become clear when equations of ratios are used.*

and richer understandings of these concepts in later grades. The possession of these understandings by the pupil will have far-reaching consequences in problem solving and in his later ability to grow in mathematical insight. Since these ideas have never been adequately clarified, failure to understand fractions and these related basic ideas is natural and not surprising. Instructional programs in arithmetic have been so eager to rush on to computation with fractions that they have neglected these basic insights. In contrast, *Seeing Through Arithmetic 6* provides learning materials that develop, by visual and other methods, understanding and insight concerning these ideas. (See pages 32-46 of the pupils' book.) Then, and not until then, is instruction in computation provided.

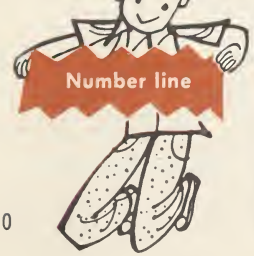
These learning materials on computation also emphasize the development of insight, by visual and other methods, as an essential element in understanding and remembering the various operations (reducing, adding, etc.) with fractions and mixed numbers.

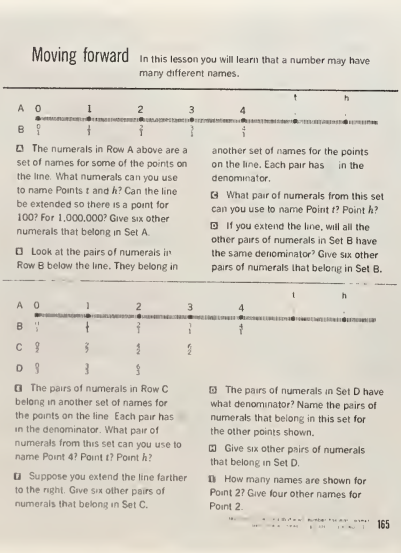
In this brief introduction, it is not possible to discuss at length these and other similar features of *Seeing Through Arithmetic 6*. A careful reading of the pupils' textbook and of the more extended discussions in later sections of this *Teaching Guide* (for example, the notes for pages 32-46 and 52-58) will reveal the care that has been taken to provide th best possible materials for learning to understan fractions, rates, and comparisons and the way sym bols for them are used to deal with problem situa tions. We must, however, call attention to certai features that are new, distinctive, and mathematicall up to date. Among these are the introduction t finding common denominators (pages 66-69 of th pupils' book), the use of a "number line" to illustrat various sets of number names (pages 165-168 an 175-176), the introduction to the "distributive law' (pages 107-109), and the introduction to simpl inequalities (pages 104-106).

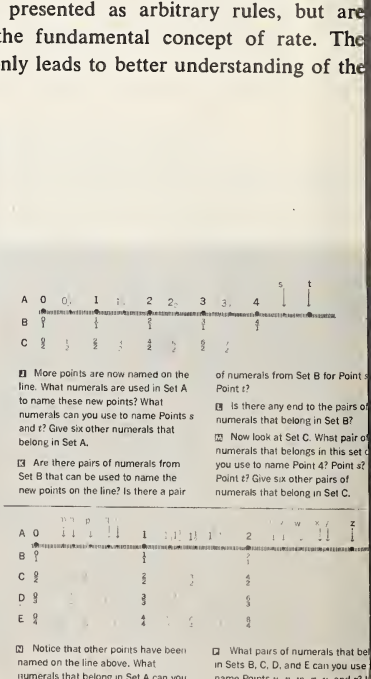
Finally, it should be observed that the meanin of multiplication (and division) must be drastically extended when fractions are involved. Meanings based solely on the combining of equal groups (o separating into equal subgroups) are no longer ade quate. In *Seeing Through Arithmetic 6* this necessary extension is made in a way that is mathematically sound and that does not rely upon superficial and meaningless verbal expressions.

Similarly, the methods of computing area and vol ume are not presented as arbitrary rules, but are based upon the fundamental concept of rate. The method not only leads to better understanding of the

*The number line is a powerful device for extending the child's ideas of the number system to include fractions.*





Moving forward
In this lesson you will learn that a number may have many different names.
The numerals in Row A above are a set of names for some of the points on the line. What numerals can you use to name Points t and h? Can the line be extended so there is a point for 100? For 1,000,000? Give six other numerals that belong in Set A.
Look at the pairs of numerals in Row B below the line. They belong in another set of names for the points on the line. Each pair has in the denominator.
What pair of numerals from this set can you use to name Point t? Point h?
If you extend the line, will all the other pairs of numerals in Set B have the same denominator? Give six other pairs of numerals that belong in Set B.
The pairs of numerals in Row C belong in another set of names for the points on the line. Each pair has in the denominator. What pair of numerals from this set can you use to name Point 4? Point t? Point h?
Suppose you extend the line farther to the right. Give six other pairs of numerals that belong in Set C.
The pairs of numerals in Set D have what denominator? Name the pairs of numerals that belong in this set for the other points shown.
Give six other pairs of numerals that belong in Set D.
How many names are shown for Point 2? Give four other names for Point 2.
165


More points are now named on the line. What numerals are used in Set A to name these new points? What numerals can you use to name Points s and t? Give six other numerals that belong in Set A.
Are there pairs of numerals from Set B that can be used to name the new points on the line? Is there a pair of numerals from Set B for Point s? Point t?
Is there any end to the pairs of numerals that belong in Set B?
Now look at Set C. What pair of numerals that belongs in this set can you use to name Point 4? Point s? Point t? Give six other pairs of numerals that belong in Set C.
Notice that other points have been named on the line above. What numerals that belong in Set A can you use to name Points m, n, p, q, and r? Points u, v, w, x, y, and z?
What pairs of numerals that be in Sets B, C, D, and E can you use t name Points u, v, w, x, y, and z? I there a pair of numerals in each se each point?
Give six other numerals that belong in Set A.
Give six other pairs of numerals that belong in Sets D and E.
166

›pic at the time it is being studied, but also provides far sounder foundation than has hitherto been vailable for the mathematics to be studied in the ınior and senior high schools.

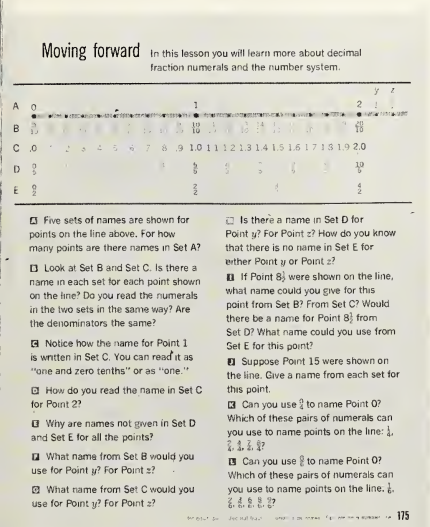
## 2 Analysis and communication

The *Seeing Through Arithmetic* program puts heavy mphasis upon helping pupils learn effective methods ›f mathematical analysis and communication. In ›roblem solving such methods are needed.

### Analyzing the problem situation

n the past, little has been done to help pupils develop kill in analyzing problem situations. The instruc- ional methods have relied heavily upon "cue" words. (Examples: How many *in all?* How many are *left?*) They also have relied on advice that is of no real elp to the pupil. (Examples: Read the problem care- fully. Think what to do with the numbers.) The diffi- culty lay in the pupil's inability to analyze.

Only the simplest mathematical problems can be solved without the help of written symbols. One im-


Moving forward In this lesson you will learn more about decimal fraction numerals and the number system.

1. Five sets of names are shown for points on the line above. For how many points are there names in Set A?
2. Look at Set B and Set C. Is there a name in each set for each point shown on the line? Do you read the numerals in the two sets in the same way? Are the denominators the same?
3. Notice how the name for Point 1 is written in Set C. You can read it as "one and zero tenths" or as "one."
4. How do you read the name in Set C for Point 2?
5. Why are names not given in Set D and Set E for all the points?
6. What name from Set B would you use for Point *y*? For Point *z*?
7. What name from Set C would you use for Point *y*? For Point *z*?
8. Is there a name in Set D for Point *y*? For Point *z*? How do you know that there is no name in Set E for either Point *y* or Point *z*?
9. If Point $8\frac{1}{2}$ were shown on the line, what name could you give for this point from Set B? From Set C? Would there be a name for Point $8\frac{1}{2}$ from Set D? What name could you use from Set E for this point?
10. Suppose Point 15 were shown on the line. Give a name from each set for this point.
11. Can you use $\frac{0}{2}$ to name Point 0? Which of these pairs of numerals can you use to name points on the line: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$?
12. Can you use $\frac{0}{2}$ to name Point 0? Which of these pairs of numerals can you use to name points on the line: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{6}{6}$?

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portant use of the symbols is to make a *record of the thinking* that occurs in trying to solve the problem. For example, the pupil may quite properly see the following as an additive situation.

> Sam had some marbles. A friend gave him 18 more marbles. After accepting them, Sam said, "Now I have 29 marbles." How many marbles did Sam have before he received the gift?

In the *Seeing Through Arithmetic* program the pupil is taught to record his thinking about such a problem in the form of an equation. Suppose he uses the letter $n$ to hold a place for the unknown. He can write the equation $n+18=29$ to show that Sam's original supply of marbles was increased by 18 and the result was a total of 29 marbles. This equation shows that the structure of the problem situation is seen as additive, and serves as a record of this analysis. It thus also provides a simple way to *communicate* the thinking to others. Moreover, since it is clear to the pupil that he cannot add 18 to a number he does not yet know, the equation suggests that another procedure must be found. The equation guides him to a new analysis. Thus he can focus on the 29 marbles that Sam now has, and mentally take away the 18 that Sam received as a gift. This transformation suggests that the answer may be found by subtracting 18 from 29.

The teaching of problem solving in *Seeing Through Arithmetic* is based on clear distinctions among: (1) the recognition of the structural nature of the situations, (2) the use of symbols to record the structure and communicate thinking, and (3) the processes to be used, i.e., addition, subtraction, multiplication, and division. These distinctions are important because, as in the example above, the process used is not always what the structure of the situation seems to suggest. It is not surprising that pupils see the situation mentioned above as additive, because it involves the combining of two groups. Too often this difficulty is not recognized, and the pupil is told only that this is a subtraction problem. Confusion and lack of understanding naturally result. This type of problem situation is only one among many.

Under traditional methods of teaching, the pupil has not learned to use available tools of analysis and communication. The emphasis has been put upon "the answer" and the computational process used to get it. Deliberate analysis of the structure of the

situation has been neglected, or if not, has sometimes been actually misleading. Teachers often say the difficulty in problem solving is that "the pupils cannot read the problems." The pupils usually recognize the individual words; what the teacher means is that the pupils cannot see the structure of the *situations* and symbolize them appropriately. Dependence on "cue" words or other superficial techniques leaves the pupils in a weak and helpless position. They must learn to base their decisions about which process to use on an understanding of the total situation. The development of such an understanding on the part of the child has been of primary concern to the authors of this text.

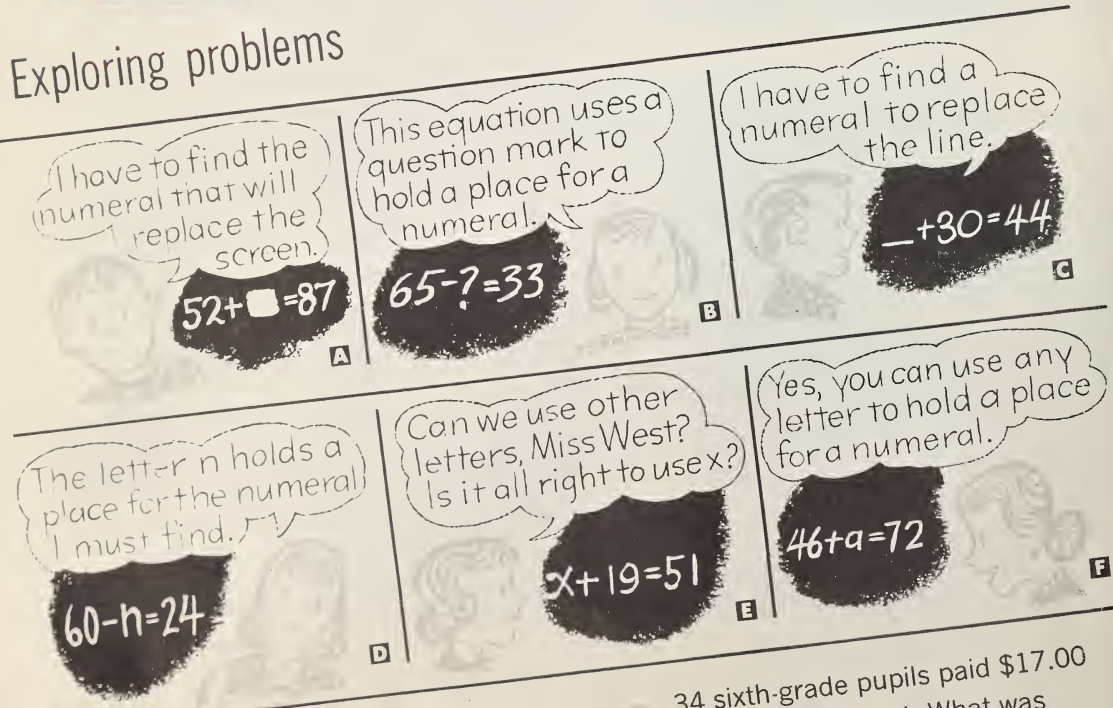
In *Seeing Through Arithmetic,* lessons entitled "Exploring problems" show the pupil how to write equations to describe problem situations. The lessons lead him to *analyze the situation* instead of rushing on to a computational process that may or may no be appropriate. The equation focuses attention o understanding the situation and serves as a record o how the pupil sees and thinks about the situation. I the pupil cannot analyze the situation, any attemp to do the computational step is a blind and ofte futile procedure.

In Books 3 and 4 of the *Seeing Through Arithmeti* program a "screen" (▒) has been used as a symbo to hold a place for an unknown numeral. This "screen" could always be read "what number." In *Seeing Through Arithmetic 5* the symbol *n* for an unknown numeral was introduced, and it also can be read "what number." In *Seeing Through Arithmetic 6* the pupil learns that he can use any symbol or letter to hold the place for the unknown numeral. He also learns to use more than one such place holder in an equation. The use of these symbols introduces no dif-

*By carefully planned stages, the child learns to make equations for the problems solved by arithmetic.*

**The equation**

## Exploring problems


I have to find the numeral that will replace the screen.
52+▒=87
A
This equation uses a question mark to hold a place for a numeral.
65-?=33
B
I have to find a numeral to replace the line.
_+30=44
C
The letter n holds a place for the numeral I must find.
60-n=24
D
Can we use other letters, Miss West? Is it all right to use x?
x+19=51
E
Yes, you can use any letter to hold a place for a numeral.
46+a=72
F

34 sixth-grade pupils paid $17.00

ficulties that are not also encountered when a blank such as in $5+\_\_=8$, or a question mark, as in $5+?=8$, or any similar device is used. Such devices have long been commonplace in arithmetic.

In the simplest situations the appropriate computational process involves the direct use of the addition, subtraction, multiplication, or division process as indicated by the equation. In other situations, as was noted above, the process is not the one suggested by the structure of the situation. In these cases, the pupil must be helped to see why another process can be used. In *Seeing Through Arithmetic,* he is shown how to think about the situation in a way that reveals the appropriate process. He is then ready to compute because he has insight into the total situation.

The basic types of problems introduced in each book are selected from a complete list of such types. A table of these types appears on pages 388-389 of this *Teaching Guide*. This list, which is the result of an analysis by the authors, provides a guide for a consistent and comprehensive treatment of problem solving. The eight simplest types of problems were presented in *Seeing Through Arithmetic 3*. These were reviewed and three new types were introduced in *Seeing Through Arithmetic 4*. Several new types were also introduced in *Seeing Through Arithmetic 5,* and the list is completed in *Seeing Through Arithmetic 6*.

The reader should not infer from this organization for teaching purposes that the pupil is expected to recognize each problem by its type and follow a routine procedure for that type. Some problems can be correctly viewed in more than one way. What is essential is that the pupil who adopts a possible view knows what to do with the situation as he sees it.

### Analyzing more complex situations

In the work of the middle and upper grades, the problem situations become more complex. In many cases they involve such fundamental concepts as rate and comparison by ratio. In these situations an equation in the form of a "proportion" naturally results. The neglect of these concepts and the associated computational techniques is undoubtedly the cause of much of the prevalent difficulty in problem solving. Such basic concepts can be ignored only when rules and similar formal methods of attack on problems are supplied in their place. As noted earlier, in *Seeing Through Arithmetic* the fundamental ideas are carefully taught. As a consequence, equations in the form of proportions become available as powerful tools of analysis. Once available, they become the key to understanding a great variety of supposedly different situations. For example, they become the basis for explaining the familiar rule for finding the area of a rectangle ("multiply the width by the length"). This rule, and many others of similar type, may be regarded as "short-cut" ways of solving an equation in proportion form. Prominent among the other occurrences of this form are those which arise in connection with *per cent* problems. The use of the proportion equation in such problems serves to simplify the ideas and to unify the various "cases" that are customarily recognized.

It should be evident that these instructional methods are not to be regarded as mere "devices" introduced for the sake of being different. On the contrary, they are employed because they are fundamental and cannot be ignored in a program that endeavors to develop genuine understanding of mathematics.

Certain other features of the way problem solving is taught are also noteworthy. One of these is that the pupil is frequently expected to use data that were given (or found by computation) in preceding problems. This resembles the situation in life outside the classroom, where problems do not always occur in isolation. Instead, several related problems are often involved in a total situation.

Another important feature is the manner in which multiple-step problems are analyzed. The pupil is encouraged to see the situation as a whole from the outset. For example, suppose the problem reads as follows: "Tony bought 2 cards of fish hooks with 10 hooks on each card. Jim bought 2 cards of fish hooks with 12 hooks on each card. How many fish hooks did the boys buy in all?" The dominant aspect of this problem is that the total of two groups of unknown size is to be found—the situation is basically additive. Another and secondary aspect of the situation is that the size of each of the groups can be found by multiplication. The pupil who sees the situation this way knows all the steps he is going to follow *before* he begins to compute. This is in contrast to the pupil who starts to compute more or less blindly with no clear idea as to what the subsequent steps are to be.

At first the screen (▤) is used in the equation to hold a place for a numeral that will be found by computation. The numerals that replace these screens are found by computation. Then the equation is rewritten with the numerals inserted. Later, parentheses as symbols of grouping are introduced. As the pupil advances in mathematical maturity, he increasingly needs to become familiar with the use of symbols to communicate information about the structure of mathematics itself as well as the structure of particular problems. For example, eventually he will be expected not only to know that the sum in addition does not depend upon the way in which the addends are grouped, but also to understand how this principle is symbolized by $(a+b)+c=a+(b+c)$. In *Seeing Through Arithmetic 6,* he is introduced to this on page 31.

In *Seeing Through Arithmetic,* pupils are taught to analyze multiple-step problems by methods that emphasize the total situation and then fit the details of the separate steps into this larger picture. Equations are used to symbolize all problem situations. This procedure becomes increasingly meaningful and useful as problems involving rates, comparisons, and fractions are all seen to involve similar equations.

# 3 Simplified organization

Organization of the learning situation is of great importance in fostering insight. For several decades the trend in arithmetic was toward increasing complexity in the organization of topics. This came about because the stimulus-response theory of learning dominated instruction in arithmetic. Every desired response was specifically identified and connected to a stimulus for it. With this theory as a basis for instructional methods, drill played a dominant role.

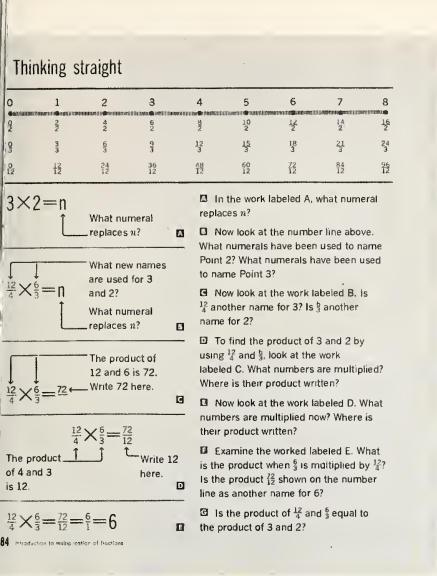
The process of multiplication with fractions, for example, was usually separated into many—a dozen or more—different "skills." Different lessons were devoted to "multiplying a fraction by a whole number" (for example, $3\times\frac{1}{2}$), to "multiplying a whole number by a fraction" (for example, $\frac{1}{2}\times3$), to "multiplying one fraction by another fraction" (for example, $\frac{1}{2}\times\frac{1}{3}$), to "multiplying a mixed number by a whole number" (for example, $3\times2\frac{1}{2}$), and so on. When the learning task is broken up in this way, instruction is spread out over weeks or even months. As a result, pupils often failed to understand the nature of the task as a whole.

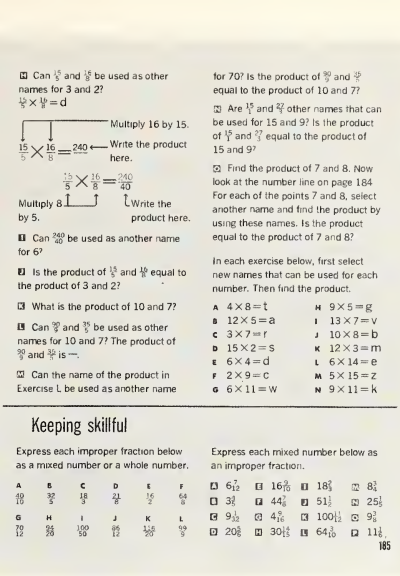
In *Seeing Through Arithmetic 6,* the multiplication of fractions and mixed numbers is taught in just two lessons. Pupils have already been taught that the same fraction can be expressed in many equivalent forms. Consequently, in these lessons, attention can be focused on the mathematical principle that is involved in multiplying fractions. Teachers who are well trained in mathematics will recognize that this principle underlies the definition given to multiplication in the system of rational numbers, but the basic idea can easily be grasped by children.

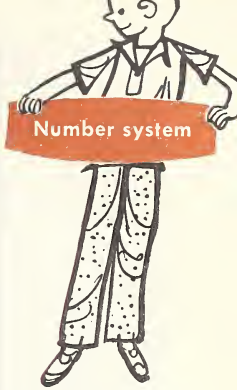
The first lesson develops readiness for the general case by multiplying whole numbers expressed in fraction numeral form. The pupil knows that $3\times2=6$. Then 3 is expressed as $1\frac{1}{4}$ and 2 is expressed as $\frac{8}{4}$. The product is then expressed as $1\frac{1}{4}\times\frac{8}{4}$. By multiplying $12\times6$ (i.e., the numbers expressed by the numerators) and $4\times3$ (i.e., the numbers expressed by the denominators), the result $7\frac{2}{12}$ is obtained, and this is another way of expressing 6. The pupil is thus helped to see that the rule makes sense and is valid (i.e., "it works") when the fraction numerals really express only whole numbers.

Mathematically speaking, the same principle is applied in the general case where the two factors are each actual fractions. "Improper fractions," which can, of course, be expressed as "mixed numbers," are included. Consequently, in the second lesson, the pupil is taught to multiply one mixed number by another. It is possible to verify by practical examples that the rule gives sensible answers in such cases also. Although practical examples can give support to the rule in this way, they should not be considered an "explanation" of it. Essentially, it is a rule which is adopted because it gives results that are consistent when the symbols actually represent whole numbers instead of fractions.

In similar fashion, other topics in *Seeing Through Arithmetic 6* are organized in terms of basic mathematical ideas, rather than in terms of an elaborate analysis of "separate" computational skills. This organization is not only mathematically sound, but the authors firmly believe that it is psychologically superior to traditional organizations.


Thinking straight
3×2=n
What numeral replaces n?
What new names are used for 3 and 2?
What numeral replaces n?
The product of 12 and 6 is 72. Write 72 here.
The product of 4 and 3 is 12.
Write 12 here.
Introduction to wider notion of fractions
84


Multiply 16 by 15.
Write the product here.
Multiply 8 by 5.
Write the product here.
Keeping skillful
Express each improper fraction below as a mixed number or a whole number.
Express each mixed number below as an improper fraction.
185




*The child learns through the use of the number line the procedures he will use in multiplying fractions.*

The method of teaching long division is especially noteworthy. It enables pupils to gain a general understanding of *the nature of the process* before they are faced with some of the difficulties inherent in it. In particular, the determination at each step of the largest possible quotient figure is not necessary. Even mature and skilled computers do not always find the "right" quotient figures immediately. The long division method used in *Seeing Through Arithmetic* enables pupils to compute correct answers with understanding, but without unnecessary stumbling over quotient difficulties. Pupils are then shown gradually how to improve their estimates of the quotient figure.

In brief, learning tasks are so organized as to develop *insight* and to enable the pupil to use it as a means to further learning. The effect is then cumulative: better understanding at each stage contributes to better understanding at following stages. The result is that fewer treatments of a topic are needed, and the organization of the subject as a whole is simplified. These are only a few of the many examples of reorganization suggested by psychological principles concerned with the development of insight. The simplification in organization discussed above is feasible in this program because new visual methods of presentation and provision of a good understanding of the number system are used to open the door to insight.

# 4 Effective methods

In *Seeing Through Arithmetic,* the learning of arithmetic is made efficient and enjoyable for most children by a variety of new methods of presentation. These methods include extensive creative use of the pictorial and typographic arts and many other features of presentation, emphasis, and organization. Introduction of these features is based on accepted principles of learning theory. These methods are used systematically, not erratically or superficially. A complete discussion of all these methods would unduly extend these introductory comments, but some general remarks will be helpful here.

### Presentation of processes

The "four-step teaching method" is an important feature of *Seeing Through Arithmetic.* This method is used in sections called "Learning how," in which the processes are taught. (See, for example, pages 80-85 of the pupils' textbook for Grade 6.) In Step 1,

## Learning how

Now you will learn how to find the product when you use mixed numbers and proper fractions.

### see

What are the dimensions of the piece of paper in Diagram A? Find its area in square inches.

A

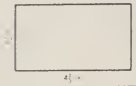


Diagram B shows why you can use the equation below to find the area.

B

Number of square inches in 1 row $\quad 4\frac{1}{2} = \frac{w}{2\frac{1}{2}}$ $\quad$ Number of square inches in $2\frac{1}{2}$ rows

When you find the numeral that replaces w, you will know the area of the piece of paper.

When you use the ratio test, you get this equation.

$$w = 4\frac{1}{2} \times 2\frac{1}{2}$$

To solve this equation, you must find the product of $4\frac{1}{2}$ and $2\frac{1}{2}$. To multiply these mixed numbers, first find other names for them. Express them with fraction numerals.

$\frac{13}{3}$ is another name for $4\frac{1}{3}$.

$$w = \frac{13}{3} \times \frac{5}{2}$$

$\frac{5}{2}$ is another name for $2\frac{1}{2}$.

Now multiply the numerators.

Multiply 5 by 13.

$$\frac{13}{3} \times \frac{5}{2} = \frac{65}{6}$$

Write the product here.

Next multiply the denominators.

$$\frac{13}{3} \times \frac{5}{2} = \frac{65}{6}$$

Multiply 2 by 3.

Write the product here.

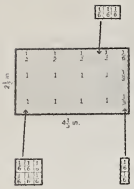
Express $\frac{65}{6}$ as a mixed number.

$$\frac{65}{6} = 10\frac{5}{6}$$

$10\frac{5}{6}$ is another name for $\frac{65}{6}$.



C



The work you have just done shows that the area is $10\frac{5}{6}$ sq. in. Diagram C also shows that the area is $10\frac{5}{6}$ sq. in.

Each square inch is how many one sixths of a square inch?

Each $\frac{1}{2}$ square inch is how many one sixths of a square inch?

Each $\frac{1}{3}$ square inch is how many one sixths of a square inch?

Now use Diagram C to find how many one sixths of a square inch there are in the area. You can either count or add. Do not forget the $\frac{5}{6}$ sq. in. in the upper right corner.

Is the total area $\frac{65}{6}$ sq. in.?

How does the diagram show that the area can also be expressed as $10\frac{5}{6}$ sq. in.? The area of the paper is $10\frac{5}{6}$ sq. in.

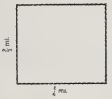
$$4\frac{1}{2} \times 2\frac{1}{2} = 10\frac{5}{6}$$

### see

A

Mr. Jackson's farm is a rectangle $\frac{3}{4}$ mi. long and $\frac{2}{3}$ mi. wide. Find its area in square miles.

You can use the equation below to find the area.



$$\frac{\frac{3}{4}}{1} = \frac{t}{\frac{2}{3}}$$

When you find the numeral that replaces t, you will know the area.

Now turn the page.



*The multiplication of fractions becomes an understandable process when properly taught.*

## Multiplication of fractions

When you use the ratio test, you get this equation.

$$t = \frac{3}{4} \times \frac{2}{5}$$

Multiply 2 by 3. The product is 6.

Write 6 here.

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{}$$

To solve the equation, you must find the product of $\frac{3}{4}$ and $\frac{2}{5}$.

Multiply 3 by 4. The product is 12.

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{12}$$

Look at the work in the next column. Since the numbers are already expressed with fraction numerals, you do not need to use new names for them.

Write 12 here.

$\frac{6}{12} = \frac{1}{2}$

$\frac{1}{2}$ is another name for $\frac{6}{12}$.

B

Now look at Diagram B. The dotted lines help show the whole square mile of which Mr. Jackson's farm is a part.

One side of the square is marked off in fourths because the farm is $\frac{3}{4}$ mi. long. The other side is marked off in thirds because the farm is $\frac{2}{3}$ mi. wide. Now the whole square is marked off in twelfths.

You can see from the diagram that the area of Mr. Jackson's farm is $\frac{6}{12}$ sq. mi.

How does Diagram B show that you can also think of the area of the farm as $\frac{1}{2}$ sq. mi.?

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, or $\frac{1}{2}$

The area of Mr. Jackson's farm is $\frac{1}{2}$ sq. mi.



### think

Jane said, "My story is $1\frac{1}{4}$ pages long." Ann said, "My story is 2 times as long as yours, Jane." Ann's story was how many pages long?

Is the equation below correct?

$$\frac{2}{1} = \frac{d}{1\frac{1}{4}}$$

How do you get the equation below?

$$d = 2 \times 1\frac{1}{4}$$

Before you multiply $1\frac{1}{4}$ and 2 with fraction

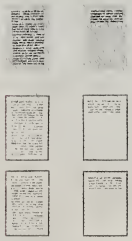
Are $\frac{2}{1}$ a corre names 2 and

$$\frac{2}{1} \times \frac{5}{4} = \frac{10}{4}$$

How ou find the 4?

Now express 1 mixed numbe

Ann's story was how many pages long?



You can also use the diagram to find how many pages Ann used for her story.

For the 1 page that Jane used, Ann used pages.

For the $\frac{1}{4}$ page that Jane used, Ann used fourths of a page.

Count or add Ann's pages by fourths. Do you get $\frac{10}{4}$?

Can you use the diagram to show that Ann used $2\frac{1}{2}$ pages?

$2 \times 1\frac{1}{4} =$ ?

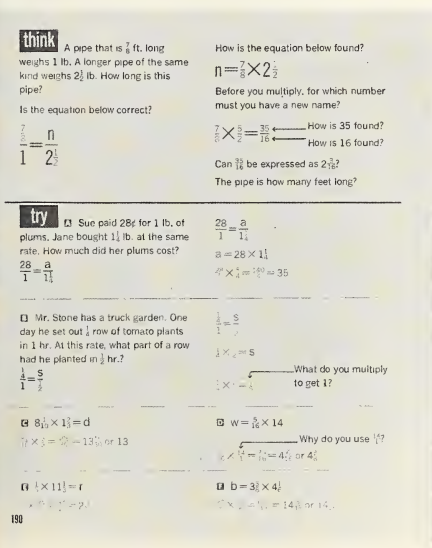
called "See," the pupil is shown in detail what to do. Pictorial representations show what happens to the objects involved. The process with the symbols is shown and is closely connected with the visual representation. Verbal explanations are included, but they are simple and brief. In Step 2, called "Think," another example is worked out and the pupil is asked some questions about important details. He is encouraged and helped to *think* about the example. In Step 3, called "Try," other completely worked examples are given, but the pupil is expected to work these himself and then compare his work with the completed examples in his book. In Step 4, called "Do," examples for practice are given.

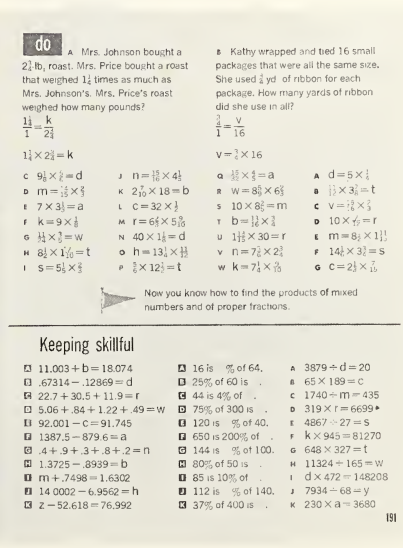
The "four-step method" has proved its effectiveness through years of use. In *Seeing Through Arithmetic,* it has been adapted to new visual techniques and modern principles of learning theory. It guides the pupil so that, literally, he can see, think, try, and do a process on his own and with confidence.

## Visual aids

In *Seeing Through Arithmetic,* all pictures are learning aids. No picture is presented for its purely decorative or presumed interest value. In many cases the pictures show both the basic data and the action —that is, the grouping and regrouping involved in the problem situation. The pupil can actually see what is happening. Moreover, because many lessons employ a sequence of related scenes, or "movies," they serve as a guide to teachers and pupils who wish to use actual objects to gain additional experience with the ideas. The Expanded Notes, which begin on page 271, contain many suggestions for this kind of enrichment.

In similar fashion, the use of colors and of many special printing devices is the result of careful study. Color is used not only to add general attractiveness or to call attention to some special point, but also to help teacher and pupils communicate with each other. For instance, the teacher may say, "Now we will do the exercises beside the red letters." Special printing devices are used to "dim out" in pictures everything that is not to be in the focus of attention. The design of every page is worked out with teaching values as a dominant criterion. The goal is to help the pupil see—visually and intellectually—what he is asked to learn.


think
A pipe that is $\frac{2}{3}$ ft. long weighs 1 lb. A longer pipe of the same kind weighs $2\frac{1}{2}$ lb. How long is this pipe?
Is the equation below correct?
$\frac{\frac{2}{3}}{1}=\frac{n}{2\frac{1}{2}}$
How is the equation below found?
$n=\frac{2}{3}\times 2\frac{1}{2}$
Before you multiply, for which number must you have a new name?
$\frac{2}{3}\times\frac{5}{2}=\frac{35}{16}$ ← How is 35 found? ← How is 16 found?
Can $\frac{35}{16}$ be expressed as $2\frac{3}{16}$?
The pipe is how many feet long?
try
Sue paid 28¢ for 1 lb. of plums. Jane bought $1\frac{1}{4}$ lb. at the same rate. How much did her plums cost?
$\frac{28}{1}=\frac{a}{1\frac{1}{4}}$
$a=28\times 1\frac{1}{4}$
$a=\frac{28}{1}\times\frac{5}{4}=35$
Mr. Stone has a truck garden. One day he set out $\frac{1}{3}$ row of tomato plants in 1 hr. At this rate, what part of a row had he planted in $\frac{2}{3}$ hr.?
$\frac{\frac{1}{3}}{1}=\frac{s}{\frac{2}{3}}$
$\frac{1}{3}\times\frac{2}{1}=s$
What do you multiply to get 1?
$\frac{2}{3}\times\frac{3}{2}=1$
$8\frac{1}{10}\times 1\frac{3}{5}=d$
$w=\frac{8}{10}\times 14$
Why do you use $\frac{1}{10}$?
$\frac{8}{10}\times\frac{3}{5}=\frac{24}{50}=13\frac{1}{50}$ or 13
$\frac{8}{10}\times\frac{14}{1}=\frac{112}{10}=11\frac{2}{10}$ or $11\frac{1}{5}$
$1\times 11\frac{1}{5}=r$
$b=3\frac{1}{2}\times 4\frac{1}{2}$
$1\times\frac{56}{5}=r$
$\frac{7}{2}\times\frac{9}{2}=\frac{63}{4}=15\frac{3}{4}$
$r=11\frac{1}{5}$ or $11.2$
$b=14\frac{1}{4}$ or $14.25$
190


do
A Mrs. Johnson bought a $2\frac{1}{2}$ lb. roast. Mrs. Price bought a roast that weighed $1\frac{1}{4}$ times as much as Mrs. Johnson's. Mrs. Price's roast weighed how many pounds?
$\frac{1\frac{1}{4}}{1}=\frac{k}{2\frac{1}{2}}$
$1\frac{1}{4}\times 2\frac{1}{2}=k$
B Kathy wrapped and tied 16 small packages that were all the same size. She used $\frac{3}{4}$ yd of ribbon for each package. How many yards of ribbon did she use in all?
$\frac{\frac{3}{4}}{1}=\frac{v}{16}$
$v=\frac{3}{4}\times 16$
C $9\frac{1}{8}\times\frac{2}{5}=d$
D $m=\frac{15}{16}\times\frac{8}{5}$
E $7\times 3\frac{1}{2}=a$
F $k=9\times\frac{1}{8}$
G $1\frac{1}{2}\times\frac{3}{5}=w$
H $8\frac{1}{2}\times 1\frac{7}{10}=t$
I $s=5\frac{1}{8}\times\frac{2}{5}$
J $n=1\frac{15}{16}\times 4\frac{1}{2}$
K $2\frac{7}{10}\times 18=b$
L $c=32\times\frac{1}{2}$
M $r=6\frac{4}{5}\times 5\frac{5}{16}$
N $40\times 1\frac{1}{4}=d$
O $h=13\frac{1}{2}\times 1\frac{1}{13}$
P $\frac{3}{5}\times 12\frac{1}{2}=t$
Q $\frac{15}{16}\times\frac{4}{5}=a$
R $w=8\frac{1}{2}\times 6\frac{2}{3}$
S $10\times 8\frac{2}{5}=m$
T $b=1\frac{1}{16}\times\frac{1}{2}$
U $1\frac{11}{12}\times 30=r$
V $n=7\frac{1}{2}\times 2\frac{1}{3}$
W $k=7\frac{1}{4}\times\frac{2}{15}$
A $d=5\times\frac{1}{4}$
B $\frac{11}{12}\times 3\frac{1}{5}=t$
C $v=1\frac{2}{3}\times\frac{3}{5}$
D $10\times\frac{1}{2}=r$
E $m=8\frac{1}{3}\times 1\frac{1}{11}$
F $14\frac{1}{2}\times 3\frac{1}{2}=s$
G $c=2\frac{1}{5}\times\frac{5}{11}$
Now you know how to find the products of mixed numbers and of proper fractions.
Keeping skillful
A $11.003+b=18.074$
B $.67314-.12869=d$
C $22.7+30.5+11.9=r$
D $5.06+.84+1.22+.49=w$
E $92.001-c=91.745$
F $1387.5-879.6=a$
G $.4+.9+.3+.8+.2=n$
H $1.3725-.8939=b$
I $m+.7498=1.6302$
J $14.0002-6.9562=h$
K $z-52.618=76.992$
A 16 is ___% of 64.
B 25% of 60 is ___.
C 44 is 4% of ___.
D 75% of 300 is ___.
E 120 is ___% of 40.
F 650 is 200% of ___.
G 144 is ___% of 100.
H 80% of 50 is ___.
I 85 is 10% of ___.
J 112 is ___% of 140.
K 37% of 400 is ___.
A $3879\div d=20$
B $65\times 189=c$
C $1740\div m=435$
D $319\times r=6699$
E $4867\div 27=s$
F $k\times 945=81270$
G $648\times 327=t$
H $11324\div 165=w$
I $d\times 472=148208$
J $7934\div 68=y$
K $230\times a=3680$
191

C Diagram C represents a rectangular rug 12 ft. long and 9 ft. wide. Find the area in square feet.

D You can also find the area in square yards. One way to do this is to change 108 sq. ft. to square yards.

The rate of square feet per square yards is always the same. Why does 9 per 1 express this rate? Look at Diagram D. In Problem B, the rate of square feet per square yards can also be expressed as 108 per *t*. What does *t* represent?

$$\frac{9}{1} = \frac{108}{t}$$

The numeral that replaces *t* will tell you how many square yards there are in 108 sq. ft.

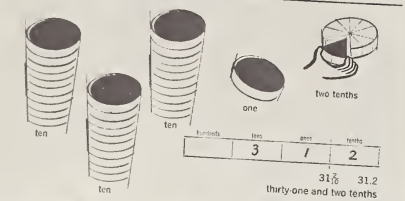
you use the ratio test to solve is equation, you get $9t = 1 \times 108$. ow do you find the numeral that places *t*?

8 sq. ft. = sq. yd.

E Diagram E represents a rectangular

## Moving forward

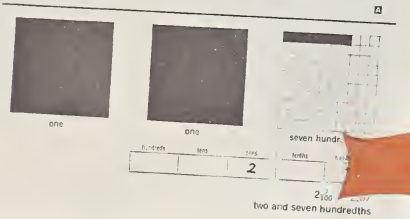
In this lesson you will learn how to use decimal fraction numerals as names for numbers.




$31\frac{2}{10}$ 31.2

thirty-one and two tenths

D




$2\frac{7}{100}$

two and seven hundredths

D

169

## Looking back

If you have a remainder when you divide, sometimes you should use it to make a fraction numeral in the answer.

Sue, Ann, and Kathy have 17 yd. of cloth to share equally. Each girl's share will be how many yards?

$17 \div n = 3$ Divide 17 by 3.

5 yd. for Sue

The answer is 5 and 2 remainder.

Each girl can have 5 whole yards.

2 yards will be left over to be shared.

5 yd. for Ann

5 yd. for Kathy

Sue Ann Kathy

Think of each leftover yard as marked off into 3 equal parts.

Each girl can have 1 of the equal parts of each leftover yard. Each girl can have $\frac{1}{3}$ of each leftover yard.

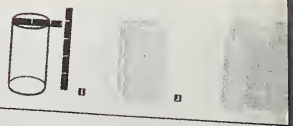
Since there are 2 leftover yards, each girl can have 2 of the equal parts, or $\frac{2}{3}$ yd.

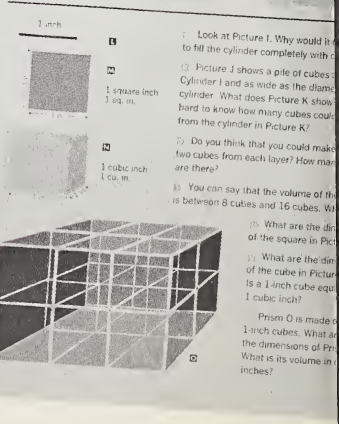
2 yd. left over to be shared

59

**Visual aids**

*Pictures of the proper kind give the child the help he needs in understanding difficult ideas.*






Look at Picture I. Why would it to fill the cylinder completely with

Picture J shows a pile of cubes Cylinder I and as wide as the diam cylinder. What does Picture K show hard to know how many cubes coul from the cylinder in Picture K?

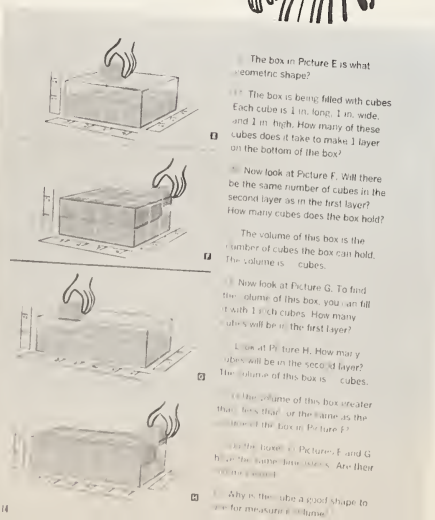
Do you think that you could make two cubes from each layer? How ma are there?

You can say that the volume of th is between 8 cubes and 16 cubes. Wh

What are the dim of the square in Pict

What are the dim of the cube in Pictur is a 1-inch cube equ 1 cubic inch?

Prism O is made o 1-inch cubes. What a the dimensions of Pri What is its volume in inches?



The box in Picture E is what geometric shape?

The box is being filled with cubes Each cube is 1 in. long, 1 in. wide, and 1 in. high. How many of these cubes does it take to make 1 layer on the bottom of the box?

Now look at Picture F. Will there be the same number of cubes in the second layer as in the first layer? How many cubes does the box hold?

The volume of this box is the number of cubes the box can hold. The volume is cubes.

Now look at Picture G. To find the volume of this box, you can fill it with 1-inch cubes. How many cubes will be in the first layer?

Look at Picture H. How many cubes will be in the second layer? The volume of this box is cubes.

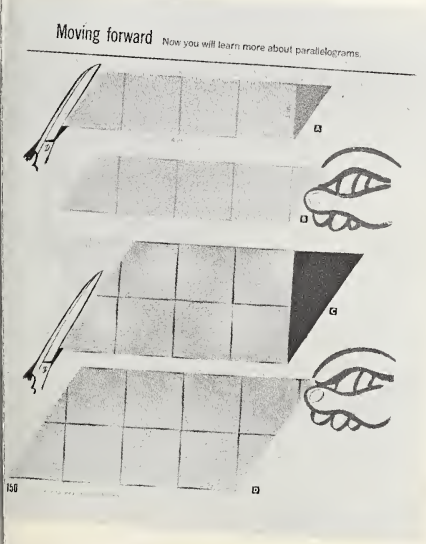
Is the volume of this box greater than, less than, or the same as the volume of the box in Picture F?

Is the volume of Pictures E and G the same? How many cubes are there in each? Are their volumes equal?

Why is the cube a good shape to use for measuring volume?

The value of visual learning aids is now well established. Film strips and moving pictures are increasingly popular. The visual aids in this book have a distinct advantage, however, since teachers and pupils can study them as long and as often as necessary. Arithmetic requires study. Pictures and other visual devices, when they are properly planned, make this study possible and fruitful. In his study time, each pupil can take as much time as he needs on each scene. This contrasts sharply with visual aids that move from scene to scene at a speed supposedly suitable for the average pupil, but which is certainly too slow for the able pupil and much too fast for the slow pupil.

Illustrations of the kind used in this program are an important reading aid. Much of the difficulty encountered in reading arithmetic materials results from the pupil's inability to visualize. Pictures carefully related to the text aid in visualizing. Moreover, since visual aids reduce the reading load and aid comprehension, the pupils can return to earlier lessons for review and restudy and use the materials effectively. The value of an ordinary illustration is soon used up, but the value of carefully planned visual devices can continue long after they are first used.




The methods of presentation used in *Seeing Through Arithmetic 6* are designed to satisfy the intellectual needs of pupils as well as to help them develop computational skill. This concern for intellectual needs finds expression in the unreserved effort to develop understanding of mathematical principles. Interest is by no means ignored, but neither is it debased through superficial devices. Concern for intellectual stimulation and growth is not to be limited to the "bright" pupils only. The pictorial and other methods of presentation and the provisions for individual differences suggested throughout this *Guide* are designed to reach all levels of ability. It is not expected that all children will gain the same values from each lesson, but it is believed that each child can gain some value.

### Vocabulary control

The purpose of studying arithmetic is to learn how to solve quantitative problems. A fairly extensive vocabulary and the ability to read well are of course important. However, since the task of learning arithmetic is sufficiently difficult for the child, it is foolish to add extra burdens in the form of unnecessary vocabulary and reading difficulties. For this reason the vocabulary of *Seeing Through Arithmetic 6* is carefully controlled.

In *Seeing Through Arithmetic 6*, it is assumed that the children know only the 841 reading words (both arithmetical and general) that appeared in the preceding books of the *Seeing Through Arithmetic* program. These words are listed alphabetically on pages 392-395 of the *Teaching Guide* for *Seeing Through Arithmetic 5*. All other words are considered new words and are listed, for special attention by the teacher, at the beginning of the Lesson Briefs for each block of work. New words are introduced at a cautious rate. The total vocabulary of *Seeing Through Arithmetic 6* is 1176 words. See pages 412-416 of this *Teaching Guide* for a complete alphabetical list of the words used and for detailed statistics on the vocabulary.

### Lesson headings

*Seeing Through Arithmetic 6* uses headings that describe the nature of the various types of lessons. They serve to alert the pupil immediately as to the nature of the lesson—learning a new computational skill, practicing problem solving, maintaining skills, and so forth. In connection with these headings, there is, whenever necessary, a statement to the teacher (in small type opposite the page number) that gives a technical description of the content of the lesson. With some of these lessons, too, there is a statement opposite the heading giving the pupil more information about the nature of the lesson.

These various types of lessons provide a wealth of material for the teaching of concepts, for practice, for maintenance, for review, and for testing achievement. Teachers who want suggestions on the varied uses of these materials will find them in the Lesson Briefs (pages 21-269) and in the Expanded Notes, which begin on page 271, for each section. These materials provide the basis for effective lesson planning by teachers and effective learning by pupils.

All material in *Seeing Through Arithmetic 6* is organized under nine headings:

| | |
|---|---|
| Learning how | lessons using the four-step teaching method to develop or review computational skills |
| Exploring problems | lessons developing specific problem-solving techniques |
| Moving forward | lessons developing other new material, including use of ratios to express rates and comparisons, fractions, measurement, area, and volume |
| Using arithmetic | maintenance and practice in solving verbal problems |
| Keeping skillful | maintenance and practice in computation with whole numbers and fractions |
| Checking up | tests of various types: inventory, achievement, diagnostic (end-of-block), and problem-solving |
| Thinking straight | lessons involving special aspects of quantitative thinking |
| Looking back | lessons involving reteaching or review of previously taught skills |
| Side trip | lessons on various topics related to the arithmetic program |

# Lesson briefs

This section of the *Teaching Guide* contains the aids that every teacher will need in using *Seeing Through Arithmetic 6.* Collected together in one place for your convenience you will find

**Objectives**

A very brief statement of the purpose of the lesson to help you see its relation to the program

**Vocabulary**

For each lesson a list of all words not used in preceding books of the program

**Comments**

Helpful information about the arithmetic itself and about methods of teaching

**Answers**

Answers for all exercises except those for which answers are obvious

**Reproduced pages**

A reproduction of each page from the pupils' book for your easy reference

**Keyed notes**

Brief directions for teaching each page, keyed by numbers to the reproduced page

# Lesson briefs

**A productive teaching situation**
includes
interested children,
an inspired teacher,
and
good materials.
In using the materials
of this program,
do not forget
the Expanded Notes
that begin
on page 271.

## 3-5 Moving forward

Expanded Notes for this lesson ore on pages 272-275.

**Objectives**

The pupil develops an awareness of three-dimensional geometric shapes that are around him. He learns how perspective is used in representing these shapes in drawings.

**Vocabulary**

A word list like the one below is given in this *Teaching Guide* for each lesson in *Seeing Through Arithmetic 6*. These words are new to this arithmetic program and appear for the first time in the sixth-grade book. Words marked with asterisks are already familiar to the pupils who have completed The New Basic Reading Program through *More Days and Deeds* (5/2). On pages 412-416 you will find a complete alphabetical list of all words used in *Seeing Through Arithmetic 6*.

The words *prism, cylinder, sphere, pyramid,* and *cone* are not used in the textbook until page 8, but you should use them orally in teaching pages 3-5.

New words page 3 familiar*; page 4 artist*, waiting*, suggest*, perspective

**Comments**

If you have not already read the introductory pages in this *Teaching Guide*, you will want to do so before proceeding with these comments and the keyed notes. To use the *Teaching Guide* effectively, you should be familiar with the aims and principles underlying the *Seeing Through Arithmetic* program. See pages 6-20.

For each lesson that is entitled "Moving forward" and "Learning how" there is a statement beside the heading that tells the pupils what they are going to learn. You will want to begin each of these lessons by having the pupils read these sentences. At the bottom of the first page of every major lesson there is a description of the content of the lesson for the teacher.

No specific time schedule can be set for the lessons in *Seeing Through Arithmetic 6*. Different classes will need different lengths of time for the various lessons in the book, depending upon the ability and preparation of the pupils. Let the needs of your pupils govern the length of time devoted to a lesson. Spend enough time for the pupils to assimilate the ideas presented before they are asked to go on to something new. On page 409 you will find an approximate time schedule for the work in this book.

In the lesson on pages 3-5 the boys and girls are introduced to various three-dimensional geometric shapes that they see daily. Through discussion they learn that these familiar shapes have names, and through observation they note how the shapes differ. They also learn how to make objects look real in drawings and to suggest three dimensions on the flat surface of the paper.

On page 3 two pictures that illustrate three-dimensional shapes are provided for discussion. At this time the pupils should not be required to use the precise names (such as, *cone*, *prism*) of these shapes. They may talk about them as being round, having flat or curved sides, and call the shapes "balls" and "boxes." However, you should begin to use the correct names in your discussion. A display of geometric shapes made of paper, cardboard, or wood can be helpful here. Identify the shapes, and let the pupils point to them in making comparisons with the objects in the pictures. The correct terms to use for some three-dimensional shapes are presented in the reading material on page 8.

On page 4 some of the devices used in drawing three-dimensional objects are illustrated. The picture showing two railroad stations and a railroad track should give the pupils a good understanding of the use of perspective in an outdoor scene. They see here that pictures of objects are made smaller to show that they are farther away and that a pair of lines drawn so that they come closer and closer together may suggest increasing distance. By examining the picture of a block, they learn what is needed to make a picture appear real. They learn to accept certain angles as representing right angles, even when they do not appear to be right angles in the drawing, and to understand that lines that come to a point may really be parallel.

The purpose of page 5 is to draw the pupils' attention to the way artists show whether or not a shape has flat or curved surfaces and how many sides a shape has. The use of dotted lines and colored lines to indicate the actual number of sides a shape has and the use of high lights to suggest curved surfaces are illustrated.

You will find helpful material for teaching this lesson (and other lessons) in the section entitled "Expanded Notes." A specific page reference to this section is given at the beginning of the Lesson Briefs for each lesson.

**Answers**

The answers given here are representative of the kind of answers you might expect. No particular wording should be insisted upon. For exercises like these the answer may be an explanation or a discussion.

Page 4:

A Yes; because in real life they appear to be meeting in the distance

B No. The artist has suggested distance by drawing a shorter pole. Telegraph poles within a given area are usually all about the same height.

C To show that they are closer

D The difference in the size of the two stations, of the two wagons, of the two signals, and of the two water towers. Also, the strips of grass, the rows of trees, and the telegraph wires along the tracks appear to meet.

E Picture C; Picture C

F On a real block Lines o and p appear to be closer together at the back when the top of the block is viewed from the side lettered "T." To represent this, the artist must draw lines which actually do come closer together.

G These corners (or angles) in the drawing are not square corners (or right angles). In the real block they are.

Page 5:

A No; 3; yes

B Lines that indicate other sides have been added. 6 sides

C A rectangle

D The rectangle outlined by the heavy blue line

E 5 sides F A part of a circle

G The dotted line indicates the circular base that is not visible in Picture F and makes it clear that the picture does not represent a flat surface.

H Nothing in the drawing makes clear what it represents. Picture H could represent either a circle or a sphere.

I Picture I is shaded except for a portion that looks as if a light were shining on it. Yes

J He has put high lights on them to indicate the parts that reflect light. Such high lights are used by artists to suggest curved surfaces.

# 3

1 Have the pupils note that this lesson is entitled "Moving forward." Explain that a heading such as this anywhere in the textbook means that they are going to learn something new. Ask them to read the text after this title to find out what they are going to learn.

2 Before working with this picture, point out to the pupils that there are objects of many different shapes around them. Then ask them what kinds of shapes they see in the picture. They may use terms such as "box," "ball," "flat side," and "curved side." Some of the boys and girls may know words like *cylinder* and *solid,* but do not insist on the use of such terms. If you have models of a prism, pyramid, cone, sphere, and cylinder, display them with identifying labels. The pupils may point to the models as they describe corresponding objects in the picture.

3 Treat this picture in the same way.

B

A Are the rails of a railroad track always the same distance apart, or parallel? Why did the artist draw the rails in Picture A as if they meet in the distance?

B Measure the picture of the pole marked A. Then measure the picture of the pole marked B. Would Pole B itself really be shorter than Pole A? Why?

C Why did the artist draw the people who are waiting for the train at Y larger than those waiting at Z?

C

D When an artist draws a picture so as to suggest how things look, he is using perspective. Do you see any other examples of perspective in Picture A?

E Both Pictures B and C show a wooden block with square corners. Which picture better suggests the shape of the block? Which picture is drawn in perspective?

D

F Now look at the same block in Picture D. On the block itself, the edges marked o and p are parallel. Why are Lines o and p not parallel in the drawing?

G What does perspective do to the corners marked f and g?

A

B

A Look at the block in Picture A. Can you see all of its sides? How many sides can you see? Is the block drawn in perspective?

C D E

B How is Picture B different from Picture A? The blue lines help you know that the block has ■ sides in all.

C What does Picture C look like?

D Look at Picture D. What part of it is like Picture C? The dotted lines show that it is a block with 6 sides. Find the 6 sides.

F G

E Now look at Picture E. The dotted lines show that this shape has ■ sides.

F What does Picture F look like?

H I

G How does the dotted line in Picture G make the shape look like a cone?

H Why is it impossible to tell whether Picture H shows a circle or a ball?

I How is Picture I different from Picture H? Does Picture I suggest the shape of a ball?

J K

J What has the artist done to suggest that the shapes in Pictures J and K are rounded and not flat?

Now you know why pictures of objects are drawn in perspective. You also know how parts that you do not see may be suggested in drawings.



# 4

1 Ask the pupils what they see in this picture. Have them note the difference in the sizes of the two stations. Get them to see that all the objects in the background of the picture are smaller than those in the foreground and that the railroad track comes to a point.

2 Have the class discuss how an actual railroad track appears in the distance. Then have the pupils answer the questions in Exercise A.

3 Use Exercises B, C, and D to emphasize that an artist must use special techniques to indicate relative position and distance in a drawing.

4 Display a real block, if possible. Then have the pupils study Picture B. Ask how many sides of the block they can see. Then have them compare this block with the one in Picture C and answer the questions in Exercise E.

5 Use Exercises F and G to show how three dimensions are represented on a flat surface.

# 5

1 Display a block and have the pupils see that it has six sides, but that only part of the block can be seen from any one point. Then have them look at the block in Picture A. They should realize that they can see three sides because the block is drawn as one really sees it. Have them look next at Picture B and note that the blue lines help them "see" all sides of the block. Let them answer Exercises A and B.

2 By comparing Pictures C, D, and E, the pupils should note that in Picture C they cannot tell if there is a third dimension, such as depth or height. Use Exercises C, D, and E to bring out that Pictures D and E, by the use of dotted lines, show two objects with different shapes. Have models available.

3 Adapt Note 2 to Exercises F and G.

4 Adapt Note 2 to Exercises H and I and ask why Picture I suggests the shape of a ball.

5 Discuss the highlighting on Pictures J and K.

# 6-9 Moving forward

Expanded Notes for this lesson are on pages 275-277.

**Objectives**

The pupil learns about the distinguishing features of prisms, cylinders, pyramids, cones, and the sphere.

**Vocabulary**

New words page 6 geometric, faces*, rectangular, triangular, appear*, circular; page 7 surfaces*, curved*, perpendicular; page 8 prism, cylinder, sphere, pyramid; page 9 cube, recognize*

**Comments**

On these four pages the pupils learn through observation and comparison the characteristics of some important three-dimensional geometric shapes. The work should be kept informal; that is, the pupils should not be required to memorize definitions of the geometric shapes. However, pupils should acquire correct impressions so that they will not be handicapped in later grades.

On page 6 some geometric shapes are shown without their names. The pupils learn that the flat sides are called *faces*. The material on page 7 helps develop the meaning of *plane surface*, *curved surface*, *perpendicular planes*, and *parallel planes*. On page 8 the pupils learn the names of the geometric shapes and more about their distinguishing features. For example, they see that prisms and cylinders have two bases that are the same shape and size and also parallel. On page 9 the pupils' ability to identify geometric shapes is tested. The cube is defined on this page.

Be sure the pupils understand that the word *side* may be used to refer to both plane sides (including the bases) and curved sides. Get them to accept the term *face* as meaning only *plane side* (including the bases).

**Answers**

Page 6:

A 6

B Yes. It has 6 sides that are rectangles. Pupils may observe that its measurements are not the same as those for the shape in Picture A.

C 2 D 6; 2 E No; 2

F It has a circular base. It comes to a point above the center of the base.

G Triangles H It has no bases or faces.

I Refrigerator, butter, canisters, milk carton, van of moving truck, wagon

J Picture E: sifter, jar, oil tank on truck, silo (without the roof), water tank, corncribs
Picture F: sifted flour, corncrib top, top of water tank
Picture H: apples

Page 7:

A [B, D, N] C [E, K, O] E [Q, U]

B [C, H, J, L] D [T, V] F [P, W]

Page 8:

A They all have parallel bases whose edges are straight lines, and all their other faces are rectangles perpendicular to the bases.

B 6

C Rectangles

D Triangles; yes; 3

E Polygons (hexagons); 6

F Yes; yes

G 2 (the bases); 1

H No; no

I A half sphere is a curved surface. A cylinder has a curved surface and two bases that are plane surfaces.

J Circle; 1 (the base); 1

K Triangular

L The base of Pyramid I is a triangle. The pyramid has three sides, all of which are triangles. The base of Pyramid H is a square. This pyramid has four sides, all triangles. Pyramid I has four faces. Pyramid H has five faces.

M Pyramid H has only one base, which is a rectangle. It has four other faces, all triangles, which come to a point above the base. Prism B has two parallel bases, both triangles. It has three other faces, all rectangles.

Page 9:

A [A] sphere [F] prism [K] cone
[B] prism [G] prism [L] cylinder
[C] cone [H] cylinder [M] half sphere
[D] cylinder [I] pyramid [N] prism
[E] pyramid [J] prism

B [B]

C [B, E, F, G, I, J, N]

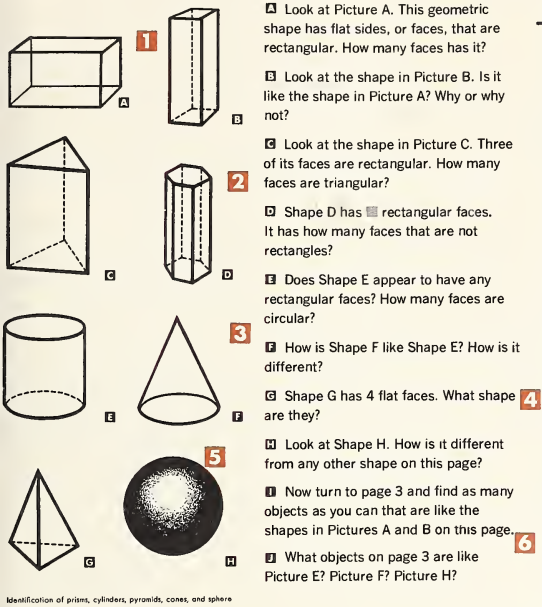
D [C, D, H, K, L]

E [B, E, F, G, I, J, N; Sphere A has no plane surfaces.]

F [C, D, H, K, L, M]; The cones and the half sphere have only one plane surface.

## Moving forward

In this lesson you will learn the names of some important geometric shapes.




A Look at Picture A. This geometric shape has flat sides, or faces, that are rectangular. How many faces has it?

B Look at the shape in Picture B. Is it like the shape in Picture A? Why or why not?

C Look at the shape in Picture C. Three of its faces are rectangular. How many faces are triangular?

D Shape D has ▪ rectangular faces. It has how many faces that are not rectangles?

E Does Shape E appear to have any rectangular faces? How many faces are circular?

F How is Shape F like Shape E? How is it different?

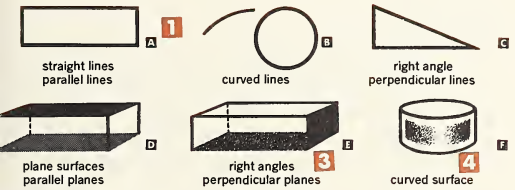
G Shape G has 4 flat faces. What shape are they?

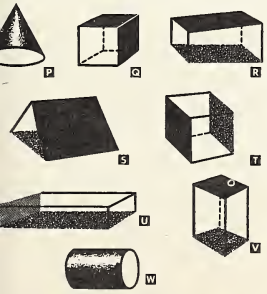
H Look at Shape H. How is it different from any other shape on this page?

I Now turn to page 3 and find as many objects as you can that are like the shapes in Pictures A and B on this page.

J What objects on page 3 are like Picture E? Picture F? Picture H?

Identification of prisms, cylinders, pyramids, cones, and sphere






A Find each set of red parallel lines in Pictures A to O above.

B Find each set of red perpendicular lines. 5

C Find each red curved line.

D Find each set of yellow parallel planes in Pictures P to W at the left.

E Find each set of yellow perpendicular planes. 6

F Find each yellow curved surface.



## 6

1 First be sure the pupils know what a rectangle is. Then have them look at Pictures A and B and answer the questions in Exercises A and B.

2 Exercises C and D and Pictures C and D should be used to bring out that some geometric shapes have faces that are not rectangles.

3 Point out in connection with Exercises E and F that some shapes have circular faces.

4 In considering this exercise and Picture G, the pupils should note that all the faces can be triangular. Have them compare Picture G with Picture C.

5 Help the pupils see that Shape H does not have any bases or faces.

6 Treat Exercises I and J orally, or have the pupils list the names of objects on page 3 that are like these shapes.

## 7

1 Discuss the terms below Pictures A, B, and C, and have the pupils point out examples of each term in the pictures.

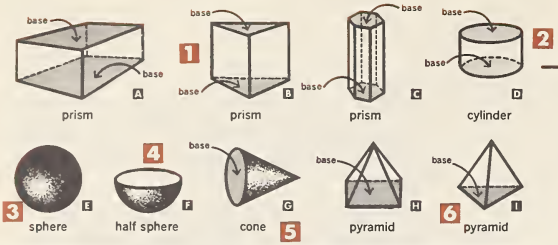
2 Discuss the meaning of *plane surface*. Have the pupils point out six plane surfaces. Tell them that planes as well as lines can be parallel, as illustrated by the blue plane surface and shaded blue plane surface. Have other parallel planes pointed out.

3 Have the pupils point out right angles. Then explain that planes, as well as lines, can be perpendicular to each other, as shown by the colored plane surfaces.

4 Discuss the curved surface of this cylinder and relate it to the curved lines in Picture B.

5 Use Exercises A to C in connection with Pictures A to O.

6 Use Exercises D to F in connection with Pictures P to W at the left.




A The geometric shapes in Pictures A, B, and C are called prisms. How are these prisms alike? 1

B Prisms have plane sides, or faces. How many plane sides, or faces, in all has Prism A?

C The bases of Prism A are rectangular in shape. They are also parallel and of the same size. What shape are the other faces of Prism A?

D What shape are the bases of Prism B? Has Prism B any rectangular faces? How many does it have?

E What shape are the bases of Prism C? How many rectangular faces has Prism C?

F The bases of Cylinder D are in the shape of circles. Do the circles have equal diameters? Do the bases of Cylinder D appear to be parallel? 2

G Cylinder D has how many plane sides? How many curved sides?

H Does Sphere E have a base? Does it have any plane sides? 3

I How is the half sphere in Picture F different from Cylinder D? 4

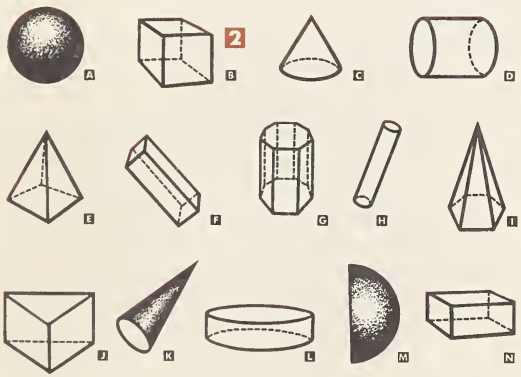
J What shape is the base of Cone G? How many plane sides has Cone G? How many curved sides does it have? 5

K The base of Pyramid H is square. What shape is each of the other faces?

L How is Pyramid I different from Pyramid H? 6

M In what ways is Pyramid H different from Prism B?

8




A Name the shape in each picture above. The names below will help you.

prism — sphere
cylinder — pyramid
cone — half sphere 1

B One of the pictures shows a prism with square faces that are the same size. This prism is a cube. It is also a rectangular prism. Which picture shows a cube? 2

C Which pictures show geometric shapes that have polygons as bases?

D Which pictures show shapes with circular bases? 3

E Which shapes have only plane surfaces? Which shape has no plane surfaces? 4

F Which shapes have both plane and curved surfaces?

Now you should be able to recognize and name some important geometric shapes. 5

9

28

# 8

1 Use Prisms A to C with Exercises A to E. Be sure the pupils understand that the two bases of each of these prisms have the same shape, are parallel, and are the same size.

2 Use Cylinder D with Exercises F and G. Help the pupils see that the bases are parallel and have diameters that are equal.

3 Pupils should see that a sphere has no base and no plane sides, or faces.

4 Bring out that a half sphere has one plane surface circular in shape.

5 Point out that a cone has a circular base and one curved side.

6 Use Pyramids H and I with Exercises K to M. Bring out that a pyramid has a polygon for a base and that its other faces are always triangles.

# 9

1 If you have models of the geometric shapes listed in Exercise A, display them and help the pupils name them. Then call on pupils to identify the shapes in Pictures A to N above.

2 Help the pupils identify Prism B as a cube. Be sure the pupils understand what distinguishes the cube from other rectangular prisms.

3 Use Exercises C and D to check on the pupils' abilities in recognizing bases that are polygons and those that are circles.

4 Have the pupils answer the questions in Exercises E and F. Help them distinguish between plane surfaces and curved surfaces.

5 Have this statement read aloud. Then, if you think the pupils are not quite sure of the names of some of the shapes, spend a little more time on having them identify Pictures A to N.

# 10-12 Moving forward

Expanded Notes for this lesson are on pages 277-281.

**Objectives**

The pupil learns how to recognize and measure the important dimensions of various geometric shapes.

**Vocabulary**

New words page 11 passes*, center*

**Comments**

Although pupils presumably already know how to measure certain simple geometric shapes, this lesson begins with a review of important dimensions of the circle and the triangle. The diameter of a circle and the base and altitude of a triangle are considered here. The pupils are reminded that the diameter of a circle is a straight line that passes through the center of the circle and touches the circle twice. They also see that the altitude of a triangle is a straight line that lies at right angles to the base and passes through the point of the triangle that is farthest from the base.

On page 10, after the review of the circle and the triangle, measurement of three-dimensional shapes is introduced. The pupils learn how to measure the height and diameter of a cylinder. Page 11 provides for discussion of how to measure prisms, spheres, cones, and pyramids. On page 12 the pupils are tested on what they have learned about dimensions of plane and three-dimensional shapes.

You may find that *height* and *altitude* are confusing terms to students. In this text *altitude* is used with plane shapes, and *height* is used with three-dimensional shapes. Do not expect the pupils to use the terms correctly at all times.

What constitutes a base may also give the class some difficulty. In rectangles and triangles any side may be used as a base, although usually the side on which the polygon appears to rest is considered to be the base. In prisms, pyramids, and cylinders, faces that may be called bases are more restricted. For example, in the case of a triangular prism, the triangular faces are always considered to be the bases, even if the prism is resting on one of its rectangular faces.

The work in the book should be supplemented by actual measuring experiences in the classroom. A collection of objects shaped like the three-dimensional shapes taken up in the lesson should be provided. The measuring activities should include experiments to discover what happens when measuring instruments are used incorrectly.

**Answers**

Page 10:

A [A]; because the measurement must be made through the center of the circle

B [D] C [y]

D From the point of the triangle farthest away from the base, draw a line to the base. Yes

E [F]; The height of a cylinder is measured on a line that is perpendicular to its base.

F Yes

G Through the center of the top circular base; Diameter must be measured through the center.

Page 11:

H Three; They are length, width, and height.

I s, o, or m for length; q, n, or l for width; t, r, or p for height

J Its bases are triangles. Its sides are rectangles.

K [z] L [u or x]

M [J]; The distance between the book ends is about equal to the length of a line through the center of the sphere.

N The diameter of its base

O The cardboard must be held parallel to the table top so that the distance between the table top and the cardboard will be the same as the distance between the table top and the point of the cone.

P Since all the sides of a square are the same length, it is necessary to measure only one side.

Q Height

Page 12:

A [A] y; [B] d; [C] z; [D] a

B [E] line m or line o; [F] z; [G] u C [K, L]; z, h

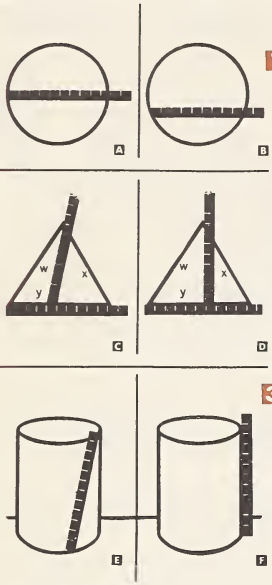
D [H, I]; for Prism H, lines c and d, or lines c and a; Allow a and b if pupil makes clear he is using that side as the base. For Prism I, lines x and z, or lines x and w. Allow x and y if that side is used as the base.

E [J]; base, e; altitude, d

F [H] b; [I] y or v; [J] g; [K] y; [L] p

## Moving forward

Now you will study the important dimensions of different geometric shapes.




10 Recognizing the important dimensions of various geometric shapes; correct and incorrect ways of measuring

**1** A Suppose you want to measure the diameter of a circle. Which picture shows the correct way of measuring a diameter? Why?

B Which picture shows the correct way to measure the base and the altitude of a triangle?

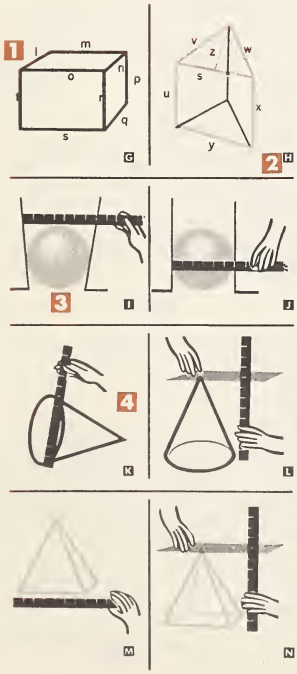
**2** C Which line is being used as the base of the triangle in Pictures C and D?

D How would you draw a line to represent the altitude of the triangle? Should the line be perpendicular to the base?

**3** E Which picture, Picture E or Picture F, shows the correct way to measure the height of a cylinder? Why do you think so?

F Is the height of a cylinder measured at right angles to the base?

G Where would you put your ruler to measure the diameter of this cylinder? Why?




**1** H Picture G shows a rectangular prism. How many dimensions do you need to measure before you can describe the size of this prism?

I Which edges of the prism would you measure? Why?

J How is Prism H different from Prism G? How is it like Prism G?

**2** K Prism H has triangular bases. If you think of Line S as the base of a triangle, which line represents the altitude of the triangle?

L On which line would you measure the height of Prism H?

**3** M The diameter of a sphere passes through the center of the sphere. Which picture, I or J, shows a correct way to find the approximate diameter of a sphere? Why do you think so?

N What dimension of the cone is being measured in Picture K?

**4** O Picture L shows a way to find the height of a cone. Why must the sheet of cardboard be held parallel to the table top?

P The base of the pyramid in Picture M is a square. How would you measure the base?

**5** Q What dimension of the pyramid is being measured in Picture N?





# 10

1 Before the pupils answer the questions in Exercise A, be sure they know that a diameter is a straight line that passes through the center of a circle between two points where the line cuts the circle. Ask them how a ruler should be held when measuring. Have them note the beginning point of the measuring scale on the ruler in the picture.

2 In discussing Exercises B, C, and D, bring out that the side upon which a triangle rests is usually considered to be the base and that the altitude is a straight line drawn at right angles from the base to the point farthest from the base.

3 For Exercises E, F, and G, the pupils should be able to deduce that the height of a cylinder is the perpendicular distance from base to base and that the diameter is measured as in the circle in Exercise A.

# 11

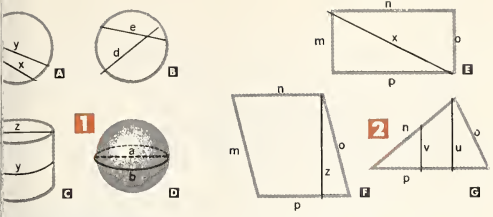
1 Draw a rectangle. Discuss the fact that the base and altitude are important dimensions of a rectangle. Ask the pupils what other dimension a rectangular prism has. [Height] Then ask them to identify by letter the dimensions of the prism in Picture G.

2 Bring out that Picture H shows a triangular prism. Then discuss Exercises J to L. For Exercise L, explain that the triangular prism is resting on one of its bases.

3 Have the pupils contrast how the diameter of the sphere is being measured in Pictures I and J.

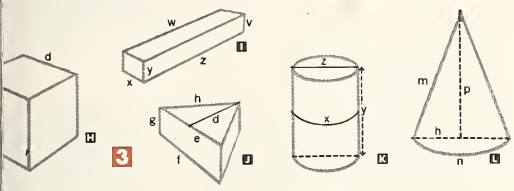
4 For Exercise N, help the pupils see that the diameter of the base is being measured in Picture K. For Exercise O, point out that the cardboard must be parallel to the table top so that the height of the cone may be measured at any point on the cardboard.

5 The pupils should use what they have already learned to answer Exercises P and Q.




For each circle, cylinder, or ...ere above, which line represents ...iameter?

B For each polygon above, if line p represents the base, which line represents the altitude?




Look at Pictures H to L. Which ...pes have circular bases? For each ..., which line shows the diameter?

Which shapes have rectangular ...es? For each one, which lines show ... dimensions of the bases?

E One shape has triangular bases. Which one is it? Which lines show the base and altitude of a triangle?

F Now for each shape in Pictures H to L, name a line that represents the height.

Now you should be able to recognize the important dimensions of the geometric shapes you have studied.

## 12

1 Mention again that a diameter is a straight line that passes through the center of a circle or of a sphere and lies within the circle or sphere. Then have the pupils select the diameters in Pictures A to D. Be sure they realize that Picture D shows a sphere, not a circle. Be sure the pupils realize that the curved line represents the circumference of the sphere, not the diameter.

2 Discuss how to find a line that represents the altitude of a parallelogram and of a triangle. Then have the pupils choose the correct line in each of Pictures E to G.

3 Use Pictures H to L with Exercises C to F to review the important dimensions of these geometric shapes.

# ...3-16 Moving forward

...ponded Notes for this lesson ore on pages 281-282.

**...bjectives**

...he pupil learns about the measurement of volume. He ...arns what a cubic inch is and how to find the volume ...f various shapes by counting the number of cubic ...ches they contain.

**...ocabulary**

...lew words page 13 volume; page 14 layer*; page 15 ...ubic, completely*; page 16 solid*

**...omments**

...n this lesson the pupils are introduced to the concept ...f volume and to the cubic inch as a measure of volume. The pupils have had sufficient work with linear ...easure to know that the inch is a standard unit of ...easure for length. If they used Book 5 of this series, ...ey learned that the square inch is a standard unit of ...easure for area. Here they learn that the cubic inch ... a standard unit of measure for volume. The pupils also learn that three-dimensional, or "solid," shapes that have different dimensions may have the same volume.

By beginning this lesson with illustrations of linear and square measurement, it is possible to show how the measurement of volume is related to these other two types of measurement. This should help the pupils understand the meaning of volume and realize how the method of measuring volume is related to the method of measuring length and area.

At this grade level, the measurement of volume is usually restricted to the rectangular prism, and the pupils may thus get the impression that only rectangular prisms have volume. To prevent this misconception the authors have included cylinders and cones in the study of volume in this lesson. The pupils are not expected to compute, or even estimate, the volume of cylinders and cones precisely, but they are made aware that all such "solid" shapes *do* have measurable volume.

Pupils' understanding of the idea of volume will be increased if they are made aware of the usefulness of

cubic measure. They know that with linear measure it is possible to find out not only that one table, for example, is longer than another, but also how much longer it is. Similarly, with cubic measure, it is possible to find not only that one container is larger, or has more capacity, than another, but also how much larger it is. Thus, a standard unit of measure—for example, the cubic inch—is used in measuring volume, just as the linear inch is used in measuring length and the square inch is used in measuring area.

If the slow pupils have difficulty in understanding how the cubic inch is a standard unit of cubic measure, you can supplement the text and illustrations with work with objects. In explaining volume and the measuring of capacity, you can use water or sand, for example, to compare the volumes of various sizes of containers. Also one-inch cubes, which can be made from cardboard or obtained commercially, can be used to fill rectangular and cylindrical boxes and cans to help the pupils understand the meaning of volume. Cubes can also be made by sawing a long piece of wood 1" x 1" into one-inch pieces.

The computation of the volume of rectangular prisms will be taught later in this book; this lesson is intended only to introduce the pupil to the meaning of volume and to show him what is involved in finding volume.

**Answers**

Pages 13-15:

A 9 B 9 in. C 20 D 20 E 20 sq. in.

F The circle in Picture D. It takes about 6 one-inch squares to cover the triangle and about 7 one-inch squares to cover the circle.

G No. A 1-inch square is a square whose sides are each 1 in. in length. 1 sq. in. is an amount of *area*, and it can have any shape. (Pupils can illustrate this for themselves by cutting a 1-inch square into pieces which, when put together into various shapes, still have a total area of 1 sq. in.)

H Rectangular prism

I 12 J Yes; 24

K 24 L 12 M 12; 24

N The same O No; yes

P Any rectangular prism might conveniently be used to measure the volume of any other rectangular prism that is larger, because there will be n unmeasured space. For purposes of demonstration the cube is most convenient.

Q Since cubes have flat sides, they do not fit th curved surface of a cylinder.

R The outside corners of the cubes in Picture J hav been cut off to make the shape of the cylinder i Picture I. It is hard to know how many cubes coul be made from the cylinder in Picture K because i is difficult to tell how much of each cube has bee cut off.

S Yes; 4

T Since there are at least 2 cubes in each layer and there are 4 layers, the volume of the cylinder is a least 8 cubes. However, the whole pile of cubes i Picture J has a volume of 16 cubes, and not all o them are needed to fill the cylinder. So the volume of the cylinder must be less than 16 cubes.

U 1 in. long, 1 in. wide

V 1 in. long, 1 in. wide, 1 in. high; yes

W 3 in. long, 2 in. wide, 2 in. high; 12 cu. in.

Page 16:

A 30 cu. in.; There are 15 cu. in. in each layer, and there are 2 layers.

B 12; 4; yes; 48

C 4 in. long, 3 in. wide, 4 in. high; 48 cu. in.

D By finding the number of cubes in one layer and then adding or multiplying to get the number of cubes in all four layers

E 6 in. long, 2 in. wide, 4 in. high; 48 cu. in.

F The same as

G Yes. Each of its faces is a square. No

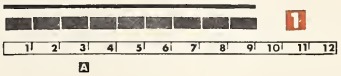
H 27 cu. in.

I Less than; To make Cylinder E, the corners of Cube D would have to be cut off.

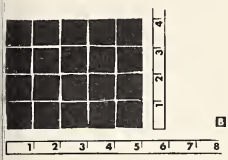
J Less than

## Moving forward

In this lesson you will learn about the measurement of area and volume.



A



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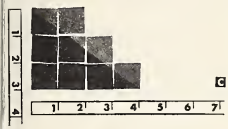
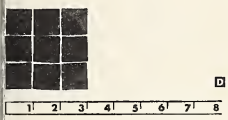


C



D

A How many strips does it take to measure the length of the blue stick in Picture A? 1

B Each strip represents 1 inch. The stick is ▒ inches long.

C How many squares does it take to cover the rectangle in Picture B?

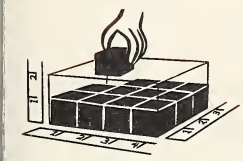
D Each square represents a 1-inch square. It takes ▒ 1-inch squares to cover the rectangle. 2 3

E Each 1-inch square is 1 square inch. The area of the rectangle shown in Picture B is ▒ square inches.

F Which do you think has the larger area, the triangle in Picture C or the circle in Picture D? Why do you think so? 4

G Does an area of 1 square inch always have the shape of a 1-inch square? Explain your answer. 5

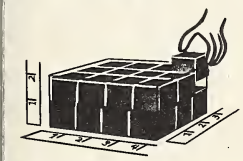




E

H The box in Picture E is what geometric shape? 1

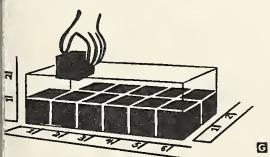
I The box is being filled with cubes. Each cube is 1 in. long, 1 in. wide, and 1 in. high. How many of these cubes does it take to make 1 layer on the bottom of the box?



F

J Now look at Picture F. Will there be the same number of cubes in the second layer as in the first layer? How many cubes does the box hold? 2

K The volume of this box is the number of cubes the box can hold. The volume is ▒ cubes. 3

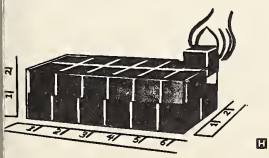


G

L Now look at Picture G. To find the volume of this box, you can fill it with 1-inch cubes. How many cubes will be in the first layer? 4

M Look at Picture H. How many cubes will be in the second layer? The volume of this box is ▒ cubes.

N Is the volume of this box greater than, less than, or the same as the volume of the box in Picture F? 5



H

O Do the boxes in Pictures E and G have the same dimensions? Are their volumes equal? 6

P Why is the cube a good shape to use for measuring volume? 7

# 13

1 In connection with Exercises A and B, have the pupils note that each strip corresponds to one unit on the ruler below the strips.

2 Let someone count the number of squares to answer Exercises C and D.

3 Some of the pupils may recall how to find the area of a rectangle by computation. Let them talk about it if they wish, but do not make a point of discussing the computation of area at this time. This topic is retaught later.

4 The pupils should be able to count the number of 1-inch squares needed to cover the triangle and to approximate the number needed to cover the circle.

5 Use this exercise for class discussion.

# 14

1 From previous work, pupils should recognize that this box is a rectangular prism. Have them answer questions in Exercises H and I.

2 To answer these questions, help pupils see that the vertical ruler shows that the prism is 2 inches high and 2 layers of cubes are needed.

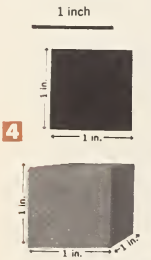
3 Relate volume (filling a box with 1-inch cubes) to area (covering a surface with 1-inch squares) and to length (using 1-inch strips).

4 Discuss Exercises L and M and Pictures G and H in much the same way as Exercises H to K and Pictures E and F.

5 Pupils should see that, if the number of cubes that the boxes in Pictures F and H hold is the same, the volumes are the same.

6 Be sure the pupils understand that rectangular prisms with different dimensions can have the same volume.

7 Use this exercise for general discussion. The convenience of using the cube to measure volume is discussed in the answer key.

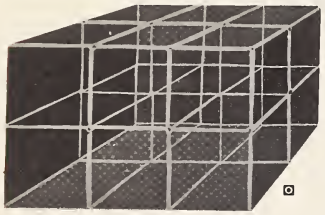



C Look at Picture I. Why would it be hard to fill the cylinder completely with cubes? 1

R Picture J shows a pile of cubes as high as Cylinder I and as wide as the diameter of the cylinder. What does Picture K show? Why is it hard to know how many cubes could be made from the cylinder in Picture K? 2

S Do you think that you could make at least two cubes from each layer? How many layers are there? 3

T You can say that the volume of the cylinder is between 8 cubes and 16 cubes. Why?

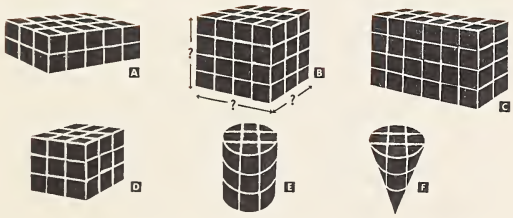



U What are the dimensions of the square in Picture M? 4

V What are the dimensions of the cube in Picture N? Is a 1-inch cube equal to 1 cubic inch?

W Prism O is made of 1-inch cubes. What are the dimensions of Prism O? What is its volume in cubic inches? 5

15




A Each shape shown above has been made of cubes, or parts of cubes. Imagine that the cubes are all 1-inch cubes. Is the volume of Prism A 15 cubic inches or 30 cubic inches? How do you know? 1

B How many cubes are in the top layer of Prism B? How many layers of cubes are there? Does each layer have the same number of cubes? How many cubes in all are in Prism B?

2 G What are the dimensions of Prism B? The volume of Prism B is ■ cu. in.

3 D How can you find the number of cubes in Prism C?

4 E What are the dimensions of Prism C? Its volume is ■ cu. in.

F Is the volume of Prism C greater than, less than, or the same as the volume of Prism B? 5

G Is Prism D a cube? How do you know? Are Prisms A, B, and C also cubes? 6

H The volume of Cube D is ■ cu. in.

I Cylinder E has the same height as the cube in Picture D. Its diameter is 3 in. Do you think that the volume of Cylinder E is greater than, less than, or the same as the volume of Cube D? Why do you think so? 7

J Cone F has the same diameter and the same height as Cylinder E. Is its volume greater than, less than, or the same as the volume of Cylinder E? 8

Now you should know what is meant by the volume of a prism or cylinder or of other solid shapes.



16

# 15

1 Tell the pupils that they are going to try to find the volume of this cylinder. Have them answer the question in Exercise Q.

2 Be sure the pupils realize that there are more cubes in the pile of cubes in Picture J than Cylinder I would hold. Have them note that Picture K shows that part of each cube has been cut away so that the parts left fit into Cylinder I.

3 To help the pupils answer Exercises S and T, be sure they see that part of each cube has been cut off. Have someone explain why the volume cannot be less than 8 cubes and then why it cannot be as much as 16 cubes.

4 In discussing Exercises U and V, relate Pictures L, M, and N.

5 Have the pupils count the cubes in each layer of Prism O and give its volume. Then relate the number of cubes to its dimensions.

# 16

1 By now the pupils should see that in a rectangular prism there are the same number of cubes in each layer, and that to find how many cubes there are in all they can count the number in a layer and then add or multiply. Have the pupils answer the questions in Exercises A and B.

2 The pupils can determine the dimensions from the number of cubes in a row, the number of rows, and the number of layers.

3 Use Note 1.

4 Use Note 2.

5 Help the pupils see that if there are the same number of cubes in Prism C as in Prism B, the volumes are the same.

6 The pupils should recall that a cube is a prism all of whose faces are equal squares.

7 Relate Exercise I to what the pupils learned about the cylinder on the preceding page.

8 Ask pupils to give reasons for their answers.

# 17 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil sees that various words can be used to describe the dimensions of geometric shapes.

**Vocabulary**

New words page 17 hole*, thick*, wastebasket*, Henry*

**Comments**

The purpose of this lesson is to show the pupils that the words used in describing dimensions depend upon what is being described as well as its shape. For example, a food freezer in the shape of a rectangular prism may be described as 5 ft. long, 4 ft. wide, and 3 ft. high. A hole in the ground, also in the shape of a rectangular prism, may be described as 5 ft. long, 4 ft. wide, and 3 ft. deep. A wall may be described as 10 ft. long, 4 ft. high, and 1 ft. thick.

The convention of writing dimensions in the form 8 ft. x 5 ft. x 3 ft. is introduced here. Also, the pupils are told that when dimensions are written in this way they are read "8 ft. by 5 ft. by 3 ft."

You might permit the pupils to practice using the various words that describe dimensions in different ways by drawing on the board a rectangular prism, with dimensions shown. Then ask the pupils to describe it as if it represents a cardboard carton, a hole in the ground, a book, etc. Give the pupils as much practice as possible in using different words to describe the dimensions of various objects.

**Answers**

A Both girls were right. Ruth thought of the trunk as being 20 in. deep from front to back and 13 in. high from the floor to the top of the lid. Carol thought of the trunk as 13 in. deep from the top to the floor.

B Yes. All its sides appear to be rectangular.

C A rectangular prism; because all its sides are roughly rectangular

D Yes. Let the pupils explain why this is a natural way to describe a hole.

E Probably not; If you were in the hole, you might possibly describe the side of the hole as 3 ft. high.

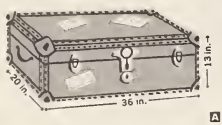
F The wall can be described as 12 ft. long, 4 ft. high, and 1 ft. wide; the 1-foot width

G 12 ft. x 4 ft. x 1 ft.

H 12 in. in diameter and 4 in. high or deep; The box can be thought of as either high or deep, as with the trunk in Exercise A. The wastebasket is 9 in. in diameter and 12 in. high or deep.

I Yes. The altitude of a triangle is sometimes called the height. This terminology arises from the fact that when the base is horizontal, the height is measured from the highest point above the base to the base. This dimension is, however, usually called *altitude*.

## Thinking straight




A

A Ruth said that the trunk in Picture A is 36 in. long, 20 in. deep, and 13 in. high. Carol said it is 36 in. wide, 20 in. long, and 13 in. deep. Which girl was right? Why do you think so? 1

B Would you say that the trunk has the shape of a rectangular prism? Why?

B

C The hole in the ground in Picture B has the shape of a prism. What kind of prism is it? Why? 2

D You can describe the hole as being 15 ft. by 9 ft. by 3 ft., which is often written like this: 15 ft. x 9 ft. x 3 ft. Could you also describe the hole as 15 ft. long, 9 ft. wide, and 3 ft. deep? Why? 3

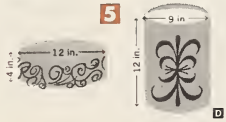
E Would you say the hole is 3 ft. high? Why or why not?

C

F Describe the size of the stone wall in Picture C. For which dimension could you use the word "thick"? 4

G The wall is ■ ft. x ■ ft. x ■ ft.

H How would you describe the size of the box in Picture D? Why? How would you describe the size of the wastebasket? 5




D

I Henry said the altitude of a certain triangle was 15 inches. John said the height of the same triangle was 15 inches. Do you think both boys were talking about the same dimension? Why? 6



# 17

1 Help the pupils see that it does not matter how the dimensions are described just as long as we know what is being described and there is no misunderstanding.

2 The pupils should see that the open top of the hole can be thought of as the other base of the prism. Discuss the perspective in the drawing.

3 Explain that dimensions written in this way usually indicate that a rectangular prism is being described.

4 The pupils will probably describe the wall in several ways; be sure they understand that there is no one correct way.

5 Ask the pupils what geometric shape the box and the wastebasket have.

6 Explain that in this case there is no difficulty in understanding which dimension both boys are talking about, but that the dimension from the base to the point of the triangle farthest from it is usually called *altitude*.

# 18 Checking up

Expanded Notes for this lesson are on pages 283-284.

**Objectives**

The pupil tests his knowledge of addition, subtraction, multiplication, and division of whole numbers, and methods of changing measures from one unit to another. He also tests his ability to choose the correct computational process (addition or subtraction) to find the numeral that replaces *n* in an equation when *n* is on the left side of the equals sign instead of by itself at the right.

**Vocabulary**

There are no new words.

**Comments**

In this book the sections called "Checking up" are tests. Such tests may be inventory tests, diagnostic tests, or achievement tests. The tests on page 18 are inventory tests that will tell how well pupils remember computational processes learned in previous years.

Test 1 is on addition of two-, three-, and four-figure numbers; Test 2 is on subtraction of three-, four-, and five-figure numbers; Test 3 is on multiplication of two-, three-, and four-figure numbers by one-figure multipliers (Exercises C and G should be computed by using the one-figure number as the multiplier); Test 4 is on multiplication by two-figure multipliers; Test 5 is on multiplication by three-figure multipliers; Test 6 is on division of two-, three-, and four-figure numbers by one-figure divisors; Test 7 requires the pupil to solve an equation by choosing the correct computational process (addition or subtraction) to find the numeral that replaces *n*. For example, the answer to Exercise A will be found by subtraction, while the answer to Exercise C will be found by addition. Test 8 is on reduction of measures.

Note that the examples given in the tests have not been arranged in computational form. This has been left for the pupils to do. The authors believe that pupils need practice in arranging the numerals for con-

venient computation, since it will be necessary for them to do so with arithmetic problems outside the classroom. Notice also that *n* or the screen (▭) has been used to hold a place for the numeral that must be found in each equation. Pupils who have used earlier books in this program will be familiar with the (▭) and with the equation form as well.

The grouping of the examples in the tests according to the skills required should help reveal the weaknesses of each child and should help you decide what topics need to be retaught either to the whole class or on an individual basis. The tests are not speed tests, and the pupils should be given ample time to complete their work.

Remember that a pupil's success in arithmetic depends upon his knowing all the basic facts. If there is any evidence that he does not know all of them, special steps should be taken to help him. Suggestions are given under the heading "Helping the slow learner" in the Expanded Notes. Much can be learned about the pupil's proficiency by noting his work habits and the procedures he uses to get his answers. Of particular significance are speed and assurance, signs of frustration and hesitation, counting on fingers or tallying, and erasures that show faulty knowledge of the basic facts or a change of mind about the procedure. All these things should be taken into account in evaluating the results of these tests, and appropriate remedial work should be planned.

To help you in any reteaching that is necessary, references to pages in *Seeing Through Arithmetic 4* and *Seeing Through Arithmetic 5** are given in the Reteaching Chart below for each test. You will find it advantageous to have a few copies of these texts and the *Teaching Guides* as well in your own classroom.

**Reteaching chart**

| Test | Pages |
|---|---|
| 1 | Book 4: 20-27 |
| 2 | Book 4: 32-41, 45, 54 |
| 3 | Book 4: 142-146, 147-153; Book 5: 48-59, 164 |
| 4 | Book 4: 161-164; Book 5: 61-64, 164 |
| 5 | Book 4: 161-164; Book 5: 77 |
| 6 | Book 4: 182-191; Book 5: 68-74, 91 |
| 7 | Book 4: 92-94; Book 5: 20 (A, F)<br>Book 4: 130-133; Book 5: 43-44 (B, H)<br>Book 4: 134-137; Book 5: 40-42 (C, E, I)<br>Book 4: 88-90; Book 5: 20 (D, G) |
| 8 | Book 5: 92-95 |

**Answers**

Whenever there is a remainder as part of the answer to a division exercise (Test 6), the answer is shown both as a mixed number and as a whole number with a remainder. Accept either form of the answer as correct.

| Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|
| A 145 | A 245 | A 288 | A 2236 |
| B 1791 | B 49 | B 292 | B 1206 |
| C 1488 | C 56 | C 430 | C 5929 |
| D 1707 | D 294 | D 843 | D 15,792 |
| E 9777 | E 1734 | E 2940 | E 15,750 |
| F 17,810 | F 2152 | F 1746 | F 8175 |
| G 14,376 | G 18,441 | G 5056 | G 4992 |
| H 18,108 | H 57,202 | H 10,138 | H 7650 |
| | I 35,828 | I 15,120 | I 75,510 |

Test 5

A 28,080
B 161,616
C 139,975
D 465,976
E 102,051
F 410,254
G 416,970
H 287,280
I 427,923

Test 6

A 39 and 1 rm. or $39\frac{1}{2}$
B 42
C 29
D 20 and 3 rm. or $20\frac{3}{4}$
E 47
F 36 and 1 rm. or $36\frac{1}{3}$
G 354 and 3 rm. or $354\frac{3}{7}$
H 678
I 307 and 2 rm. or $307\frac{2}{5}$

Test 7

A 374
B 282
C 1935
D 1625
E 7035
F 186
G 11,929
H 4145
I 15,373

Test 8

A 48 in.
B 15 qt.
C 144 oz.
D 13 wk.
E 24 yd.
F 40 pk.
G 24 fl. oz.
H 16,000 lb.
I 420 min.
J 11 pt.
K 60 mo.
L 12 gal.
M 72 things
N 9 fl. oz.
O 144 in.
P 5 pk.
Q 144 hr.
R 15,840 ft.

* *Seeing Through Arithmetic 4* and *Seeing Through Arithmetic 5* by Maurice L. Hartung, Henry Van Engen, and Lais Knowles. Fourth- and fifth-grade books in The Basic Mathematics Program. Scott, Foresman and Company.

## Checking up

**Test 1** 1
Find the sum.

A 15, 38, 26, 19, 47
B 274, 501, 386, 630
C 432, 195, 548, 313
D 169, 74, 280, 455, 729
E 6318, 2467, 992
F 7526, 4008, 6276
G 5109, 1378, 4629, 3260
H 8321, 796, 3554, 5437

**Test 2** 2

A $473 - 228 = n$
B $116 - 67 = n$
C $705 - 649 = n$
D $500 - 206 = n$
E $3499 - 1765 = n$
F $6002 - 3850 = n$
G $27134 - 8693 = n$
H $94500 - 37298 = n$
I $85623 - 49795 = n$

**Test 3** 2

A $6 \times 48 = n$
B $4 \times 73 = n$
C $86 \times 5 = n$
D $3 \times 281 = n$
E $7 \times 420 = n$
F $9 \times 194 = n$
G $632 \times 8 = n$
H $2 \times 5069 = n$
I $4 \times 3780 = n$

**Test 4** 2

A $52 \times 43 = n$
B $18 \times 67 = n$
C $77 \times 77 = n$
D $48 \times 329 = n$
E $25 \times 630 = n$
F $545 \times 15 = n$
G $64 \times 78 = n$
H $102 \times 75 = n$
I $839 \times 90 = n$

**Test 5** 2

A $117 \times 240 = n$
B $336 \times 481 = n$
C $509 \times 275 = n$
D $628 \times 742 = n$
E $493 \times 207 = n$
F $806 \times 509 = n$
G $565 \times 738 = n$
H $684 \times 420 = n$
I $729 \times 587 = n$

**Test 6** 2

A $79 \div 2 = n$
B $252 \div 6 = n$
C $145 \div 5 = n$
D $83 \div 4 = n$
E $376 \div 8 = n$
F $109 \div 3 = n$
G $2481 \div 7 = n$
H $6102 \div 9 = n$
I $1537 \div 5 = n$

**Test 7** 2

A $n + 326 = 700$
B $561 - n = 279$
C $n - 608 = 1327$
D $1463 + n = 3088$
E $n - 2729 = 4306$
F $n + 816 = 1002$
G $7205 + n = 19134$
H $9365 - n = 5220$
I $n - 6994 = 8379$

**Test 8** 3

A 4 ft. = ■ in.
B 30 pt. = ■ qt.
C 9 lb. = ■ oz.
D 91 da. = ■ wk.
E 72 ft. = ■ yd.
F 10 bu. = ■ pk.
G 3 c. = ■ fl. oz.
H 8 T. = ■ lb.
I 7 hr. = ■ min.
J 22 c. = ■ pt.
K 5 yr. = ■ mo.
L 48 qt. = ■ gal.
M 6 doz. = ■ things
N 18 tbsp. = ■ fl. oz.
O 4 yd. = ■ in.
P 40 qt. = ■ pk.
Q 6 da. = ■ hr.
R 3 mi. = ■ ft.

4

18 Inventory tests on addition, subtraction, multiplication, division, and measures

# 18

1 Ask the pupils to write the numerals in the proper form for computation, label each exercise with its identifying letter, and compute.

2 Have the pupils write the numerals in the proper form and compute, then rewrite each equation using a numeral to replace *n*.

3 For this test the pupils may not need to compute on paper for all the exercises. Ask them to write each answer after its identifying letter.

4 When the pupils have been given ample time to compiete the eight tests, provide answers so that they may verify their work. Then do whatever reteaching is necessary. (See the Reteaching Chart.)

# 19 Thinking straight

Expanded Notes for this lesson are on page 284.

**Objectives**

The pupil reviews rounding off numbers. He reviews the principle that a divisor and a dividend can be divided by the same number without affecting the quotient.

**Vocabulary**

There are no new words in this lesson.

**Comments**

The main purpose of this lesson is to give the pupil a review of the techniques he will need to use in estimating partial quotients in division. One important idea he should get from this lesson is that numbers can be rounded either up or down and to any multiple of ten (hundreds, thousands, etc.) as may be desired.

Some time should be spent in discussing the value of rounding off numbers. For example, talk about situations in which numbers should be rounded up and situations in which they should be rounded down. Discuss the degree of approximation desired; that is, whether a number rounded to the nearest hundred, thousand, etc., is close enough to the original number to be useful. Point out, too, that sometimes no rounding off should be done, but exact quantities should be used.

Another way of stating the principle that both the divisor and the dividend can be divided by the same number without affecting the quotient is to say that if the divisor and dividend are changed by dividing both by the same number, and division is again performed, the quotient will be the same as if the original numbers had been divided. (This idea was introduced in *Seeing Through Arithmetic 5*, pages 136-137.) In the example $224 \div 32$, a common factor for both divisor and dividend is 4. (Of course, there are other common factors, such as 2 or 8.) When you divide both 224 and 32 by 4, you get 56 and 8. The quotient for $56 \div 8$ is the same as the quotient for $224 \div 32$.

In this lesson the pupils also review how such changes can be made when both dividend and divisor end in

one or more zeros. They are shown that the dividend and the divisor may be divided by either 10 or 100, depending upon whether both numerals end in one or two zeros. The pupils should see that the result of dividing by either 10 or 100 can be shown by crossing out one or two zeros.

**Answers**

(block 1)

| | higher hundred | nearer hundred | lower hundred |
|---|---|---|---|
| A | 2600 | 2600 | 2500 |
| B | 500 | 500 | 400 |
| C | 400 | 300 | 300 |
| D | 1200 | 1100 | 1100 |
| E | 900 | 900 | 800 |
| F | 200 | 100 | 100 |
| G | 4600 | 4600 | 4500 |
| H | 1000 | 900 | 900 |
| I | 7100 | 7000 | 7000 |
| J | 1000 | 1000 | 900 |
| K | 600 | 500 | 500 |
| L | 6300 | 6200 | 6200 |

| | higher thousand | nearer thousand | lower thousand |
|---|---|---|---|
| M | 4000 | 3000 | 3000 |
| N | 6000 | 6000 | 5000 |
| O | 10,000 | 9000 | 9000 |
| P | 10,000 | 10,000 | 9000 |
| Q | 13,000 | 13,000 | 12,000 |
| R | 31,000 | 30,000 | 30,000 |
| S | 72,000 | 72,000 | 71,000 |
| T | 93,000 | 92,000 | 92,000 |
| U | 20,000 | 20,000 | 19,000 |
| V | 42,000 | 42,000 | 41,000 |
| W | 87,000 | 87,000 | 86,000 |
| X | 21,000 | 20,000 | 20,000 |

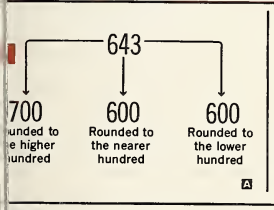
(block 2)

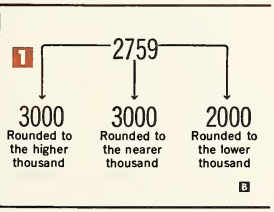
A Yes

B Because you can divide both the dividend and the divisor by the same number (10) without changing the quotient.

| | | | | |
|---|---|---|---|---|
| C 8 | D 50 | E 7 | F 70 | G 9 |
| H 90 | I 80 | J 9 | K 5 | L 8 |
| M 70 | N 60 | O 60 | P 42 | |

## Thinking straight




Round each of the following numbers to the higher hundred, the nearer hundred, and the lower hundred. For Example A you would have 2600, 2600, and 2500.

| | | | | | |
|---|---|---|---|---|---|
| A | 2573 | E | 885 | I | 7024 |
| B | 467 | F | 138 | J | 962 |
| C | 312 | G | 4550 | K | 533 |
| D | 1108 | H | 941 | L | 6247 |

Round each of the following numbers to the higher thousand, the nearer thousand, and the lower thousand.

| | | | | | |
|---|---|---|---|---|---|
| M | 3180 | Q | 12578 | U | 19738 |
| N | 5674 | R | 30294 | V | 41629 |
| O | 9302 | S | 71889 | W | 86940 |
| P | 9836 | T | 92115 | X | 20070 |

L When 224 is divided by 32, the answer is 7. Now divide both 224 and 32 by 4. Next divide 56 by 8. Is the answer the same as when 224 is divided by 32?

B $240 \div 10 = 24$, and $30 \div 10 = 3$. Now divide 24 by 3. Why does $24 \div 3$ have the same answer as $240 \div 30$? 4

C The short-cut way to divide 240 by 30 is first to divide both 240 and 30 by 10. $240 \div 30 = 24\not{0} \div 3\not{0}$. The answer is ▭. 4

D The short-cut way to divide 35000 by 700 is first to divide each number by 100. $35000 \div 700 = 350\not{0}\not{0} \div 7\not{0}\not{0}$. The answer is ▭. 5

Use the short cut and then divide. 6

| | | | |
|---|---|---|---|
| E | $560 \div 80 = n$ | K | $2000 \div 400 = n$ |
| F | $2100 \div 30 = n$ | L | $4800 \div 600 = n$ |
| G | $810 \div 90 = n$ | M | $63000 \div 900 = n$ |
| H | $3600 \div 40 = n$ | N | $42000 \div 700 = n$ |
| I | $4000 \div 50 = n$ | O | $36000 \div 600 = n$ |
| J | $720 \div 80 = n$ | P | $8400 \div 200 = n$ |



# 19

1 Have pupils study these two charts to see examples of rounding to the higher, nearer, and lower hundred and thousand. They should be able to explain why the nearer hundred is the same as the *lower* hundred in Chart A, while the nearer thousand is the same as the *higher* thousand in Chart B.

2 Let the pupils take turns in answering these exercises.

3 Let pupils discover that changing the dividend and divisor in this way and then dividing will give the same quotient as dividing 224 by 32.

4 Have pupils compare this exercise with Exercise A. Help them see that 240 and 30 are divided by 10 (because they end in 0) before doing the final division and that 224 and 32 are divided by 4 in Exercise A.

5 Pupils should see why each number can be divided by 100. Let someone give the answer.

6 Assign these exercises as written work.

# 20-23 Learning how

Expanded Notes for this lesson are on pages 284-286.

**Objectives**

The pupil reviews his knowledge of division. His ability to divide is refined by techniques of estimating partial quotients.

**Vocabulary**

New words page 20 chose*; page 21 well*

**Comments**

The process of division has always been one of the greatest stumbling blocks in arithmetic. This has continued to be so even though certain changes have been introduced in the teaching of division from time to time during the past fifty years. The division process is now customarily introduced by the "long-division" method—the method in which many details of the thinking (except that of estimating partial quotients) are written down—instead of the "short division" method. But since the traditional long-division method also involves several short cuts, in that only fragmentary parts of the complete numbers are worked upon at a time, real understanding has been difficult for pupils.

Division is a process of finding the number of equal groups that can be formed in an original group (the dividend) until that original group is used up. Thus division is performed by a process of repeated subtractions. When this understanding is used as the basis for teaching long division, certain features in the arrangement of the work become desirable because they help the pupil to understand what he is doing at every step of the way. In a modern program the decision as to what is a good arrangement of numerical work on paper depends not upon tradition but upon whether the arrangement helps or hinders an understanding of the process.

The arrangement of work used in this program differs in only a few ways from what has been done in the recent past. Partial quotients (numbers that will be combined to make the answer) are put at the right in a column. The final answer, which is their total, is put at the bottom of this column instead of above the dividend (the number to be divided). This arrangement is used so that at each step the partial quotients can be written out in full, rather than just one figure of the number, as has been customary. When the pupil thinks of 10, he writes 10, not just the figure 1; when he thinks of 200, he writes 200, not just the figure 2. Moreover, the traditional method requires the pupil to find the maximum partial quotient at each step. With the new arrangement he can use several partial quotients that are smaller than the maximum partial quotient. The final, or maximum quotient is then found by adding all the partial quotients. This enables the pupil to work within the limits of his ability much better than the usual method does.

In this arrangement, the pupil is permitted to write complete products (partial quotient $\times$ divisor) in all parts of the computation—that is, he subtracts 40 from 58 instead of "4 from 5" as in the old method. He writes the numerals so that they describe the thought processes. This enables him to understand the reasons for each step of the process. The amount that remains to be divided each time is shown completely, not in fragments as in the traditional method. This helps the pupil to think of the division process as it really is—a process of repeated subtraction.

This method of division was introduced in *Seeing Through Arithmetic 4* and retaught in *Seeing Through Arithmetic 5*. It is reviewed here in a complete lesson, which includes techniques for estimating partial quotients.

An efficient method of estimating partial quotients is shown in Exercise B on page 20 and in Exercise B on page 21. By using the rounding-off technique of estimating partial quotients shown here, the pupil can never get a partial quotient that is too large. He may occasionally estimate a partial quotient that is smaller than the maximum he could use, but by continuing the process he can obtain the correct answer without erasing or going back to correct his work.

The method of rounding off presented in this lesson involves these steps:

(1) For divisors less than 100, round up the divisor to the next multiple of 10; for divisors more than 100 but less than 1000, round them up to the next multiple of 100. Round down the dividend

to a lower multiple of 100, or of 1000, etc., so that the first one or two figures form a number that is divisible without remainder by the first figure of the rounded-up divisor. For example, suppose you want to divide 2963 by 84. Round 84 up to 90, which becomes the trial divisor. Round 2963 down to 2700, because you want a number divisible by 9. Then 2700 is used as a trial dividend.

(2) In order to arrive at a partial quotient more easily by working with smaller numbers, divide both trial divisor and trial dividend by 10, 100, 1000, etc. Thus, in the example above, it is easier to think of 270÷9 than to think of 2700÷90.

(3) Now a good partial quotient for the original division problem is easily found. For this example, 30 is a good partial quotient to start with.

Some of the pupils will be able to use this method without pencil and paper; others will need to use scratch paper to write the rounded numbers and find the partial quotient. In some division problems, no rounding off is necessary, since a good partial quotient will be immediately apparent. Or in some cases, either the dividend or the divisor will be in usable form already, and only one of the numbers will have to be rounded off.

The work on page 19 of the pupils' book reviews rounding numbers to the higher hundred, the nearer hundred, the lower thousand, etc. The page also reviews the principle that dividing both dividend and divisor by the same number does not alter the quotient. Pupils who understand the work on page 19 are prepared for the method of estimating quotients taught on pages 20 and 21.

It is usually the case that the pupils who need to use a very large number of partial quotients are the very pupils who would be unable to succeed at all with the traditional method of doing long division. The method shown here may take them longer than appears at first glance to be necessary, but it enables them to achieve unprecedented success in working division problems. In comparing methods, one should not forget the time and effort spent by pupils trying to find partial quotients in the usual division process.

Pupils who have learned the traditional arrangement of division computation and are able to use it successfully will have their understanding of the nature of division broadened by watching a demonstration of this new method. Such pupils may be permitted to continue using the arrangement they have learned. However, those who are unable to divide competently with the traditional method should be retaught by the method presented here.

The traditional arrangement of long division is shown in a "Side trip" lesson on pages 162-164 of the pupils' book.

The four-step teaching method is used in this lesson as it is throughout this program to develop all computational skills taught in lessons entitled "Learning how." Steps are provided to help the pupil SEE (by means of illustrations and explanatory text); THINK (by means of illustrations and questions about a problem); TRY (by means of sample problems and examples worked out with answers for comparison); and DO (by means of practice material without close guidance). Special material is provided for each of the four learning steps. For example, in the SEE step two examples of long division are shown worked out. In the TRY step on pages 22 and 23, three different division examples are presented, each with four solutions. The fourth solution (in blue) is the most efficient of these four ways to do the computation.

These four steps are based on a teaching method that successful teachers have used regularly in presenting new material. In this book the steps are systematically employed and are implemented by pictures, explanations, questions, examples, and problems. The SEE and THINK steps are intended to be taught under the guidance of the teacher with the aid of the book. The TRY and DO steps are intended to be used independently by pupils. They also furnish a means of providing for individual differences in pupils. The teacher should work with those who need assistance in the TRY step, at the time when they most need help. The better students may work as many problems or examples in the TRY step as they need, then go immediately to the DO step.

**Answers**

For each answer in which a remainder is involved, any of three forms may be accepted as correct—a whole number and a remainder, a mixed number whose fraction is expressed in simplest form, or a mixed number whose fraction is not expressed in simplest form. Expressing fractions in simplest form (lowest terms) and use of the remainder in division to express the answer as a mixed number are retaught on pages 52-55 and 59-62 of the pupils' book.

A 91 and 25 rm. or $91\frac{25}{91}$

B 62 and 23 rm. or $62\frac{23}{28}$

C 53 and 32 rm. or $53\frac{32}{56}$ or $53\frac{4}{7}$

D 35 and 15 rm. or $35\frac{15}{18}$ or $35\frac{5}{6}$

E 93 and 3 rm. or $93\frac{3}{32}$

F 80 and 10 rm. or $80\frac{10}{65}$ or $80\frac{2}{13}$

G 53 and 20 rm. or $53\frac{20}{87}$

H 124

I 379 and 3 rm. or $379\frac{3}{43}$

J 508

K 701 and 41 rm. or $701\frac{41}{67}$

L 49 and 269 rm. or $49\frac{269}{525}$

M 90 and 585 rm. or $90\frac{585}{806}$

N 93 and 276 rm. or $93\frac{276}{419}$

O 70 and 381 rm. or $70\frac{381}{738}$ or $70\frac{127}{246}$

P 48 and 455 rm. or $48\frac{455}{900}$ or $48\frac{91}{180}$

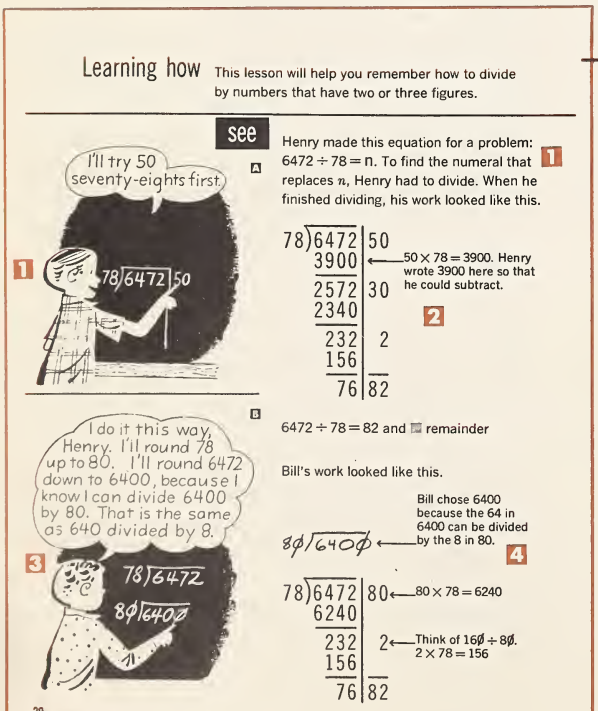
Q 69 and 320 rm. or $69\frac{320}{379}$

R 330 and 5 rm. or $330\frac{5}{600}$ or $330\frac{1}{120}$

S 111 and 159 rm. or $111\frac{159}{240}$ or $111\frac{53}{80}$

T 847 and 128 rm. or $847\frac{128}{858}$ or $847\frac{64}{429}$

U 340 and 96 rm. or $340\frac{96}{927}$ or $340\frac{32}{309}$



Learning how This lesson will help you remember how to divide by numbers that have two or three figures.

see

A

I'll try 50 seventy-eights first.

78)6472 50

Henry made this equation for a problem: $6472 \div 78 = n$. To find the numeral that replaces $n$, Henry had to divide. When he finished dividing, his work looked like this.

```
78)6472 | 50
   3900   <-- 50 × 78 = 3900. Henry wrote 3900 here so that he could subtract.
-------
   2572 | 30
   2340
-------
    232 |  2
    156
-------
     76 | 82
```

B

I do it this way, Henry. I'll round 78 up to 80. I'll round 6472 down to 6400, because I know I can divide 6400 by 80. That is the same as 640 divided by 8.

78)6472

8Ø)64ØØ

6472 ÷ 78 = 82 and ▩ remainder

Bill's work looked like this.

Bill chose 6400 because the 64 in 6400 can be divided by the 8 in 80.

8Ø)64ØØ

```
78)6472 | 80   <-- 80 × 78 = 6240
   6240
-------
    232 |  2   <-- Think of 16Ø ÷ 8Ø. 2 × 78 = 156
    156
-------
     76 | 82
```

20 Division by 2-figure and 3-figure divisors (reteaching)



# 20

1 Let a pupil read this paragraph aloud. Let pupils discuss the kind of problem that may have produced the equation. Then direct attention to the picture and say, "Let's see how Henry thought about the division."

2 Discuss the computation. Make sure the pupils understand that, first, 50 groups of 78 are subtracted from 6472. Ask how much is left. Then 30 groups of 78 are subtracted from 2572, and for the next partial quotient only 2 groups can be subtracted. The sum of the partial quotients is 82. The remainder is 76.

3 Let the pupils examine the picture and read what Bill is saying. Be sure the pupils understand Bill's method for arriving at 80 and 2 for partial quotients.

4 Discuss the computation, and ask if Bill got the same answer as Henry. Discuss which solution seems easier.

## see

Ruth made this equation for a problem: 11286 ÷ 283 = n. Her work looked like this when she finished dividing.

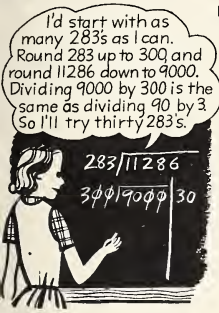
A

1

```
283)11286 | 10
    2830
    8456 | 20
    5660
    2796 |  5
    1415
    1381 |  4
    1132
     249 | 39
```

11286 ÷ 283 = 39 and ▪ remainder

B


I'd start with as many 283's as I can. Round 283 up to 300 and round 11286 down to 9000. Dividing 9000 by 300 is the same as dividing 90 by 3. So I'll try thirty 283's.
283)11286
300)9000 30

Sally's work looked like this.

2

300)9000 ← Sally chose 9000 because the 9 in 9000 can be divided by the 3 in 300.

```
283)11286 | 30   ← 30 × 283 = 8490
    8490
    2796 |  9   ← Think of 2700 ÷ 300. Try 9.
    2547
     249 | 39   ← Is Sally's answer the same as Ruth's answer?
```

21

## think

7843 ÷ 82 = n. Look below at the two ways of working this example. The second way is no shorter than the first way, but the second way does help you decide what numbers to use to find the answer.

First way

```
82)7843 | 50   ← Why can you use 50 here?
  4100
  3743 | 40
  3280
   463 |  5
   410
    53 | 95   ← How do you get 95?
```

↑ The remainder is ▪.

1

Second way

Round the divisor up to 90.

82)7843 ← Why do you then round 7843 down to 7200? 7200 ÷ 90 = ▪

```
82)7843 | 80   ← Why can you try 80 groups of 82?
  6560
  1283 | 10   ← Think: 900 ÷ 90.
   820
   463 |  5   ← Why can you try 5 groups of 82?
   410
    53 | 95
```

## try

A 34020 ÷ 486 = n

2

```
486)34020 | 10
   4860
  29160 | 20
   9720
  19440 | 20
   9720
   9720 | 20
   9720
      0 | 70
```

```
486)34020 | 30
  14580
   19440 | 30
  14580
   4860 | 10
   4860
      0 | 70
```

Round 486 up to 500.

Round 34020 down to 30000.

```
486)34020 | 60
  29160
   4860 | 10
   4860
      0 | 70
```

```
486)34020 | 70
   34020
```

3

2

# 21

1 Have the pupils read the text and examine the picture. Let them check the computation in the example. Point out that Ruth did not use any systematic method for choosing partial quotients.

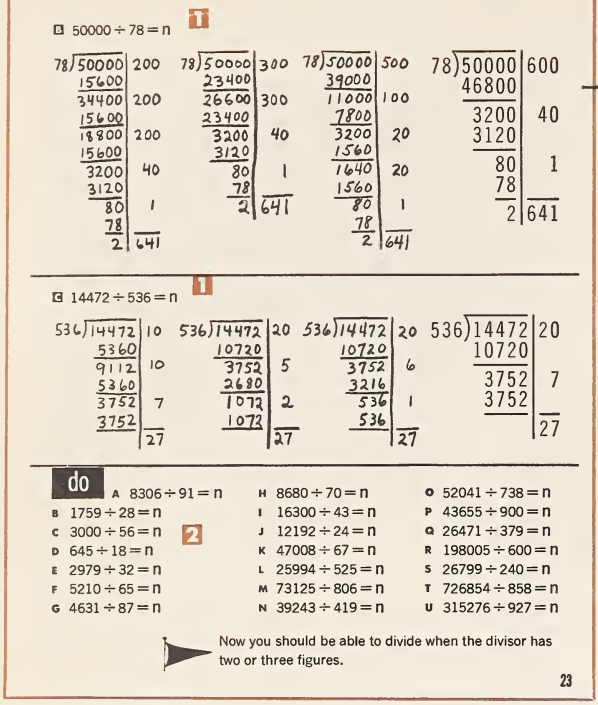
2 Point out that Sally is working the same problem as Ruth, but that she has a method for finding partial quotients. Call attention to the picture, and let a pupil read aloud what Sally is saying. Explain that Sally was able to find a large partial quotient by rounding up the divisor and rounding down the dividend. Discuss the computation. Then have the pupils compare this solution with the solution Ruth used.

# 22

1 Discuss each step in the two ways of working the example.

2 Assign Example A as written work. Tell the pupils not to look at the book until they have worked the example. When the pupils have finished their work, discuss the four solutions in the book. Then let them compare their work with the four solutions. If any pupil has a solution different from the ones in the book, let him present it. Point out that the last solution (in blue) is the most efficient, but do not make the pupils feel that other correct solutions are unacceptable. [Note that the quotient 70 is not obtained by estimating, but might be called a lucky guess. Division exercises with 3-figure divisors are more easily worked by the method taught in this book than by the traditional method.]

3 Let able pupils proceed immediately to the work in the DO step.




# 23

1 Assign Examples B and C as written work for those pupils who find it hard to estimate partial quotients. Encourage them to use the technique of estimating partial quotients that is shown in this lesson. Ask them to compare each answer with the one in the book as they work each example. Tell them that when they have finished Examples B and C, they should proceed to the work labeled DO.

2 When Exercises A to U have been worked by all the pupils, provide answers for Exercises A to U, and let the pupils put some of their solutions on the board. Discuss the different ways of getting the answers.

# 24 Checking up; Keeping skillful

Exponded Notes are not considered necessary for these lessons.

**Objectives**

The pupil tests his knowledge of division with divisors that have two figures and three figures. He reviews his knowledge of multiplying and dividing numbers that represent amounts of money.

**Vocabulary**

There are no new words.

**Comments**

These tests are end-of-block, or diagnostic, tests on division. The first test is confined to division with 2-figure divisors. Test 2 is made up of exercises with 3-figure divisors. Since these are not speed tests, the pupils should be given ample time to complete their work. Many of them should now be able to use the estimating techniques reviewed in the two previous lessons.

In the two tests at the top of the page, the division is of the type in which the size of the group is known, but the number of equal groups is to be found. In "Keeping skillful," another type of division is illustrated in Example B. We know the number of equal groups contained in $6.80, but we do not know the size of each group (*n*). (In a division equation, the numeral after the division sign always represents the size of the equal groups into which the total group is to be divided; the numeral after the equals sign represents the number of equal groups.) If the pupils understand the problem situation and know that it is not the number of groups that is to be found, but the size of the groups, they will know that the numerals in the answer represent money.

In Exercises C to R following, there are examples of both types of division. The pupils must be able to distinguish between them to interpret the answers when they have done the computation.

An activity which may be used for the able pupils in connection with this lesson is described in Activity 4, page 374.

**Answers**

## Checking up:

Whenever there is a remainder as part of the answer to a division exercise, the answer is shown both as a mixed number and as a whole number with a remainder. Accept either form of the answer as correct. Use of the remainder in division to express the answer as a mixed number will be retaught on pages 59-62 of the pupils' book.

Test 1

A 40 and 9 rm. or $40\frac{9}{21}$ or $40\frac{3}{7}$

B 45 and 7 rm. or $45\frac{7}{14}$ or $45\frac{1}{2}$

C 59

D 66 and 15 rm. or $66\frac{15}{40}$ or $66\frac{3}{8}$

E 24 and 18 rm. or $24\frac{18}{37}$

F 64 and 22 rm. or $64\frac{22}{75}$

G 73 and 14 rm. or $73\frac{14}{82}$ or $73\frac{7}{41}$

H 183 I 131 and 49 rm. or $131\frac{49}{60}$

J 307 K 439

L 420 and 50 rm. or $420\frac{50}{89}$

M 643 and 68 rm. or $643\frac{68}{93}$

N 895 and 56 rm. or $895\frac{56}{67}$

Test 2

A 23 and 60 rm. or $23\frac{60}{400}$ or $23\frac{3}{20}$

B 55 and 34 rm. or $55\frac{34}{135}$ C 73

D 50 and 10 rm. or $50\frac{10}{578}$ or $50\frac{5}{289}$

E 68 F 92

G 239 and 781 rm. or $239\frac{781}{849}$

## Keeping skillful:

(block 1)

C \$.75 D 7 and \$.14 rm. E 18 and \$.01 rm.

F \$.52 G \$.43 H 13 and \$.06 rm.

I \$.09 J 11 and \$.40 rm. K \$.40

L \$.37 and \$.29 rm. M 69 and \$.06 rm. N \$2.86

O 125 and \$.10 rm. P \$1.93 and \$.09 rm.

Q 18 and \$.15 rm. R \$3.59 and \$.04 rm.

(block 2)

A \$45.50 B \$10.71 C \$2.16 D \$63.35

E \$13.44 F \$223.80 G \$53.01 H \$584.72

## Checking up

Test 1 **1**

A $849 \div 21 = n$
B $637 \div 14 = n$
C $1062 \div 18 = n$
D $2655 \div 40 = n$
E $906 \div 37 = n$
F $4822 \div 75 = n$
G $6000 \div 82 = n$
H $5124 \div 28 = n$
I $7909 \div 60 = n$
J $17192 \div 56 = n$
K $13609 \div 31 = n$
L $37430 \div 89 = n$
M $59867 \div 93 = n$
N $60021 \div 67 = n$

Test 2 **2**

A $9260 \div 400 = n$
B $7459 \div 135 = n$
C $15038 \div 206 = n$
D $28910 \div 578 = n$
E $47600 \div 700 = n$
F $35512 \div 386 = n$
G $203692 \div 849 = n$

## Keeping skillful

A $\$8.40 \div \$.36 = n$ **3**

Divide just as you do with any numbers.

$$\begin{array}{r|l} \$.36\overline{)\$8.40} & 20 \\ 7\ 20 & \\ \hline 1\ 20 & 3 \\ 1\ 08 & \\ \hline 12 & 23 \end{array}$$

How do you know that this 23 does not mean \$.23?

This 12 means \$.12.

The answer is 23 and \$.12 remainder.

D $\$6.80 \div n = 15$ **4**

$$\begin{array}{r|l} 15\overline{)\$6.80} & 40 \\ 6\ 00 & \\ \hline 80 & 5 \\ 75 & \\ \hline 5 & 45 \end{array}$$

This 45 means \$.45. How do you know?

What does the 5 mean?

The answer is \$.45 and \$.05 remainder.

G $\$6.00 \div n = 8$ **5**
D $\$1.89 \div \$.25 = n$
E $\$.73 \div \$.04 = n$
F $\$8.84 \div n = 17$
G $\$2.58 \div n = 6$
H $\$5.00 \div \$.38 = n$
I $\$3.15 \div n = 35$
J $\$7.00 \div \$.60 = n$
K $\$10.00 \div n = 25$
L $\$12.50 \div n = 33$
M $\$6.27 \div \$.09 = n$
N $\$14.30 \div n = 5$
O $\$30.10 \div \$.24 = n$
P $\$23.25 \div n = 12$
Q $\$12.75 \div \$.70 = n$
R $\$50.30 \div n = 14$

A $2 \times \$22.75 = n$ **6**
B $17 \times \$.63 = n$
C $27 \times \$.08 = n$
D $5 \times \$12.67 = n$
E $56 \times \$.24 = n$
F $30 \times \$7.46 = n$
G $19 \times \$2.79 = n$
H $8 \times \$73.09 = n$

4 End-of-block tests on division

# 24

**1** Have each pupil label his work for each exercise with its identifying letter. Supply answers and decide what review he may need.

**2** See Note 1.

**3** Let pupils make up problems for the equation. Make sure they understand that the answer in this exercise represents the number of groups of 36 cents; thus it means 23 equal groups of cents and not 23¢. On the other hand, the remainder is what is left from an amount of money, and does represent cents.

**4** Let pupils make up problems for this equation. Discuss how this equation differs from that in Example A. Stress the fact that the numeral to replace *n* represents the size of the equal groups of money. Make sure the pupils understand that what they are trying to find out in this exercise is how much money there will be in each of the 15 equal groups.

**5** See Note 1.

**6** See Note 1.

# 25 Checking up

Expanded Notes for this lesson are on page 286.

**Objectives**

The pupil tests his ability to solve some of the basic types of problems that have been taught in Books 3, 4, and 5 of this program. He writes an equation to tell the story of the problem situation and uses the letter *n* to hold a place for the numeral representing the answer.

**Vocabulary**

New words page 25 American, Danish, Latin, Mexican*, Philip*, piano, racks, traded*, treasury*

**Comments**

This is an inventory test on problem solving. The purpose of this test is to enable you to discover how much reteaching needs to be done on problem solving. Let the pupils use as much time as they need for the test, and supply answers so that each pupil may verify his own work. To facilitate the verifying of answers, have them written on a sheet of paper, and let the two or three pupils who finish the work first make copies of the correct answers for other pupils to use. Determine which problems have caused the most trouble, and do whatever reteaching is necessary.

For the pupils who have not previously studied the problem-solving method used throughout this program, some time should be spent in familiarizing them with equation making as a means of telling the story in problem situations. These pupils will have to be taught to *think* about the situation in a problem before they begin to compute; i.e., to visualize the action that takes place, represent the action with an equation, and solve the equation. The equation which each pupil should be required to make for each problem is a written indication of his thinking, and it will assist you in appraising what he has learned. To help these pupils, it is suggested that you provide objects to use in showing the groupings and the action indicated in the various problem types. The section entitled "Analysis and communication" in the Introduction to this *Teaching Guide* (pages 11-14) will be of help to you. See also the chart in the Reference Materials section at the back of this *Teaching Guide*. This chart gives a description of each problem type, a sample of each, the equation that describes each sample problem, and the process to be used in computation.

Each of the problem types on page 25 was taught in *Seeing Through Arithmetic 4* and reviewed in *Seeing Through Arithmetic 5*. The table below shows you where to look in Books 4 and 5 for reteaching material for each of the problems in this inventory test.

**Reteaching chart**

| Problem | Pages |
|---|---|
| A | Book 4: 52-53; Book 5: 25 |
| B | Book 4: 86-87; Book 5: 21 |
| C | Book 4: 120-123; Book 5: 24, 87 |
| D | Book 4: 66-67; Book 5: 25 |
| E | Book 4: 134-137; Book 5: 40-42 |
| F | Book 4: 28-29; Book 5: 19 |
| G | Book 4: 42-43; Book 5: 19 |
| H | Book 4: 52-53; Book 5: 25 |
| I | Book 4: 92-94; Book 5: 20 |
| J | Book 4: 88-90; Book 5: 20 |
| K | Book 4: 130-133; Book 5: 43-45 |
| L | Book 4: 120-123; Book 5: 24 |
| M | Book 4: 86-87; Book 5: 21 |
| N | Book 4: 134-137; Book 5: 40-42 |

**Answers**

The brackets represent the place in which the letter *n* should be written in the equation. The numeral within the brackets is the answer for the problem.

A $7 \times 35 = [245]$

B $931 - 863 = [68]$

C $\$3.84 \div [\$.12] = 32$

D $600 \div 24 = [25]$

E $[\$6.50] - \$3.98 = \$2.52$

F $\$5.07 + \$3.89 + \$3.78 = [\$12.74]$

G $\$25.00 - \$19.49 = [\$5.51]$

H $6 \times 54 = [324]$

I $[\$1.74] + \$.65 = \$2.39$

J $\$3.47 + [\$3.48] = \$6.95$

K $200 - [37] = 163$

L $1000 \div [250] = 4$

M $\$1.30 - \$.75 = [\$.55]$

N $[200] - 46 = 154$

## Checking up 1

A Kathy practices on the piano for 35 minutes a day. How many minutes does she practice in a week?

B This September there are 931 pupils at the Castle School. Last September the Castle School had 863 pupils. This September the school has how many more pupils than it had last September?

C Thirty-two sixth graders bought a plant for their room and shared the cost equally. The plant cost $3.84. How much did each sixth grader pay?

D The sixth graders were decorating the gym for a party. How many 24-ft. lengths of crepe paper ribbon could they get from a 600-ft. roll of ribbon?

E After Philip had spent $3.98 of his birthday money to buy baseball shoes, he had $2.52 left. How much money did Philip get for his birthday?

F One month the boys in Philip's ball club spent $5.07 for baseballs, $3.89 for a catcher's mask, and $3.78 for bats. What was the total cost of this equipment?

G There was $25 in the club treasury before the boys bought some equipment that cost $19.49. How much money was left in the treasury after the equipment was bought?

H The Castle School has 6 new bicycle racks. Each rack holds 54 bicycles. There is room for how many bicycles in the 6 new racks?

I John is saving money to buy a basketball. Saturday he earned 65¢. Then he had $2.39. How much money did he have before he earned the 65¢?

J The parakeet Ann wants to buy costs $6.95. Ann has $3.47. How much more money does Ann need to buy the parakeet?

K Ruth bought 200 Mexican stamps. She traded some of them for United States stamps. Then she had 163 Mexican stamps. How many Mexican stamps had she traded?

L Ruth and three other girls bought a package of 1000 stamps and shared them equally. How many stamps did each girl get?

M Henry bought a package of Danish stamps for $1.30 and a package of Latin American stamps for 75¢. The Latin American stamps cost how much less than the Danish stamps?

N After Henry had traded 46 of his Danish stamps, he had 154 Danish stamps left. He had how many Danish stamps to begin with?

2

Inventory test on problem solving 25

## 25

1 Give this test without a time limit. Tell pupils that for each problem they are to make an equation that tells the story and to use *n* to hold a place for the numeral they must find. Explain that they are then to compute to find the answer and rewrite the equation, putting in the answer to replace *n*.

2 Provide answers and do whatever reteaching is necessary. (See the Reteaching Chart on page 46, which gives page references to the lessons in *Seeing Through Arithmetic 5* in which the various types of problems are taught.)

# 26-30 Exploring problems

Exponded Notes for this lesson ore on poges 287-288.

**Objectives**

This lesson reviews the multiple-step problem. The pupil learns how to write and solve one equation that fits the problem situation.

**Vocabulary**

New words page 28 jelly; page 29 plums; page 30 parentheses, Alice, station*

**Comments**

Pupils who have studied the previous books in this program will be familiar with the method of solving multiple-step problems that is reviewed in this lesson. The procedure of first making one equation and then doing the necessary computation to complete this equation is used. The purpose in having the pupils make one equation for each multiple-step problem is to teach them to first find out what basic question is asked in the problem. It is only if they know what the basic question is that they can make an equation. The fact that some preliminary work must be done before the main question can be answered does not affect the form of the equation. Thus, in the problem on pages 26 and 27, the fact that the 26 pints must be changed to quarts does not change the fact that the two amounts of peaches must be added to find the answer.

This approach to multiple-step problems is in keeping with the learning theory of this entire program—that the pupils should be helped to understand the whole situation from the beginning. In other words, by making one equation the pupils learn to analyze the problem situation and see what the problem as a whole actually asks, instead of rushing into computation. This method will give the pupils a good preparation for more complex problems, such as those in algebra in which two or more unknowns are involved.

Problems on finding the average are included in this lesson, since they are considered as a special type of multiple-step problem. The equation for finding an

average expresses division of the partitive type in which the number in each equal group is to be found. In many problems the total number that is to be divided into equal groups is not known at first and must be found before the number in each equal group can be found (see the example in A to D on pages 28 and 29). In cases where the total number is known, only one computation is necessary (see Problem I on page 30).

In the first three problems in this lesson, a "screen" has been used in the equation to hold a place for a numeral to be found by computation before the equation can be solved. (A screen was also used for this purpose in *Seeing Through Arithmetic 5*.) In Problem B on page 29, however, the data that need to be used are enclosed in parentheses and included in the equation instead of a screen. At this point the pupils are to discontinue the use of the screen in the equation and write the data in parentheses. Note that the inclusion of (17+9) in the equation in Problem B makes it apparent that 9 must be added to 17 before the division can be performed and the answer found.

**Answers**

Pages 29-30:

In this answer key the numeral that replaces *n* in the equation is bracketed. The numeral found by completing the work shown in parentheses in the equation is enclosed in parentheses here also.

A ($5.50)+$3.39=[$8.89]

B $3.45÷[$.13]=(26) and 7¢ rm.; 13¢; The remainder is too small to add another cent to the answer.

C $.35+($1.50)=[$1.85]

D (120)−(70)=[50]

E ($159.44)÷[$39.86]=4

F (36)÷[6]=6

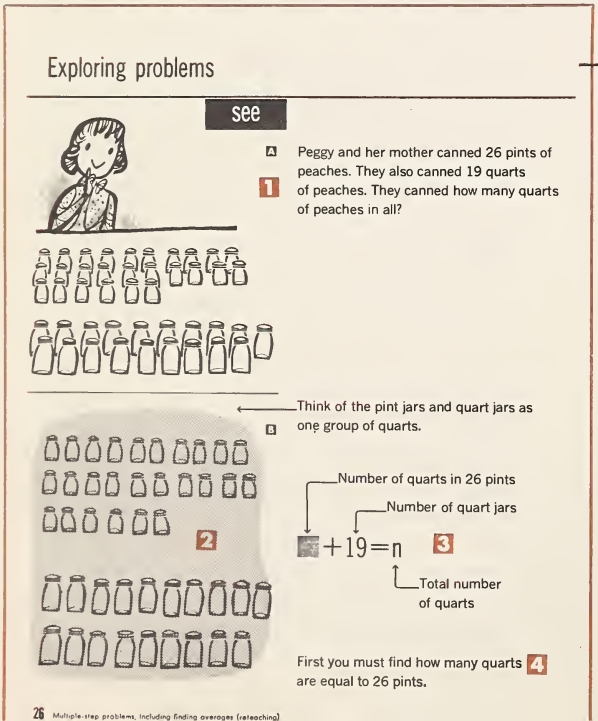
G (24)−[16]=8

H (394)÷[35]=11 and 9 rm.; about 36 pages

I 60÷[12]=5

J (256)÷[64]=4; 64°

K $5.00−($2.98)=[$2.02]


Exploring problems
see
A
1
Peggy and her mother canned 26 pints of peaches. They also canned 19 quarts of peaches. They canned how many quarts of peaches in all?
B
Think of the pint jars and quart jars as one group of quarts.
2
Number of quarts in 26 pints
Number of quart jars
+19=n
3
Total number of quarts
First you must find how many quarts are equal to 26 pints.
4

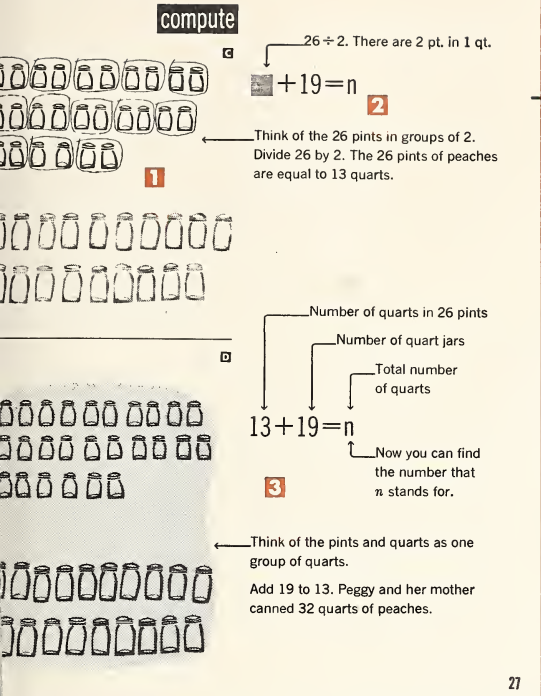
## 26

1 Have a pupil read the problem and then look at Picture A. Ask someone to explain why you cannot add 26 to 19 to find the answer.

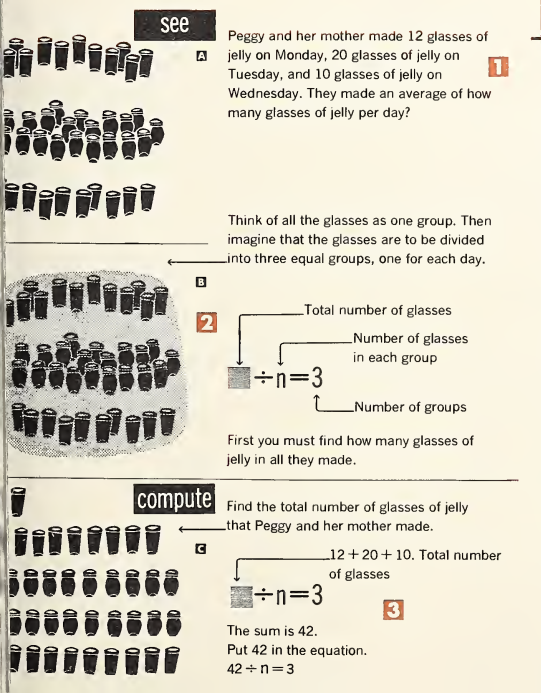
2 Tell the pupils that the dotted background in Picture B indicates that the pints and quarts are to be thought of as combined.

3 Call attention to the equation and discuss each symbol in it. Explain that the screen is used to hold a place for a numeral that must be found before the answer, *n*, can be found. Discuss reasons why this equation fits the problem.

4 Ask the pupils if the problem will be solved when they find how many quarts there are in 26 pints, and have them explain why or why not.


compute
C
26 ÷ 2. There are 2 pt. in 1 qt.
□+19=n
2
Think of the 26 pints in groups of 2. Divide 26 by 2. The 26 pints of peaches are equal to 13 quarts.
1
Number of quarts in 26 pints
Number of quart jars
Total number of quarts
13+19=n
Now you can find the number that n stands for.
3
D
Think of the pints and quarts as one group of quarts.
Add 19 to 13. Peggy and her mother canned 32 quarts of peaches.



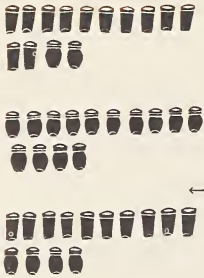

see
A
Peggy and her mother made 12 glasses of jelly on Monday, 20 glasses of jelly on Tuesday, and 10 glasses of jelly on Wednesday. They made an average of how many glasses of jelly per day?
1
Think of all the glasses as one group. Then imagine that the glasses are to be divided into three equal groups, one for each day.
B
2
Total number of glasses
Number of glasses in each group
□÷n=3
Number of groups
First you must find how many glasses of jelly in all they made.
compute
Find the total number of glasses of jelly that Peggy and her mother made.
C
12+20+10. Total number of glasses
□÷n=3
3
The sum is 42.
Put 42 in the equation.
42÷n=3

# 27

1. Have the pupils look at the picture. Explain that the 19 quarts are dimmed off because they are to think only about the 26 pints right now. They must find how many quarts there are in 26 pints. They should note that rings are used to divide the 26 pint jars into groups of two because it takes 2 pints to make 1 quart. Ask how many groups of two pints, or quarts, there are.
2. Next, call attention to the equation and ask the pupils to read the text. Have them note how the number of quarts in 26 pints is found by computation. Ask where the 13 should be put in the equation.
3. Ask if the numeral that replaces *n* in the equation can now be found. Have the pupils read the remaining text and verify the computation in the book. Then have them read the problem on page 26 again to see if the question asked has been answered.

# 28

1. Have someone read the problem. Explain to the pupils, if they have forgotten, that finding the average means finding how many glasses of jelly Peggy and her mother would make per day if they made the same number of glasses each day.
2. Ask the pupils to look at Picture B, to explain the use of the dotted background, and then to read the text opposite the picture. Have someone explain how the equation describes the problem situation. Then ask how the numeral that will replace the screen can be found.
3. Make sure the pupils understand why the 42 is put in place of the screen and *not* in place of *n*.



D

1

Total number of glasses
Number of glasses made per day
Number of days
$42 \div n = 3$
Now you can find the number that $n$ stands for.

Think of the 42 glasses as divided into 3 equal groups. How many glasses will there be in each group?

Divide 42 by 3. The average number of glasses of jelly made per day was 14.

A Peggy's mother bought 2 bushels of peaches for $2.75 a bushel and $\frac{1}{2}$ bushel of plums for $3.39. How much did she pay for all this fruit?

Amount paid for peaches.
Amount paid for plums.
$\square + \$3.39 = n$
Total amount paid.

2

$(2 \times \$2.75) + \$3.39 = n$
First find how much Peggy's mother paid for the peaches. To do this, multiply $2.75 by 2. Put $5.50 in the equation.
$\$5.50 + \$3.39 = n$
Peggy's mother paid ■ for the fruit.

B Henry was paid $3.45 for washing 17 large screens and 9 small screens. He was paid an average of how much per screen?

3

Amount paid for all the screens.
Amount paid per screen.
$\$3.45 \div n = (17 + 9)$
Number of screens.

First you must find how many screens Henry washed. To do this, add 9 to 17. Put 26 in the equation.
$\$3.45 \div n = 26$
Is 13¢ or 14¢ the better answer? Why?



1 C One Saturday George raked leaves for 3 hr. and also walked a neighbor's dog. He was paid 35¢ for walking the dog and 50¢ an hour for raking leaves. How much did George earn that day?

THINK $.35 + (3 \times \$.50) = n$
Always do the work in the parentheses first.

D Alice practiced on the piano for 20 min. on Monday, 35 min. on Tuesday, and 15 min. on Wednesday. Her sister Ruth practiced 40 min. a day on these three days. Ruth practiced how much longer than Alice?

THINK $(3 \times 40) - (20 + 35 + 15) = n$
What should you do first? What should you do next?

E Mrs. Bell spent $46.40 for groceries the first week of September, $30.72 the second week, $43.08 the third week, and $39.24 the fourth week. She spent an average of how much per week for groceries in September?

THINK $(\$46.40 + \$30.72 + \$43.08 + \$39.24) \div n = 4$

F Mrs. Bell used 3 doz. eggs in 6 days. She used an average of how many eggs per day?
$(3 \times 12) \div n = 6$

G Carol baked 2 doz. brownies one morning. After lunch only 8 brownies were left. How many brownies had been eaten?
$(2 \times 12) - n = 8$

H David read two books in 11 days. One book had 266 pages, and the other had 128 pages. David read an average of about how many pages per day?
$(266 + 128) \div n = 11$

I David's older brother built a ham radio station. He said that he spent about 60 hr. working on the station and that he finished it in 5 weeks. On the average, he spent how many hours per week working on the station?
$60 \div n = 5$

2

J One day David read the temperature of the air outdoors at four different times. At 8 A.M. the temperature was 58°. At noon it was 74°. At 4 P.M. it was 70°. At 8 P.M. it was 54°. What was the average of the temperatures?
$(58 + 74 + 70 + 54) \div n = 4$

K Alice bought 2 lb. of salted nuts at $1.49 per pound and paid for them with a 5-dollar bill. How much money should she have received in change?
$\$5.00 - (2 \times \$1.49) = n$

3

In most of these problems you had to do more than one thing to find the answer to the question in the problem.





# 29

1 Be sure the pupils realize that they should divide 42 by 3 because 3 glasses will be used each time 1 glass of jelly is put into each of 3 groups. Show this with objects if necessary.

2 Discuss this problem with the class. Point out that the equation $(2 \times \$2.75) + \$3.39 = n$ is more useful than the original equation with the screen because it shows one thing they must do before they can find the numeral to replace $n$. Tell them that the computation indicated within parentheses must always be done first.

3 Work through this problem with the class. Point out that in this equation the computation to be done before the numeral to replace $n$ can be found is indicated within parentheses. Discuss the remainder in the answer.

# 30

1 Have the pupils work these problems independently. Since the equations are given, the pupils need show only their computation and the completed equations on their papers.

2 Point out to the pupils that in this problem the total is already given and they only have to divide to find the answer.

3 Supply the pupils with completed equations and let them verify their work. Indicate with parentheses and brackets the numerals so shown in the answer key. Discuss any problems that a significant number of pupils did not understand.

# 31 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns that when several numbers are to be added or multiplied, he must group by twos to add or multiply, and he may do this grouping in a number of ways.

**Vocabulary**

There are no new words.

**Comments**

This page shows that when three or more numbers are added or multiplied, they are grouped by twos, but it does not matter in what order they are grouped. The principle involved is the associative law, but you should not use this term with the pupils.

For the first time in this program, the pupils are taught that more than two numbers may be multiplied in one problem and that they may be associated, or grouped, for multiplication in any order without affecting the product.

**Answers**

A 59

B By adding 16 and 29; 59

C By adding 29 and 14; 59

D They are all the same. No

For Exercises E to H, only the sums are given below; any way of grouping the addends with parentheses should be accepted as correct.

E 110 F 146 G 214 H 224

I Pupils may group the factors in several ways, but the product will be the same. 2496

J By multiplying 6 by 13; 2496

K By multiplying 32 by 6; 2496

L They are all the same. No

For Exercises M to T only the answers are given below; any way of grouping the factors with parentheses should be accepted as correct.

M 1512 N 3450 O 1476 P 1120

Q 5760 R 6000 S 2940 T 3816

## Thinking straight

A $16+29+14$ 1

B $(16+29)+14$ (45) 2

C $16+(29+14)$ (43) 3

D $13\times6\times32$

E $(13\times6)\times32$ (78) 5

F $13\times(6\times32)$ (192) 6

Look at Example A. What is the sum of 16, 29, and 14?

Look at Example B. How do you get 45? What is the sum of 45 and 14?

In Example C, how do you get 43? What is the sum of 16 and 43?

What do you notice about the sum in Examples A, B, and C? Does it make any difference which two numbers are added first?

Each exercise below, first use parentheses to group the numerals in different ways. Then add in different ways.

G $4+18+16+52$

H $9+32+28+47$

I $6+15+34+109$

J $7+22+40+125$

Look at Example D. How would you find the answer? What is the answer?

Look at Example E. How do you get 78? What is the answer when you multiply 32 by 78? 5

In Example F, how do you get 192? Multiply 192 by 13. 6

What do you notice about the answer in Examples D, E, and F? Does it make any difference which two numbers are multiplied first?

Use parentheses to group the numerals in different ways. Then multiply. 7

M $7\times9\times6\times4$

N $30\times23\times5$

O $2\times41\times18$

P $8\times5\times4\times7$

Q $20\times9\times32$

R $8\times50\times15$

S $14\times10\times21$

T $36\times2\times53$



## 31

1 The pupils should put these numerals in a column and add.

2 Have the pupils add $16+29$ first and then add 14 to the sum.

3 Have the pupils add $29+14$ first and then add 16. Then they should compare Example B with Example C.

4 The pupils should group the numerals and add in at least two different ways.

5 Have the pupils multiply 6 by 13 and then multiply 32 by this product. Ask pupils to compare Examples E and F with Examples B and C to see that the methods of grouping are alike, although the processes are different.

6 Have the pupils multiply 32 by 6 and then multiply this product by 13.

7 The numerals should be grouped in at least two different ways for multiplying.

# 32-37 Looking back

Expanded Notes for this lesson are on pages 290-292.

**Objectives**

The pupil reviews the method of expressing a rate by using two numerals to make a ratio. He reviews how to read a ratio when it expresses a rate. He also reviews the fact that the same rate may be expressed by different ratios.

**Vocabulary**

New words page 32 batteries*; page 35 serve*

**Comments**

The material in this lesson was introduced in *Seeing Through Arithmetic 5* on pages 112-117. A complete reteaching of the ideas presented on those pages is provided here.

The mathematical terms *rate* and *ratio* are used in connection with situations that have to do with common everyday experiences. Since these terms may be entirely new to some pupils, the difference between them should be carefully taught. The terms *rate* and *ratio* need not be confused if *rate* is used to describe the physical situation and if *ratio* is used as the name of a pair of numerals, written one over the other, to express the rate.

For mechanical reasons, ratios are written with a diagonal line, as 4/56, in the text of this *Teaching Guide*. Do not, however, at this time write the ratios in this way for the pupils; use the horizontal line. It should also be mentioned here that the expressions *first term* and *second term*, which are not introduced in the pupils' book until page 52, may be used orally in referring to the two numerals in a ratio. Call the upper numeral the first term and the lower numeral the second term.

On page 32, a situation in which batteries were sold at a *rate* of 4 batteries for 56¢ is presented. The numeral 4 shows the number of batteries that were sold. The numeral 56 shows the number of cents for which they were sold. There is a pairing, or correspondence, of 4 batteries with 56 cents. The text explains how the numerals 4 and 56 may be written together, one over the other, to express the rate. A pair of numerals used in this way is called a *ratio*, and it is read "4 per 56."

On pages 33 and 34, the pupils see that when smaller and larger quantities of batteries are bought at the same rate, other pairs of numerals, or ratios—for example, 1/14, 2/28, 6/84, etc.—can be written to express this relationship, or rate. This principle is taught by means of pictures which enable the pupils to understand what is happening without being obliged to compute. They are also shown how to express the rate as a ratio when the relationship of the two numbers is thought of in the opposite way—that is, when the amount of money is thought of before the number of batteries. Again, different ratios express the same rate and they are illustrated by pictures of items and coins in equal groups. It is extremely important that the student realizes that these different ratios which express the same rate are *equal*. It is this fact that enables him to use ratios in solving problems.

On pages 35 and 36, another situation is described in detail. On page 37, the pupils are given an opportunity to decide what rates the pictures illustrate and then to write ratios for some statements.

**Answers**

Page 37:

(block 1)

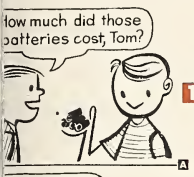
| | | | |
|---|---|---|---|
| A [B] | F [D] | K [A] | P [A |
| B [B] | G [C] | L [B] | Q [C |
| C [F] | H [B] | M [D] | R [F |
| D [C] | I [E] | N [F] | S [D |
| E [E] | J [A] | O [E] | T [A |

(block 2)

| | |
|---|---|
| A 12/4, 4/12 | F 1/52, 52/1 |
| B 3/29, 29/3 | G 2/40, 40/2 |
| C 59/2, 2/59 | H 79/16, 16/79 |
| D 250/57, 57/250 | I 1/12, 12/1 |
| E 4/1, 1/4 | J 5/1, 1/5 |

## Looking back

This lesson will help you remember how to use rates and how to write ratios that express these rates.




A

When Tom answered David's question, he told him the rate at which the batteries were sold. He used two numbers to do this. The rate at which the batteries were sold can be expressed in different ways. 1

One way to express the rate is to use the numerals 4 and 56. You can say that the batteries were sold at the rate of 4 for 56 cents.

2

B

Here is the way to write 4 and 56 together to express this rate.

4 shows the number of batteries that were sold. 3 → $\frac{4}{56}$ ← 56 shows the number of cents needed.

4 per 56 4

A pair of numerals used in this way is called a ratio. This ratio expresses a rate. The rate is expressed as 4 per 56. 5

g ratios to express rates (reteaching)

C

You can use other numerals to express the rate at which the batteries were sold.

In Picture C the 4 batteries and the 56 cents are separated into 4 equal groups. The money in each new group will buy the battery in that group. 1

1 battery and 14 cents are in each group. 2

Think of only 1 of these 4 new groups.
1 battery will sell for 14 cents.

D

Now you can express the rate as 1 battery per 14 cents. Write the ratio this way. 4

1 shows the number of batteries. → $\frac{1}{14}$ ← 14 shows the number of cents needed.

1 per 14

You can now express the rate as 1 per 14.

You can use two other numerals to express the rate. Think of 2 of the 4 new groups.
2 batteries will sell for 28 cents.

E

Now you can express the rate as 2 batteries per 28 cents. Write the ratio this way. 6

2 shows the number of batteries. → $\frac{2}{28}$ ← 28 shows the number of cents needed.

2 per 28

Now the rate is expressed as 2 per 28.

5

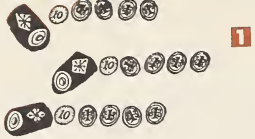


# 32

1 Let two pupils act out the parts of Tom and David and have a third pupil read the text for Movie A. Be sure the pupils understand that two numerals, 4 and 56, are used to express the rate at which the batteries were sold.
2 Have the pupils first note the number of batteries in Picture B, then count the money. Let a pupil read the text opposite the arrow pointing to the picture and relate it to the picture.
3 Ask the pupils to read the text and note the position of the two numerals when they are written together to express the rate.
4 Have the pupils note that in reading a rate written in this way, the word "per" is used in preference to the word "for."
5 Point out again that it takes two numerals to express a rate. Emphasize that a pair of numerals written one over the other to express a rate is called a *ratio*.

# 33

1 Let a pupil read the text. Help the pupils understand that the batteries and coins have been divided into equal groups to determine the price for fewer than 4 batteries.
2 Have the pupils verify the text by counting the batteries and money in each group in Picture C.
3 Tell the pupils to study Picture D and imagine they are going to buy only 1 battery. Ask them how much 1 battery would cost. Have them note that the other batteries and coins are dimmed off because they are not thinking of them now.
4 Have someone read the text for Picture D. Be sure that the pupils understand what each numeral in the ratio means.
5 Ask the pupils to look at Picture E and imagine they are going to buy 2 batteries. Ask them to count the money to find out how much 2 batteries would cost.
6 See Note 4.



1

F

Picture F shows what happens when more than 4 batteries are sold at the rate Tom paid. It shows that for 6 batteries, 84 cents would be needed. This time you can express the rate as 6 per 84.

2

What does the 6 show? → $\frac{6}{84}$ ← What does the 84 show?

6 per 84

A Does 5 per 70 also express the rate shown in Picture F? Does 3 per 42 express the same rate?

B Now you see that 1 per 14, 2 per 28, 3 per 42, 4 per 56, 5 per 70, and 6 per 84 all express the same rate.

$\frac{1}{14}=\frac{\blacksquare}{28}=\frac{\blacksquare}{42}=\frac{\blacksquare}{56}=\frac{\blacksquare}{70}=\frac{\blacksquare}{84}$ 3

When you expressed the rate as 6 batteries for 84 cents, you thought of the number of batteries first. Look at Picture F again. Now think of the amount of money first. When you think this way, you can express the rate as 84 cents for 6 batteries. Write a ratio for the rate this way.

4

84 shows the number of cents needed. → $\frac{84}{6}$ ← 6 shows the number of batteries.

84 per 6

C Now look at 1 group in Picture F. Think of the money first. You can express the rate as ■ cents for ■ battery. Write this ratio for the rate.

What does the 14 show? → $\frac{14}{1}$ ← What does the 1 show? 5

14 per 1

D If you express the rate as 42 per 3, are you thinking first of the money or of the batteries?

E 14 per 1, 28 per 2, 42 per 3, 56 per 4, 70 per 5, and 84 per 6 all express the same rate.

$\frac{14}{1}=\frac{\blacksquare}{2}=\frac{\blacksquare}{3}=\frac{\blacksquare}{4}=\frac{\blacksquare}{5}=\frac{\blacksquare}{6}$ 6

1

G Jane also expressed a rate when she told Nancy how much cocoa she was going to make. This rate, too, can be expressed in different ways.

H

You can say that Jane will make cocoa at the rate of 15 cups for 6 persons.

2

To show the rate, you can write the numerals 15 and 6 as a ratio.

15 shows the number of cups of cocoa. → $\frac{15}{6}$ ← 6 shows the number of persons.

■ per 6

You can express the rate as ■ per ■.

I

3

You can use other numerals to express the rate at which Jane will make cocoa.

The 15 cups and the 6 persons are now separated into 3 equal groups. The cups of cocoa in each new group will serve the persons in that group.

There are ■ cups and ■ persons in each new group.

Now turn the page.





# 34

1 Have the pupils note that while this picture shows more batteries and coins than the pictures on page 33, the rate of 1 battery per 14 cents is still true. Have them express the rate for one of the six groups and for two of the six groups.

2 Ask one pupil to read the text and explain what each numeral of the ratio means.

3 Let the pupils show, by referring to the groups in Picture F, that all these ratios express the same rate that 1 per 14 expresses. Let several pupils supply the missing numerals.

4 Use Note 2. Be sure the pupils understand that the way numerals are placed in a ratio depends upon whether you think of the number of batteries per cents or cents per number of batteries.

5 Use Note 2.

6 Adapt the procedures in Note 3.

# 35

1 Let the pupils read the text and dialogue for Picture G.

2 Have the pupils count the cups of cocoa and the stick figures in Picture H. Have them relate the text to the picture and explain the meaning of both numerals in the ratio.

3 Ask a pupil to read the text for Picture I, then count the cups of cocoa and stick figures in each group in the picture and fill in the missing numerals.

J

Think of only 1 of these 3 new groups. 5 cups of cocoa will serve 2 persons.

You can say that Jane will make cocoa at a rate of ▩ cups for ▩ persons. You can write the ratio that shows the rate this way.

**1**

What does the 5 show? $\frac{5}{2}$ What does the 2 show?

▩ per 2

Now the rate is expressed as 5 per ▩.



K

Picture K shows how many persons could be served if Jane made more cocoa at the same rate. It shows that if Jane made 20 cups, she could serve 8 persons.

**2**

You can express the rate now as 20 per 8.

What does the 20 show? $\frac{20}{8}$ **3** What does the 8 show?

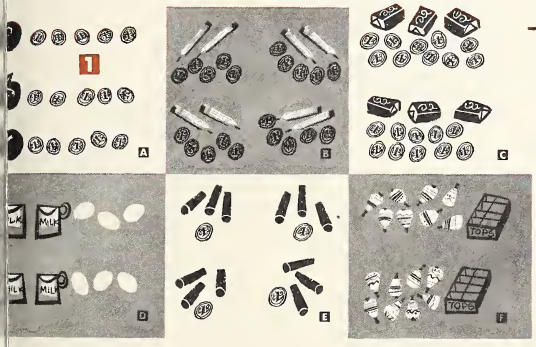
▩ per ▩

Does 10 per 4 also express the rate shown in Pictures J and K?

Do you see that 15 per 6, 5 per 2, 20 per 8, and 10 per 4 all express the same rate?

$$\frac{15}{6}=\frac{5}{2}=\frac{20}{8}=\frac{10}{4}$$ **4**

You can also express the rate at which Jane will make cocoa as 2 per 5. What do the 2 and 5 mean? **5**



**1**

A B C D E F

Each ratio below expresses a rate. For each ratio there is a picture. Find the picture that shows each rate.

| | | | |
|---|---|---|---|
| $\frac{2}{5}$ | F $\frac{3}{2}$ **2** | K $\frac{3}{15}$ | P $\frac{5}{1}$ |
| $\frac{10}{4}$ | G $\frac{6}{20}$ | L $\frac{20}{8}$ | Q $\frac{10}{3}$ |
| $\frac{1}{8}$ | H $\frac{6}{15}$ | M $\frac{2}{3}$ | R $\frac{16}{2}$ |
| $\frac{3}{10}$ | I $\frac{3}{1}$ | N $\frac{8}{1}$ | S $\frac{6}{4}$ |
| $\frac{1}{3}$ | J $\frac{15}{3}$ | O $\frac{3}{9}$ | T $\frac{2}{10}$ |

For each statement below, write the numerals as a ratio to show the rate.

A 12 sandwiches for 4 boys
B 3 bars of soap for 29¢ **3**
C 59¢ for 2 cans of meat
D 250 sheets of paper for 57¢
E 4 cookies per person
F 1 lb. of cheese for 52¢
G 2 lb. of salted peanuts for 40 people
H 79¢ for 16 birthday cards
I 1 cake for 12 people
J 5 people to a car

Now you know how numerals are used to express rates. Later on you will use rates to solve problems.



# 36

1. Tell the pupils to study Picture J and imagine they will make only 5 cups of cocoa. Ask them how many persons these 5 cups will serve. Have them study the ratio and explain what each numeral means.
2. Let someone tell how many groups there are in Picture K and how many cups of cocoa and stick figures in each group.
3. Have a pupil read the text for Picture K and explain the meaning of each numeral in the ratio.
4. Help the pupils relate the groups in Picture K to these ratios.
5. Be sure the pupils understand that 2 stands for the number of persons and 5 for the number of cups of cocoa.

# 37

1. Let the pupils have time to study the pictures and decide what ratios can be written for each one.
2. This group of exercises may be discussed orally or assigned as written work.
3. Let the pupils use these exercises as written work. Tell them that the numerals in each statement can be used to write two ratios to show a rate. In doing this, some of the pupils may need help. They should understand clearly what each term of the ratio stands for. Make sure the pupils correct any errors made in their work.

# 38-42 Looking back

Exponded Notes for this lesson are on pages 292-293.

**Objectives**

The pupil reviews how to use a ratio to compare one quantity with another. He reviews the meaning of each term in the ratio and how to write the terms together as one symbol.

**Vocabulary**

New words page 38 silver*, Joan; page 41 cactus*, violet*

**Comments**

Work with ratios is extended in this lesson to include the *comparison* of a quantity with another quantity. Making comparisons of quantities is a common, everyday experience. Newspapers and magazines are full of statistical data and illustrations using comparisons of all kinds. The pupils will have occasion to use ratios and the related terminology the rest of their lives. Also, these lessons on ratio are very important because ratios will be used in equations later. See pages 112-124 of the pupils' book, for example.

In the previous lesson the pupils were shown that more than one ratio can be written for a rate. In this lesson they are shown that more than one ratio can be written to express a comparison.

On pages 38-40 two sets of objects are used to show how to read, write, and interpret a comparison by using ratios. First the two sets of objects are compared. Then each of the two sets of objects is separated into the same number of equal subgroups. A subgroup of one kind of object is then put with a subgroup of the other kind to form a new combined group which becomes the basis for writing new ratios. It is shown that several ratios may be written for the same comparison. The picture on page 40 shows what happens when the number of each kind of object is increased, and the accompanying text explains how still more equivalent ratios can be written for the same comparison.

Also on page 40, it is shown that, just as ratios have been written to compare Mary's spoons with Joan's, ratios can be written to compare Joan's spoons with Mary's—that is, we may think either of black spoons compared with blue spoons or blue spoons compared with black spoons. It is important for the pupils to realize that either the numeral that represents the black spoons or the numeral that represents the blue spoons can be the first term of a ratio.

On page 41 three pictures are used for making comparisons. Exercises A to E are detailed steps that can be followed by you to explain writing ratios for Picture A. These same steps can be used in studying Pictures B and C. If you let the pupils in your class use objects for counting and grouping, they will gain a better understanding of the work on these pages. Using a different number of objects than shown here and forming other groups, will give the pupils still further practice in visualizing comparisons and making ratios to express these comparisons.

**Answers**

Page 41:

A 21 stamps; 14 coins; 21/14
B 3 stamps and 2 coins; 3/2; 2/3
C 9 stamps and 6 coins; 9/6; 6/9
D 6/4, 12/8, 15/10, 18/12, 21/14
E 4/6, 8/12, 10/15, 12/18, 14/21
F 10/25
G 2/5, 4/10, 6/15, 8/20
H 4/2, 8/4, 12/6, 2/1, 6/3, 10/5
I 5/10, 3/6, 6/12, 2/4, 1/2, 4/8

Page 42:

(block 1)

| | | | |
|---|---|---|---|
| A [B] | E [F] | I [C] | M [F] |
| B [A] | F [D] | J [B] | N [A] |
| C [E] | G [E] | K [A] | O [F] |
| D [C] | H [E] | L [D] | P [B] |

(block 2)

| | | |
|---|---|---|
| A 8/3, 3/8 | E 1/5, 5/1 | I 48/32, 32/48 |
| B 3/8, 8/3 | F 16/12, 12/16 | J 50/45, 45/50 |
| C 2/1, 1/2 | G 15/27, 27/15 | K 20/25, 25/20 |
| D 15/8, 8/15 | H 27/15, 15/27 | |

## Looking back

This lesson will help you remember how ratios are used to compare one group with another. 1

A

Mary compared her spoons with Joan's spoons by using two numbers, 9 and 12.

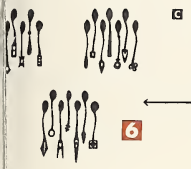
Picture B shows both girls' spoons.

B

You can express the comparison that Mary made by writing the numerals 9 and 12 as a ratio. Your ratio will look like this. 4

Number of Mary's spoons that are compared with Joan's spoons → 9/12 ← Number of Joan's spoons that Mary's spoons are compared with

9 to 12 5




C

You can use other numbers to compare Mary's spoons with Joan's.

Now Mary's 9 spoons and Joan's 12 spoons are separated into 3 equal groups. 3 of Mary's spoons and 4 of Joan's are in each group.

ratios to show comparison (reteaching)

Think of one of these groups. Now you can compare Mary's spoons with Joan's by saying that there are 3 of Mary's spoons to 4 of Joan's.

D

The ratio that shows this comparison is written this way.

Number of Mary's spoons that you are thinking about now → 3/4 ← Number of Joan's spoons that Mary's spoons are compared with 2

3 to 4

You can now express the comparison of Mary's spoons with Joan's as 3 to 4.

E

When you think of two of the groups, you can use other numerals to express the comparison of Mary's spoons with Joan's.

Now you can express the comparison as 6 spoons to 8 spoons. Write the ratio this way.

Number of Mary's spoons that you are thinking about → 6/8 ← Number of Joan's spoons that Mary's spoons are compared with 4

6 to 8

The comparison of Mary's spoons with Joan's is now expressed as 6 to 8.



# 38

1 Explain to the pupils that in the preceding lesson they used ratios to express rate. In this lesson they will use ratios to express a comparison between two quantities.

2 Let two pupils read the dialogue between Mary and Joan. Be sure pupils observe that the two girls are comparing the number of spoons they have.

3 Have the pupils count the spoons in each group. Let someone tell which group belongs to Mary and which to Joan.

4 Have a pupil read the text for Picture B and explain what each numeral in the ratio means.

5 Tell the pupils to note that the word *to* can be used in reading ratios that express comparisons. Ask them how this differs from reading ratios that express rates.

6 Tell the class to look at the picture and note the equality of the groups and the number of Mary's spoons and Joan's spoons in each group.

# 39

1 Have the class study the picture. Be sure they observe that each group of spoons includes 3 of Mary's and 4 of Joan's. Explain that since we are especially interested in one of these groups now, the others are dimmed off. Ask questions to get the pupils to see that the ratio 3/4 expresses the comparison. Emphasize the idea that the relationship between the groups is the same as it was on page 38, but that different numerals are used to express it. Stress the idea that the ratio 9/12 equals the ratio 3/4.

2 Call on several pupils to read the entire text for Picture D and to explain the numerals in the ratio.

3 Ask: "How many groups are we considering now? How many of the spoons are Mary's? How many are Joan's?"

4 Have the pupils read the text and explain the numerals in the ratio in terms of the picture.

F

Picture F shows that Mary and Joan collected more spoons. Now there are 15 of Mary's spoons to 20 of Joan's. Write a ratio for the comparison this way.

What does the 15 show? → $\frac{15}{20}$ ← What does the 20 show?

15 to 20

A Does 12 to 16 also express the comparison of Mary's spoons with Joan's? What part of Picture F shows this?

B 3 to 4, 6 to 8, 9 to 12, 12 to 16, and 15 to 20 all express the same comparison.

$\frac{3}{4}=\frac{\square}{8}=\frac{\square}{12}=\frac{\square}{16}=\frac{\square}{20}$

When you expressed the comparison as 15 spoons to 20 spoons, you compared Mary's 15 spoons with Joan's 20 spoons. Now look at Picture F again. This time compare Joan's 20 spoons with Mary's 15 spoons.

Number of Joan's spoons that you are thinking about → $\frac{20}{15}$ ← Number of Mary's spoons that Joan's spoons are compared with

20 to 15

Look at one group in Picture F and compare Joan's spoons with Mary's. You can express the comparison now as 4 spoons to 3 spoons. Write a ratio for the comparison.

What does the 4 show? → $\frac{4}{3}$ ← What does the 3 show?

4 to 3

C If you express the comparison as 16 to 12, are you comparing Joan's spoons with Mary's or Mary's spoons with Joan's?

D 4 to 3, 8 to 6, 12 to 9, 16 to 12, and 20 to 15 all express the same comparison.

$\frac{4}{3}=\frac{\square}{6}=\frac{\square}{9}=\frac{\square}{12}=\frac{\square}{15}$

A

B

C

A Look at Picture A. There are ■ stamps. There are ■ coins. Finish the ratio at the right to compare all the stamps with all the coins. $\frac{21}{\square}$

B Think of one group of stamps and coins. There are ■ stamps and ■ coins. Use a ratio to compare stamps with coins. Use a ratio to compare coins with stamps.

C Think of three groups of stamps and coins. There are ■ stamps and ■ coins. Use a ratio to compare stamps with coins. Use a ratio to compare coins with stamps.

D Use three other ratios to compare the stamps with the coins.

E Use three other ratios to compare the coins with the stamps.

F For Picture B, use a ratio to compare all the arrowheads with all the coins.

G Use two other ratios to compare the arrowheads with the coins.

H Which ratios below express the comparison of cactus plants with violet plants in Picture C?

$\frac{4}{2}$ $\frac{2}{3}$ $\frac{8}{4}$ $\frac{12}{6}$ $\frac{6}{12}$ $\frac{2}{1}$ $\frac{3}{5}$ $\frac{6}{3}$ $\frac{10}{5}$

I Which ratios below express the comparison of violet plants with cactus plants?

$\frac{2}{1}$ $\frac{5}{10}$ $\frac{3}{6}$ $\frac{4}{3}$ $\frac{6}{12}$ $\frac{2}{4}$ $\frac{1}{2}$ $\frac{6}{3}$ $\frac{4}{8}$



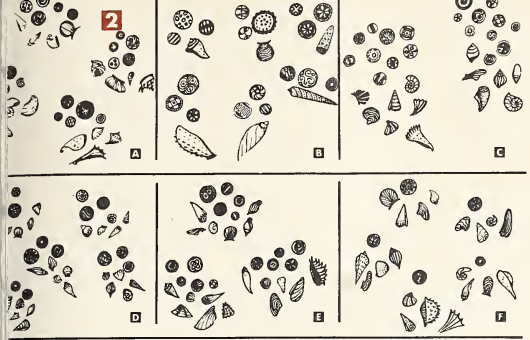


# 40

1 Ask the pupils how Picture F differs from Picture E [two more groups of 7 spoons]. Determine the total number of each girl's spoons. Develop the idea that as long as Mary has 3 spoons to 4 of Joan's, the relationship is the same, but a different ratio may be used.
2 Relate the ratio to Picture F.
3 Let the pupils answer the questions in Exercise A.
4 Have the pupils use the groups in Picture F to complete the ratios in Exercise B.
5 The class should understand that Joan's spoons can also be compared with Mary's.
6 Tell the class to count all the blue spoons in Picture F and then all the black ones. Have someone explain the new ratio.
7 Read the text and relate the ratio to Picture F.
8 Let the pupils answer the question in Exercise C and complete the ratios in Exercise D.

# 41

1 Discuss Exercise A in connection with this picture. Have the pupils observe that each group of stamps and coins contains 3 stamps and 2 coins.
2 Call on a pupil to read this exercise and supply the missing numerals. Then ask all the pupils to write the two ratios for one group. Discuss the ratios.
3 Proceed in the way you did for Exercise B.
4 Discuss the other ratios that can be written for Exercises D and E.
5 Discuss Exercise F in connection with this picture. Pupils should observe that each group of 7 objects contains 2 coins and 5 arrowheads.
6 In connection with this exercise, discuss the ratios that can be written for Picture B.
7 Exercises H and I may be discussed orally or assigned as written work.




ch ratio below expresses a comparison.
nd a picture that belongs with each ratio.
r example, Ratio A can be used to express
e comparison shown in Picture B.

4

| | | | |
|---|---|---|---|
| $\frac{1}{3}$ | E $\frac{1}{4}$ | I $\frac{18}{14}$ | M $\frac{12}{3}$ |
| $\frac{10}{8}$ | F $\frac{6}{10}$ | J $\frac{9}{3}$ | N $\frac{12}{15}$ |
| $\frac{12}{10}$ | G $\frac{5}{6}$ | K $\frac{5}{4}$ | O $\frac{4}{16}$ |
| $\frac{7}{9}$ | H $\frac{15}{18}$ | L $\frac{12}{20}$ | P $\frac{18}{6}$ |

For each comparison given below, use the numerals to write a ratio.

5

A 8 goldfish with 3 goldfish
B 3 goldfish with 8 goldfish
C 2 turtles with 1 turtle
D 15 shells with 8 shells
E 1 horse with 5 cows
F 16 plants with 12 plants
G 15 boys with 27 girls
H 27 girls with 15 boys
I 48 cookies with 32 cookies
J 50 cents with 45 cents
K 20 cards with 25 envelopes

Now you should be able to write a ratio to express a comparison.

## 42

1 This page provides the pupils with an opportunity to practice what they have learned on pages 38-41.

2 Tell the pupils that each of these six pictures shows a comparison of buttons with shells. Give the pupils time to study the pictures and think about the various ratios that can be used to compare the buttons with the shells in each picture.

3 Read the text to the class and make sure the pupils understand the example that is given.

4 Discuss these exercises orally or assign them as written work.

5 Assign this group of exercises as written work.

## 43-45 Looking back

xpanded Notes for this lesson are on pages 293-296.

**)bjectives**

he pupil reviews proper fractions, improper fractions, nd mixed numbers and how to read and write fraction numerals. He reviews the fact that different pairs f numerals may be used to represent the same fraction and that mixed numbers and improper fractions ay be equal.

**'ocabulary**

lew words page 43 terms

**omments**

his is the third lesson in which the pupils review the ses of number pairs. On pages 32-37 they reviewed airs of numerals used to express rates. On pages 8-42 they reviewed pairs of numerals used to make omparisons. Now they are going to review pairs of umerals that express fractions. In this lesson the pupils ecall that a fraction is part of a whole, that a fraction numeral is a way to symbolize a fraction, and that more than one fraction numeral can be used to symbolize or express the same fraction. At this point no attempt is made to review the computational procedure for the reduction (or changing) of fraction numerals. This will be done later.

The pupils are reminded in this lesson that it is possible to use a pair of numerals to express a quantity equal to or larger than the quantity that 1 represents. They learn to use the expression *improper fraction*. The pictures and text on page 44 show that some pairs of numerals may represent whole numbers ($\frac{12}{12}=1$) and other pairs may represent a whole number and a proper fraction ($\frac{5}{3}=1\frac{2}{3}$).

By the time the pupils have finished this lesson, they should be able to distinguish among proper fractions, improper fractions, and mixed numbers, and to read and write numerals that represent them. They should also know that the numerator and denominator of a fraction numeral are called its *terms*.

**Answers**

Page 44:

J $2\frac{1}{3}$, $\frac{21}{9}$, $\frac{7}{3}$, $2\frac{3}{9}$

Page 45:

(block 1)

Proper fractions: B, D, E, G, H

Improper fractions: J, K, M, N, P

Mixed numbers: R, T, W, X

(block 3)

A $1\frac{3}{8}$, $\frac{11}{8}$

B $1\frac{3}{6}$, $1\frac{1}{2}$, $\frac{9}{6}$, $\frac{3}{2}$

C $\frac{4}{8}$, $\frac{2}{4}$, $\frac{1}{2}$

D $2\frac{3}{5}$, $\frac{13}{5}$

E $2$, $\frac{8}{4}$, $\frac{4}{2}$

F $\frac{4}{12}$, $\frac{2}{6}$, $\frac{1}{3}$

## Looking back

This lesson will help you remember what fractions and mixed numbers are.

1

A Look at Picture A. The paper disk is marked off in ■ equal parts. You can use the numerals 2 and 3 to tell how much of the disk is blue.

The 3 shows the number of equal parts. → $\frac{2}{3}$ ← The 2 shows you are thinking of 2 of the equal parts.

A

two thirds

3

B

$\frac{2}{3}$ is a fraction numeral. 3 is the denominator. 2 is the numerator. The numerator and the denominator are the terms of a fraction numeral. Is $\frac{2}{3}$ less than 1?

2

B In Picture B the same disk has been marked off in 6 equal parts. The same amount is blue, but now you can use the numerals 4 and 6 to tell how much is blue. Is $\frac{4}{6}$ equal to $\frac{2}{3}$? Is $\frac{4}{6}$ less than 1?

4

5

C In Picture C each disk has been cut into 3 equal parts. Each of these parts is $\frac{1}{3}$ of a whole disk. How many of these one thirds are blue? You can use the numerals 5 and 3 to tell how much of these disks is blue.

5

The 3 shows the number of equal parts in each disk. → $\frac{5}{3}$ ← What does the 5 show?

five thirds

C

The denominator of $\frac{5}{3}$ is ■. The numerator is ■. What are the terms of $\frac{5}{3}$? Is $\frac{5}{3}$ more than 1?

6





# 43

1 Discuss the division of the disk into three equal parts. Note the two parts that are colored blue. Ask someone to read the explanation beside this picture, and be sure the pupils can relate each of the numerals to the picture.

2 Read and discuss these statements. The identification of numerator and denominator as the *terms* of a fraction numeral is new here.

3 Have the pupils tell how many equal parts this disk is divided into and the number of these parts that are blue. Be sure they see how the 4 sixths and the 2 thirds coincide.

4 Have someone read the text and answer the questions.

5 Have the pupils read the text at the right of this picture and answer the questions. Ask someone to relate the fraction numerals to the disks.

6 Call on pupils to supply the missing numerals and answer these questions.

D You know that $\frac{5}{3}$ of the disks in Picture D are blue. You can also use $1\frac{2}{3}$ to tell how much of the disks in Picture D is blue. 1

What does the 1 show? → $1\frac{2}{3}$ ← What does the 2 show?

What does the 3 show? 2

one and two thirds

Now look at Picture C on page 43. You can see that the blue parts of the disks in Pictures C and D are equal. $\frac{5}{3} = 1\frac{2}{3}$ 3

E In Picture E the same disks have been cut into 6 equal parts. Can you say that $\frac{10}{6}$ are blue? Can you say that $1\frac{4}{6}$ of the disks are blue? $\frac{10}{6} = 1\frac{4}{6}$ 4

F Is $\frac{10}{6}$ equal to $\frac{5}{3}$? Is $1\frac{4}{6}$ equal to $1\frac{2}{3}$? 5

G The rectangular piece of paper in Picture F has been marked off in ▦ equal parts. Each part is $\frac{1}{▦}$ of the paper. How much of the paper is blue? 6

H The fraction numeral $\frac{12}{12}$ tells you how much of the paper is blue. Is $\frac{12}{12}$ equal to 1?

I The denominator of $\frac{12}{12}$ is ▦. The numerator is ▦. What are the terms of $\frac{12}{12}$?

J Which numerals can you use to tell how much of the paper in Picture G is blue? 7

$2\frac{1}{3}$ 3 $\frac{21}{9}$ $1\frac{3}{9}$ $\frac{-7}{3}$ $2\frac{3}{9}$ $\frac{27}{9}$

# 44

1 Let the pupils read these sentences and relate them to the picture.

2 Have the class read what is said about each of the numerals. Ask them to find the quantity in the picture that each numeral represents. Then discuss why $\frac{5}{3}$ equals $1\frac{2}{3}$.

3 Now have the pupils compare these two disks with the ones in Picture C. The thirds in Picture D are not separated, but the pupils should see that the same amount of two disks is blue in both pictures.

4 Have the pupils read Exercise E and answer the questions by referring to Picture E.

5 Tell pupils to use Pictures D and E to find the answers to these questions.

6 Have the pupils refer to Picture F to get the answers to Exercises G, H, and I.

7 Explain that the square pieces of paper in this picture can be thought of as divided into both thirds and ninths.

actions which are used to tell that mething is less than a whole, or ss than 1, are proper fractions. hich numerals below represent oper fractions?

| B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|
| $\frac{5}{6}$ | $\frac{8}{8}$ | $\frac{7}{12}$ | $\frac{1}{9}$ | $\frac{13}{10}$ | $\frac{2}{5}$ | $\frac{6}{10}$ |

actions which are used to tell that mething is equal to a whole, or more an a whole, are improper fractions. hich numerals represent improper actions?

| J | K | L | M | N | O | P |
|---|---|---|---|---|---|---|
| $\frac{9}{8}$ | $\frac{3}{3}$ | $\frac{1}{4}$ | $\frac{6}{5}$ | $\frac{7}{7}$ | $\frac{4}{12}$ | $\frac{20}{15}$ |

expresses a mixed number. It is ade up of a numeral for a whole number and a fraction numeral. Which numerals represent mixed numbers?

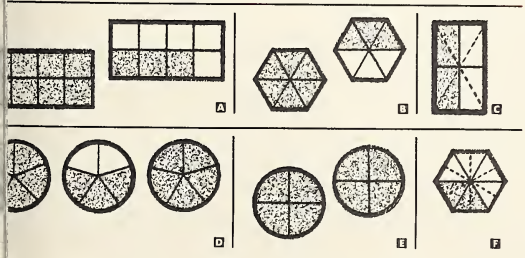
| Q | R | S | T | U | V | W | X |
|---|---|---|---|---|---|---|---|
| 9 | $2\frac{1}{2}$ | $\frac{6}{8}$ | $13\frac{3}{8}$ | 5 | $\frac{5}{12}$ | $6\frac{1}{10}$ | $8\frac{3}{4}$ |

A Write 5 numerals that represent proper fractions. 4

B Write 5 numerals that represent improper fractions.

C Write 5 numerals that represent mixed numbers.

For each picture below, write two numerals that tell how much of the pieces of paper is blue. You can write $1\frac{3}{8}$ and $\frac{11}{8}$ for Picture A. 5



Now you should be able to read and write any fraction numeral or mixed number.



# 45

1 First explain that in a proper fraction the number expressed in the numerator is smaller than the number expressed in the denominator. Then have a pupil name the proper fractions in Exercises A to H.

2 Be sure the pupils understand that if the number expressed in the numerator is the same as, or is greater than, the number expressed in the denominator, the fraction is called an improper fraction. Call on a pupil to name in Exercises I to P the numerals that represent improper fractions.

3 Discuss the term *mixed number*. Then have the pupils find the numerals that represent mixed numbers in Exercises Q to X.

4 Assign Exercises A to C as written work.

5 Assign this exercise as written work. Be sure the pupils see that Pictures C and F illustrate proper fractions and Pictures A, B, D, and E illustrate mixed numbers, whole numbers, and improper fractions.

# 46 Thinking straight

Expanded Notes for this lesson are on page 296.

**Objectives**

The pupil learns to distinguish among situations involving fractions, rates, and comparisons.

**Vocabulary**

New words page 46 jam*

**Comments**

The pupils have just reviewed three uses of pairs of numerals that are written in the same way. A pair of numerals written in this way is commonly thought of as representing a fraction. Yet such a pair of numerals can also be a ratio that represents a rate or a comparison.

Be sure to tell the pupils that unless they know whether the numerals represent a rate, a comparison, or a fraction, they cannot be sure how to read them. Suggest that a pair of numerals like $\frac{3}{14}$ be read "three over fourteen" if the idea the pair expresses is not known. If the pair of numerals expresses a fraction, it should be read "three fourteenths." If it expresses a comparison, the pupils should read it "three to fourteen." If it expresses a rate, it should be read "three per fourteen."

**Answers**

(block 1)

A Three fourths of the cup is filled.

B Four bunches of flowers per $3.00

C Three blue pencils to four red pencils

D Three fourteenths

(block 2)

A $\frac{6}{8}$, six eighths

B 55/10, 55 per 10 or 10/55, 10 per 55

C $\frac{1}{2}$, one half

D 7/5, 7 to 5 or 5/7, 5 to 7

E 4/6, 4 to 6 or 6/4, 6 to 4

F 3/2, 3 per 2 or 2/3, 2 per 3

G 800/3, 800 per 3

H 3/25, 3 per 25 or 25/3, 25 per 3

I 2/3, 2 to 3 or 3/2, 3 to 2

J 1/20, 1 per 20

K 10/1, 10 per 1 or 1/10, 1 per 10

## Thinking straight

A Look at Picture A. Using the fraction numeral, make a statement about the picture.

B Make a statement about Picture B, using the ratio to express a rate.

C Make a statement about Picture C, using the ratio to express a comparison.

2 D Until you know what the pair of numerals at the right means, read it "3 over 14." If it is a ratio expressing a rate such as 3 miles in 14 minutes, read it "3 per 14." If it is a ratio expressing 3 books compared with 14 books, read it "3 to 14." How can you read the pair of numerals if it expresses a fraction?

$\frac{3}{14}$

3 For each exercise in the next column, write a ratio or a fraction numeral. Decide whether it expresses a rate, a comparison, or a fraction. Then write the words you would use to read the ratio or fraction numeral. For Exercise A write, "$\frac{6}{8}$, six eighths."

A Jim cut a watermelon into 8 equal pieces. He and his friends ate 6 of the pieces.

B Ann paid 55¢ for 10 yd. of ribbon.

C Mrs. Stone cut a coffee cake into 2 equal parts. She gave 1 part to Mrs. Hill.

D Last month Tony earned $7. Bill earned $5.

E Mrs. Downs made 4 jars of blueberry jam and 6 jars of strawberry jam.

F When Mrs. Hill makes cocoa, she uses 3 teaspoonfuls of cocoa for 2 cups of milk.

G The airplane traveled 800 miles in 3 hours.

H Ellen bought 3 cans of tomato juice for 25¢.

I Jane pays $2 for a piano lesson. Sally pays $3 for a piano lesson.

J Andy can walk 1 mile in 20 minutes.

K Don paid 10¢ for a birthday card.



## 46

1 Have the pupils study Pictures A, B, and C carefully and then use Exercises A, B, and C in connection with them. You can refer by color to the pairs of numerals in the pictures.

2 Ask a pupil to read this paragraph and answer the question.

3 Tell the pupils to read this paragraph carefully. Then assign Exercises A to K as written work. When the pupils have finished their work, discuss each exercise and have them tell why they interpreted the statements as they did.

# 47-48 Using arithmetic

ponded Notes are not considered necessary for this lesson.

**bjectives**

ne pupil solves problems of various types, some of hich involve using information or answers from preous problems.

**cabulary**

ew words page 47 Soo, canals, ships*, locks*, ore; age 48 countries*

**mments**

his lesson is the first of a series of problem sets anned to give the pupils practice in recognizing varus problem types and in choosing the correct method r solving each of them. (An inventory test on probm solving was provided in the lesson entitled "Checkg up" on page 25 of the pupils' book. "Exploring probms" on pages 26-30 reviewed multiple-step problems.)

This problem set reviews the solving of problems at come in "chains," or sequences. In some of the oblems in this set, information given in previous probms or as answers to other problems must be used. In al life situations, problems do not always come singly, d the pupils need practice in solving such interrelated oblems.

These problems should be assigned without any preinary discussion. By insisting that the pupils make e proper equation for each of the problems on pages and 48, you will be helping them maintain the habit analyzing the problem situation and making a record their thinking before they start to compute. As the dent progresses in his study of mathematics, this derly method of problem solving becomes increasly important.

The "Keeping skillful" exercises should be treated parately.

**swers**

ing arithmetic:

e brackets represent the place where the pupils write e letter *n* in the equation. The numeral within the ackets is the answer for the problem.

In the equations for multiple-step problems, the rentheses represent the place or places where the pils must do some preliminary computation before they can find the final answer. The numeral within the parentheses is the result of such preliminary work.

A [63]+51=114
B 114÷[38]=3
C ($4)+($3)=[$7]
D 250×90=[22,500]
E 1895+[48]=1943 or 1943−1895=[48]
F 5280−1350=[3930]
G 90÷[3]=24 and 18 ships rm.; about 4
H 70×15,000=[1,050,000]
I 70×300=[21,000]
(Problem H tells that 1 freighter carries 15,000 T.)
Also allow: 300/1=[21,000]/70 or
15,000/300=1,050,000/[21,000]
J 98,347,000+30,163,000=[128,510,000]
K 98,347,000−62,245,000=[36,102,000]
L Less than; 128,510,000−83,616,000=[44,894,000]
M 128,510,000−107,301,000=[21,209,000]
N $75−[$46.68]=$28.32

Keeping skillful:

For the division exercises, if there is a remainder in the answer, it may be left as a whole number. If some pupils recall how to use the remainder with the divisor to make a fraction numeral in the answer, accept such answers. It is usually best to express a fraction numeral in reduced form, but do not insist on such reduction at this point.

(block 1)

| | | |
|---|---|---|
| A 143 | H 767 | O 9740 |
| B 191 | I 700 | P 16,608 |
| C 106 | J 2684 | Q 11,874 |
| D 761 | K 1379 | |
| E 1431 | L 1975 | |
| F 1859 | M 2240 | |
| G 1549 | N 16,403 | |

(block 2)

| | | |
|---|---|---|
| A 565 | G 1719 | M 30,070 |
| B 182 | H 4304 | N 27,938 |
| C 407 | I 7167 | O 69,117 |
| D 245 | J 4034 | P 15,133 |
| E 11,182 | K 6087 | Q 18,623 |
| F 1359 | L 27,227 | R 43,358 |

(block 3)

A 618
B 1316
C 22 and 7 rm. or $22\frac{7}{16}$

D 3 and 29 rm. or $3\frac{29}{48}$

E 2052

F 6

G 3700

H 33,336

I 22 and 18 rm. or $22\frac{18}{45}$ or $22\frac{2}{5}$

J 197,730

K 46

L 55

M 108,528

N 106,750

O 83 and 69 rm. or $83\frac{69}{81}$ or $83\frac{23}{27}$

P 168 and 3 rm. or $168\frac{3}{14}$

Q 415,332

R 48 and 116 rm. or $48\frac{116}{830}$ or $48\frac{58}{415}$

## Using arithmetic 1

A One morning the Baker family started out for the Soo Canals, which were 114 mi. away. After lunch, they still had 51 mi. to drive. How many miles had they driven in the morning?

B It took the Bakers 3 hr. to get to the Soo Canals. They traveled at an average rate of how many miles per hour?

C The Bakers took a boat trip through the Soo Canals. Tickets for grown-ups cost $2 apiece. Tickets for children cost $1. How much did the Baker family of 2 grown-ups and 3 children spend for tickets?
(?) + (?) = n

D Ships cannot go through the Soo Canals part of the year because of ice. The locks in the canals can be used about 250 days of the year. About 90 ships per day go through the locks. About how many ships go through the locks in a year?

E The oldest of the five locks was finished in 1895. The newest lock was finished in 1943. The oldest lock is how many years older than the newest lock?

THINK 1943 was how many years later than 1895?

F Each of the two longest locks is 1350 ft. long. One of these locks is how many feet longer than or shorter than 1 mile?

G The locks are used 24 hr. a day. If 90 ships go through the locks per day, about how many ships go through the locks per hour, on an average?

THINK Which answer is better, about 3 ships per hour or about 4 ships per hour? Why?

H About 70 of the 90 ships are freighters. A freighter of today can carry about 15,000 T. About how many tons of freight could 70 fully loaded freighters carry through the canals?

I It would take 300 railway freight cars to carry 15,000 T. of freight. How many railway freight cars would be needed to carry as much freight as can be shipped in 70 freighters?

J Most of the iron ore found in the United States is shipped through the canals. About 98,347,000 T. of iron ore were shipped through the canals in one year. About 30,163,000 T. of other goods were shipped through the canals that year. What was the total number of tons of freight shipped through the canals that year?





# 47

1 Ask the pupils to work independently on Pro lems A to N (K to N are on page 48). Dire them first to write an equation with the lett *n* to hold a place for the numeral that will b the answer. In the case of a multiple-ste problem, such as Problem C, tell them to wri within parentheses in the equation the pr liminary work they must do. Then ask th pupils to show their computation and, finall to rewrite the equation with the numera found by computation enclosed in parenthes and the answer in place of the letter *n*. Hav them label their work for each problem wi its identifying letter.

The next year the number of tons iron ore shipped through the canals opped to 62,245,000 T. How many wer tons of ore were shipped in this ar than in the year before it?

83,616,000 tons of freight were ipped through the canals in the ar described in Problem K. Was this ore than or less than the total nount of freight shipped in the year scribed in Problem J? How much ore or less was it?

M One year United States ships carried 107,301,000 tons of the total amount of 128,510,000 tons. The rest of the freight was carried by ships of other countries. How many tons of freight were carried by ships of other countries?

N Mr. Baker had $28.32 when the Bakers got back from their trip to the Soo Canals. Mr. Baker had started the trip with $75. How much money had he spent on the trip? 1

## Keeping skillful 2

| | | | |
|---|---|---|---|
| id the sum. | A $214+n=779$ | A $3\times206=n$ |
| 36, 14, 72, 21 | B $n+318=500$ | B $47\times28=n$ |
| 8, 34, 57, 29, 63 | C $683-n=276$ | C $359\div16=n$ |
| 19, 7, 25, 9, 46 | D $728-483=n$ | D $173\div48=n$ |
| 206, 118, 437 | E $n-5063=6119$ | E $36\times57=n$ |
| 590, 386, 455 | F $n+2836=4195$ | F $204\div n=34$ |
| 723, 535, 601 | G $7603-n=5884$ | G $25\times148=n$ |
| 892, 78, 579 | H $3216+n=7520$ | H $463\times72=n$ |
| 366, 24, 291, 86 | I $n-5036=2131$ | I $1008\div45=n$ |
| 68, 405, 173, 54 | J $n+3968=8002$ | J $390\times507=n$ |
| 913, 874, 258, 639 | K $15730-9643=n$ | K $1242\div n=27$ |
| 468, 237, 150, 524 | L $2848+n=30075$ | L $3465\div63=n$ |
| 212, 781, 305, 677 | M $53467-n=23397$ | M $228\times476=n$ |
| 475, 827, 653, 285 | N $65994-38056=n$ | N $35\times3050=n$ |
| 9037, 4016, 3350 | O $n-41253=27864$ | O $6792\div81=n$ |
| 6116, 745, 2879 | P $75000-59867=n$ | P $2355\div14=n$ |
| 5234, 8710, 2664 | Q $n+6308=24931$ | Q $83\times5004=n$ |
| 3175, 2998, 1696, 4005 | R $52975+n=96333$ | R $39956\div830=n$ |

# 48

1 Provide the class with equations and answers for Problems A to N (A to J are on page 47), and let them verify their work. Discuss thoroughly the problems that caused trouble. Give individual help where it is necessary.

2 Assign only as many of the exercises in each of these three blocks of work as you feel the pupils can do in the time they have. Be sure they understand that they are to write the numerals in each exercise in the proper form for computing. Provide answers so that the pupils can verify their work, and discuss any exercises that have caused trouble.

# 9-51 Exploring problems

panded Notes for this lesson are on pages 296-298.

**bjectives**

ne pupil learns that he can use other letters of the phabet, besides *n*, to hold a place for the answer in n equation. He learns that in an equation for a ultiple-step problem he can use as many different tters of the alphabet as he needs to hold places for merals that are to be found.

**cabulary**

ew words page 49 coat*, concert*, toads, yes*; age 51 fare*, glue*, thread*

**mments**

the third-grade book of this program, the pupil was roduced to equation-making by being shown how use a symbol called a screen (▤) to hold a place r the answer in an equation written to describe a oblem situation. In Grade 5 he learned to use the tter *n* to stand for the answer and a screen to indicate preliminary computation in multiple-step problems. On page 29 of *Seeing Through Arithmetic 6*, the use of the screen to indicate preliminary computation was discontinued, and the pupil learned to put these numerals within parentheses.

In the lesson to be studied now, another stage in the development of making equations is presented. On page 49 the pupil learns that he can use other letters besides *n* to hold a place for the answer. Examples are given, and the pupil has an opportunity to use various letters in writing equations. On pages 50 and 51, he learns that from the outset he can use different letters for different numerals to be found.

Thus, in the equation for Problem J on page 51, the letter *x* stands for the final answer. Another letter, *a*, holds a place for a numeral that must be supplied before the final answer can be found. This is indicated in the problem by the statement that two teachers bought tickets at $.50 each. The letter *b* holds a place for another numeral that must also be supplied before

the final answer can be found. This is indicated by the statement that 34 pupils' tickets cost $.24 each. The basic equation, then, is $a+b=x$.

The equation is the model for the problem situation under consideration. The method of problem solving is concerned with the construction of this model. Obtaining a solution to the problem consists of finding the proper numerical replacements for $a$, $b$, and $x$.

Although this use of letters is characteristic of algebra, the purpose here is not to teach algebra. Symbols such as the question mark (?), or blank space, have long been used in arithmetic. The equation $a+b=x$ could be written ?+?=?, or in some other form, but letters serve as convenient place-holders. They also suggest that different numbers are expected from different situations in the problem. The purpose here is to provide the pupil with a powerful tool for aiding and recording his thinking. He should not be denied the use of this tool until he studies algebra. Furthermore, working with more than one letter should prepare the pupil for the sort of thinking he will do later on.

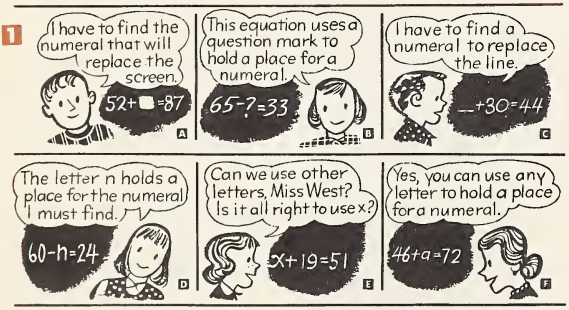
**Answers**

See the note following "Answers" on page 63 for a explanation of the use of parentheses and brackets i the answer key.

A [$1.80]−$.65=$1.15
B $1.82+[$2.18]=$4.00
C $32.50−$12.95=n; $32.50−$12.95=[$19.55]
D $17.00÷g=34; $17.00÷[$.50]=34
E 12×$3.50=r; 12×$3.50=[$42.00]
F 396÷t=11; 396÷[36]=11
G ($.74)+$.49=[$1.23]
H (96)÷12=[8]
I ($1.60)÷[$.40]=4
J ($1.00)+($8.16)=[$9.16]
K $5.00−($3.97)=[$1.03]
L ($5.56)+($.20)+$.89=[$6.65]
M (24)+14=[38]

## Exploring problems

**1**


I have to find the numeral that will replace the screen.
52+■=87
A
This equation uses a question mark to hold a place for a numeral.
65-?=33
B
I have to find a numeral to replace the line.
__+30=44
C
The letter n holds a place for the numeral I must find.
60-n=24
D
Can we use other letters, Miss West? Is it all right to use x?
x+19=51
E
Yes, you can use any letter to hold a place for a numeral.
46+a=72
F

**2**

A After Henry had bought a movie ticket for 65¢, he counted the money he had left. He found that he had $1.15. How much money did he have before he bought the movie ticket?
a−$.65=$1.15

B A book about frogs and toads that Jack wants costs $4.00. Jack has saved $1.82. How much more money does he need before he can buy the book?
$1.82+s=$4.00

C Ellen bought a coat for $32.50 and a dress for $12.95. The coat cost how much more than the dress?
Use the letter $n$ in your equation.

D 34 sixth-grade pupils paid $17.00 for tickets to a concert. What was the cost of each pupil's ticket?
Use the letter $g$ in your equation.

E Kathy's piano lessons cost $3.50 per hour. Each week she takes a 1-hr. lesson. What is the cost of her piano lessons for 12 weeks?
Use the letter $r$ in your equation.

F There are 11 classrooms in the Cook School. In September there were 396 pupils in the school. Find the average number of pupils per classroom in September.
Use the letter $t$ in your equation.

**3**





# 49

**1** Let various pupils take turns reading what i being said in Pictures A to F and reading th equation for each. Point out that a differen symbol holds a place for a numeral in eac equation, and discuss each. Have the pupil show their understanding by using each sym bol in writing the other equations shown o the chalkboard. In Pictures E and F, have th pupils notice the use of other letters than $n$ t hold a place for the numeral that is to b found.

**2** Have Problems A and B read aloud. Discus each equation with the class. Let someon compute at the board to find what the letter represent.

**3** Assign Problems C to F as written work. Hav the equation and solution for each put on th board and discussed.

Joan bought 2 packages of notebook paper at 37¢ each and a package of pencils for 49¢. How much did she spend for all these things?

THINK In this equation there will be two numerals that you do not know. One stands for the amount spent for paper, and the other stands for the total amount spent. You can use letters to hold places for these numerals.

Holds a place for the amount spent for paper

$+\$.49=d$ — Holds a place for the total amount spent

$(2\times\$.37)$

$+\$.49=d$

Now you may write this to replace the letter $a$.

$\times\$.37)+\$.49=d$

$74+\$.49=d$

What numeral will replace $d$?

Mrs. Parks made 6 lb. of candy for school sale. She packed it in bags, putting 12 oz. of candy into each bag. How many bags of candy did she pack?

THINK You do not know how many ounces of candy Mrs. Parks made, or how many bags she packed. Use letters in the equation to hold places for the two unknown numerals.

Holds a place for the total number of ounces

$m\div 12=t$ — Holds a place for the number of bags

You may write this to replace $m$.

$(6\times 16)\div 12=t$

$96\div 12=t$

What numeral will replace $t$?

I Jim earned 45¢ on Monday, 30¢ on Tuesday, 35¢ on Wednesday, and 50¢ on Thursday. Find the average amount he earned per day. 3

THINK You do not know the total amount that Jim earned. You do not know the average amount he earned per day. You do know that he worked 4 days.

Holds a place for the total amount earned

$s\div m=4$

Holds a place for the average amount earned

Write this to replace $s$.

$(\$.45+\$.30+\$.35+\$.50)\div m=4$

$\$1.60\div m=4$

What numeral will replace $m$?

Two teachers took 34 pupils on a trip. The fare was $.50 for each teacher and $.24 for each pupil. What was the total amount of all the fares?

THINK In Problem J there are three things you do not know. So you can use $b=x$ for your equation. What does $b$ hold a place for? If $a$ holds a place for the total amount of the teachers' fares, what does $b$ hold a place for?

$(2\times\$.50)$

$(34\times\$.24)$

$+b=x$

$\$.50)+(34\times\$.24)=x$

$00+\$8.16=x$

What numeral will replace $x$?

Tony bought a model boat for $2.89. a kit for 10¢, and a set of paints for $1. He paid for all this with a dollar bill. How much change should he have received?

K How do you know that you will need to use two letters in the equation?

The letter $z$ holds a place for what amount?

$\$5.00-z=r$

holds a place for what amount?

This replaces $z$.

$\$5.00-(\$2.89+\$.10+\$.98)=r$

$\$5.00-\$3.97=r$

L Mrs. Long bought 4 yd. of cloth at $1.39 a yard, 2 spools of thread at 10¢ a spool, and a card of buttons for 89¢. How much did all these materials cost?

THINK How many letters will you use in the equation? You can use the equation $d+t+\$.89=y$. If $d$ holds a place for the total cost of the cloth, what does $t$ hold a place for? What does $y$ hold a place for?

$(4\times\$1.39)$

$(2\times\$.10)$

$d+t+\$.89=y$

$(4\times\$1.39)+(2\times\$.10)+\$.89=y$

$\$5.56+\$.20+\$.89=y$

M Kathy bought three 8-oz. packages of candy corn and a 14-oz. package of another kind of candy. How many ounces of candy did she buy in all?

THINK There will be ▦ letters in the equation. To find the numeral that replaces one of them, why do you multiply 8 by 3? Now find the numeral that replaces the other letter.

Now you can use different letters to hold places for numerals in the equations you write for problems.



# 50

1 Have a pupil read Problem G. Establish the fact that the equation for it should indicate a combining of two groups. Then have the pupils read the hints for solving the problem.

2 Let the class study the first equation. Then remind them that $a$ stands for the cost of 2 packages of notepaper and that the cost of one package is $.37. Ask them to figure out the cost of 2 packages mentally or on paper. Tell them this thinking is suggested below in black ink. Then have them look at the third equation printed in blue ink and note that $(2\times\$.37)$ has been substituted for $a$ in the equation. The final equation $(\$.74+\$.49=d)$ shows the result of the preliminary computation. Now they can add to find $d$.

3 Adapt the steps suggested in Notes 1 and 2.

# 51

1 Treat this problem as you did the ones on page 50, but call attention to the fact that there are three letters in the equation. Establish that $x$ holds a place for the final answer (as $n$ or any other letter could). Remind the class that the work indicated in parentheses above the basic equation shows what they must do before they can find the answer. Be sure the pupils understand what the numerals stand for. Tell them to do any necessary computation at the side, then to rewrite the equation using the numerals they have found. Then let them find the numeral that replaces $x$.

2 Go through the same steps in discussing and solving Problems K to M. For each of the equations set up in the book, ask the pupils to suggest other letters that can be used.

# 52-55 Looking back

Expanded Notes for this lesson are on pages 299-301.

**Objectives**

The pupil reviews how to compute to replace a ratio or fraction numeral with an equal ratio or fraction numeral. He also reviews reduction of these pairs of numerals to lowest terms.

**Vocabulary**

There are no new words.

**Comments**

Up to this time the pupils have reviewed reading and writing ratios and fraction numerals. On page 46 they learned to distinguish among the three types of situations. Now they are ready to review the computational procedures for reducing ratios and fraction numerals. Explain to the pupils that the word *reduce* is used when a fraction numeral is replaced by an *equal fraction numeral* and when a ratio is replaced by an *equal ratio*. A pair of numerals is reduced to lower terms when the terms of the new pair are smaller than the terms of the original pair. It is reduced to higher terms when the terms of the new pair are larger than the terms of the original pair.

Pages 52 and 53 of this lesson deal with reduction of ratios. An important point to stress in teaching the computational procedures is that *both* terms of a ratio must be multiplied or divided by the same number. For example, if John buys pencils at the rate of 4 for 10 cents, it is not the 4 or the 10 that is important, but the relation between 4 and 10, or the rate 4 per 10. If John buys only 2 pencils, he pays only 5 cents; if he buys 8 pencils, he pays 20 cents.

The same thing is true for comparisons. If Tim's 12 planes are being compared to Ned's 10 planes, we are concerned with the *relation* of 12 to 10. As long as we are concerned with the comparison of 12 to 10, the numeral in one term cannot be replaced unless the numeral in the other term is also properly replaced. Since the 12 and 10 are used together to express a comparison, to express the same relation based on only 6 of Tim's planes we should think: "12 divided by 2 equals 6. So we must also divide 10 by 2 to get the other number. It will be 5."

Although the procedures in this lesson focus on computational methods, you will still want to use pictures or objects to help the pupils understand that multiplying or dividing both terms of a ratio by the same number produces a new ratio that expresses the same rate or comparison.

Pages 54 and 55 deal with reduction of fraction numerals. When we reduce a fraction numeral, we replace it with another fraction numeral that is equivalent and represents the same fraction. Make sure the pupils understand that the various equal fraction numerals are only different ways of expressing the same fraction. The fraction itself remains the same. Only the numerals are different. In other words, the pupils should see that $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{8}{12}$ are simply different ways of writing the same fraction. They are different names for the same fraction.

To clarify this idea for the pupils, draw a rectangular shape and shade part of it. With only this information, you cannot tell how much of it is shaded. You can only point to the shaded portion and say that "this much" of the rectangle is shaded. To describe "this much," it is necessary, first, to divide the rectangle into equal parts. The part that is shaded then can be expressed by a fraction numeral that expresses the number of equal parts (the denominator) and the number of these parts that are being considered (the numerator). Note that no matter how many or how few divisions have been made, the shaded portion remains the same. However, many fraction numerals may be written to describe it.

This is not the only lesson in which reduction of fractions is considered. Reduction of improper fractions to mixed numbers is reviewed on pages 70-71 of the pupils' book, and reduction of mixed numbers to improper fractions is reviewed on pages 87-88.

**Answers**

**Page 53:**

(block 2)

| | |
|---|---|
| A 1/6, 3/18, 2/12 | E 7/5 |
| B 4/5 | F 3/1, 6/2 |
| C 2/5, 4/10 | G 3/2, 9/6 |
| D 10/3, 20/6 | H 5/3, 10/6, 25/15 |

I to P: Innumerable ratios can be written.

Page 55:

Since the answers are obvious, answers are not given for Blocks 1 (blue letters A to D) and 3 (black letters A to N).

block 2)

A $\frac{8}{12}, \frac{4}{6}, \frac{2}{3}$

B $\frac{6}{2}, \frac{3}{1}$

C $\frac{9}{18}, \frac{6}{12}, \frac{3}{6}, \frac{2}{4}, \frac{1}{2}$

D $\frac{7}{5}$

E $\frac{6}{9}, \frac{4}{6}, \frac{2}{3}$

F $\frac{3}{4}$

G $\frac{20}{6}, \frac{10}{3}$

H $\frac{5}{3}$

I $\frac{5}{8}$

J $\frac{4}{5}$

K $\frac{4}{10}, \frac{2}{5}$

L $\frac{3}{18}, \frac{2}{12}, \frac{1}{6}$

M to X: Innumerable fraction numerals can be written.

block 4)

| | | | | |
|---|---|---|---|---|
| A $\frac{3}{4}$ | E $\frac{1}{2}$ | I $\frac{2}{3}$ | M $\frac{3}{4}$ | Q $\frac{7}{8}$ |
| B $\frac{4}{5}$ | F $\frac{5}{6}$ | J $\frac{1}{2}$ | N $\frac{2}{3}$ | R $\frac{8}{15}$ |
| C $\frac{3}{5}$ | G $\frac{1}{2}$ | K $\frac{5}{7}$ | O $\frac{1}{3}$ | S $\frac{3}{4}$ |
| D $\frac{1}{3}$ | H $\frac{2}{3}$ | L $\frac{4}{5}$ | P $\frac{1}{4}$ | T $\frac{3}{4}$ |

## Looking back

Now you will learn how to find ratios and fraction numerals equal to other ratios and fraction numerals. 1

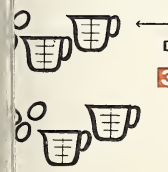
A

When Mrs. Stone makes custard, she uses 6 eggs for 4 cups of milk. Does the ratio at the right express the rate at which Mrs. Stone uses eggs and milk? Read the ratio ■ per ■. 2

$\frac{6}{4}$

6 and 4 are the terms of the ratio. 6 is the first term. 4 is the second term.

B



Now the 6 eggs and the 4 cups of milk have been divided into 2 equal groups. Think of 1 of the equal groups. Does the ratio at the right also express the rate at which Mrs. Stone uses eggs and milk? What is the first term of this ratio? What is the second term? 3

$\frac{3}{2}$

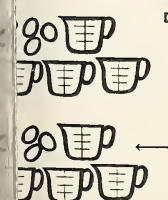
You can use arithmetic to get this other ratio that expresses the same rate. You can divide both the 6 and the 4 by 2. 4

$6 \div 2 = 3$

$4 \div 2 = 2$

$\frac{6}{4} = \frac{3}{2}$ 6 per 4 = 3 per 2

C



The ratio 3 over 2 expresses the rate in lower terms than the ratio 6 over 4 does.

One day Mrs. Stone made a larger amount of custard. 5

She used 2 × 6 eggs, or ■ eggs.

She used 2 × 4 cups of milk, or ■ cups of milk.

ction of number pairs (ratios and fraction numerals);

# 52

1 Remind the pupils that they have learned how to find different equal ratios and different equal fraction numerals by using pictures or objects. Now they will find them by computing.

2 Ask the pupils to read the first five lines, look at Picture A and the ratio in blue. Stress that 6/4 expresses the rate of eggs per cups of milk.

3 Call on someone to read the text and explain Picture B. From their previous work with ratios, the pupils will see that 3/2 expresses the same rate as 6/4.

4 Be sure the pupils understand that the computation produces the same result as dividing the original objects into 2 equal groups.

5 Point out that Mrs. Stone used two groups like the group in Picture A. Ask, "How many eggs did she use? How many cups of milk?" Then have the pupils replace the screens with numerals.

Think of all the eggs as one group. Think of all the cups of milk as one group. Does the ratio at the right also express the rate at which Mrs. Stone uses eggs and milk? $\frac{12}{8}$ 1

You can use arithmetic to get this new ratio that expresses the same rate. You can multiply both 6 and 4 by 2.

$2 \times 6 = \blacksquare$ $\quad \frac{6}{4} = \frac{\blacksquare}{\blacksquare}$
$2 \times 4 = \blacksquare$ 1

The ratio 12 over 8 expresses the rate in higher terms than 6 over 4 does.

A You can find many ratios equal to the ratio at the right. One way is to divide both terms by the same number. Can both 16 and 20 be divided by 4? $\frac{16}{20}$

$16 \div 4 = \blacksquare$ $\quad \frac{16}{20} = \frac{\blacksquare}{\blacksquare}$
$20 \div 4 = \blacksquare$ 2

B By what other number can both 16 and 20 be divided?

$16 \div 2 = \blacksquare$ $\quad \frac{16}{20} = \frac{\blacksquare}{\blacksquare}$
$20 \div 2 = \blacksquare$ 3

You can also find other ratios equal to the ratio 16 over 20 by multiplying both terms by the same numbers.

C $3 \times 16 = \blacksquare$ $\quad \frac{16}{20} = \frac{\blacksquare}{\blacksquare}$
$3 \times 20 = \blacksquare$

D $4 \times 16 = \blacksquare$ $\quad \frac{16}{20} = \frac{\blacksquare}{\blacksquare}$
$4 \times 20 = \blacksquare$ 4

E You can use arithmetic to find a new ratio that is equal to the ratio at the right, but in lower terms. You can divide both terms by 2. $\frac{40}{8}$

$40 \div 2 = \blacksquare$ $\quad \frac{40}{8} = \frac{\blacksquare}{\blacksquare}$
$8 \div 2 = \blacksquare$ 5

F Find two more ratios equal to the ratio 40 over 8, but in lower terms.

G You can also find other ratios equal to the ratio 40 over 8, but in higher terms. You can multiply both terms by the same number.

$5 \times 40 = \blacksquare$ $\quad \frac{40}{8} = \frac{\blacksquare}{\blacksquare}$
$5 \times 8 = \blacksquare$

H Find two more ratios equal to the ratio 40 over 8, but in higher terms.

For each ratio below, find a ratio equal to it, but in lower terms. For Exercise A you could have 1 over 6. 6

A $\frac{6}{36}$ C $\frac{8}{20}$ E $\frac{35}{25}$ G $\frac{27}{18}$

B $\frac{12}{15}$ D $\frac{40}{12}$ F $\frac{12}{4}$ H $\frac{50}{30}$

Now find a ratio equal to each ratio below, but in higher terms. 6

I $\frac{2}{3}$ K $\frac{3}{4}$ M $\frac{11}{8}$ O $\frac{7}{1}$

J $\frac{6}{5}$ L $\frac{5}{9}$ N $\frac{4}{6}$ P $\frac{8}{3}$

53

# 53

1 The same rate is now expressed by a third ratio. Relate the computation to the two equal groups of eggs and cups in Picture C.
2 Show pictorially or with objects that dividing by 4 gives a ratio that is equal to 16/20.
3 Adapt the procedures in Note 2.
4 Have the pupils replace the screens with numerals. Be sure they understand that when they have multiplied, the new ratio will be equal to the original ratio.
5 Discuss Exercises E to H. Make sure the pupils understand why dividing both terms of a ratio by the same number gives an equal ratio in lower terms and why multiplying both terms by the same number gives an equal ratio in higher terms.
6 Let the pupils work independently to find the ratios for Exercises A to H and I to P.

1

2

A The paper disk in Picture A is marked off in ■ equal parts. What fraction of the disk is yellow?

Now imagine that the disk has been marked off in only 5 equal parts. Each part is $\frac{1}{5}$ of the disk. How many one fifteenths are in each $\frac{1}{5}$?

Can you still say that $\frac{9}{15}$ of the disk is yellow?
Is $\frac{3}{5}$ of the disk yellow?

3

B You can express $\frac{9}{15}$ as fifths by dividing both terms of $\frac{9}{15}$ by 3. The terms of a fraction numeral are its ~~ and its ~~

$9 \div 3 = 3$
$15 \div 3 = 5$ $\quad \frac{9}{15} = \frac{3}{5}$

Now imagine that the same disk has been marked off in 30 equal parts. Each part is $\frac{1}{30}$ of the disk. How many one thirtieths are in each $\frac{1}{15}$?

4

Is $\frac{18}{30}$ of the disk yellow?
Is $\frac{9}{15}$ of the disk yellow?

You can express $\frac{9}{15}$ as thirtieths by multiplying both terms of $\frac{9}{15}$ by 2.

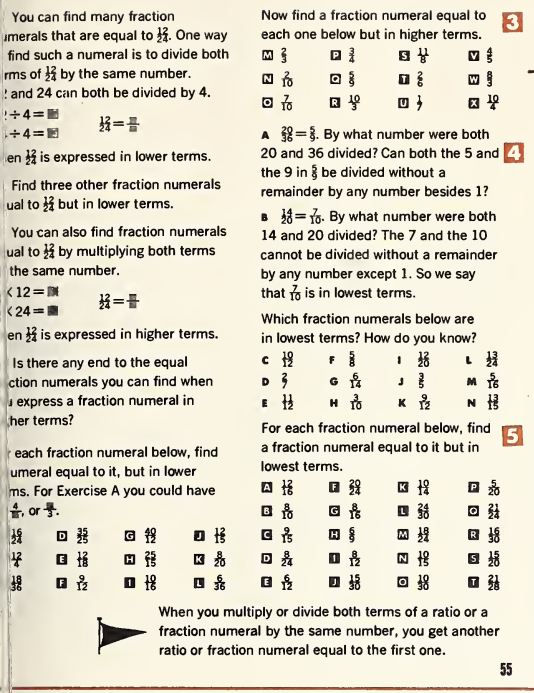
$2 \times 9 = \blacksquare$
$2 \times 15 = \blacksquare$ $\quad \frac{9}{15} = \frac{\blacksquare}{\blacksquare}$ 5

$\frac{9}{15}$, $\frac{3}{5}$, and $\frac{18}{30}$ all express the fraction of the disk that is yellow.

54

# 54

1 Have the pupils observe that they are working with fractions. Have them note that the shaded portion of the paper disk is the same in all three pictures. Have them observe that the light, the heavy, and the broken lines mark off different fractional parts.
2 Point out that the yellow portion in Picture A is the part they are interested in.
3 Emphasize that now the disk is marked off in fifths, and each fifth contains three fifteenths. Show how this fact is used in reducing $\frac{9}{15}$ to $\frac{3}{5}$ by computation.
4 Have the class note that the shaded portion is now divided into still smaller parts. Each one fifteenth is made up of two thirtieths.
5 Explain that since it takes two thirtieths to make one fifteenth, it is possible to replace fifteenths by thirtieths. The new fraction numeral can be found by multiplying both the 9 and the 15 of $\frac{9}{15}$ by 2.



You can find many fraction
imerals that are equal to $\frac{12}{24}$. One way
find such a numeral is to divide both
rms of $\frac{12}{24}$ by the same number.
and 24 can both be divided by 4.

÷ 4 = ■
÷ 4 = ■ $\frac{12}{24} = \frac{\blacksquare}{\blacksquare}$

en $\frac{12}{24}$ is expressed in lower terms.

Find three other fraction numerals
ual to $\frac{12}{24}$ but in lower terms.

You can also find fraction numerals
ual to $\frac{12}{24}$ by multiplying both terms
the same number.

× 12 = ■
× 24 = ■ $\frac{12}{24} = \frac{\blacksquare}{\blacksquare}$

en $\frac{12}{24}$ is expressed in higher terms.

Is there any end to the equal
ction numerals you can find when
express a fraction numeral in
her terms?

each fraction numeral below, find
umeral equal to it, but in lower
ms. For Exercise A you could have
$\frac{4}{12}$, or $\frac{\blacksquare}{3}$.

| | | | |
|---|---|---|---|
| $\frac{16}{24}$ | D $\frac{35}{25}$ | G $\frac{40}{12}$ | J $\frac{12}{15}$ |
| $\frac{12}{4}$ | E $\frac{12}{18}$ | H $\frac{25}{15}$ | K $\frac{8}{20}$ |
| $\frac{18}{36}$ | F $\frac{9}{12}$ | I $\frac{10}{16}$ | L $\frac{6}{36}$ |

Now find a fraction numeral equal to each one below but in higher terms.

| | | | |
|---|---|---|---|
| M $\frac{2}{3}$ | P $\frac{3}{4}$ | S $\frac{11}{8}$ | V $\frac{4}{5}$ |
| N $\frac{7}{10}$ | Q $\frac{5}{6}$ | T $\frac{5}{6}$ | W $\frac{8}{9}$ |
| O $\frac{7}{10}$ | R $\frac{10}{3}$ | U $\frac{1}{7}$ | X $\frac{10}{4}$ |

A $\frac{20}{36} = \frac{5}{9}$. By what number were both 20 and 36 divided? Can both the 5 and the 9 in $\frac{5}{9}$ be divided without a remainder by any number besides 1?

B $\frac{14}{20} = \frac{7}{10}$. By what number were both 14 and 20 divided? The 7 and the 10 cannot be divided without a remainder by any number except 1. So we say that $\frac{7}{10}$ is in lowest terms.

Which fraction numerals below are in lowest terms? How do you know?

| | | | |
|---|---|---|---|
| C $\frac{10}{12}$ | F $\frac{8}{8}$ | I $\frac{16}{20}$ | L $\frac{13}{24}$ |
| D $\frac{2}{9}$ | G $\frac{6}{14}$ | J $\frac{3}{5}$ | M $\frac{5}{16}$ |
| E $\frac{11}{12}$ | H $\frac{3}{10}$ | K $\frac{9}{12}$ | N $\frac{13}{15}$ |

For each fraction numeral below, find a fraction numeral equal to it but in lowest terms.

| | | | |
|---|---|---|---|
| A $\frac{12}{16}$ | F $\frac{20}{24}$ | K $\frac{10}{14}$ | P $\frac{5}{20}$ |
| B $\frac{8}{10}$ | G $\frac{8}{16}$ | L $\frac{24}{30}$ | Q $\frac{21}{24}$ |
| C $\frac{9}{15}$ | H $\frac{6}{9}$ | M $\frac{18}{24}$ | R $\frac{16}{30}$ |
| D $\frac{8}{24}$ | I $\frac{8}{12}$ | N $\frac{10}{15}$ | S $\frac{15}{20}$ |
| E $\frac{6}{12}$ | J $\frac{15}{30}$ | O $\frac{10}{30}$ | T $\frac{21}{28}$ |

When you multiply or divide both terms of a ratio or a fraction numeral by the same number, you get another ratio or fraction numeral equal to the first one.

55

# 55

1 Discuss the two ways of finding equal fraction numerals that the pupils have studied [by multiplication and by division]. Work through Exercises A to C orally. If necessary, use diagrams to demonstrate how $\frac{12}{24}$ can be reduced to $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{6}{12}$, $\frac{24}{48}$, etc.

2 Help the pupils understand that the number of reductions by division is limited, but that any number of reductions by multiplication can be performed.

3 Allow the pupils to do Exercises A to L and M to X independently, but be prepared to give help where it is needed.

4 Have Exercises A and B read and discussed. Be sure the pupils get the meaning of *lowest terms*. Remind the pupils that last year they learned to reduce to "simplest form," and tell them that reducing to lowest terms is the same thing. Then discuss Exercises C to N.

5 Assign Exercises A to T as written work.

# 56-58 Looking back

Expanded Notes for this lesson are on pages 301-302.

**Objectives**

The pupil reviews finding the missing term in a ratio that is equal to another ratio. He also reviews finding the missing numerator in a fraction numeral that is equal to another fraction numeral.

**Vocabulary**

New words page 56 Martha*, socks*

**Comments**

In the lesson preceding this one, the pupils reviewed the reduction of ratios and fraction numerals by computation. They were asked to find pairs of numerals that are equal to other pairs of numerals; they were asked to replace pairs of numerals by others with "higher terms," with "lower terms," and with "lowest terms."

This lesson reteaches the material presented in the Grade 5 textbook on pages 141-142 and 145-146. The pupils are asked to find the missing numeral in pairs of equal ratios. They are asked to find the missing numeral in one numerator of two equal fraction numerals.

The same procedures are used for finding missing numerals in ratios and fraction numerals. As the pupils have already seen, both terms of a ratio or a fraction numeral can be divided or multiplied by the same number without affecting the rate or comparison or fraction. By using the known term of the new ratio or fraction numeral and the corresponding term in the original ratio or fraction numeral, the pupils can discover whether multiplication or division can be used. They then use the same process to find the missing numeral. Thus, in the example on page 56, the pupils can see that 3 can be multiplied by 2 to get 6. Therefore, 117 must also be multiplied by 2 to find the numeral that replaces *a*.

In Grade 5, screens were used to indicate missing numerals in both ratios and fraction numerals. In this book, a screen is used to indicate a missing term in

a fraction numeral, but a letter is used to indicate a missing term in a ratio. Fraction numerals are also usually distinguished from ratios by the use of smaller type. No effort should be made, however, to have the pupils observe these typographic distinctions.

Do not expect the pupils to use the word "reduction" with complete understanding. Many may think of reduction as meaning only to make smaller. Let them use, instead, an expression such as "changing to an equal ratio or fraction numeral."

**Answers**

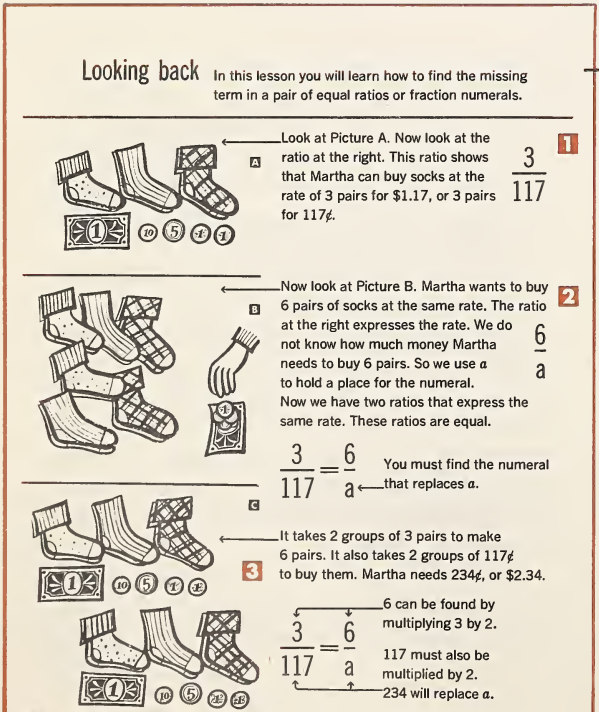
Page 58:

(block 1)

| | | | |
|---|---|---|---|
| C 10 | I 15 | O 6 | |
| D 14 | J 3 | P 1 | |
| E 4 | K 5 | Q 10 | |
| F 1 | L 16 | R 2 | |
| G 7 | M 4 | S 15 | |
| H 12 | N 5 | T 3 | |

(block 2)

| | | |
|---|---|---|
| C 95 | J 7 | |
| D 9 | K 52 | |
| E 10 | L 3 | |
| F 20 | M 4 | |
| G 168 | N 40 | |
| H 18 | O 90 | |
| I 3 | P 13 | |
| | Q 1 | |


Looking back
In this lesson you will learn how to find the missing term in a pair of equal ratios or fraction numerals.
Look at Picture A. Now look at the ratio at the right. This ratio shows that Martha can buy socks at the rate of 3 pairs for $1.17, or 3 pairs for 117¢.
3/117
1
Now look at Picture B. Martha wants to buy 6 pairs of socks at the same rate. The ratio at the right expresses the rate. We do not know how much money Martha needs to buy 6 pairs. So we use a to hold a place for the numeral. Now we have two ratios that express the same rate. These ratios are equal.
6/a
2
3/117 = 6/a
You must find the numeral that replaces a.
It takes 2 groups of 3 pairs to make 6 pairs. It also takes 2 groups of 117¢ to buy them. Martha needs 234¢, or $2.34.
3
3/117 = 6/a
6 can be found by multiplying 3 by 2.
117 must also be multiplied by 2.
234 will replace a.
72
56 Finding the missing term in a pair of equal ratios or fraction numerals (reteaching)

# 56

1 Ask the pupils to relate the written ratio t the cost of 3 pairs of socks. Point out that th first term represents the number of pairs o socks and the second term represents the cost.

2 Have the class read the text and look a Picture B. Have the pupils note that Martha wants to buy 6 pairs of socks at that same *rate*. Tell them to find a new ratio that i equal to 3/117. Since the first term in th original ratio represents the number of pair of socks, 6 is used for the first term in th new ratio. Since the second term represent the cost of the socks, a is used to hold place for the cost of 6 pairs.

3 In examining Picture C and reading the text the pupils should see that since 3 in the firs ratio can be multiplied by 2 to get 6, 11 can be multiplied by 2 to find the numera to replace a.

D Here is a rectangular piece of paper marked off in thirds. The fraction numeral at the right expresses how much of the rectangle is blue. $\frac{2}{3}$

1

E Now suppose you want to mark off the same rectangle in twenty-fourths. You will use a new fraction numeral to express how many twenty-fourths will be blue. The denominator will be 24. You do not know what numeral to use for the numerator, but you can use a screen to hold a place for the numerator. $\frac{\square}{24}$

2

Since the same amount of paper is blue, $\frac{\square}{24}$ and $\frac{2}{3}$ express the same fraction.

F $\frac{2}{3} = \frac{\square}{24}$ ← You must find the numerator.

You can see that since there are 8 twenty-fourths in each one third, there will be 8 × 2, or 16, twenty-fourths in 2 of these thirds. 3

4 $\frac{2}{3} = \frac{\square}{24}$ Because 24 can be found by multiplying 3 by 8, you must multiply 2 by 8 also. 16 will replace the screen.

57

$\frac{3}{5} = \frac{\square}{20}$

find the missing numerator, first
nd the number by which both the
merator and the denominator of
must be multiplied. How do you
d this number? What must you
next?
× 5 = 20
× 5 = 20 $\frac{3}{5} = \frac{12}{20}$
× 3 = 12

1

$\frac{27}{36} = \frac{\square}{4}$

find the missing numerator, first
d the number by which both the
merator and the denominator of
must be divided. How do you
d this number? What must you
next?
÷ ■ = 4
÷ 9 = 4 $\frac{27}{36} = \frac{3}{4}$
÷ 9 = 3

2

each pair of fraction numerals
low, find the missing numerator.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\frac{2}{3}=\frac{\square}{15}$ | I | $\frac{5}{8}=\frac{\square}{24}$ | O | $\frac{3}{5}=\frac{\square}{10}$ |
| $\frac{7}{8}=\frac{\square}{16}$ | J | $\frac{12}{16}=\frac{\square}{4}$ | P | $\frac{9}{18}=\frac{\square}{2}$ |
| $\frac{1}{6}=\frac{\square}{24}$ | K | $\frac{10}{24}=\frac{\square}{12}$ | Q | $\frac{5}{6}=\frac{\square}{12}$ |
| $\frac{8}{32}=\frac{\square}{4}$ | L | $\frac{5}{6}=\frac{\square}{36}$ | R | $\frac{6}{9}=\frac{\square}{3}$ |
| $\frac{14}{20}=\frac{\square}{10}$ | M | $\frac{12}{15}=\frac{\square}{5}$ | S | $\frac{3}{4}=\frac{\square}{20}$ |
| $\frac{4}{7}=\frac{\square}{21}$ | N | $\frac{25}{30}=\frac{\square}{6}$ | T | $\frac{18}{24}=\frac{\square}{4}$ |

A $\frac{17}{12} = \frac{51}{c}$ 4

To find the missing term, first find the number by which both terms of the ratio 17 over 12 must be multiplied. How do you find this number? Then what must you do to get the numeral that replaces c?

■ × 17 = 51
3 × 17 = 51 $\frac{17}{12} = \frac{51}{36}$
3 × 12 = 36

B $\frac{10}{25} = \frac{a}{5}$ 5

By what number must 25 be divided to get 5? What must you do to get the numeral that replaces a?

25 ÷ ■ = 5
25 ÷ 5 = 5 $\frac{10}{25} = \frac{2}{5}$
10 ÷ 5 = 2

In each pair of ratios below, find the missing term. 6

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| C | $\frac{6}{19}=\frac{30}{e}$ | H | $\frac{6}{5}=\frac{d}{15}$ | M | $\frac{75}{12}=\frac{25}{n}$ |
| D | $\frac{27}{15}=\frac{m}{5}$ | I | $\frac{33}{44}=\frac{v}{4}$ | N | $\frac{4}{9}=\frac{c}{90}$ |
| E | $\frac{10}{50}=\frac{2}{s}$ | J | $\frac{21}{3}=\frac{x}{1}$ | O | $\frac{8}{18}=\frac{40}{f}$ |
| F | $\frac{4}{5}=\frac{16}{a}$ | K | $\frac{5}{13}=\frac{20}{b}$ | P | $\frac{30}{65}=\frac{6}{h}$ |
| G | $\frac{42}{9}=\frac{b}{36}$ | L | $\frac{17}{51}=\frac{1}{n}$ | Q | $\frac{49}{7}=\frac{7}{y}$ |

Now you should be able to find the missing term in a pair of equal ratios or fraction numerals.

# 57

1. Have the pupils explain why the fraction numeral $\frac{2}{3}$ represents the part of the rectangle that is blue.
2. Explain that when the whole rectangle is marked off, it will be divided into twenty-fourths. Therefore, the new fraction numeral will have 24 as the denominator. A screen is used to hold a place for the numerator, which will express the number of twenty-fourths that will be blue.
3. Ask a pupil to verify the statement in the first three lines of text by counting the twenty-fourths in one third, then in two thirds.
4. Have the pupils study the equation to see what numeral will replace the screen. Ask, "Do we multiply or divide the 3 in the denominator to get 24? By what number do we multiply? By what number do we multiply the 2 in the numerator? Why?"

# 58

1. Point out that the equations below the text show the necessary steps in thinking to find the missing numerator. Help the pupils see that to find the number by which both 3 and 5 are to be multiplied, they can divide 20 by 5 to get 4, then multiply 3 by 4 to find the missing numeral in the numerator.
2. To find the number by which 36 can be divided to get 4, the pupils should see that they divide 36 by 4 and get 9. Then they should divide 27 also by 9 to find the missing numeral in the numerator.
3. Assign Exercises C to T as written work.
4. Tell the pupils that to find the number by which both terms in the first ratio are to be multiplied, they can divide 51 by 17 to get 3, then multiply 12 by 3 to find the numeral that replaces c.
5. Adapt Note 2.
6. Assign Exercises C to Q as written work.

# 59-62 Looking back

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil reviews how to consider the remainder in division so that he can express the answer as a mixed number.

**Vocabulary**

New words page 61 rope*

**Comments**

In traditional methods of teaching division, the pupils are taught to use the remainder with the divisor to form a fraction numeral as soon as they have been introduced to fractions. The fraction numeral, then, together with the whole number, is used to express the answer as a mixed number. The entire procedure is carried out more or less mechanically, without any real attention to the meaning of the answer. Often the fraction in the answer in a genuine problem situation is not meaningful. In *Seeing Through Arithmetic 5* (pages 210-212), the pupils were given some idea of the significance of using a fraction numeral to express the remainder in the answer for a division problem. That lesson is reviewed and expanded here.

The work on pages 59-62 shows that the meaning of the fraction depends upon whether the problem involves partitive or quotitive division. Partitive division is involved in a *sharing* type of situation, as illustrated by Problems A and B on pages 59 and 60. In partitive division, the number of groups is known, but not their size. In the problem on page 59, a piece of cloth 17 yd. long is to be cut in 3 equal parts, or groups. Each of these parts will represent one girl's share. Obviously, there will be at least 5 yd. in each girl's share. But there will be 2 whole yards left over to be divided equally among them. If the 2 yd. are divided into thirds, each girl will get $\frac{1}{3}$ of each leftover yard, or $\frac{2}{3}$ of a yard in addition to 5 yards.

It should be brought out that the amount each girl gets should be in one piece to be useful. Therefore, the division must be worked out before any cutting of cloth is done. In other words, each girl should get $5\frac{2}{3}$ yd. *all in one piece*—not 5 yd. and 2 pieces of two other yards, or 5 yd. and another piece $\frac{2}{3}$ of a yard long. For this reason all work is done with the measuring unit (represented by yardsticks) instead of with the cloth.

In quotitive division, on the other hand, the fraction represents a part of one of the groups into which the objects are divided. Thus, in the example on page 61, the numeral $\frac{5}{12}$ represents $\frac{5}{12}$ of a group of 12 inches, or $\frac{5}{12}$ of a foot.

The pupils must understand that they should not show both a remainder and a fraction numeral in the answer. Have them note in the division examples on pages 60-62 that the remainder is crossed out when a fraction numeral is used to express the answer as a mixed number.

Division exercises involving answers that are mixed numerals are included at this time so that the pupil will not get the idea that division always involves "whole number" answers. The teacher should realize that at this point in his work the pupil will not be able to check his answer by multiplication when the answer is a mixed numeral, because he does not yet know how to compute with such numbers.

**Answers**

Page 62:

F $8\frac{1}{2}$ hr. The $\frac{1}{2}$ means $\frac{1}{2}$ hr. because the 25¢ remainder represents his pay for $\frac{1}{2}$ hr. of work.

G $11\frac{1}{2}$ gal. You divide by 4 because there are 4 qt. in 1 gallon. The remainder represents quarts. 2 qt. are $\frac{2}{4}$ gal. or $\frac{1}{2}$ gal.

H $2\frac{3}{4}$ lb.

(block 2)

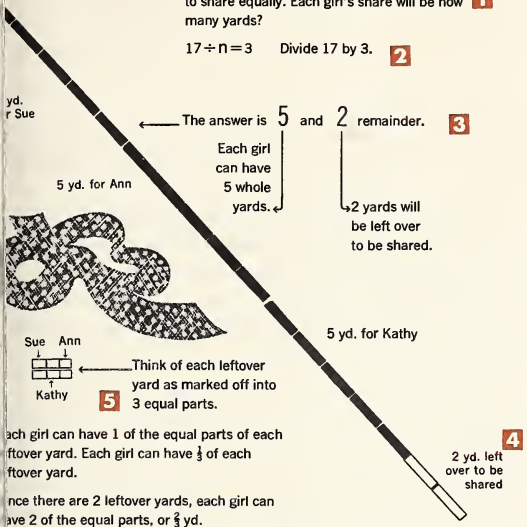
| | |
|---|---|
| A $5\frac{2}{3}$ | J $7\frac{1}{4}$ |
| B $14\frac{1}{2}$ | K $16\frac{3}{10}$ |
| C $4\frac{3}{4}$ | L $16\frac{1}{2}$ |
| D $14\frac{1}{6}$ | M 15 |
| E $13\frac{1}{7}$ | N $32\frac{1}{6}$ |
| F $8\frac{3}{7}$ | O $20\frac{1}{5}$ |
| G 5 | P 13 |
| H $6\frac{2}{5}$ | Q $11\frac{1}{3}$ |
| I $6\frac{1}{8}$ | R $10\frac{1}{2}$ |

## Looking back

If you have a remainder when you divide, sometimes you should use it to make a fraction numeral in the answer.

A Sue, Ann, and Kathy have 17 yd. of cloth to share equally. Each girl's share will be how many yards? 1

$17 \div n = 3$ Divide 17 by 3. 2


yd.
r Sue
The answer is 5 and 2 remainder. 3
Each girl can have 5 whole yards.
2 yards will be left over to be shared.
5 yd. for Ann
5 yd. for Kathy
Sue
Ann
Kathy
Think of each leftover yard as marked off into 3 equal parts. 5
4
2 yd. left over to be shared

ach girl can have 1 of the equal parts of each ftover yard. Each girl can have $\frac{1}{3}$ of each ftover yard.

nce there are 2 leftover yards, each girl can ave 2 of the equal parts, or $\frac{2}{3}$ yd.

Before cutting the cloth, measure 5 whole yards and $\frac{2}{3}$ of another yard for each girl's share. 1

When you divide 17 by 3, you can express the answer as $5\frac{2}{3}$. Notice that the divisor, 3, tells what denominator to use in the fraction numeral. The remainder, 2, tells what numerator to use. 2

Number of whole yards in each girl's share → $5\frac{2}{3}$ ← Number of parts of the leftover yards each girl will get

Number of parts into which each leftover yard is marked → $5\frac{2}{3}$

Each girl's share will be $5\frac{2}{3}$ yd.

Twelve boys picked 129 lb. of ples to share equally. What was ch boy's share of the apples?

$12 \div a = 12$ Divide 129 by 12.

$12\overline{)129}$ $10\frac{9}{12}$ ← The divisor, 12, tells you what denominator to use. The remainder, 9, tells you what numerator to use. 4

120

9

Number of whole pounds in each boy's share → $10\frac{9}{12}$ ← Number of parts of the leftover pounds in each boy's share 5

Number of parts into which each leftover pound is divided → $10\frac{9}{12}$

ch boy's share was $10\frac{9}{12}$ lb., or $10\frac{3}{4}$ lb.

# 59

1 Ask the pupils to study the problem. They should see that this is the sharing type of divisive situation.

2 Discuss the equation.

3 Discuss this statement and relate it to the picture. Explain that the red, yellow, blue, and white strips are to be thought of as yardsticks. Discuss the reasons for working with the yardsticks (measuring unit) instead of with the cloth.

4 Ask for suggestions as to how 2 yards can be shared by the three girls.

5 Let someone read the text and explain what the picture shows. Be sure to relate this picture with the 2 yards shown as left over at the lower right of the page.

# 60

1 Ask the pupils why the cloth should not be cut until the exact amount of each girl's share is known. [Cloth may be more advantageously used if it is all in one piece.] Pupils should see how sharing a continuous substance like cloth, string, sugar, etc. differs from sharing things that can be counted, such as marbles, apples, books, etc.

2 Let someone read the text and relate it to the answer, $5\frac{2}{3}$. Have another pupil explain the meaning of each numeral in the answer.

3 Let the pupils read the problem. Then discuss the computation. By asking questions, bring out these facts: that the pounds are divided into 12 equal groups; that one group represents one boy's share; that the answer tells how many pounds each boy's share is.

4 Discuss why the remainder has been crossed out.

5 Discuss the meaning of the numerator and the denominator in the fraction numeral.

C Don has a rope 41 in. long. How many feet long is the rope?

1 $41 \div 12 = g$ ← What does the 12 represent?

2
$$12\overline{)41}\ \Big|\ 3 \quad \begin{array}{l} 36 \\ \hline 5 \end{array}$$

← There are 3 whole feet in the piece of rope.

← There are 5 inches more than 3 ft. in the piece of rope.

3 Think of each inch as $\frac{1}{12}$ of a whole foot. Think of the 5 inches as $\frac{5}{12}$ of a foot.

Number of whole feet in 41 inches

$$12\overline{)41}\ \Big|\ 3\frac{5}{12} \quad \begin{array}{l} 36 \\ \hline 5 \end{array}$$

Number of inches more than 3 ft.

Number of inches in 1 ft.

4 When you use the fraction numeral $\frac{5}{12}$ as part of the answer, the divisor, ■, tells what denominator to use. The remainder, ■, tells what numerator to use.

Don's piece of rope is $3\frac{5}{12}$ ft. long.



# 61

1 Let the pupils read the problem. Have someone explain why 41 must be divided by 12 and why the number of groups of 12 inches will show the number of whole feet in 41 inches. Let another pupil tell what the answer will show.

2 Have the pupils look at the second picture and verify the results of the computation. Note that the blue strips represent 1-foot rulers.

3 Let the pupils read the text and discuss the meaning of the remainder. Have them relate the text to the picture.

4 When the pupils have read the text, have someone explain the meaning of each numeral in the answer.

D Mrs. Cook bought a tablecloth that was 120 in. long. Alice said, "The tablecloth is $3\frac{1}{3}$ yd. long." Was Alice right?

$120 \div 36 = b$

1
$$36\overline{)120}\ \Big|\ 3\frac{12}{36} \quad \begin{array}{l} 108 \\ \hline 12 \end{array}$$

The divisor, 36, tells you what denominator to use. The remainder, 12, tells you what numerator to use.

Number of whole yards in 120 in.

2 $3\frac{12}{36}$ ← Number of inches more than 3 yd.

Number of inches in 1 yd.

3 Can the mixed number $3\frac{12}{36}$ also be expressed as $3\frac{1}{3}$? How do you know? The tablecloth was $3\frac{1}{3}$ yd. long.

E Philip cut 25 ft. of screening into 4 equal pieces. Each piece was how many feet long?

4 $25 \div n = 4$ Divide 25 by 4.

What is the remainder? The answer is $6\frac{1}{4}$ ft. How do you get the $\frac{1}{4}$?

F Don earned $4.25 one week. He was paid 50¢ an hour. How many hours did he work that week?

What is the remainder? You may write $\frac{25}{50}$ or $\frac{1}{2}$ as part of the answer. Does the $\frac{1}{2}$ mean hours or money? 4

G A milkman sold 46 qt. of milk one morning. This amount was equal to how many gallons?

Why do you divide by 4? What does the remainder represent? 2 qt. are $\frac{1}{4}$ gal. or $\frac{2}{8}$ gal. 46 qt. $= 11\frac{2}{8}$ gal.

H Mrs. Long bought 4 chickens that weighed a total of 11 lb. What was the average weight of the chickens?

The average weight was $2\frac{3}{4}$ lb.

If you have a remainder in any exercise below, use a fraction numeral in lowest terms in the answer. 5

| | | | |
|---|---|---|---|
| A | $34 \div d = 6$ | J | $145 \div 20 = n$ |
| B | $116 \div 8 = n$ | K | $163 \div n = 10$ |
| C | $76 \div 16 = c$ | L | $396 \div 24 = a$ |
| D | $85 \div f = 6$ | M | $750 \div m = 50$ |
| E | $92 \div w = 7$ | N | $579 \div n = 18$ |
| F | $118 \div 14 = n$ | O | $505 \div 25 = b$ |
| G | $145 \div a = 29$ | P | $611 \div n = 47$ |
| H | $32 \div 5 = g$ | Q | $510 \div 45 = n$ |
| I | $196 \div v = 32$ | R | $252 \div c = 24$ |

Now you should be able to use a remainder to make a fraction numeral in an answer when you divide.





# 62

1 Have the pupils read the text and study the computation. Ask such questions as "Why do we divide by 36? Will the answer mean yards and a fraction of another yard, or inches and a fraction of another inch? Is an inch a fraction of a yard? What fraction of a yard is one inch?"

2 Be sure the pupils understand what these numerals represent.

3 The pupils should see that when the remainder is used to make a fraction numeral in the quotient, the fraction numeral expresses part of a *yard*, not part of an inch.

4 Discuss Problems E to H in detail. In Problem F, make clear that the number of *groups* of 50 cents contained in 425 cents corresponds to the number of hours of work.

5 Use these exercises as written work. Provide answers.

# 63-65 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to make judgments in the use of remainders in division problems.

**Vocabulary**

New words page 63 leap*; page 64 grain*, laws*, oats*, probably*; page 65 items, permissible

**Comments**

In the last lesson the pupils reviewed how to use the remainder in division to make a fraction numeral in the answer. Now they need practice in using judgment as to when they should use the remainder to make a fraction numeral and when they should not.

This lesson should help the pupils see that when the division situation requires the finding of the number of equal groups, they can sometimes use the remainder to make a fraction numeral in the answer. Such situations are solved by equations in which the number of equal groups is to be found (for example, $21 \div 3 = n$). Then the fraction numeral will represent a part of one of the equal groups into which the objects are being divided. The pupils should realize, however, that there may be instances when it is not sensible to use the remainder to make a fraction numeral. When the problem situation is one of sharing and involves objects not measured, but counted, the pupils will have to be very cautious about using a fraction numeral in the answer. Some kinds of objects cannot be divided into fractional parts; others can. For example, candy bars can be divided into fractional parts and shared equally; balloons cannot. Mention and discuss objects found in the classroom and the home which can or cannot be divided into fraction parts.

There are two other situations in which good judgment is needed. One situation concerns what should be done if the remainder is not to be used to make a fraction numeral in the answer. Should the remainder be viewed as another whole unit (rounded off to the next higher number), or should it be dropped and disregarded? The other situation concerns the form of the fraction numeral. Should the exact fraction be expressed, or should it be rounded off? When the answer is rounded off, the pupil should be cautioned that the product of the divisor and the answer will not produce the dividend. It is not possible to check by multiplication when the answer has been rounded off.

On page 64, the hundredweight is introduced as a unit of measurement used in commerce and farming activities. The pupils should look for examples of its use in magazines and newspapers.

In this lesson the background is also laid for learning that the bushel is a measure of capacity and that the weight of the commodity measured by the bushel varies. Bushels of different things can weigh different amounts. In discussing the bushel, you might bring in the so-called level bushel, or struck bushel (compare this with the level teaspoonful).

Be sure to bring out in connection with exercises on page 65 that the fraction numeral can be used in averages in a way that it cannot sensibly be used in other situations. The other exercises (T to W) on page 65 help the pupils make generalizations about what they have learned in this lesson.

**Answers**

Thinking straight:

A (1) days
(2) part of a day
(3) 30 days; $30\frac{5}{12}$ is not a practical equivalent.
(4) 30 days; $30\frac{1}{2}$ would be acceptable, however, since it is not difficult to work with.

B (1) day
(2) part of a week
(3) 52 weeks; $52\frac{1}{7}$ is hard to work with, and 53 weeks is almost a full week too many.
(4) 52 weeks, for the reasons given in (3) above

C (1) $\frac{1}{3}$ of a cent
(2) 89¢
(3) 90¢; In merchandising, prices are always rounded up to the nearer cent. If canned peas are available at 2 cans for 49¢, 1 can will cost 25¢.

D (1) cents
(2) $\frac{3}{4}$; the fraction of a pound that 63¢ will buy
(3) $3\frac{3}{4}$ lb.; That is the exact amount of meat that Mrs. Stevens could buy for $3.15.

E No; yes; no

F $14 \div 4 = [3\frac{1}{2}]$; yes; yes; no; 3 lb. would not make

14 hamburgers of the size she wants. She does not need 4 lb. of meat.

G $14 \div 6 = [2\frac{1}{3}]$; no; no; no; yes

H 20 cwt.; 500 lb.; $2\frac{1}{4}$ cwt.

I Yes; The weight of different kinds of grain varies.

J $100 \div 32 = [3\frac{1}{8}]$

K 3 bu.

L No

M $3\frac{1}{8}$ bushels

N $100 \div 56 = [1\frac{11}{14}]$

O Round it off and say "2 bushels."

P Pay for $1\frac{11}{14}$ bushels.

Q $48 \div 5 = [9\frac{3}{5}]$; no; yes; no

R 9 bags; Keep the apples or sell them by the pound.

S Yes; yes, when stating an average; no

T No; You must always add 1 cent to the answer.

U Yes

V Yes; yes

W Yes; Round it to the next higher amount or to the next lower amount, or state the remainder in terms of the number of objects left over.

**Keeping skillful:**

(block 1)

| | | | | |
|---|---|---|---|---|
| A $\frac{4}{7}$ | F $\frac{3}{4}$ | J $\frac{1}{6}$ | N $\frac{3}{10}$ | R $\frac{1}{2}$ |
| B $\frac{2}{5}$ | G $\frac{7}{10}$ | K $\frac{1}{2}$ | O $\frac{1}{4}$ | S $\frac{5}{9}$ |
| C $\frac{1}{3}$ | H $\frac{3}{5}$ | L $\frac{3}{8}$ | P $\frac{4}{5}$ | T $\frac{1}{5}$ |
| D $\frac{4}{5}$ | I $\frac{2}{3}$ | M $\frac{3}{4}$ | Q $\frac{2}{3}$ | U $\frac{7}{10}$ |
| E $\frac{5}{8}$ | | | | |

(block 2)

| | | | |
|---|---|---|---|
| A 32 | E 30 | I 16 | M 8 |
| B 8 | F 3 | J 7 | N 18 |
| C 5 | G 1 | K 54 | O 7 |
| D 25 | H 3 | L 2 | P 40 |

## Thinking straight

1

A To find the average number of days per month in 1 year, use the equation $365 \div n = 12$.

1 Will the remainder represent days or months?

2 If you use the remainder to make a fraction, will the fraction numeral represent part of a day or part of a month?

3 Which do you think is the best answer, 30 days, $30\frac{5}{12}$ days, $30\frac{1}{2}$ days, or 31 days? Why?

4 Which answer above would you use if you were finding the average number of days per month in a leap year of 366 days? Why?

2

B To find the number of weeks in a year of 365 days, use the equation $365 \div 7 = y$.

1 Will the remainder represent weeks or days?

2 If you use the remainder to make a fraction, will the fraction numeral represent part of a day or part of a week?

3 Which do you think is the best answer, 52 weeks, $52\frac{1}{2}$ weeks, or 53 weeks? Why?

4 What answer would you use if you were finding the number of weeks in a leap year? Why?

3

C Mrs. Beach paid $8.04 for 9 yd. of material. To find the cost of the material per yard, use the equation $\$8.04 \div b = 9$.

1 What will the fraction numeral in your answer represent?

2 What is the cost of the material per yard to the nearer cent?

3 If Mrs. Beach had bought only 1 yd. of the material, how much would she have had to pay for it? Why?

4

D Mrs. Stevens paid $3.15 for meat that cost 84¢ per pound. Use the equation $\$3.15 \div \$.84 = s$ to find the number of pounds of meat that Mrs. Stevens bought.

1 What will the remainder represent?

2 What fraction numeral can you write in your answer? What does it represent?

3 Which do you think is the best answer, 3 lb., $3\frac{63}{84}$ lb., $3\frac{3}{4}$ lb., or 4 lb.? Why?

5

E Jim has 28 tadpoles that he is going to give to 5 of his friends. Can Jim give away all of the tadpoles by giving each of the 5 boys the same number? Will some of the boys get more than 5 tadpoles? Should you use a fraction numeral in the answer to a problem like this?





# 63

1 Bring out that in this division problem the size of the equal groups is to be found and that the remainder represents days. When the remainder is used to make a fraction numeral, it represents a part of a day. Explain why the remainder represents days but the fraction numeral represents part of a day.

2 Point out that in this type of division equation the number of groups is to be found. The remainder represents a day. When the remainder is used to make a fraction numeral, it represents part of a group of seven days or part of a week.

3 Have the pupils decide what type of division problem this is; then discuss the questions.

4 Bring out that the remainder can be used to make a fraction numeral which represents the part of a pound that can be bought for the amount of money left over.

5 Let pupils use objects to illustrate the problem.

**F** Mrs. Beach can make 4 hamburgers from 1 lb. of meat. How would you find how many pounds of meat she needs to make 14 hamburgers? Can Mrs. Beach buy a part of a pound of meat? Should you use the remainder to make a fraction numeral in the answer? Or should you give the answer as 3 lb.? As 4 lb.?

**G** If each package of hamburger buns contains 6 buns, how would you find the number of packages Mrs. Beach needs for 14 hamburgers? Can she buy part of a package? Should you use a fraction numeral in your answer? Or should you give the answer as 2 packages? As 3 packages?

**H** Grain is often measured by the hundredweight. 1 hundredweight equals 100 pounds. For the word hundredweight you can use the abbreviation cwt.
1 T. = ▦ cwt. 5 cwt. = ▦ lb.
225 lb. = ▦ cwt.

**I** Grain also is sold by the bushel. The bushel measure is the same size everywhere. But laws have been passed that tell how much a bushel of different kinds of grain must weigh. Do you think that a bushel of one kind of grain may weigh more than or less than a bushel of another kind of grain? Why?

**J** The law says that a bushel of oats must weigh 32 lb. How would you find the number of bushels of oats in 1 cwt.? 4

**K** If you were telling someone how many bushels of oats there are in 1 cwt., what would you probably say?

**L** Will 3 bushel baskets hold 1 cwt. of oats?

**M** If you were buying 1 cwt. of oats, how many bushels do you think you would have to pay for? 5

**N** One bushel of shelled corn weighs 56 lb. How would you find the number of bushels of corn in 1 cwt.? 6

**O** If you were telling someone how many bushels of corn there are in 1 cwt., what would you do with the remainder?

**P** What would you do with the remainder if you were paying by the bushel for 1 cwt. of corn?

**Q** One bushel of apples weighs 48 lb. How many 5-lb. bags of apples would you say you can get from 1 bushel of apples? Should you use a fraction numeral in the answer? Or should you give the answer as 9 bags? Or should you give the answer as 10 bags? 7

**R** If you were going to sell the 5-lb. bags of apples described in Problem Q, how many would there be to sell? What should you do with the remainder?



**S** The 32 pupils in the sixth grade have a total of 56 brothers and sisters. When they divided to find the average number of brothers and sisters per sixth grader, their answer was 1 and 24 remainder. Should the remainder be used to make a fraction? Is it permissible to say that the average number of brothers and sisters per sixth grader is $1\frac{3}{4}$? Does this mean that it is possible to have $\frac{3}{4}$ of a brother or a sister?

**T** Suppose you know how many cents several items of the same kind cost when bought together. You want to buy only 1 item. When you divide to find the price of 1 item, you may have a remainder. Should you use the remainder to make a fraction numeral in the answer? Can you forget the remainder, or must you always add 1 cent to the answer?

**U** When you find an average, is it permissible to have a fraction numeral in the answer, even though no one can have a fraction of the thing that the fraction numeral represents? 3

**V** In division problems in which you find weight or length or some other measure, can you usually use the remainder to make a fraction numeral in your answer? Is it sensible to talk about a fractional part of a pound or of an inch or of a bushel? 4

**W** Does the way you are to use the answer help you to decide whether or not to use the remainder to make a fraction numeral in the answer? What can you do with the remainder if you do not use it to make a fraction? 5

## Keeping skillful 6

Express each of these fraction numerals in lowest terms.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | $\frac{16}{28}$ | H | $\frac{18}{30}$ | O | $\frac{3}{12}$ | | |
| B | $\frac{4}{10}$ | I | $\frac{8}{12}$ | P | $\frac{16}{20}$ | | |
| C | $\frac{8}{24}$ | J | $\frac{4}{24}$ | Q | $\frac{10}{15}$ | | |
| D | $\frac{12}{15}$ | K | $\frac{5}{10}$ | R | $\frac{8}{16}$ | | |
| E | $\frac{20}{32}$ | L | $\frac{6}{16}$ | S | $\frac{20}{36}$ | | |
| F | $\frac{12}{16}$ | M | $\frac{21}{28}$ | T | $\frac{5}{25}$ | | |
| G | $\frac{14}{20}$ | N | $\frac{9}{30}$ | U | $\frac{12}{60}$ | | |

Find the missing term in each pair of ratios.

| | | | |
|---|---|---|---|
| A $\frac{7}{8}=\frac{28}{c}$ | E $\frac{6}{7}=\frac{n}{35}$ | I $\frac{3}{2}=\frac{24}{k}$ | M $\frac{25}{40}=\frac{5}{z}$ |
| B $\frac{16}{32}=\frac{t}{16}$ | F $\frac{65}{15}=\frac{13}{y}$ | J $\frac{8}{14}=\frac{4}{w}$ | N $\frac{3}{11}=\frac{n}{66}$ |
| C $\frac{22}{10}=\frac{11}{a}$ | G $\frac{14}{42}=\frac{a}{3}$ | K $\frac{9}{5}=\frac{m}{30}$ | O $\frac{28}{14}=\frac{14}{d}$ |
| D $\frac{5}{9}=\frac{b}{45}$ | H $\frac{6}{18}=\frac{x}{9}$ | L $\frac{18}{36}=\frac{g}{4}$ | P $\frac{3}{8}=\frac{15}{c}$ |



# 64

1. In discussing Problems F and G, point out that a part of a pound of meat can be bought, but packaged buns are sold by the package.
2. Have the pupils state the equations they use in finding the equivalents.
3. Emphasize that the bushel measure is a measure of capacity, not weight.
4. In discussing Exercise J, bring out that $100 \div 32 = n$ is the equation for this problem.
5. Point out for Exercise M that the fraction numeral represents a fraction of one of the groups (bushel), and that the buyer would have to pay for this part of a bushel also.
6. For the problem situation in Exercises N to P, bring out that in telling someone how many bushels of corn there are in 1 cwt., ($1\frac{11}{14}$), you could round off $\frac{11}{14}$ to 1 and say "2 bushels." If you were buying 1 cwt. of corn and paying for it by the bushel, you would want to pay for $1\frac{11}{14}$ bushels.
7. Discuss Exercises Q and R.

# 65

1. Ask the pupils if they think it is permissible to use a remainder to make a fraction numeral when finding an average. Here the average $1\frac{3}{4}$ shows that there are almost as many as 2 brothers and sisters per sixth grader.
2. Use these questions to help the pupils see that in such buying situations the remainder cannot be used to make a fraction numeral; they must always round up to the next cent.
3. Refer to Problem S in discussing this question.
4. Bring out that in measurement problems the remainder can usually be used to make a fraction numeral.
5. Use this exercise to review all the ways of using a remainder.
6. Assign these "Keeping skillful" exercises as written work, and discuss the solutions.

# 66-69 Looking back

Expanded Notes for this lesson are on pages 304-306.

**Objectives**

The pupil reviews finding common denominators. He also reviews the meaning of the expression *lowest common denominator*.

**Vocabulary**

New words page 67 chart

**Comments**

This lesson, which explains what a common denominator is and presents some techniques for finding a common denominator, is based on a thoroughly established background of understanding instead of on memorization of rules. The ideas reviewed here are needed when fractions are added and subtracted.

The chart on page 66 may be used by having pupils place markers on the numerals to show responses to questions. It should also be used in connection with the exercises on page 67. The chart enables the pupils to see which of two fractions is smaller or larger by using fraction numerals that have common denominators. Some fraction numerals are shown in color to make class discussion easier.

The chart shows several sets of fraction numerals. The name for each set is the fraction numeral given in lowest terms. A set is a group of elements that have some common characteristic. The characteristic that these sets of fraction numerals have in common is that they are equal. The simple concept of a set is used here and will be used throughout the pupils' book.

In talking about common denominators, you may use the word *least* instead of *lowest*, if you wish. *Least* has not been used in the text because its use would necessitate using the word *lesser*, which the authors wished to avoid. They felt, also, that the word *lowest*, which indicates position on a scale, was preferable to *smallest*, which denotes size.

Pupils are taught in this lesson that they can always find a common denominator by using the product of the denominators involved. To find the lowest common denominator, they must test multiples of the highest denominator by dividing the multiples by the other denominators.

**Answers**

Page 67:

A Some of the names for $\frac{2}{3}$

B No

C Yes; $\frac{10}{15}$

D Yes

E No. You cannot express $\frac{2}{3}$ as twentieths.

F Yes, $\frac{20}{30}$; Fifths and thirds can be expressed as thirtieths.

G Yes. You can tell by examining the sets of names for $\frac{3}{5}$ and $\frac{2}{3}$ that the sets can be extended indefinitely. In sets that never end, there probably are other common denominators.

H $\frac{9}{15}$; $\frac{10}{15}$; $\frac{3}{5}$

I 16 and 32

J Yes; $\frac{36}{48}$; yes

K 16; $\frac{12}{16}$; $\frac{10}{16}$; $\frac{13}{16}$

L $\frac{1}{2}$ is smallest; yes

M Yes; yes; $\frac{12}{24}$; yes

N 12; $\frac{8}{12}$; $\frac{6}{12}$; $\frac{9}{12}$; $\frac{1}{2}$

Pages 68-69:

A 80; yes; no; 40 is the lowest denomimator that appears in both sets.

B 24; yes; 24

C Yes; 6; yes; 6

D No; no; 8

E No. You cannot express sixths as sixteenths.

F Yes. You can express both fractions as twenty-fourths.

G Yes; 24

H Yes; yes

I No. You cannot express $\frac{1}{4}$ as sixths.

J 12 is the lowest number that can be divided by both 4 and 6.

K $\frac{5}{12}$

L $\frac{5}{12}$

M Yes

N Ann; yes

O 24. Generally, it is easier to work with smaller numbers.

P 20 is the first number after 10 that can be divided by 10. No

Q 30, 60, 90; $\frac{7}{10}$ is smaller.

(block 2)

A $\frac{9}{10}, \frac{8}{10}$
B $\frac{20}{30}, \frac{21}{30}$
C $\frac{5}{15}, \frac{3}{15}$
D $\frac{6}{8}, \frac{5}{8}$
E $\frac{14}{24}, \frac{9}{24}$
F $\frac{2}{6}, \frac{5}{6}$
G $\frac{20}{24}, \frac{21}{24}$
H $\frac{25}{30}, \frac{27}{30}$
I $\frac{11}{12}, \frac{8}{12}$
J $\frac{15}{18}, \frac{16}{18}$
K $\frac{16}{20}, \frac{11}{20}$
L $\frac{21}{24}, \frac{16}{24}$
M $\frac{7}{12}, \frac{9}{12}$
N $\frac{6}{16}, \frac{5}{16}$
O $\frac{5}{9}, \frac{3}{9}$
P $\frac{8}{16}, \frac{11}{16}$
Q $\frac{3}{16}, \frac{4}{16}$
R $\frac{14}{18}, \frac{9}{18}$
S $\frac{8}{12}, \frac{9}{12}$
T $\frac{5}{20}, \frac{6}{20}$

Keeping skillful:

(block 1)

A $\frac{15}{20}, \frac{4}{20}$
B $\frac{9}{24}, \frac{4}{24}$
C $\frac{8}{10}, \frac{5}{10}$
D $\frac{22}{24}, \frac{21}{24}$
E $\frac{9}{30}, \frac{25}{30}$
F $\frac{10}{12}, \frac{3}{12}$
G $\frac{16}{30}, \frac{5}{30}$
H $\frac{10}{18}, \frac{15}{18}$
I $\frac{15}{20}, \frac{14}{20}$
J $\frac{27}{36}, \frac{28}{36}$

Keeping skillful (cont.):

(block 2)

A 1764
B 15,362
C 10,102
D 25,231
E 3055
F 10,688
G 11,472
H 30,927
I 1604
J 15,727

(block 3)

A 38
B $158\frac{16}{55}$
C $54\frac{6}{48}$ or $54\frac{1}{8}$
D 95
E 87
F $79\frac{8}{64}$ or $79\frac{1}{8}$
G $177\frac{229}{236}$

## Looking back

1 When you work with two or more fractions, you often need to express them as fraction numerals with denominators that are alike. Now you will learn how to find the denominator to use.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{3}{5}$ | $\frac{6}{10}$ | $\frac{9}{15}$ | $\frac{12}{20}$ | $\frac{15}{25}$ | $\frac{18}{30}$ | $\frac{21}{35}$ | $\frac{24}{40}$ | $\frac{27}{45}$ | $\frac{30}{50}$ |
| $\frac{2}{3}$ | $\frac{4}{6}$ | $\frac{6}{9}$ | $\frac{8}{12}$ | $\frac{10}{15}$ | $\frac{12}{18}$ | $\frac{14}{21}$ | $\frac{16}{24}$ | $\frac{18}{27}$ | $\frac{20}{30}$ |
| $\frac{1}{2}$ | $\frac{2}{4}$ | $\frac{3}{6}$ | $\frac{4}{8}$ | $\frac{5}{10}$ | $\frac{6}{12}$ | $\frac{7}{14}$ | $\frac{8}{16}$ | $\frac{9}{18}$ | $\frac{10}{20}$ |
| $\frac{3}{4}$ | $\frac{6}{8}$ | $\frac{9}{12}$ | $\frac{12}{16}$ | $\frac{15}{20}$ | $\frac{18}{24}$ | $\frac{21}{28}$ | $\frac{24}{32}$ | $\frac{27}{36}$ | $\frac{30}{40}$ |
| $\frac{1}{6}$ | $\frac{2}{12}$ | $\frac{3}{18}$ | $\frac{4}{24}$ | $\frac{5}{30}$ | $\frac{6}{36}$ | $\frac{7}{42}$ | $\frac{8}{48}$ | $\frac{9}{54}$ | $\frac{10}{60}$ |
| $\frac{5}{8}$ | $\frac{10}{16}$ | $\frac{15}{24}$ | $\frac{20}{32}$ | $\frac{25}{40}$ | $\frac{30}{48}$ | $\frac{35}{56}$ | $\frac{40}{64}$ | $\frac{45}{72}$ | $\frac{50}{80}$ |
| $\frac{3}{10}$ | $\frac{6}{20}$ | $\frac{9}{30}$ | $\frac{12}{40}$ | $\frac{15}{50}$ | $\frac{18}{60}$ | $\frac{21}{70}$ | $\frac{24}{80}$ | $\frac{27}{90}$ | $\frac{30}{100}$ |
| $\frac{11}{12}$ | $\frac{22}{24}$ | $\frac{33}{36}$ | $\frac{44}{48}$ | $\frac{55}{60}$ | $\frac{66}{72}$ | $\frac{77}{84}$ | $\frac{88}{96}$ | $\frac{99}{108}$ | $\frac{110}{120}$ |
| $\frac{13}{16}$ | $\frac{26}{32}$ | $\frac{39}{48}$ | $\frac{52}{64}$ | $\frac{65}{80}$ | $\frac{78}{96}$ | $\frac{91}{112}$ | $\frac{104}{128}$ | $\frac{117}{144}$ | $\frac{130}{160}$ |

ding common denominators (reteaching)

# 66

1 Ask someone to read the "Looking back" statement. Remind the pupils that when they add and subtract fractions they will need to know how to express fraction numerals with denominators that are alike.

2 Have the pupils examine the chart after reading Exercise A on page 67. Be sure pupils see that each fraction numeral in Row A is equal to $\frac{3}{5}$. Choose a few fraction numerals from this row and have the pupils tell how each was obtained from $\frac{3}{5}$. Have them name other fraction numerals that could have been included in Row A. Explain that each fraction numeral has a definite relationship to all the others in the row and that all belong to a set.

3 Use the procedure in Note 2 to discuss Row B and several more rows of fraction numerals.

1

A Each row of fraction numerals on the chart on page 66 shows part of a set of names for the same fraction. Row A shows some of the names for $\frac{3}{5}$. What does Row B show?

2 B Now suppose you want to know which is smaller, $\frac{3}{8}$ or $\frac{5}{8}$. Look at the sets for $\frac{3}{8}$ and $\frac{5}{8}$. Find $\frac{30}{80}$ in the $\frac{3}{8}$ set. Does any fraction numeral in the $\frac{5}{8}$ set have 10 as the denominator?

C Now find $\frac{9}{15}$ in the $\frac{3}{5}$ set. Does any fraction numeral in the $\frac{2}{3}$ set have 15 as the denominator?

D Do $\frac{9}{15}$ and $\frac{10}{15}$ have denominators that are alike? 15 is a common denominator for $\frac{3}{5}$ and $\frac{2}{3}$.

3 E $\frac{12}{20}$ is also in the $\frac{3}{5}$ set. In the $\frac{2}{3}$ set, does any fraction numeral have 20 as the denominator? Why is 20 not a common denominator for $\frac{3}{5}$ and $\frac{2}{3}$?

F Now find $\frac{18}{90}$ in the $\frac{3}{5}$ set. In the $\frac{2}{3}$ set, is there a fraction numeral with 30 as the denominator? Why is 30 a common denominator for fifths and thirds?

G Do you think there may be other common denominators not on the chart that can be used to compare $\frac{3}{5}$ and $\frac{2}{3}$? Why do you think so?

H Of all the common denominators that can be used to compare $\frac{3}{5}$ and $\frac{2}{3}$, 15 is the lowest. 15 is the lowest common denominator for fifths and thirds. $\frac{3}{5}=\frac{\square}{15}$. $\frac{2}{3}=\frac{\square}{15}$. Which is smaller, $\frac{3}{5}$ or $\frac{2}{3}$?

5 I Now suppose you need to know which is largest, $\frac{3}{4}$, $\frac{5}{8}$, or $\frac{13}{16}$. On the chart find the sets of fraction numerals for these fractions. What common denominators are shown in blue for $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{13}{16}$?

J The chart shows that 48 is a common denominator for $\frac{5}{8}$ and $\frac{13}{16}$. Can $\frac{3}{4}$ be expressed by a fraction numeral with 48 for the denominator? $\frac{3}{4}=\frac{\square}{48}$. Is 48 a common denominator for fourths, eighths, and sixteenths?

K The lowest common denominator you can use to compare fourths, eighths, and sixteenths is ■. $\frac{3}{4}=\frac{\square}{16}$. $\frac{5}{8}=\frac{\square}{16}$. Which is largest, $\frac{3}{4}$, $\frac{5}{8}$, or $\frac{13}{16}$?

L Now find which is smallest, $\frac{2}{3}$, $\frac{1}{2}$, or $\frac{3}{4}$. Find the sets of fraction numerals for these three fractions on the chart. Is 12 a common denominator for $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$?

M Is 24 a common denominator for $\frac{2}{3}$ and $\frac{3}{4}$? Can $\frac{1}{2}$ be expressed by a fraction numeral with 24 for the denominator? $\frac{1}{2}=\frac{\square}{24}$. Is 24 a common denominator for $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$?

N The lowest common denominator you can use to compare thirds, halves, and fourths is ■. $\frac{2}{3}=\frac{\square}{12}$. $\frac{1}{2}=\frac{\square}{12}$. $\frac{3}{4}=\frac{\square}{12}$. Which is smallest, $\frac{2}{3}$, $\frac{1}{2}$, or $\frac{3}{4}$?

4



# 67

1 Have the class use the chart on page 66 in connection with all the exercises on this page.

2 Use Exercises B to D to develop the idea that to tell how much larger one fraction is than another, the fraction numerals must be replaced by equivalent fraction numerals that have the same, or a common, denominator.

3 Exercises E to G should help the pupils see that there can be many common denominators for two fraction numerals.

4 Discuss the meaning of *lowest common denominator*. Make sure the pupils understand that 15 is the lowest common denominator for all fraction numerals that have 5 and 3 as their denominators.

5 Use Exercises I to N to show how more than two fractions can be arranged in order of size.

1

A You can find common denominators without using sets of fraction numerals. Suppose you want to find a common denominator for $\frac{3}{8}$ and $\frac{3}{10}$. Multiply 10 by 8. $8\times 10=\square$. Does the chart on page 66 show that 80 is a common denominator for eighths and tenths? Is 80 the lowest common denominator? How does the chart show that 40 is the lowest common denominator for $\frac{3}{8}$ and $\frac{3}{10}$?

B Now think about finding a common denominator for $\frac{2}{3}$ and $\frac{3}{8}$. Multiply the 8 by the 3. $3\times 8=\square$. Is 24 a common denominator for thirds and eighths? Use the chart to find the lowest common denominator for $\frac{2}{3}$ and $\frac{3}{8}$. What is it?

C You can get a common denominator for $\frac{1}{2}$ and $\frac{1}{6}$ by multiplying 2 by 6. Does the chart show a common denominator lower than 12? What is it? Can $\frac{1}{2}$ be expressed as sixths? The lowest common denominator for halves and sixths is ■.

2 D To find the lowest common denominator for $\frac{5}{6}$ and $\frac{7}{8}$, first think: Can $\frac{5}{6}$ be expressed as eighths? Can $\frac{7}{8}$ be expressed as sixths? Which is the higher denominator, 6 or 8?

E Try 16 for a common denominator because 16 is the first number after 8 that can be divided by 8. You know that $\frac{7}{8}$ can be expressed as sixteenths. Can $\frac{5}{6}$ be expressed as sixteenths? Why is 16 not a common denominator for sixths and eighths?

F Now try 24, because the next number after 16 that can be divided by 8 is 24. You can express $\frac{7}{8}$ as twenty-fourths. Can $\frac{5}{6}$ also be expressed as twenty-fourths? 24 is a common denominator for $\frac{5}{6}$ and $\frac{7}{8}$. Why?

G Are there many other common denominators for sixths and eighths? Which is the lowest of all?

3 H Now find the lowest common denominator for $\frac{5}{6}$ and $\frac{1}{12}$. Can $\frac{5}{6}$ be expressed as twelfths? Is 12 the lowest common denominator for sixths and twelfths?

I When you find the lowest common denominator for $\frac{1}{4}$ and $\frac{1}{6}$, can you use the higher denominator, 6, as a common denominator? Why or why not?

J Try 12, the next number after 6 that can be divided by 6. How do you know that 12 is the lowest common denominator for fourths and sixths?

K When Ann compared $\frac{3}{8}$ and $\frac{5}{12}$, she used 24 for a common denominator. Here is Ann's work.
$\frac{3}{8}=\frac{9}{24}$ $\frac{5}{12}=\frac{10}{24}$
Which is larger, $\frac{3}{8}$ or $\frac{5}{12}$?

4



# 68

1 Make sure the pupils see that a common denominator can always be found by multiplying the numbers shown by the denominators. This method, however, will not necessarily produce the lowest common denominator.

2 Exercises D to J show a method of finding the lowest common denominator. Explain that the pupils should see if one of the denominators can be used as a common denominator. If not, the pupils should test multiples of the higher denominator by dividing the multiples by the other denominator.

3 Help pupils see that in Exercise H one of the denominators is the lowest common denominator.

4 Discuss how 24 was obtained as a common denominator.

When Joan compared $\frac{3}{8}$ and $\frac{5}{12}$, she used 48 as a common denominator. Here is Joan's work.

$\frac{3}{8}=\frac{18}{48}$ $\frac{5}{12}=\frac{20}{48}$

Which is larger, $\frac{3}{8}$ or $\frac{5}{12}$?

Can either 24 or 48 be used as a common denominator for $\frac{3}{8}$ and $\frac{5}{12}$?

Which girl used the lower common denominator? Is 24 the lowest common denominator for eighths and twelfths?

Which would you use, 24 or 48, to compare eighths and twelfths? Why?

Suppose you want to compare $\frac{7}{10}$ and $\frac{5}{8}$. Why might you first try 20 for a common denominator? Is 20 a common denominator for tenths and sixths?

Q From the numerals that follow, choose those that can be used as a common denominator to compare $\frac{7}{10}$ and $\frac{5}{8}$: 30, 40, 50, 60, 70, 80, 90, 100. Use one of these denominators to find which is smaller, $\frac{7}{10}$ or $\frac{5}{8}$.

Find the lowest common denominator for each pair of fraction numerals below. Express each pair with fraction numerals having this lowest common denominator. 3

A $\frac{9}{10}, \frac{4}{5}$ F $\frac{1}{3}, \frac{5}{6}$ K $\frac{4}{5}, \frac{11}{20}$ P $\frac{1}{2}, \frac{11}{16}$

B $\frac{2}{3}, \frac{7}{10}$ G $\frac{5}{6}, \frac{7}{8}$ L $\frac{7}{8}, \frac{2}{3}$ Q $\frac{3}{16}, \frac{1}{4}$

C $\frac{1}{3}, \frac{1}{5}$ H $\frac{5}{6}, \frac{9}{10}$ M $\frac{7}{12}, \frac{3}{4}$ R $\frac{7}{9}, \frac{1}{2}$

D $\frac{3}{4}, \frac{5}{8}$ I $\frac{11}{12}, \frac{2}{3}$ N $\frac{3}{8}, \frac{5}{16}$ S $\frac{5}{3}, \frac{3}{4}$

E $\frac{7}{12}, \frac{3}{8}$ J $\frac{5}{6}, \frac{8}{9}$ O $\frac{5}{8}, \frac{1}{3}$ T $\frac{1}{4}, \frac{3}{10}$

Now you should be able to find common denominators for two or more fraction numerals.

## Keeping skillful 4

Express each pair of fraction numerals with the lowest common denominator.

F $\frac{5}{6}, \frac{1}{4}$

G $\frac{5}{8}, \frac{1}{6}$

H $\frac{5}{9}, \frac{5}{6}$

I $\frac{3}{4}, \frac{7}{10}$

J $\frac{3}{4}, \frac{7}{9}$

A 8236 + a = 10000

B n − 7159 = 8203

C 6245 + 3857 = c

D 79511 − m = 54280

E n + 2056 = 5111

F 11384 − 696 = d

G w − 4938 = 6534

H 62065 − n = 31138

I z + 7508 = 9112

J 54312 + x = 70039

If there is a remainder, use it to make a fraction numeral in the answer.

A 3686 ÷ n = 97

B 8706 ÷ 55 = c

C 2598 ÷ 48 = y

D 67640 ÷ 712 = n

E 53940 ÷ e = 620

F 5064 ÷ n = 64

G 42001 ÷ 236 = a

69

# 69

1 Have the pupils notice that $\frac{3}{8}$ and $\frac{5}{12}$ are again being compared. Ask them to explain how $\frac{18}{48}$ and $\frac{20}{48}$ were obtained from $\frac{3}{8}$ and $\frac{5}{12}$. Point out that the question, "Which is larger, $\frac{3}{8}$ or $\frac{5}{12}$?" can be answered correctly by replacing both fraction numerals either by twenty-fourths or by forty-eighths.

2 In Exercise P be sure pupils see that 20 was tried first because it is the next larger multiple of 10. But 6 cannot be multiplied by a whole number to get 20, so 20 is not a common denominator for sixths and tenths. For Exercise Q, have the pupils tell how they choose the correct numerals for common denominators. Ask what the lowest common denominator is.

3 Assign Exercises A to T as written work.

4 Assign the three blocks of exercises in "Keeping skillful" as written work.

# 70-71 Looking back

Expanded Notes for this lesson are on pages 306-307.

**Objectives**

The pupil learns how division can be used to replace an improper fraction numeral with a mixed numeral or a numeral for a whole number.

**Vocabulary**

There are no new words.

**Comments**

In the lesson on pages 43-45 of the pupils' book, the pupils learned that an improper fraction numeral and a mixed numeral can both represent the same amount. In this lesson the pupils review how to replace a fraction numeral with a mixed numeral or a numeral for a whole number. This process was first taught in *Seeing Through Arithmetic 5* on pages 151-153.

No attempt has been made to give the pupils a rule to memorize for replacing improper fraction numerals with equivalent numerals. Instead, illustrations are provided to help pupils understand that the amount represented does not change; only the name of the number changes. Disks and other objects cut into fractional parts can be used to supplement the work in the book, especially with the slow pupils.

The replacement, or reduction, of improper fraction numerals is reviewed here because this skill is needed in addition of fractions. It will also be needed in multiplication and division of fractions. The reduction of a mixed numeral (such as $5\frac{11}{12}$) to its corresponding improper fraction numeral ($\frac{71}{12}$) is taught later (on pages 87-88 of the pupils' text).

**Answers**

Page 71:

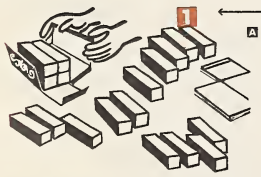
| | | | |
|---|---|---|---|
| A $3\frac{4}{5}$ | B $7\frac{1}{6}$ | C $4\frac{7}{8}$ | D 8 |
| E $5\frac{1}{2}$ | F $6\frac{1}{2}$ | G $20\frac{2}{3}$ | H 3 |
| I $3\frac{1}{3}$ | J $1\frac{2}{7}$ | K 2 | L $1\frac{1}{2}$ |
| M $4\frac{1}{2}$ | N 7 | O $1\frac{1}{2}$ | P $2\frac{11}{14}$ |
| Q 16 | R $3\frac{5}{6}$ | | |

Keeping skillful:

| (block 1) | (block 2) | (block 3) |
|---|---|---|
| A 161 | A 172 | A 2036 |
| B 177 | B 1930 | B 1825 |
| C 166 | C 677 | C 5796 |
| D 1568 | D 477 | D 30,480 |
| E 937 | E 1702 | E 4116 |
| F 1237 | F 3083 | F 18,278 |
| G 2720 | G 23,508 | G 24,816 |
| H 3035 | H 17,923 | H 11,718 |
| I 17,514 | I 32,172 | I 97,650 |
| J 14,455 | J 24,235 | J 79,764 |
| K 17,218 | K 83,708 | K 237,220 |
| L 11,038 | L 13,563 | L 245,976 |
| M 27,562 | M 7591 | M 450,072 |
| N 15,372 | N 66,851 | N 139,536 |
| O 10,220 | O 30,534 | O 522,783 |
| P 12,360 | P 94,167 | P 357,588 |
| Q 14,409 | Q 83,000 | Q 309,186 |
| | R 40,274 | R 513,911 |

## Looking back

Sometimes you will need to change improper fractions to other forms. Now you will learn how to do this.



1 A Each stick of butter in Picture A is $\frac{1}{4}$ of a whole pound. The 18 fourths are being arranged in whole pounds. it takes four $\frac{1}{4}$-lb. sticks to make 1 pound.

It takes 4 fourths to make 1 whole. $\frac{4}{4} = 1$

The 18 fourths have been arranged as whole pounds and part of another pound. There are 4 whole pounds and 2 fourths of another pound.

2 B You can use arithmetic to find how many wholes, or ones, there are in 18 fourths. Divide the 18 by 4.

3 $18 \div 4 = 4$ and 2 remainder. There are 4 ones, and 2 fourths are left over.

$\frac{18}{4} = 4$ ones and 2 fourths $\quad \frac{18}{4} = 4\frac{2}{4}$

$4\frac{2}{4}$ in lowest terms is $4\frac{1}{2}$.

4
- A How many sixths are there in 1?
- B How many thirds are there in 1?
- C How many tenths are there in 1?
- D How many fifteenths are there in 1?
- E Imagine that you have 21 thirds of equal-sized paper disks. To find how many whole disks can be made from the 21 thirds, first think: It takes ■ thirds to make 1 whole. $\frac{■}{■} = 1$
- F To find how many wholes, or ones, there are in 21 thirds, divide ■ by 3. $21 \div 3 = ■$. There are ■ ones. No thirds are left over. The 21 thirds equal ■ ones. $\frac{21}{3} = ■$





# 70

1 Ask the pupils to use the text to find what part of a whole pound each of the sticks of butter is. Let them count the fourths to verify that there are 18 fourths.

2 To find how many whole pounds of butter and leftover fourths the 18 fourths make, let the pupils separate blocks representing the 18 fourths of butter into groups of 4 fourths to represent whole pounds of butter. They can see that there are 4 whole pounds and 2 fourths of another pound.

3 Point out that dividing 18 by 4 is the same as finding how many groups of 4 fourths there are in 18 fourths. Be sure the pupils understand that $\frac{18}{4}$ is an improper fraction numeral and that $4\frac{2}{4}$ is a mixed numeral. Discuss how and why $\frac{2}{4}$ is replaced by $\frac{1}{2}$.

4 Discuss the remaining exercises. Ask for other examples of improper fraction numerals that can be replaced by numerals for whole numbers.

G To change $\frac{56}{5}$ to a whole number or a mixed number, first think: $\frac{5}{5}=1$. Now divide 56 by ■. $56 \div 5 =$ ■ ones and ■ remainder. $\frac{56}{5}=11\frac{\square}{5}$

H To change $\frac{150}{24}$ to a whole number or a mixed number, think: $\frac{24}{24}=1$. Divide 150 by ■. $150 \div 24 =$ ■ ones and ■ remainder. $\frac{150}{24}=6\frac{\square}{24}$. Write $6\frac{6}{24}$ in lowest terms. $\frac{150}{24}=6\frac{\square}{\square}$

Change each improper fraction below to a mixed number or a whole number. Be sure that each mixed number is expressed in lowest terms. 2

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| $\frac{19}{5}$ | $\frac{86}{12}$ | $\frac{39}{8}$ | $\frac{40}{5}$ | $\frac{11}{2}$ | $\frac{52}{8}$ |
| **G** | **H** | **I** | **J** | **K** | **L** |
| $\frac{62}{3}$ | $\frac{48}{16}$ | $\frac{30}{9}$ | $\frac{36}{28}$ | $\frac{128}{64}$ | $\frac{48}{32}$ |
| **M** | **N** | **O** | **P** | **Q** | **R** |
| $\frac{18}{4}$ | $\frac{56}{8}$ | $\frac{96}{64}$ | $\frac{39}{14}$ | $\frac{48}{3}$ | $\frac{115}{30}$ |

Now you can change improper fractions to mixed numbers or whole numbers when you need to.

## Keeping skillful 3

Find the sum.

| | | |
|---|---|---|
| A 27, 41, 80, 13 | A $728+n=900$ | A $4\times509=d$ |
| B 56, 9, 34, 78 | B $t-834=1096$ | B $73\times25=n$ |
| C 15, 63, 22, 26, 40 | C $1364-c=687$ | C $828\times7=r$ |
| D 689, 371, 508 | D $5210-4733=n$ | D $6\times5080=c$ |
| E 267, 155, 438, 77 | E $z+2306=4008$ | E $49\times84=f$ |
| F 326, 307, 411, 193 | F $3149+n=6232$ | F $26\times703=n$ |
| G 692, 833, 575, 620 | G $48797-25289=g$ | G $528\times47=p$ |
| H 721, 548, 900, 866 | H $a-7380=10543$ | H $62\times189=n$ |
| I 8314, 2751, 6449 | I $42130-n=9958$ | I $434\times225=w$ |
| J 5030, 974, 3189, 5262 | J $s+59106=83341$ | J $204\times391=n$ |
| K 9657, 1664, 5032, 865 | K $92483-8775=m$ | K $58\times4090=n$ |
| L 2851, 1563, 2490, 4134 | L $r-6543=7020$ | L $37\times6648=k$ |
| M 4266, 9500, 5272, 8524 | M $48391+n=55982$ | M $987\times456=v$ |
| N 5607, 4710, 1836, 3219 | N $83755-e=16904$ | N $2736\times51=c$ |
| O 6538, 465, 212, 3005 | O $q+39486=70020$ | O $6009\times87=n$ |
| P 1400, 9624, 1308, 28 | P $96524-n=2357$ | P $594\times602=q$ |
| Q 873, 7115, 91, 6330 | Q $h-10001=72999$ | Q $89\times3474=t$ |
| | R $y+35837=76111$ | R $787\times653=n$ |

71

# 71

1 Discuss Exercises G and H.
2 Assign Exercises A to R as written work.
3 Assign these sets of "Keeping skillful" exercises as time allows. Discuss the exercises that cause difficulty.

# 72 Checking up

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil tests his skill in replacing fraction numerals and ratios with equivalent fraction numerals and ratios, in expressing improper fraction numerals as mixed numerals or as numerals for whole numbers, and in finding the lowest common denominator for pairs of fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

The five tests in this lesson can be used to determine the pupils' understanding of and skill in the reduction of fraction numerals and ratios and in finding common denominators. Have the pupils work independently. Then discuss the exercises with the whole class.

The references in the Reteaching Chart may be used for any reteaching that is necessary.

**Reteaching chart**

| Test | Pages (Book 6) | Test | Pages (Book 6) |
|---|---|---|---|
| 1 | 54-55 | 4 | 70-71 |
| 2 | 52-53 | 5 | 66-69 |
| 3 | 56 and 58 | | |

**Answers**

No answers are given for Tests 1 and 2 because an unlimited variety of answers is possible.

Test 3

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | 15 | C | 7 | E | 8 | G | 1 | I | 3 | K | 35 |
| B | 5 | D | 6 | F | 9 | H | 24 | J | 7 | L | 51 |

Test 4

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| A | 5 | C | $4\frac{1}{7}$ | E | 7 | G | $2\frac{1}{12}$ | I | 2 | K | $12\frac{1}{2}$ |
| B | $9\frac{1}{3}$ | D | $2\frac{2}{15}$ | F | $2\frac{3}{8}$ | H | $4\frac{4}{9}$ | J | $3\frac{7}{8}$ | L | 4 |

Test 5

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | $\frac{8}{16}, \frac{7}{16}$ | E | $\frac{27}{30}, \frac{25}{30}$ | I | $\frac{9}{24}, \frac{4}{24}$ | M | $\frac{9}{12}, \frac{10}{12}$ |
| B | $\frac{7}{10}, \frac{8}{10}$ | F | $\frac{7}{12}, \frac{6}{12}$ | J | $\frac{5}{12}, \frac{9}{12}$ | N | $\frac{16}{30}, \frac{21}{30}$ |
| C | $\frac{12}{20}, \frac{5}{20}$ | G | $\frac{27}{30}, \frac{20}{30}$ | K | $\frac{28}{36}, \frac{15}{36}$ | O | $\frac{16}{24}, \frac{15}{24}$ |
| D | $\frac{15}{20}, \frac{6}{20}$ | H | $\frac{10}{15}, \frac{9}{15}$ | L | $\frac{25}{45}, \frac{36}{45}$ | | |

## Checking up 1

**Test 1**

Write two fraction numerals equal to each of the following. Write one with lower terms and one with higher terms.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | $\frac{8}{2}$ | G | $\frac{6}{8}$ | M | $\frac{10}{12}$ | S | $\frac{9}{36}$ |
| B | $\frac{15}{20}$ | H | $\frac{8}{10}$ | N | $\frac{14}{16}$ | T | $\frac{4}{20}$ |
| C | $\frac{12}{16}$ | I | $\frac{3}{12}$ | O | $\frac{5}{25}$ | U | $\frac{3}{9}$ |
| D | $\frac{25}{30}$ | J | $\frac{5}{15}$ | P | $\frac{6}{18}$ | V | $\frac{10}{15}$ |
| E | $\frac{6}{8}$ | K | $\frac{4}{32}$ | Q | $\frac{5}{10}$ | W | $\frac{4}{8}$ |
| F | $\frac{4}{6}$ | L | $\frac{9}{24}$ | R | $\frac{16}{24}$ | X | $\frac{3}{18}$ |

**Test 2**

Write two ratios equal to each ratio below. Write one with lower terms and one with higher terms.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | $\frac{12}{15}$ | G | $\frac{14}{24}$ | M | $\frac{14}{18}$ | S | $\frac{3}{15}$ |
| B | $\frac{4}{12}$ | H | $\frac{28}{32}$ | N | $\frac{9}{3}$ | T | $\frac{20}{25}$ |
| C | $\frac{16}{10}$ | I | $\frac{15}{30}$ | O | $\frac{30}{5}$ | U | $\frac{6}{30}$ |
| D | $\frac{18}{24}$ | J | $\frac{36}{24}$ | P | $\frac{46}{14}$ | V | $\frac{28}{21}$ |
| E | $\frac{21}{9}$ | K | $\frac{14}{35}$ | Q | $\frac{7}{21}$ | W | $\frac{11}{22}$ |
| F | $\frac{14}{7}$ | L | $\frac{18}{10}$ | R | $\frac{18}{12}$ | X | $\frac{35}{5}$ |

**Test 3**

In each exercise one ratio has a letter. Find the numeral that will replace it.

| | | | | | |
|---|---|---|---|---|---|
| A | $\frac{3}{5}=\frac{z}{25}$ | E | $\frac{12}{48}=\frac{2}{m}$ | I | $\frac{18}{27}=\frac{2}{e}$ |
| B | $\frac{10}{25}=\frac{2}{n}$ | F | $\frac{36}{4}=\frac{d}{1}$ | J | $\frac{28}{14}=\frac{14}{n}$ |
| C | $\frac{14}{2}=\frac{b}{1}$ | G | $\frac{10}{30}=\frac{t}{3}$ | K | $\frac{5}{1}=\frac{v}{7}$ |
| D | $\frac{1}{6}=\frac{r}{36}$ | H | $\frac{7}{8}=\frac{21}{f}$ | L | $\frac{17}{12}=\frac{s}{36}$ |

**Test 4**

Change each improper fraction to a mixed number or to a whole number. Write the fraction numeral in each mixed number in lowest terms.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | $\frac{40}{8}$ | D | $\frac{64}{30}$ | G | $\frac{25}{12}$ | J | $\frac{93}{24}$ |
| B | $\frac{56}{6}$ | E | $\frac{21}{3}$ | H | $\frac{40}{9}$ | K | $\frac{150}{12}$ |
| C | $\frac{29}{7}$ | F | $\frac{19}{8}$ | I | $\frac{64}{32}$ | L | $\frac{80}{20}$ |

**Test 5**

Write each pair of fraction numerals with the lowest common denominator

| | | | | | |
|---|---|---|---|---|---|
| A | $\frac{1}{2}, \frac{7}{16}$ | F | $\frac{7}{12}, \frac{1}{2}$ | K | $\frac{7}{9}, \frac{5}{12}$ |
| B | $\frac{7}{10}, \frac{4}{5}$ | G | $\frac{9}{10}, \frac{2}{5}$ | L | $\frac{5}{9}, \frac{4}{5}$ |
| C | $\frac{3}{5}, \frac{1}{4}$ | H | $\frac{2}{3}, \frac{3}{5}$ | M | $\frac{3}{4}, \frac{5}{8}$ |
| D | $\frac{3}{4}, \frac{3}{10}$ | I | $\frac{3}{8}, \frac{1}{6}$ | N | $\frac{8}{15}, \frac{7}{10}$ |
| E | $\frac{9}{10}, \frac{5}{6}$ | J | $\frac{5}{12}, \frac{3}{4}$ | O | $\frac{2}{3}, \frac{5}{8}$ |

72 End-of-block tests on reduction of fraction numerals and ratios; on finding the missing term; on finding the lowest common denominator

2

# 72

1 Discuss the instructions for each test with the pupils. Before they start to work, be sure that they understand what they are to do. Remind them to label each response with its identifying letter.

2 Discuss the work when the pupils have finished. Any reteaching needed by the whole class should be done at this time. Individual difficulties can be discussed at another time.

# 73 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil uses equations to solve various types of verbal problems.

**Vocabulary**

New words page 73 enamel, sweater*

**Comments**

For each of these problems the pupils are to write an equation that describes the problem situation, using as many letters as they need to hold places for the numbers to be found. Remind the pupils that for some problems they will have to use information or answers from preceding problems. The pupils will often meet this kind of problem outside the arithmetic class. You may need to review pages 49-51 of the pupils' book.

**Answers**

A $2 \times 675 = [1350]$ or $675/1 = [1350]/2$

B $1350 \div 450 = [3]$ or $450/1 = 1350/[3]$

C $(\$12.87) + \$1.95 = [\$14.82]$

D $\$15.00 - \$12.87 = [\$2.13]$

E $\$5.39 - \$2.98 = [\$2.41]$

F $(\$8.37) + \$14.82 = [\$23.19]$

G $(180) \div 60 = [3]$

H $\$9.04 + (\$1.50) = [\$10.54]$

I $180 \div [30] = 6$

J $\$10.54 - [\$7.98] = \$2.56$

K $(\$.56) + (\$.54) = [\$1.10]$

L $\$5 - \$1.10 = [\$3.90]$

M $(24) \div 3 = [8]$

N $16 + [8] = 24$

O $\$4.20 \div [\$.15] = 28$

## Using arithmetic 1

A When Mr. Olds planned to paint the kitchen, he found that there were 575 sq. ft. of surface to paint. How many square feet would he paint in all if he gave the kitchen 2 coats of paint?

B Mr. Olds knew that 1 gal. of paint would cover 450 sq. ft. How many gallons of paint would he need for the 2 coats?

C The paint that Mr. Olds bought cost $4.29 per gallon. How much in all did he have to pay for this paint and for 1 qt. of enamel that cost $1.95?

D If Mr. Olds had bought the paint in quart cans, it would have cost $15. How much less did he pay for the paint by buying it in gallons?

E Mr. Olds bought a large paintbrush for $5.39 and a small brush for $2.98. He spent how much more for the large brush than for the small one?

F Mr. Olds spent how much in all for the paintbrushes, paint, and enamel?

G June Olds helped her mother with the housework for 50 min. on Monday, 35 min. on Tuesday, 40 min. on Thursday, and 55 min. on Saturday. She helped with the housework for how many hours that week?

H June's mother paid her 50¢ per hour for her work. June added her earnings to her savings of $9.04. How much money did she have then?

I June worked an average of how many minutes per day for the 6 days from Monday through Saturday?

J June used part of her savings to buy a sweater. After she had bought the sweater, she still had $2.56 left. How much had the sweater cost?

K Mrs. Olds sent Philip to buy some bananas and apples. He bought 2 lb. of bananas at 28¢ per pound and 3 lb. of apples at 18¢ per pound. How much did Philip pay for all the fruit?

L How much change should Philip have received from a 5-dollar bill?

M When Mrs. Olds put shelf paper on the cupboard shelves, she needed enough paper for 6 shelves that were each 4 ft. long. How many yards of shelf paper did she need?

N Mrs. Olds used 16 ft. of shelf paper from a leftover roll. How much more shelf paper did she need from a second roll to cover all 6 shelves?

O Mr. Olds paid Philip $4.20 for washing 28 windows. How much did he pay Philip per window?

2



## 73

1 Ask the pupils to write an equation for each problem, using any letter they wish to hold a place for the answer. Tell the pupils that if more than one thing is unknown, they should use letters to hold places for the numerals they must find before getting the final answer. Then have them do the necessary computation and rewrite the equation with the missing numerals inserted.

2 When the pupils have worked the problems, supply answers and give individual help where it is needed. The pupils may work in groups and help one another with problems that are especially difficult, then verify their own work when the answers are supplied.

# 74-79 Learning how; Keeping skillful

Expanded Notes for these lessons are on pages 308-311.

**Objectives**

The pupil reviews the method of adding proper fractions.

**Vocabulary**

New words page 75 stuffed*; page 79 caramels, advertising*

**Comments**

In the *Seeing Through Arithmetic* program, the pupils are given careful preparation for addition of fractions. The procedures taught in Grade 5 have been reviewed in this book on the following pages:

| | |
|---|---|
| Meaning of fractions and mixed numbers | 43-45 |
| Reducing fraction numerals to lowest terms | 54-55 |
| Using divisor and remainder to make a fraction numeral in the quotient | 59-62 |
| Finding common denominators | 66-69 |
| Reducing improper fraction numerals | 70-71 |

By now, most pupils have had considerable experience in replacing a fraction numeral by an equal fraction numeral with a specified denominator. They should be able to replace an improper fraction numeral with a mixed numeral or a numeral for a whole number. Pupils who cannot do this or who still have difficulty in finding a common denominator for two or more fraction numerals will need special help.

Notice that the first example requires the pupil, right at the start, to choose a common denominator and find equivalent fraction numerals before he proceeds to add. He sees the need for a common denominator and the importance of the denominator much more clearly than the child who begins by adding eighths "as if they were apples" and who is, therefore, quite likely to ignore the denominator almost completely.

Notice, too, that the second example in the SEE step on page 75 has three addends. Once a pupil has learned to add two fractions, he should have no special difficulty in adding three fractions.

The addition of *proper* fractions is first shown in the horizontal form because this form is just as convenient for the addition of *proper* fractions as is the vertical form. The vertical form is shown as an alternative on page 78. This is done for two reasons. The first reason is that some teachers may prefer to have the pupils put their work on their paper in vertical form, and the second is that later on, when numbers represented by mixed numerals are added, the vertical form will be more convenient. The practice exercises in the book will continue to be shown in equation form. The pupil should know how to use each form, as well as have experience in writing his work in whatever form is convenient.

The four-step teaching method is used in this lesson. In the TRY step on page 78, you will find examples with a variety of denominators and answer forms. You will want to make sure the pupils understand thoroughly all the situations used in the TRY step, so that they will be ready to work the exercises in the DO step. These exercises may be used in several ways. You may or may not want all the pupils to do both blocks of exercises in the DO step. The second block may be used by those who need additional practice, or it may be used later. You may have the pupils use the horizontal form with the first block of exercises and the vertical form with the second block. Or you may let the pupil decide for himself which form he will use for each exercise. At the end of this lesson the class should be able to use either form with ease.

**Answers**

Page 79:

(block 1)

A $\frac{7}{8}$ F $1\frac{1}{10}$ K $\frac{4}{5}$ P $1\frac{1}{14}$

B $1\frac{5}{8}$ G $\frac{13}{16}$ L 2

C $\frac{5}{6}$ H $\frac{1}{2}$ M $1\frac{3}{5}$

D 1 I $2\frac{11}{24}$ N $\frac{8}{9}$

E $1\frac{5}{24}$ J $1\frac{4}{15}$ O $\frac{19}{24}$

(block 2)

A $\frac{13}{16}$ F $1\frac{1}{3}$ K $1\frac{9}{20}$

B $1\frac{17}{21}$ G $1\frac{1}{3}$ L $\frac{7}{24}$

C $\frac{13}{20}$ H $1\frac{5}{18}$ M $1\frac{43}{60}$

D $\frac{7}{10}$ I $1\frac{3}{8}$ N $\frac{17}{18}$

E $1\frac{1}{3}$ J $\frac{31}{48}$

**Keeping skillful:**

(block 1)

A 7868 H 135,167

B 4171 I 3152

C 20,448 J 20,369

D 2860 K 2647

E 3598 L 103,843

F 3538 M 20,861

G 4374 N 25,605

(block 2)

A 11,610 H 13,800

B 653,772 I 65,046

C 5928 J 14,527

D 10,507 K 266,202

E 7744 L 412,620

F 57,102 M 17,301

G 82,875 N 286,572

(block 3)

A $30\frac{17}{31}$ G $728\frac{15}{38}$

B 7 H 222

C 976 I $201\frac{2}{17}$

D $3947\frac{1}{5}$ J 98

E $30\frac{3}{11}$ K 144

F 125

## Learning how In this lesson you will learn how to add fractions.

**see**

A

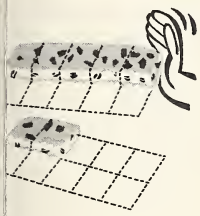
Susan gave $\frac{1}{5}$ of a candy bar to Ann and $\frac{1}{2}$ of the same candy bar to Tom. How much of the bar did she give to Ann and Tom?

1

$\frac{1}{5}+\frac{1}{2}=$ ▯ — Holds a place for the total amount given away

Imagine the $\frac{1}{2}$ bar put with the $\frac{1}{5}$ bar. You need a name for the part given away.

B



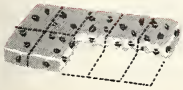
2 Picture B shows how you can find a name for the new fraction that tells how much was given away. Think of the $\frac{1}{5}$ of the bar as $\frac{2}{10}$ of the bar. Then think of the $\frac{1}{2}$ of the bar as $\frac{5}{10}$ of the bar.

You can also find the new fraction by adding the fractions $\frac{1}{5}$ and $\frac{1}{2}$. When you add, you use a common denominator. You can change $\frac{1}{5}$ to tenths. You can also change $\frac{1}{2}$ to tenths.

$\frac{1}{5}=\frac{2}{10}$ $\frac{1}{2}=\frac{5}{10}$ $\frac{1}{5}+\frac{1}{2}=\frac{2}{10}+\frac{5}{10}$



C



You can see from Picture C that $\frac{7}{10}$ of the bar was given away.

You can also get the answer $\frac{7}{10}$ by adding the fractions $\frac{2}{10}$ and $\frac{5}{10}$.

1

Add the numerators. The sum is 7.

$\frac{2}{10}+\frac{5}{10}=\frac{7}{10}$ — Write 7 as the numerator. You are adding tenths. So write 10 as the denominator.

Susan gave $\frac{7}{10}$ of the bar to Ann and Tom.
$\frac{1}{5}+\frac{1}{2}=\frac{7}{10}$

**see**

A

Alice bought cloth to make stuffed toys. She bought $\frac{2}{3}$ yd. of one kind of cloth, $\frac{3}{4}$ yd. of another kind, and $\frac{1}{2}$ yd. of a third kind. How many yards of cloth in all did she buy?

2

$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}=$ ▯ — Holds a place for the total number of yards bought

Imagine the $\frac{2}{3}$ yd., the $\frac{3}{4}$ yd., and the $\frac{1}{2}$ yd. as being put together. You need a name for the new fraction that tells how many yards there are in all.

Now turn the page.



# 74

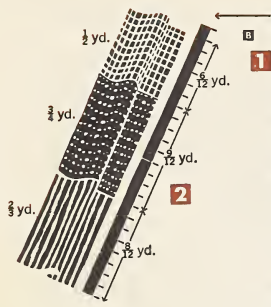
1 Have the pupils read the problem and the equation and relate the fraction numerals to the picture. Let a pupil read aloud the two lines of text below the equation. Be sure to discuss the equation. Discuss why a new name is needed for the part of a candy bar Susan gave away. Be sure the pupils understand that when they find the number that is the sum of the fractions, they are really finding a name for the total.

2 Have the pupils read the text and relate it to Picture B. [$\frac{5}{10}$ of the whole bar is being put with $\frac{2}{10}$ of the bar.] Ask someone to explain why the $\frac{1}{5}$ bar should be thought of as $\frac{2}{10}$ of the whole bar, and why the $\frac{1}{2}$ bar should be thought of as $\frac{5}{10}$ of the whole bar. Discuss the reason for choosing 10 as a common denominator. Ask: "Why not use 5?" "Could we use 15?" "Could we use 20?"

# 75

1 Let pupils relate Picture C to the equation. Compare Picture C with Pictures A and B. You might ask such questions as "What has been put together in Picture C? Is $\frac{2}{10}$ equal to $\frac{1}{5}$? Is $\frac{5}{10}$ equal to $\frac{1}{2}$? How much of the bar did Susan give away in all? Does adding the numerators in the equation give the same answer as counting the parts of the bar in Picture C?" Also ask: "Could there be any names other than $\frac{7}{10}$ for the part of a candy bar Susan gave away?"

2 Call on someone to read the problem and the equation and identify the amounts of cloth in Picture A. Let someone else read the text below the equation. You might then ask: "How can we find a name for the fraction that will tell how many yards in all Alice bought? What must be done with the fraction numerals in the equation before we can add?"

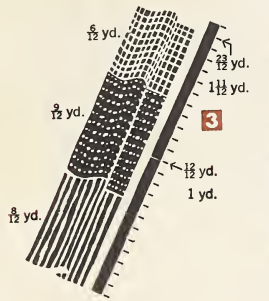



Picture B shows how to find the name for this new fraction. Think of the $\frac{2}{3}$ yd. as $\frac{8}{12}$ yd. Think of the $\frac{3}{4}$ yd. as $\frac{9}{12}$ yd. Think of the $\frac{1}{2}$ yd. as $\frac{6}{12}$ yd. You know that the new fraction will be a certain number of twelfths.

You can also find the new fraction by adding the fractions $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$. You can use 12 as a common denominator and change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to twelfths.

$\frac{2}{3}=\frac{8}{12}$ $\frac{3}{4}=\frac{9}{12}$ $\frac{1}{2}=\frac{6}{12}$

$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}=\frac{8}{12}+\frac{9}{12}+\frac{6}{12}$




Picture C shows that Alice bought a total of $\frac{23}{12}$ yd.

You can also get the answer by adding.

Add the numerators. The sum is 23.

$\frac{8}{12}+\frac{9}{12}+\frac{6}{12}=\frac{23}{12}$

Write 23 as the numerator. You are adding twelfths. So write 12 as the denominator.

Picture C shows that $\frac{23}{12}$ yd. is more than one whole yard. You can see that Alice bought 1 whole yard and $\frac{11}{12}$ of another yard.

$\frac{23}{12}=1\frac{11}{12}$

Alice bought $1\frac{11}{12}$ yd. of cloth in all.

$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}=1\frac{11}{12}$



# 76

1. Have a pupil read the text opposite Picture B. Discuss the reason for choosing 12 as a common denominator.
2. Let a pupil count the number of twelfths included by each pair of arrows in Picture B and then relate each of these groups to the corresponding quantity of cloth ($\frac{2}{3}$ yd., $\frac{3}{4}$ yd., $\frac{1}{2}$ yd.). Be sure the pupils understand the scale beside the arrows.
3. Let someone count in Picture C the number of twelfths that equal the entire length of the pieces of cloth that have been put together (23 twelfths, or 1 and 11 twelfths). Help the pupils observe that the length of the cloth is $\frac{1}{12}$ yd. shorter than two yardsticks.
4. Let a pupil explain how the answer is found by computation. Be sure everyone understands why the numerators are added and not the denominators.

think

Mrs. Cook gave $\frac{1}{3}$ of a pie to Mrs. Page. She gave Jim $\frac{1}{6}$ of the same pie for his lunch. How much of the pie did she give away?

$\frac{1}{3}+\frac{1}{6}=\square$ Holds a place for the amount of pie given away

If you put $\frac{1}{6}$ of the pie with $\frac{1}{3}$ of the pie, would you know the name of the new fraction of the pie?

Picture B shows how to find the name for this new fraction. Can you think of both $\frac{1}{3}$ and $\frac{1}{6}$ as a number of sixths? Think of one third as ■ sixths.

3

When you compute to find the new fraction, you must use a common denominator. You can change $\frac{1}{3}$ to $\frac{\square}{6}$.

$\frac{1}{3}=\frac{\square}{6}$ $\frac{1}{3}+\frac{1}{6}=\frac{\square}{6}+\frac{\square}{6}$

The new fraction is ■ sixths of the pie.

What fractions can you add to get the answer?

What do you do with the numerators?

$\frac{2}{6}+\frac{1}{6}=\frac{3}{6}$

Why is 3 written in the numerator? Why is 6 written in the denominator?

Is $\frac{3}{6}$ in lowest terms?

$\frac{3}{6}=\frac{\square}{2}$

Mrs. Cook gave $\frac{\square}{\square}$ of the pie away.

$\frac{1}{3}+\frac{1}{6}=\square$





# 77

1. Ask the pupils if the fractions in the problem correspond to the pieces of pie in Picture A. Discuss the idea of getting a new name for the part of a pie given away.
2. Let someone explain why the dotted black lines are used in Picture B. Ask: "Why do we divide a whole pie into sixths?" "What is being done with the one piece of pie?"
3. Have several pupils read the text and supply the answers.
4. Picture C and previous work in expressing fraction numerals in lowest terms should make it easy for everyone to see that $\frac{3}{6}$ is equal to $\frac{1}{2}$.

## try

A $\frac{1}{10}+\frac{7}{10}=x$

Do you need to change the fraction numerals?

$\frac{1}{10}+\frac{7}{10}=\frac{8}{10}$ or $\frac{4}{5}$

How do you get the 8 in $\frac{8}{10}$?

How do you get $\frac{4}{5}$?

You may write your work this way if you want to.

$\frac{1}{10}$
$\frac{7}{10}$
$\frac{8}{10}$ or $\frac{4}{5}$

---

B $\frac{2}{3}+\frac{7}{8}+\frac{5}{8}=f$

What common denominator do you use?

$\frac{16}{24}+\frac{21}{24}+\frac{20}{24}=f$

$\frac{16}{24}+\frac{21}{24}+\frac{20}{24}=\frac{57}{24}$ or $2\frac{9}{24}$ or $2\frac{3}{8}$

How do you get the 57 in $\frac{57}{24}$?

How do you know that $2\frac{3}{8}$ is in lowest terms?

You can also write your work for Example B in this way.

$\frac{2}{3}=\frac{16}{24}$
$\frac{7}{8}=\frac{21}{24}$
$\frac{5}{8}=\frac{20}{24}$
$\frac{57}{24}$ or $2\frac{9}{24}$ or $2\frac{3}{8}$

---

C $\frac{2}{5}+\frac{2}{3}+\frac{4}{5}=t$

What common denominator do you use?

$\frac{6}{15}+\frac{10}{15}+\frac{12}{15}=t$

$\frac{6}{15}+\frac{10}{15}+\frac{12}{15}=\frac{28}{15}$ or $1\frac{13}{15}$

How do you know that $1\frac{13}{15}$ is in lowest terms?

$\frac{2}{5}=\frac{6}{15}$
$\frac{2}{3}=\frac{10}{15}$
$\frac{4}{5}=\frac{12}{15}$
$\frac{28}{15}$ or $1\frac{13}{15}$

---

D $\frac{1}{4}+\frac{11}{12}+\frac{5}{8}=d$

What common denominator do you use?

$\frac{3}{12}+\frac{11}{12}+\frac{10}{12}=d$

$\frac{3}{12}+\frac{11}{12}+\frac{10}{12}=\frac{24}{12}$ or 2

$\frac{1}{4}=\frac{3}{12}$
$\frac{11}{12}=\frac{11}{12}$
$\frac{5}{8}=\frac{10}{12}$
$\frac{24}{12}$ or 2

## do

A Tom bought $\frac{5}{8}$ lb. of fudge and $\frac{1}{4}$ lb. of caramels. What fraction of a pound of candy did Tom buy in all?

$\frac{5}{8}+\frac{1}{4}=p$

B Bill sold $\frac{1}{2}$ page of advertising space in the yearbook, Ellen sold $\frac{3}{8}$ of a page, and Mary sold $\frac{3}{4}$ of a page. How much space did they sell in all?

$\frac{1}{2}+\frac{3}{8}+\frac{3}{4}=h$

Show your answers in lowest terms.

C $\frac{2}{3}+\frac{1}{6}=n$
D $\frac{7}{10}+\frac{3}{10}=z$
E $\frac{3}{8}+\frac{1}{4}+\frac{1}{12}=n$
F $\frac{3}{5}+\frac{1}{2}=c$
G $\frac{15}{16}+\frac{3}{8}=p$
H $\frac{1}{12}+\frac{5}{12}=n$
I $\frac{5}{6}+\frac{7}{8}+\frac{3}{4}=a$

J $\frac{2}{3}+\frac{2}{3}=n$
K $\frac{3}{10}+\frac{1}{2}=y$
L $\frac{7}{12}+\frac{2}{3}+\frac{5}{6}=n$
M $\frac{5}{8}+\frac{1}{5}=d$
N $\frac{5}{8}+\frac{1}{4}+\frac{1}{2}=n$
O $\frac{5}{12}+\frac{3}{8}=r$
P $\frac{1}{3}+\frac{1}{2}+\frac{2}{3}=n$

A $\frac{5}{16}+\frac{1}{2}=n$
B $\frac{5}{7}+\frac{2}{3}+\frac{2}{3}=n$
C $\frac{5}{6}+\frac{1}{3}=t$
D $\frac{2}{5}+\frac{3}{10}=b$
E $\frac{1}{6}+\frac{3}{4}+\frac{5}{12}=w$
F $\frac{2}{3}+\frac{2}{3}=n$
G $\frac{5}{8}+\frac{3}{10}+\frac{1}{5}=q$

H $\frac{4}{5}+\frac{1}{4}+\frac{7}{12}=f$
I $\frac{7}{8}+\frac{1}{2}=n$
J $\frac{1}{3}+\frac{5}{16}=w$
K $\frac{7}{10}+\frac{3}{4}=n$
L $\frac{1}{6}+\frac{1}{8}=x$
M $\frac{4}{5}+\frac{7}{12}+\frac{1}{3}=n$
N $\frac{1}{3}+\frac{2}{5}=s$

Now you should know how to add fractions and give the answers in different forms.

## Keeping skillful 3

A $15732+n=23600$
B $4379-208=x$
C $6519+13929=n$
D $3788-a=928$
E $n+3502=7100$
F $q-1918=1620$
G $5440-1066=b$
H $38412+96755=n$
I $5639-n=2487$
J $71861-51492=w$
K $h+9375=12022$
L $87339+16504=n$
M $27000-6139=e$
N $n+6398=32003$

A $135\times86=n$
B $724\times903=t$
C $38\times156=m$
D $553\times19=n$
E $88\times88=d$
F $614\times93=s$
G $85\times975=n$
H $600\times23=w$
I $111\times586=q$
J $199\times73=n$
K $414\times643=n$
L $529\times780=y$
M $219\times79=n$
N $4342\times66=n$

If there is a remainder, write a fraction numeral in the answer.

A $1894\div62=n$
B $2492\div b=356$
C $783728\div803=n$
D $98680\div25=h$
E $333\div k=11$
F $15625\div125=s$
G $27679\div38=n$
H $97902\div441=a$
I $3419\div c=17$
J $9604\div n=98$
K $46368\div322=f$



# 78

1. Discuss Example A carefully. Explain how to use the vertical form. Ask: "How does this example differ from those on pages 74-77?"
2. Discuss the reduction of the improper fraction numeral in the sum. Ask: "Can we find the answer by using 48 as a common denominator? Why does the book use 24?"
3. Have the pupils copy Examples C and D and find the answers. Then have them verify their work by using the computation in the book. Discuss the reduction of the improper fraction numerals.

# 79

1. Have the pupils work Exercises A to P independently. Ask them to write their work in either horizontal or vertical form (as you or they prefer) and show each step of the computation as was done in blue on page 78. Encourage the pupils to give their answers in lowest terms. Examine each pupil's work and make sure he understands how to correct any errors he has made.
2. Exercises A to N may be used for the pupils who need more practice, or they may be used at a later time.
3. These three blocks of work may be used separately, or they may be used together whenever such practice seems to be required.

# 80-85 Learning how

Expanded Notes for this lesson are on pages 311-312.

**Objectives**

Pupils review the addition of numbers expressed as mixed numerals and fraction numerals.

**Vocabulary**

New words page 81 computation; page 82 restaurant, spice*; page 84 steak; page 85 Jackson*, beyond*

**Comments**

The addition of numbers expressed as mixed numerals was first taught in *Seeing Through Arithmetic 5*. In this review lesson, as in the original presentation, pictures are used to show what is taking place in each step of the computation. The problems in the SEE and THINK steps involve choosing a common denominator and "carrying." The selection of examples of this kind for teaching is in keeping with the authors' belief that processes should be taught as a whole and not in disjointed parts.

The form in which the examples are written for computation avoids unnecessary rewriting. Note that when the fractions are added and the sum is represented by an improper fraction numeral, the pupils are not required to write this numeral (see page 81 of the pupils' book). Instead, they immediately replace it with a mixed numeral. Then they add the whole number part of it to the other whole numbers. Some pupils may be able to reduce the improper fraction numeral mentally. Others may need to write this computation on paper.

Be sure pupils understand why it is necessary to have a common denominator before fractions are added. Some pupils may need a review of the methods of finding a common denominator. Do not place too much emphasis on the importance of using the *lowest* common denominator, but help the pupils see that computation is often easier and faster when the lowest common denominator is used.

At this grade level, pupils can be expected to reduce the answer to a mixed numeral, if necessary, and to reduce a fraction numeral in the answer to lowest terms.

The four-step teaching method is used here. The SEE step, with full explanations and pictures, appears on pages 80-81. The THINK step, with pictures and questions to guide the pupils' thinking, appears on pages 82-84. The TRY step, also on page 84, and continuing on page 85, provides problems and examples without illustrations, but with computation and answers for final comparison. The last half of page 85 presents the DO step, made up of problems and exercises for the pupils to work independently.

**Answers**

Page 85:

(block 1)

A $5\frac{7}{12}$
B $20\frac{1}{20}$
C $9\frac{19}{24}$
D $6\frac{1}{15}$
E $12\frac{2}{5}$
F $5\frac{5}{12}$
G $16\frac{5}{12}$
H $6\frac{33}{35}$
I $3\frac{3}{16}$
J $8\frac{1}{2}$
K $23\frac{8}{9}$
L $40\frac{5}{24}$
M $20\frac{1}{10}$
N $28\frac{1}{12}$
O $33\frac{29}{30}$
P $21\frac{3}{14}$
Q $18\frac{2}{3}$
R 42
S $8\frac{1}{8}$
T 23
U $13\frac{1}{8}$
V $11\frac{2}{15}$
W $25\frac{5}{21}$

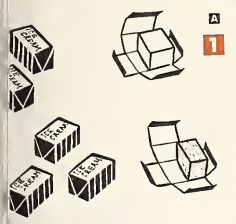
(block 2)

A 22
B $36\frac{1}{2}$
C $11\frac{19}{20}$
D $16\frac{5}{36}$
E $14\frac{1}{2}$
F $10\frac{29}{30}$
G $15\frac{1}{16}$

## Learning how

Now you will learn how to add mixed numbers and fractions.

### see



A

1 Mrs. Newton has $2\frac{2}{3}$ pt. of vanilla ice cream and $3\frac{1}{2}$ pt. of blueberry ice cream in her freezer. She has how much ice cream in her freezer?

$2\frac{2}{3}+3\frac{1}{2}=n$ Holds a place for the total number of pints

To find the total number of pints of ice cream, put the $\frac{2}{3}$ pt. and the $\frac{1}{2}$ pt. together before you put the whole pints together. You need a new name for this fractional part of the ice cream.

B

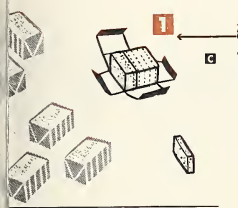
2 Picture B shows how to find a name for the new fraction that tells how much ice cream there is when you put $\frac{2}{3}$ pt. and $\frac{1}{2}$ pt. together. Think of the $\frac{2}{3}$ pt. as $\frac{4}{6}$ pt. and the $\frac{1}{2}$ pt. as $\frac{3}{6}$ pt. There are $\frac{7}{6}$ pt. in all.

You can use arithmetic to find the total number of pints. You add the numbers $2\frac{2}{3}$ and $3\frac{1}{2}$. First you add $\frac{2}{3}$ and $\frac{1}{2}$. You must find a common denominator. You can change $\frac{2}{3}$ to $\frac{4}{6}$, and you can change $\frac{1}{2}$ to $\frac{3}{6}$.

The sum of $\frac{4}{6}$ and $\frac{3}{6}$ is $\frac{7}{6}$.

$2\frac{2}{3}=2\frac{4}{6}$
$3\frac{1}{2}=3\frac{3}{6}$ 3

Addition of mixed numbers (reteaching)



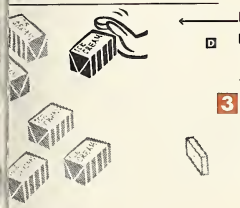
1 $\frac{7}{6}$ pt. will make 1 whole pint and $\frac{1}{6}$ of a pint.

C When you compute, change $\frac{7}{6}$ to a mixed number.

The sum is $1\frac{1}{6}$.

$2\frac{2}{3}=2\frac{4}{6}$
$3\frac{1}{2}=3\frac{3}{6}$
$\frac{1}{6}$ ← Write $\frac{1}{6}$ here.

Remember the 1. 2



D Put the new whole pint with the other whole pints.

In your computation, you put the new 1 with the whole numbers.

3

1 ← Write 1 in the ones' column to show that there is 1 more whole.

$2\frac{2}{3}=2\frac{4}{6}$
$3\frac{1}{2}=3\frac{3}{6}$
$\frac{1}{6}$

E Put the whole pints together.

When you compute, you add the whole numbers.

Add the ones. The sum is 6.

1

$2\frac{2}{3}=2\frac{4}{6}$ 5
$3\frac{1}{2}=3\frac{3}{6}$
$6\frac{1}{6}$ ← Write 6 here.

$6\frac{1}{6}$ pt. of ice cream are in the freezer.

$2\frac{2}{3}+3\frac{1}{2}=6\frac{1}{6}$ 6



# 80

1 Ask the pupils to read the problem and then look at Picture A. Help them see that some way must be found to describe how much ice cream there is when the whole pints and parts of pints are combined. Discuss the equation.

2 Have pupils note that the fractions of pints are to be combined first. Let everyone examine the part of Picture B that is in strong color and note that the $\frac{2}{3}$ pt. is now replaced by $\frac{4}{6}$ pt. and the $\frac{1}{2}$ pt. by $\frac{3}{6}$ pt. Be sure pupils understand why this replacement has been made. Discuss how the dotted black lines in the picture show this replacement.

3 Relate $\frac{4}{6}$ and $\frac{3}{6}$ (in black) to Picture B. Have someone explain how Picture B shows that the sum of the two fractions is $\frac{7}{6}$.

# 81

1 Help the pupils relate the text to Picture C.

2 Be sure the pupils note where the $\frac{1}{6}$ is written. Relate the statements "Remember the 1" and "Write $\frac{1}{6}$ here" to the picture.

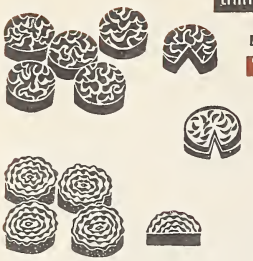
3 Have pupils examine Picture D. Then ask someone to explain why the new whole pint is being put with the other whole pints. Then point out that in the computation, the 1 is put with the other whole numbers. Have someone explain why this is done.

4 In Picture E, get the pupils to observe that all the whole pints are being put together. Let a pupil give the total number of pints.

5 Have the pupils add the column of whole numbers and observe where the 6 is written.

6 Direct the pupils to refer to the problem and equation on page 80. Ask if the answer seems reasonable.

think



A

**1** One day in his restaurant Mr. Miner sold $5\frac{4}{5}$ chocolate cakes, $\frac{9}{10}$ of a banana cake, and $4\frac{1}{2}$ spice cakes. Mr. Miner sold how much cake in all that day?

$5\frac{4}{5}+\frac{9}{10}+4\frac{1}{2}=n$

Holds a place for the number that tells how much cake was sold

First put together the parts of cakes. Do you know what name to use for the fractional part of the cakes that you will have?

B

Picture B shows how to find a name for the new fraction that the parts of cakes will make. Why can you think of each fraction of cake as marked off to show tenths of a whole cake?

Think of $\frac{4}{5}$ of a cake as $\frac{\square}{10}$ of a cake, and $\frac{1}{2}$ of a cake as $\frac{\square}{10}$. There will be $\frac{\square}{10}$ in all.

**2** When you use arithmetic, what will you add first? What common denominator will you use?

$\frac{4}{5}=\frac{\square}{10}$ $\frac{1}{2}=\frac{\square}{10}$

$5\frac{4}{5}=5\frac{8}{10}$ The sum of the fractions is $\frac{\square}{10}$. **3**

$\frac{9}{10}=\frac{9}{10}$

$4\frac{1}{2}=4\frac{5}{10}$

$\frac{22}{10}$ cakes are more than 1 cake.



## 82

1. Have the pupils identify in Picture A the three groups of cakes mentioned in the problem. Ask someone to relate the equation to the picture and to the problem.
2. Get the pupils to see that the fractional parts of the cakes are put together first. Get them to explain why they can think of these parts as marked off into tenths of a whole cake. Talk about the reason for replacing the fraction numerals by equal fraction numerals with a common denominator. Discuss why 10 can be used as a common denominator. Ask: "Why may 10 be better to use than 50?"
3. Let pupils discuss the combining of tenths and ask them how many tenths there are in all. Ask if more than one cake can be made from the three parts of cakes.

C

There are ▪ new whole cakes. There is also $\frac{\square}{10}$ of another cake.

When you compute, change $\frac{22}{10}$ to a mixed number.

**1** $5\frac{4}{5}=5\frac{8}{10}$ The sum is $2\frac{\square}{10}$.

$\frac{9}{10}=\frac{9}{10}$

$4\frac{1}{2}=4\frac{5}{10}$

$\frac{2}{10}$ Why is $\frac{2}{10}$ written here? What must you remember?

**2**

D

Think of the ▪ new cakes as being put with the other whole cakes.

In your computation, what should you do with the 2 that you were to remember?

**3**

2 Why may 2 be written here?

$5\frac{4}{5}=5\frac{8}{10}$

$\frac{9}{10}=\frac{9}{10}$

$4\frac{1}{2}=4\frac{5}{10}$

$\frac{2}{10}$

Now turn the page.



## 83

1. For Picture C, the pupils should explain where the two new whole cakes come from. They should also explain why $\frac{2}{10}$ of a cake is left over.
2. Discuss the $\frac{2}{10}$ in the computation and its position in the answer.
3. Discuss what is to be done with the 2 that represents the 2 new whole cakes. The pupils should observe that the 2 in the computation corresponds to what is being done with the 2 new cakes in Picture D.

What do you put together now? There are ▦ whole cakes.

When you compute, what do you add now?

What numbers do you add? The sum is ▭.

$$5\tfrac{4}{5} = 5\tfrac{8}{10}$$
$$\tfrac{9}{10} = \tfrac{9}{10}$$
$$4\tfrac{1}{2} = 4\tfrac{5}{10}$$
$$11\tfrac{2}{10}$$

1

Why do you write 11 here?

F

How much cake in all did Mr. Miner sell?

$5\frac{8}{10} + \frac{9}{10} + 4\frac{1}{2} = n$

Is the answer in lowest terms? $\frac{2}{10} = \frac{?}{5}$

Mr. Miner sold ▦ cakes.

2

## try

A Mrs. Philips bought $2\frac{5}{8}$ lb. of hamburger and a steak that weighed $1\frac{2}{3}$ lb. How much in all did the hamburger and the steak weigh?

$2\frac{5}{8} + 1\frac{2}{3} = n$

$$2\tfrac{5}{8} = 2\tfrac{15}{24}$$
$$1\tfrac{2}{3} = 1\tfrac{16}{24}$$
$$4\tfrac{7}{24}$$

1

Either remember this 1 or write it.

$3\frac{3}{8} + 6\frac{1}{8} = y$

$$3\tfrac{3}{8} = 3\tfrac{9}{24}$$
$$6\tfrac{1}{8} = 6\tfrac{3}{24}$$
$$9\tfrac{12}{24}$$

2

C $1\frac{1}{5} + \frac{2}{3} = a$

$$1\tfrac{1}{5} = 1\tfrac{3}{15}$$
$$\tfrac{2}{3} = \tfrac{10}{15}$$
$$1\tfrac{13}{15}$$

2

D $9\frac{1}{2} + 1\frac{3}{4} = m$

$$9\tfrac{1}{2} = 9\tfrac{2}{4}$$
$$1\tfrac{3}{4} = 1\tfrac{3}{4}$$
$$11\tfrac{1}{4}$$

Why do you write 11 here instead of 10?

$4\frac{1}{3} + 2\frac{5}{9} = z$

$$4\tfrac{1}{3}$$
$$2\tfrac{5}{9}$$
$$6\tfrac{8}{9} \text{ or } 6\tfrac{2}{3}$$

2

3

F $\frac{3}{10} + 7\frac{2}{5} + 5\frac{3}{4} = c$

$$\tfrac{3}{10} = \tfrac{6}{20}$$
$$7\tfrac{2}{5} = 7\tfrac{8}{20}$$
$$5\tfrac{3}{4} = 5\tfrac{15}{20}$$
$$13\tfrac{17}{20}$$

G $11\frac{5}{12} + 23\frac{1}{4} + 8\frac{1}{3} = n$

$$11\tfrac{5}{12} = 11\tfrac{5}{12}$$
$$23\tfrac{1}{4} = 23\tfrac{3}{12}$$
$$8\tfrac{1}{3} = 8\tfrac{4}{12}$$
$$43$$

Why is there no fraction numeral in the answer?

4

## do

A One Saturday Tony spent $2\frac{3}{4}$ hours working in the yard, $1\frac{1}{3}$ hours on his homework, and $1\frac{1}{2}$ hours on his hobby. How much time did he spend on these three things?

$2\frac{3}{4} + 1\frac{1}{3} + 1\frac{1}{2} = n$

5

B The distance from Stone City to Canton is $7\frac{3}{10}$ miles. Jackson is $12\frac{3}{4}$ miles beyond Canton on the same road. How far is it from Stone City to Jackson?

$7\frac{3}{10} + 12\frac{3}{4} = v$

Show any fractions in your answers in lowest terms.

| | | | |
|---|---|---|---|
| E $6\frac{1}{8} + 3\frac{5}{8} = d$ | J $3\frac{1}{4} + 5\frac{1}{4} = p$ | O $\frac{11}{12} + 17\frac{3}{8} = k$ | A $16\frac{1}{2} + 5\frac{1}{2} = n$ |
| D $5\frac{4}{5} + \frac{9}{10} = y$ | K $8\frac{5}{8} + 15\frac{1}{3} = e$ | R $22\frac{5}{8} + 19\frac{1}{8} = n$ | B $23\frac{1}{6} + 12\frac{5}{8} = s$ |
| B $2\frac{5}{8} + 9\frac{3}{8} = n$ | L $13\frac{7}{12} + 26\frac{5}{8} = n$ | S $3\frac{1}{3} + \frac{5}{12} + 4\frac{3}{8} = r$ | C $9\frac{3}{4} + 2\frac{1}{5} = f$ |
| I $1\frac{7}{12} + 3\frac{5}{8} = k$ | M $\frac{1}{2} + 19\frac{3}{8} = t$ | T $15\frac{1}{2} + \frac{3}{4} + 6\frac{1}{4} = c$ | D $\frac{5}{8} + 15\frac{7}{12} = n$ |
| G $8\frac{3}{4} + 7\frac{1}{3} = a$ | N $7\frac{1}{4} + 20\frac{3}{8} = b$ | U $9\frac{7}{16} + 2\frac{3}{8} + 1\frac{7}{16} = n$ | E $13\frac{3}{8} + \frac{7}{8} = v$ |
| H $6\frac{1}{2} + \frac{4}{5} = z$ | O $28\frac{3}{10} + 5\frac{3}{8} = n$ | V $\frac{2}{3} + 6\frac{5}{8} + 3\frac{9}{10} = w$ | F $4\frac{9}{10} + 6\frac{1}{15} = g$ |
| L $\frac{13}{16} + 2\frac{3}{8} = n$ | P $17\frac{5}{7} + 3\frac{1}{2} = x$ | W $8\frac{2}{3} + 5\frac{1}{7} + 11\frac{1}{3} = t$ | G $7\frac{13}{16} + 7\frac{1}{4} = n$ |

7

6

Now you should be able to add mixed numbers and fractions.



# 84

1 Have someone explain why all the whole cakes are put together in Picture E. Ask another pupil to explain why 11 is written where it is in the answer.

2 Ask the pupils if the problem on page 82 has been solved. Let another pupil give the answer in lowest terms.

3 Assign Examples A to G as written work (Examples B to G are on page 85). Tell the pupils that when they have completed their work, they can verify their selection of common denominators, reducing of fraction numerals, and the final answers by referring to the book.

# 85

1 Have pupils continue working the TRY step. (See Note 3 for page 84.)

2 Have someone explain why we do not add an extra number in the whole-number column.

3 Discuss the reduction to lowest terms of the proper fraction numeral in the sum.

4 Let someone explain why the answer is not a mixed number.

5 Remind pupils to use the vertical form for their computation in Exercises A to W.

6 When the pupils have completed Exercises A to W, have them put the solutions on the chalkboard for any examples that caused great difficulty.

7 This block of work may be assigned to pupils who require additional practice.

# 86 Using arithmetic

Exponded Notes ore not considered necessary for this lesson.

**Objectives**

The pupils solve problems that require the addition of numbers expressed as fraction numerals and mixed numerals.

**Vocabulary**

New words page 86 leather*, webbing, blanket*, mess*, soil*, wheat*

**Comments**

In this problem set, the pupils meet addition of fractions in two types of problem-solving situations. One type is described by the familiar additive equation ($\frac{3}{4}+\frac{1}{2}=t$). The other type is described by equations such as $n-2\frac{1}{2}=3\frac{1}{4}$. The pupils should have had enough experience by now with this second type of problem to know that the answer is found by addition.

**Answers**

A $\frac{3}{4}+2\frac{1}{8}+1\frac{3}{4}=[4\frac{5}{8}]$

B $4\frac{5}{8}+\frac{3}{8}+1=[6]$

C $\frac{1}{6}+\frac{5}{8}+\frac{2}{3}+\frac{3}{4}=[2\frac{5}{24}]$

D $[32\frac{1}{16}]-(28\frac{3}{4})=3\frac{5}{16}$ or $10\frac{1}{2}+9\frac{1}{2}+8\frac{3}{4}+3\frac{5}{16}=[32\frac{1}{16}]$

E $3\frac{1}{3}+2\frac{3}{8}+1\frac{1}{2}=[7\frac{5}{24}]$; 8 ft.

F $3\frac{3}{4}+7\frac{5}{8}+2\frac{7}{8}+\frac{1}{3}=[14\frac{7}{12}]$

G $1\frac{1}{2}+2\frac{3}{8}+2\frac{5}{8}=[6\frac{1}{2}]$

H $14\frac{7}{12}+6\frac{1}{2}+2\frac{1}{8}=[23\frac{5}{24}]$

I $[8\frac{1}{3}]-5\frac{2}{3}=2\frac{2}{3}$

J $6\frac{1}{2}+9\frac{3}{4}+11\frac{5}{8}+2\frac{1}{8}=[30]$

## Using arithmetic 1

A On a trip to the Soo Canals, John used $\frac{3}{4}$ of a roll of film on the way, $2\frac{1}{8}$ rolls when he went through the locks, and $1\frac{3}{4}$ rolls on the way home. He had used how much film?

B When John arrived home, he had $\frac{3}{8}$ of a roll of unused film in his camera and 1 full roll of film that he had not used. How many rolls of film had John taken with him?

C When Tony's Boy Scout troop made Indian headdresses, they first cut leather into strips and then sewed the ends of the strips together. They used one strip $\frac{1}{6}$ yd. long, one strip $\frac{5}{8}$ yd. long, one strip $\frac{2}{3}$ yd. long, and one strip $\frac{3}{4}$ yd. long. About how long was the piece of leather after all the strips had been sewn together?

D George made a carrier for his knapsack. He cut 3 pieces from a board. One piece of board was $10\frac{1}{2}$ in. long, one piece was $9\frac{1}{2}$ in. long, and the third piece was $8\frac{3}{4}$ in. long. A piece of board $3\frac{5}{16}$ in. long was left over. How long was the board before George cut it up?

E George also needed 3 pieces of webbing. He needed one piece $3\frac{1}{3}$ ft. long, one piece $2\frac{3}{8}$ ft. long, and a third piece $1\frac{1}{2}$ ft. long. How many feet of webbing did he need in all?

THINK What would be a sensible amount of webbing for him to buy?

F George began to pack his knapsack for a camping trip. His blanket weighed $3\frac{3}{4}$ lb. He had $7\frac{5}{8}$ lb. of food to pack. His flashlight and first aid kit together weighed $2\frac{7}{8}$ lb. His canteen weighed $\frac{1}{3}$ lb. How much in all did these things weigh?

G He then put in his mess kit, which weighed $1\frac{1}{2}$ lb., extra shoes that weighed $2\frac{3}{8}$ lb., and other small things that weighed $2\frac{5}{8}$ lb. How much in all did these things weigh?

H George's knapsack, with the carrier, weighed $2\frac{1}{8}$ lb. The total weight of the knapsack and all the things George put into it was how much?

I Sue had a bag of potting soil. After she had put $5\frac{2}{3}$ cups of the soil in a flower pot, she had $2\frac{2}{3}$ cups left. How much soil did she have to begin with?

J Ruth mixed different kinds of seed to put in her bird feeder. She used $6\frac{1}{2}$ lb. of wheat, $9\frac{3}{4}$ lb. of corn, $11\frac{5}{8}$ lb. of sunflower seed, and $2\frac{1}{8}$ lb. of nuts. How many pounds of mixed seed did she have?

2



# 86

1 Assign Problems A to J as written work. Remind pupils to write the proper equation for each, show the computation, and rewrite the equation with the answer inserted.

2 When the pupils have completed this problem set, supply equations with answers so that they may verify their work. Discuss the problems that caused the most difficulty.

# 87-88 Looking back

Exponded Notes for this lesson ore on pages 312-313.

**Objectives**

The pupil reviews certain ideas he will need in the subtraction, multiplication, and division of mixed numbers. He learns, for example, that $2\frac{1}{3}$ can be replaced with $\frac{7}{3}$ or $1\frac{4}{3}$.

**Vocabulary**

There are no new words.

**Comments**

Since the pupils are already familiar with regrouping in connection with subtraction of whole numbers, this lesson should be taught as an extension of the regrouping principle. If necessary, have the pupils review how to regroup a number. For example, have them regroup 53 into 4 tens and 13 ones so that 9 can be subtracted from it. To illustrate this, use 53 sticks arranged as 5 groups of 10 sticks and 3 single sticks. When the attempt is made to take away 9 sticks, the class will discover it cannot be done unless one group of 10 is broken up so that there are 10 single sticks that can be combined with the original 3. Now there are four groups of 10 and 13 single sticks, from which 9 sticks may be taken.

This principle of regrouping or "borrowing" in whole-number computation can be extended to regrouping a mixed number. For instance, $2\frac{1}{2}$ can become $\frac{5}{2}$ or $1\frac{3}{2}$. The number itself does not change, only the numeral used to write it. The pupils will understand that in replacing the mixed numeral with an improper fraction numeral, all the wholes are replaced by their fractional equivalent and added to the fractional part already there. When $2\frac{1}{2}$ is replaced by $1\frac{3}{2}$, only one whole is replaced by its fractional equivalent and added to the fractional part already there. Disks and fractional parts of disks may be used to illustrate this regrouping.

**Answers**

Page 87:

(block 2)

| | | | |
|---|---|---|---|
| A $\frac{29}{3}$ | D $\frac{29}{4}$ | G $\frac{79}{10}$ | J $\frac{35}{16}$ |
| B $\frac{11}{6}$ | E $\frac{53}{12}$ | H $\frac{31}{3}$ | K $\frac{27}{5}$ |
| C $\frac{55}{8}$ | F $\frac{9}{2}$ | I $\frac{62}{9}$ | L $\frac{103}{16}$ |
| M $\frac{263}{12}$ | O $\frac{61}{2}$ | Q $\frac{197}{10}$ | S $\frac{453}{16}$ |
| N $\frac{68}{5}$ | P $\frac{143}{4}$ | R $\frac{1155}{24}$ | T $\frac{175}{12}$ |

Page 88:

(block 2)

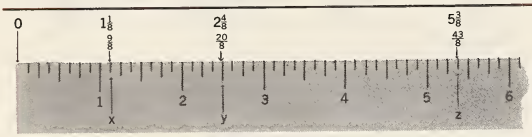
| | | | |
|---|---|---|---|
| A 10 | E 17 | I 15 | M 31 |
| B 7 | F 7 | J 17 | N 5 |
| C 5 | G 6 | K 18 | O 13 |
| D 26 | H 11 | L 6 | P 31 |

Keeping skillful:

| (block 1) | (block 2) | (block 3) |
|---|---|---|
| A $1\frac{1}{4}$ | A $9\frac{1}{24}$ | A 2403 |
| B $1\frac{23}{30}$ | B $7\frac{11}{12}$ | B 13,791 |
| C $1\frac{23}{24}$ | C $4\frac{11}{12}$ | C 305 |
| D $1\frac{21}{40}$ | D $11\frac{1}{8}$ | D 125,644 |
| E $2\frac{1}{24}$ | E $16\frac{1}{12}$ | E $59\frac{267}{421}$ |
| F $2\frac{1}{3}$ | F $12\frac{1}{5}$ | F 722,376 |
| G $1\frac{11}{15}$ | G $39\frac{13}{32}$ | G 7338 |
| H $1\frac{7}{24}$ | H $38\frac{23}{48}$ | H $94\frac{19}{36}$ |
| I $2\frac{1}{8}$ | I 108 | I 8771 |
| | | J 103,103 |

## Looking back

Sometimes you will need to change mixed numbers to other forms. Now you will learn how to do this.




A The picture above shows part of a ruler. Each inch on this ruler is marked off into how many equal parts?

B Why can zero be used at the beginning point of the ruler? Find the point that is marked X on the ruler. How far is it from 0 to Point X? **1**

C The mixed number $1\frac{1}{8}$ expresses a length of 1 whole inch and $\frac{1}{8}$ in. more. If you want to, you can express this length by using an improper fraction. In 1 inch there are ▪ eighths. In $1\frac{1}{8}$ in. there are 8 eighths + 1 eighth, or ▪ eighths. You can think of $1\frac{1}{8}$ as how many eighths? **2**

$1 = \frac{8}{8}$ $\quad 1\frac{1}{8} = \frac{?}{8}$

D The length from 0 to Point Y on the ruler is $2\frac{?}{8}$ in. You can express this length by using an improper fraction. **3** There are ▪ eighths in each inch. In the 2 inches there are $2 \times 8$, or ▪ eighths. In the $2\frac{4}{8}$ inches there are 16 eighths + 4 eighths, or ▪ eighths. You can think of $2\frac{4}{8}$ as how many eighths?

$1 = \frac{?}{8}$ $\quad 2 = \frac{?}{8}$ $\quad 2\frac{4}{8} = \frac{?}{8}$

E The length from 0 to Point Z on the ruler is $5\frac{?}{8}$ in. You can express this length, too, by using an improper fraction. **3** Each whole inch = ▪ eighths. Then 5 inches $= 5 \times$ ▪, or ▪ eighths. $5\frac{3}{8}$ in. = ▪ eighths + 3 eighths, or ▪ eighths.

$5\frac{3}{8} = \frac{?}{8}$

F Change $11\frac{4}{5}$ to an improper fraction.

$1 = \frac{5}{5}$ $\quad 11 = \frac{?}{5}$ $\quad 11\frac{4}{5} = \frac{?}{5}$

Change each mixed number below to an improper fraction. **4**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | $9\frac{5}{8}$ | F | $4\frac{1}{2}$ | K | $5\frac{6}{7}$ | P | $35\frac{3}{4}$ |
| B | $1\frac{5}{8}$ | G | $7\frac{9}{10}$ | L | $6\frac{7}{16}$ | Q | $19\frac{7}{10}$ |
| C | $6\frac{6}{8}$ | H | $10\frac{1}{3}$ | M | $21\frac{1}{12}$ | R | $48\frac{3}{24}$ |
| D | $7\frac{1}{4}$ | I | $6\frac{8}{9}$ | N | $13\frac{3}{5}$ | S | $28\frac{5}{16}$ |
| E | $4\frac{5}{12}$ | J | $2\frac{3}{16}$ | O | $30\frac{1}{2}$ | T | $14\frac{7}{12}$ |

Changing mixed numbers to equivalent forms (reteaching) 87

A Look again at the ruler on page 87. The distance from 0 to Point Y is $2\frac{4}{8}$ in. **1** You can also think of part of this length as 1 in. Then think of the rest of it as eighths of an inch. If you begin at 1 in. on the ruler and count the eighths to Point Y, you can think of the distance as 1 in. and ▪ eighths in.

$2\frac{4}{8} = 1\frac{?}{8}$

B To change $2\frac{4}{8}$ to $1\frac{?}{8}$, you use 1 of the 2 wholes and think of it as $\frac{8}{8}$. **2** Then you add the $\frac{8}{8}$ and the $\frac{4}{8}$. Why do you write 1 for the whole number instead of 2?

C The length from 0 to Point Z on the ruler is $5\frac{?}{8}$ in. Does $4\frac{11}{8}$ in. also express this length? **3**

D To change $5\frac{3}{8}$ to $4\frac{11}{8}$, first use 1 of the 5 wholes and think of it as $\frac{?}{8}$. Why do you next add $\frac{8}{8}$ and $\frac{3}{8}$? Why do you write 4 instead of 5 as the whole number?

$5\frac{3}{8} = 4\frac{?}{8}$

In each exercise below, what is the missing numerator? **4**

| | | | |
|---|---|---|---|
| A | $7\frac{?}{8} = 6\frac{?}{8}$ | I | $7\frac{?}{8} = 6\frac{?}{8}$ |
| B | $2\frac{1}{?} = 1\frac{?}{?}$ | J | $8\frac{?}{12} = 7\frac{?}{12}$ |
| C | $5\frac{?}{3} = 4\frac{?}{3}$ | K | $5\frac{?}{16} = 4\frac{?}{16}$ |
| D | $8\frac{10}{?} = 7\frac{?}{?}$ | L | $2\frac{?}{4} = 1\frac{?}{4}$ |
| E | $2\frac{?}{10} = 1\frac{?}{10}$ | M | $6\frac{15}{?} = 5\frac{?}{?}$ |
| F | $4\frac{3}{4} = 3\frac{?}{?}$ | N | $9\frac{1}{?} = 8\frac{?}{?}$ |
| G | $3\frac{1}{5} = 2\frac{?}{5}$ | O | $8\frac{?}{10} = 7\frac{?}{10}$ |
| H | $9\frac{3}{6} = 8\frac{?}{6}$ | P | $10\frac{11}{20} = 9\frac{?}{20}$ |

Now you should be able to change mixed numbers to other forms.

## Keeping skillful **5**

| | | | | | |
|---|---|---|---|---|---|
| A | $\frac{7}{8} + \frac{3}{8} = x$ | A | $6\frac{3}{4} + \frac{5}{8} + 1\frac{2}{3} = n$ | A | $7643 - n = 5240$ |
| B | $\frac{1}{2} + \frac{3}{5} + \frac{3}{5} = a$ | B | $5\frac{1}{12} + 2\frac{2}{8} = a$ | B | $m + 13824 = 27615$ |
| C | $\frac{7}{8} + \frac{5}{8} + \frac{1}{4} = r$ | C | $3\frac{1}{2} + 1\frac{1}{4} + \frac{1}{8} = t$ | C | $20130 \div r = 66$ |
| D | $\frac{9}{10} + \frac{5}{8} = b$ | D | $\frac{7}{8} + 2\frac{3}{4} + 7\frac{1}{2} = c$ | D | $a - 57211 = 68433$ |
| E | $\frac{3}{8} + \frac{3}{4} + \frac{11}{12} = s$ | E | $4\frac{11}{12} + 8\frac{2}{3} + 2\frac{1}{3} = x$ | E | $50212 \div c = 842$ |
| F | $\frac{7}{10} + \frac{5}{6} + \frac{4}{5} = t$ | F | $1\frac{9}{10} + \frac{1}{2} + 9\frac{4}{5} = d$ | F | $948 \times 762 = f$ |
| G | $\frac{13}{15} + \frac{1}{5} + \frac{2}{3} = n$ | G | $17\frac{1}{2} + 21\frac{29}{32} = s$ | G | $3749 + d = 11087$ |
| H | $\frac{11}{12} + \frac{3}{8} = m$ | H | $24\frac{9}{16} + 13\frac{11}{12} = b$ | H | $3403 \div 36 = x$ |
| I | $\frac{2}{3} + \frac{5}{8} + \frac{5}{8} = g$ | I | $76\frac{7}{12} + 30\frac{8}{8} + \frac{23}{24} = m$ | I | $68470 - 59699 = t$ |
| | | | | J | $98325 + 4778 = n$ |





# 87

1 Be sure that the pupils understand that when 0 occurs at the beginning of a scale, a measurement can be read directly. The distance from 0 to Point X can be read immediately as $1\frac{1}{8}$.

2 Let the pupils verify the $\frac{9}{8}$ inches by counting the eighths between 0 and Point X. Help them see that now the length is expressed as an *improper fraction numeral*. Only the name, not the length has changed.

3 Adapt the procedures in Note 2 when discussing Exercises D and E.

4 Assign Exercises A to T as written work. Discuss the answers afterwards.

# 88

1 Have the pupils read Exercise A and use the ruler pictured on page 87 to see that $2\frac{4}{8}$ can be thought of as $1\frac{12}{8}$.

2 Discuss Exercise B thoroughly to show how to make this change by computation. Be sure the pupils realize that 1 of the 2 wholes is changed to eighths and added to the four eighths.

3 Adapt Notes 1 and 2 to Exercises C and D.

4 Assign these exercises as written work.

5 Assign these exercises as written work as time permits.

# 89-92 Learning how

Expanded Notes for this lesson are on pages 313-315.

**Objectives**

The pupil reviews how to subtract numbers expressed as proper fraction numerals.

**Vocabulary**

New words page 90 Peters*; page 92 metal*

**Comments**

Subtraction of numbers expressed by proper fraction numerals was introduced in *Seeing Through Arithmetic 5*. This lesson brings together ideas already reviewed in the present book and applies them to the subtraction of such numbers. The pupils understand the meaning of the subtraction process. They have learned how to find common denominators and how to replace fraction numerals with equivalent ones. In the lesson on pages 87-88 they learned that mixed numerals can be replaced with equivalent ones, an idea basic to an understanding of borrowing in subtraction. Now the pupils review the application of these previously-learned ideas in subtraction involving fractions.

If you wish, you may demonstrate the subtraction of fractions by using fractional parts of disks or other shapes made of oaktag, flannel, or other materials. Plan your demonstrations in such a way as to make it clear that when one fraction is subtracted from another, the remainder cannot be named by a single fraction numeral unless the fractions have been expressed by fraction numerals having the same denominator.

Pupils who know how to add fractions should have no trouble subtracting them. Notice that the first example (page 89 of the pupils' book) requires the pupil to choose a common denominator and find equivalent fraction numerals before he proceeds to subtract. This should convince pupils of the need for a common denominator and the importance of the denominator in both addition and subtraction of fractions. Point out to the pupils the convenience of using the lowest common denominator.

The four-step teaching method is used to present the material in this lesson. Note that, as usual, pupils are required to write an equation to show the problem situation ($\frac{2}{3}-\frac{1}{2}=n$). The vertical arrangement of computation is shown as an alternative procedure in the TRY step on page 91. There is no advantage in using the column arrangement when proper fractions are to be subtracted, but the vertical form is introduced so that the pupils will be familiar with it when they subtract numbers expressed as mixed numerals.

In the DO step on page 92, a second set of practice exercises (A to L) has been provided so that you may assign extra work when needed.

**Answers**

Page 92:

(block 1)

| | | | |
|---|---|---|---|
| A $\frac{3}{8}$ | F $\frac{1}{2}$ | K $\frac{1}{2}$ | P $\frac{7}{20}$ |
| B $\frac{9}{16}$ | G $\frac{3}{4}$ | L $\frac{5}{8}$ | Q $\frac{5}{6}$ |
| C $\frac{1}{12}$ | H $\frac{3}{16}$ | M $\frac{7}{16}$ | R $\frac{7}{12}$ |
| D $\frac{3}{8}$ | I $\frac{1}{15}$ | N $\frac{5}{24}$ | S $\frac{3}{40}$ |
| E $\frac{9}{20}$ | J $\frac{5}{16}$ | O $\frac{3}{8}$ | T $\frac{1}{6}$ |

(block 2)

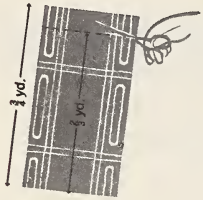
| | |
|---|---|
| A $\frac{1}{8}$ | G $\frac{3}{8}$ |
| B $\frac{1}{2}$ | H $\frac{1}{12}$ |
| C $\frac{7}{24}$ | I $\frac{11}{24}$ |
| D $\frac{1}{20}$ | J $\frac{3}{10}$ |
| E $\frac{4}{15}$ | K $\frac{3}{4}$ |
| F $\frac{5}{24}$ | L $\frac{4}{15}$ |

## Learning how

Now you will learn how to subtract fractions.

### see

A




Joan has $\frac{3}{4}$ yd. of material. She is cutting $\frac{2}{3}$ yd. from the piece to make place mats. What fraction of a yard will be left?

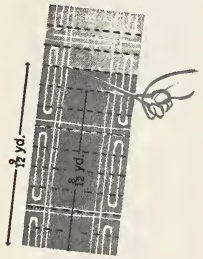
1

$\frac{3}{4} - \frac{2}{3} = d$

Holds a place for the fraction of a yard left

Think of the $\frac{2}{3}$ yd. as taken away from the $\frac{3}{4}$ yd. You need a name for the fraction of a yard that will be left.

B




Picture B shows how you must think about the $\frac{3}{4}$ yd. and the $\frac{2}{3}$ yd. before you can find a name for the fraction of a yard that will be left.

2

Think of the $\frac{3}{4}$ yd. that Joan had as $\frac{9}{12}$ of the whole yard. Think of the $\frac{2}{3}$ yd. that is being cut off as $\frac{8}{12}$ of the whole yard.

When you use arithmetic to find what fraction of a yard will be left, you subtract $\frac{2}{3}$ from $\frac{3}{4}$. But before you can do this, you must replace the fraction numerals $\frac{2}{3}$ and $\frac{3}{4}$ by other fraction numerals with a common denominator. You will use 12 as a common denominator.

3

$\frac{3}{4} = \frac{9}{12}$ $\quad \frac{2}{3} = \frac{8}{12}$

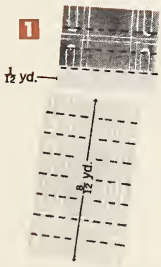
$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12}$

Now turn the page.



C

1




Picture C shows that Joan will have $\frac{1}{12}$ yd. of material left when she has cut off $\frac{8}{12}$ yd., or $\frac{2}{3}$ yd.

You can also get the answer by subtracting $\frac{8}{12}$ from $\frac{9}{12}$.

Subtract the numerators. $9 - 8 = 1$

2

$\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Write 1 as the numerator. You are subtracting twelfths. So write 12 as the denominator.

Joan will have $\frac{1}{12}$ yd. of material left.

$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

### think

A

3

After breakfast Mrs. Peters had $\frac{7}{8}$ of a coffee cake left. She used $\frac{1}{2}$ of the whole coffee cake for supper. How much of the whole coffee cake did she have left then?

$\frac{7}{8} - \frac{1}{2} = a$

4

Holds a place for the amount of coffee cake left

You know that $\frac{4}{8}$ of the coffee cake was taken away from $\frac{7}{8}$ of the coffee cake. Do you know what name to use for the fraction of the coffee cake that was left?





# 89

1 Tell the pupils to read the problem and examine the picture. They should see that $\frac{2}{3}$ of a yard is being cut from $\frac{3}{4}$ of a yard. Ask a pupil why it is hard to describe how much of a yard is left. Be sure the pupils understand that *d* in the equation holds a place for the numeral that represents the fraction of a yard that is left.

2 Have the pupils read the first seven lines of text and look at Picture B. They should notice that the dimmed-off part completes the whole yard, which has been marked off into twelfths. Now it is easy to see how many twelfths there are in $\frac{3}{4}$ of a yard and in $\frac{2}{3}$ of a yard.

3 Ask why 12 is selected as the common denominator. Ask someone to explain how $\frac{3}{4}$ was replaced by $\frac{9}{12}$, and how $\frac{2}{3}$ was replaced by $\frac{8}{12}$.

# 90

1 Have someone explain how the picture shows that $\frac{8}{12}$ of a yard of material has been cut off and that $\frac{1}{12}$ of a yard is left.

2 Point out that the numerators are subtracted, and have someone explain why, in terms of the picture.

3 Call on someone to read the problem. Get the pupils to relate Picture A to the problem. Then ask someone to point out the part of the coffee cake that will be left.

4 The pupils should explain the equation, relating it to the problem and to the picture. Then someone should read the remaining text and supply the numerators. Discuss why it is difficult to know what fraction of the coffee cake was left.

**1** Before you can find a name for the fraction of the coffee cake that was left, think of the $\frac{1}{2}$ taken away as $\frac{4}{8}$ of the whole coffee cake.

When $\frac{4}{8}$ of the coffee cake is taken away from $\frac{7}{8}$ of the coffee cake, $\frac{\square}{8}$ is left.

When you use arithmetic to get the answer, you subtract $\frac{1}{2}$ from $\frac{7}{8}$. You must use a common denominator. Why can you use 8 as a common denominator? $\frac{1}{2}=\frac{\square}{8}$ **2**

$\frac{7}{8}-\frac{1}{2}=\frac{7}{8}-\frac{4}{8}$

Subtract ▭ from ■. What do you write as the numerator of the answer? What do you write as the denominator?

$\frac{\square}{8}$ of the coffee cake was left.

$\frac{7}{8}-\frac{1}{2}=\frac{\square}{8}$

## try

A $\frac{5}{6}-\frac{1}{8}=t$

Why do you use 24 as a common denominator when you subtract?

$\frac{1}{4}-\frac{3}{24}=t$

$\frac{1}{4}-\frac{3}{24}=\frac{17}{24}$

**3** You may write your work this way if you want to.

$\frac{5}{6}=\frac{20}{24}$

$\frac{1}{8}=\frac{3}{24}$

$\frac{17}{24}$

$\frac{15}{16}-\frac{3}{16}=b$

Do you need to change the fraction numerals? Why?

$\frac{15}{16}-\frac{3}{16}=\frac{12}{16}$ or $\frac{3}{4}$ **4**

Is $\frac{3}{4}$ in lowest terms?

You may write your work this way.

$\frac{15}{16}$

$\frac{3}{16}$

$\frac{12}{16}$ or $\frac{3}{4}$



$\frac{9}{10}-\frac{3}{4}=m$ **1**

What common denominator do you use?

$\frac{9}{10}-\frac{15}{20}=m$

$\frac{3}{4}-\frac{15}{20}=\frac{3}{20}$

Your work may be written this way.

$\frac{9}{10}=\frac{18}{20}$

$\frac{3}{4}=\frac{15}{20}$

$\frac{3}{20}$

$\frac{11}{24}-\frac{1}{12}=c$

What common denominator do you use?

$\frac{11}{24}-\frac{2}{24}=c$

$\frac{11}{24}-\frac{2}{24}=\frac{9}{24}$ or $\frac{3}{8}$

You may write your work this way.

$\frac{11}{24}=\frac{11}{24}$

$\frac{1}{12}=\frac{2}{24}$

$\frac{9}{24}$ or $\frac{3}{8}$

$\frac{5}{6}-\frac{3}{10}=n$

What common denominator do you use?

$\frac{5}{6}-\frac{9}{30}=n$

$\frac{25}{30}-\frac{9}{30}=\frac{16}{30}$ or $\frac{8}{15}$

Your work may be written this way.

$\frac{5}{6}=\frac{25}{30}$

$\frac{3}{10}=\frac{9}{30}$

$\frac{16}{30}$ or $\frac{8}{15}$

## do

A June bought $\frac{5}{8}$ lb. of candy. She gave away $\frac{1}{4}$ lb. of the candy. How much candy was left?

$\frac{5}{8}-\frac{1}{4}=r$ **2**

B Paul sawed $\frac{3}{16}$ in. from one end of a piece of metal that was $\frac{3}{4}$ in. long. How long was the piece of metal then?

$\frac{3}{4}-\frac{3}{16}=x$ **3**

Be sure your answers are in lowest terms.

| | | | | | |
|---|---|---|---|---|---|
| $\frac{1}{2}-\frac{5}{12}=n$ | $\frac{2}{3}-\frac{3}{5}=d$ | $\frac{9}{16}-\frac{3}{16}=m$ | $\frac{11}{24}-\frac{1}{3}=s$ | $\frac{19}{24}-\frac{5}{12}=c$ |
| $\frac{3}{4}-\frac{3}{8}=t$ | $\frac{11}{16}-\frac{3}{8}=k$ | $\frac{3}{4}-\frac{2}{8}=h$ | $\frac{9}{10}-\frac{2}{5}=t$ | $\frac{11}{12}-\frac{5}{8}=r$ |
| $\frac{3}{4}-\frac{3}{10}=a$ | $\frac{3}{4}-\frac{1}{4}=e$ | $\frac{11}{12}-\frac{1}{12}=b$ | $\frac{2}{3}-\frac{3}{8}=v$ | $\frac{5}{8}-\frac{1}{8}=m$ |
| $\frac{5}{6}-\frac{1}{6}=g$ | $\frac{23}{24}-\frac{1}{2}=x$ | $\frac{3}{4}-\frac{1}{8}=s$ | $\frac{3}{4}-\frac{7}{10}=x$ | $\frac{3}{4}-\frac{1}{2}=d$ |
| $\frac{7}{8}-\frac{1}{8}=m$ | $\frac{5}{8}-\frac{3}{16}=r$ | $\frac{7}{8}-\frac{5}{8}=w$ | $\frac{3}{5}-\frac{1}{3}=a$ | $\frac{17}{20}-\frac{1}{10}=e$ |
| $\frac{9}{16}-\frac{3}{8}=b$ | $\frac{1}{3}-\frac{1}{8}=g$ | $\frac{2}{3}-\frac{1}{2}=x$ | $\frac{7}{8}-\frac{2}{3}=b$ | $\frac{14}{15}-\frac{2}{3}=n$ |

Now you should know how to subtract one fraction from another.

# 91

1. Ask why the cake is thought of as marked off into eighths. Be sure the pupils see that 7 eighths of a whole cake are shown in the picture, and that the part being taken away ($\frac{1}{2}$) can be considered as $\frac{4}{8}$. Let the pupils count the remaining eighths to see how many are left.
2. Discuss why 8 was chosen as a common denominator, and how $\frac{1}{2}$ is replaced by $\frac{4}{8}$. Ask the pupils why only numerators are subtracted. [The denominators show that the parts are eighths of the whole cake, but since 4 of the 7 parts of a cake are being removed, the only subtraction is 4 from 7.]
3. Discuss this exercise. Call attention to the computation shown in vertical form.
4. Assign this exercise and Examples C to E on page 92 as written work. Have the pupils verify their work by comparing it with the computation in the book.

# 92

1. Have the pupils copy Examples C to E and work them without looking at their books. Let them verify their work by comparing it with the computation in the book.
2. Have the pupils work Exercises A to T independently.
3. Use these exercises for pupils who need additional practice.

# 93-97 Learning how

Expanded Notes for this lesson are on pages 315-316.

**Objectives**

The pupil learns to subtract numbers expressed as mixed numerals.

**Vocabulary**

New words page 97 unnecessary*, nails*

**Comments**

This lesson begins with an example that requires borrowing ($9\frac{1}{2}-4\frac{7}{8}$). This type of example presents the only real difficulty that the well-prepared pupil is likely to encounter when subtracting mixed numbers. The lesson on pages 87-88 of the pupils' book has shown him that he can replace a mixed numeral like $9\frac{1}{2}$ with $8\frac{3}{2}$, or $8\frac{12}{8}$, or whatever equivalent he needs in a particular situation. This replacement can be compared to the regrouping that the pupil is familiar with in the subtraction of whole numbers. Help the pupils see the similarity between thinking of 1 ten as 10 ones and thinking of 1 (taken from the whole number) as 2 halves, or 8 eighths, or whatever fraction numeral is required in the computation.

**Answers**

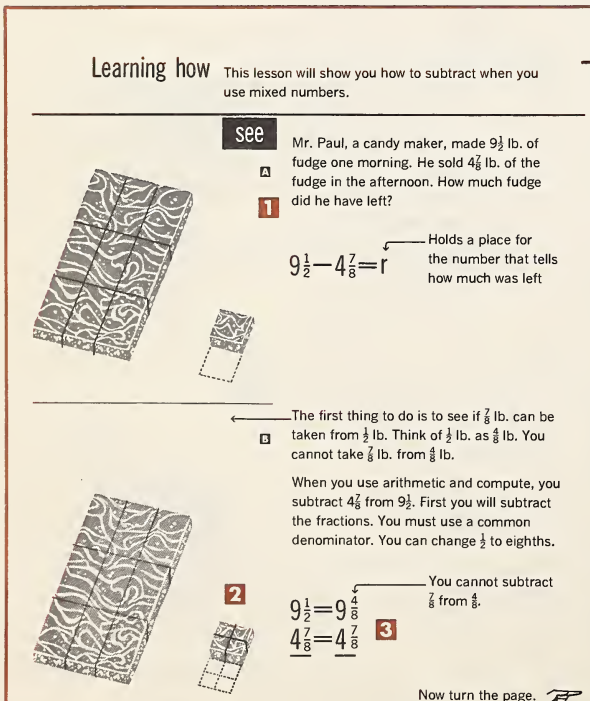
Page 97:

(block 1)

| | | | |
|---|---|---|---|
| A $\frac{3}{8}$ | G $5\frac{7}{12}$ | | |
| B $1\frac{5}{6}$ | H $6\frac{9}{16}$ | | |
| C $1\frac{2}{5}$ | I $8\frac{8}{15}$ | | |
| D $2\frac{7}{24}$ | J $7\frac{1}{12}$ | | |
| E $1\frac{14}{15}$ | K $5\frac{3}{10}$ | | |
| F $3\frac{3}{5}$ | L $31\frac{5}{12}$ | | |

(block 2)

| | |
|---|---|
| A 1 | F $7\frac{3}{8}$ |
| B $5\frac{1}{4}$ | G $11\frac{7}{12}$ |
| C $\frac{17}{24}$ | H $9\frac{5}{8}$ |
| D $5\frac{1}{2}$ | I $4\frac{1}{6}$ |
| E $3\frac{1}{16}$ | J $24\frac{5}{12}$ |



Learning how This lesson will show you how to subtract when you use mixed numbers.

see

A Mr. Paul, a candy maker, made $9\frac{1}{2}$ lb. of fudge one morning. He sold $4\frac{7}{8}$ lb. of the fudge in the afternoon. How much fudge did he have left?

1

$9\frac{1}{2}-4\frac{7}{8}=r$ ← Holds a place for the number that tells how much was left

B ← The first thing to do is to see if $\frac{7}{8}$ lb. can be taken from $\frac{1}{2}$ lb. Think of $\frac{1}{2}$ lb. as $\frac{4}{8}$ lb. You cannot take $\frac{7}{8}$ lb. from $\frac{4}{8}$ lb.

When you use arithmetic and compute, you subtract $4\frac{7}{8}$ from $9\frac{1}{2}$. First you will subtract the fractions. You must use a common denominator. You can change $\frac{1}{2}$ to eighths.

2

$9\frac{1}{2}=9\frac{4}{8}$ ← You cannot subtract $\frac{7}{8}$ from $\frac{4}{8}$.

$4\frac{7}{8}=4\frac{7}{8}$ 3

Now turn the page.

Subtraction involving mixed numbers (reteaching) 93



# 93

1 Call on a pupil to read the problem aloud. Direct attention to Picture A. The pupils should understand that each block in the larger portion represents 1 lb. of fudge and that the small block represents $\frac{1}{2}$ lb. Discuss with the class the dotted lines that help identify the $\frac{1}{2}$ lb. Also discuss how the equation correctly describes the problem situation.

2 Discuss the reason for marking off the $\frac{1}{2}$ lb. into eighths of a whole pound. Draw attention to the black lines. Then observe that the rest of the $\frac{7}{8}$ lb. that is to be subtracted must come from one of the pounds in the big piece.

3 Note how the change in the numerals expresses what has been done with the $\frac{1}{2}$ lb. of fudge in the picture. Discuss the impossibility of subtracting $\frac{7}{8}$ from $\frac{4}{8}$.

C

1

You need more eighths before you can take away $\frac{7}{8}$ lb. So think of 1 of the whole pounds as cut up into eighths. You can think of this pound as $\frac{8}{8}$ lb. Put the $\frac{8}{8}$ lb. with the $\frac{4}{8}$ lb. Now you have 8 whole pounds and $\frac{12}{8}$ lb. more. You can take $\frac{7}{8}$ lb. from $\frac{12}{8}$ lb.

When you compute, you take 1 from the 9 ones and think of it as $\frac{8}{8}$. Put $\frac{8}{8}$ with $\frac{4}{8}$. You now have $\frac{12}{8}$.

$$9\tfrac{1}{2} = 9\tfrac{4}{8} \quad (8\tfrac{12}{8})$$
$$4\tfrac{7}{8} = 4\tfrac{7}{8}$$

2

You have changed $9\frac{4}{8}$ to $8\frac{12}{8}$. You can subtract $\frac{7}{8}$ from $\frac{12}{8}$.

D

When you take $\frac{7}{8}$ lb. from $\frac{12}{8}$ lb., the amount left is $\frac{5}{8}$ lb.

3

In your computation you subtract the fractions.

$$9\tfrac{1}{2} = 9\tfrac{4}{8} \quad (8\tfrac{12}{8})$$
$$4\tfrac{7}{8} = 4\tfrac{7}{8}$$
$$\tfrac{5}{8}$$

Subtract $\frac{7}{8}$ from $\frac{12}{8}$. $\frac{5}{8}$ is left.

Write $\frac{5}{8}$ here.

E

1

Now the 4 whole pounds can be taken from the 8 whole pounds. 4 lb. will be left.

In your computation the next step is to subtract the whole numbers.

$$9\tfrac{1}{2} = 9\tfrac{4}{8} \quad (8\tfrac{12}{8})$$
$$4\tfrac{7}{8} = 4\tfrac{7}{8}$$
$$4\tfrac{5}{8}$$

Subtract 4 from 8.

Write the 4 here.

Mr. Paul had $4\frac{5}{8}$ lb. of fudge left.

$9\frac{1}{2} - 4\frac{7}{8} = 4\frac{5}{8}$

think

2 A

Mrs. Price had $7\frac{1}{4}$ sticks of butter. She used $6\frac{3}{4}$ sticks for baking. How much butter did she have left?

$7\frac{1}{4} - 6\frac{3}{4} = S$

Holds a place for the number of sticks left

Can the $\frac{3}{4}$ stick be taken from the $\frac{1}{4}$ stick? How do you know right away that you will have to think of cutting up another stick?

B

3

$7\frac{1}{4}$

$6\frac{3}{4}$

When you compute, do you have to find a common denominator? Why?

How do you know you need more fourths before you can subtract?

Now turn the page.



# 94

1 Have the pupils read this paragraph. Let someone point out the pound of fudge that has been marked off into eighths. Ask: "What is being done with this pound?" Have someone count to verify that there are now $\frac{12}{8}$ lb. of fudge in the smaller portion and 8 whole pounds in the larger. Be sure the pupils understand that the picture now illustrates the mixed numeral $8\frac{12}{8}$.

2 Explain that the changes that occurred in Picture C are now made in the numerals. Have someone explain where the $\frac{12}{8}$ came from. Ask: "Why are 9 and $\frac{4}{8}$ crossed off?"

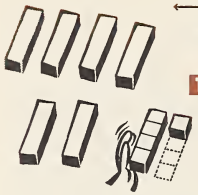
3 Ask a pupil to explain what the hand is removing in Picture D. Ask: "How many eighths of a pound are left in the small portion?" Then call attention to the corresponding work in the computation. Ask the pupils to verify the computation.

# 95

1 Have the pupils relate the computation for the whole numbers to the action shown in the picture. Have them compare the amount of fudge Mr. Paul has left, as shown in the picture, with the final answer obtained by computation.

2 Have the pupils read the problem and relate Picture A and the equation to it. Ask: "Which number from the problem is illustrated?"

3 Ask: "Why is the stick of butter being cut into fourths? What will have to be done with $7\frac{1}{4}$ in the computation?"



C Why is the stick thought of as cut into fourths? How many one fourths of a stick will there be in all? How many whole uncut sticks will there be?

1

$6\frac{3}{4}$
$7\frac{1}{4}$
$6\frac{3}{4}$

When you compute, how do you get $6\frac{3}{4}$? What do you subtract first?

D When you take $\frac{3}{4}$ stick from $\frac{5}{4}$ stick, $\frac{2}{4}$ of a stick is left.

2

$6\frac{3}{4}$
$7\frac{1}{4}$ 3
$6\frac{3}{4}$
$\frac{2}{4}$

In your computation, you subtract $\frac{3}{4}$ from $\frac{5}{4}$. $\frac{2}{4}$ is left.

Why do you write $\frac{2}{4}$ here?

Now take away the 6 whole sticks. How many whole sticks will be left?

E

4

$6\frac{3}{4}$
$7\frac{1}{4}$
$6\frac{3}{4}$
$\frac{2}{4}$

In your computation, you subtract ■ from ■. What is left?

Why do you not write a numeral here?

Mrs. Price had $\frac{2}{4}$, or $\frac{1}{2}$, of a stick left.

$7\frac{1}{4} - 6\frac{3}{4} = \frac{2}{4}$ or $\frac{1}{2}$



## try

A $5 - 1\frac{1}{16} = t$ 1

$5 = 4\frac{16}{16}$
$1\frac{1}{16} = 1\frac{1}{16}$
$3\frac{15}{16}$

Why do you change 5 to $4\frac{16}{16}$?

B $3\frac{4}{5} - \frac{9}{10} = d$

$3\frac{4}{5} = 2\frac{18}{10}$
$\frac{9}{10} = \frac{9}{10}$
$2\frac{9}{10}$ 2

Why is it unnecessary to subtract from the 2?

C $11\frac{7}{10} - 5\frac{7}{10} = x$

$11\frac{7}{10}$
$5\frac{7}{10}$
6

Why is there no fraction numeral in the answer?

D $9\frac{7}{8} - 7\frac{2}{3} = r$

$9\frac{7}{8} = 9\frac{21}{24}$
$7\frac{2}{3} = 7\frac{16}{24}$
$2\frac{5}{24}$

Why do you not have to change $9\frac{21}{24}$ before you subtract?

E $9\frac{5}{8} - 2 = c$

$9\frac{5}{8}$
2 3
$7\frac{5}{8}$

F $13\frac{1}{6} - 12\frac{5}{6} = n$

$13\frac{1}{6} = 12\frac{7}{6}$
$12\frac{5}{6} = 12\frac{3}{6}$ 4
$\frac{2}{6}$ or $\frac{1}{3}$

G $4 - \frac{7}{12} = k$

$4 = 3\frac{12}{12}$
$\frac{7}{12} = \frac{7}{12}$
$3\frac{5}{12}$

H $8\frac{5}{6} - 3\frac{1}{6} = s$

$8\frac{5}{6}$
$3\frac{1}{6}$ 4
$5\frac{4}{6}$ or $5\frac{2}{3}$

## do

A Don bought $1\frac{1}{4}$ lb. of nails. He used $\frac{7}{8}$ lb. How many pounds of nails did he have left?
$1\frac{1}{4} - \frac{7}{8} = m$ 5

B Mrs. Peters made $4\frac{1}{2}$ doz. cookies. She gave $2\frac{2}{3}$ doz. to friends. How many dozen cookies did she have left?
$4\frac{1}{2} - 2\frac{2}{3} = a$

Show any fraction numerals in your answers in lowest terms.

C $5\frac{7}{10} - 4\frac{3}{10} = d$
D $3\frac{1}{6} - \frac{7}{8} = x$
E $7\frac{3}{5} - 5\frac{2}{5} = m$
F $9\frac{9}{10} - 6\frac{3}{10} = c$
G $6\frac{1}{2} - 1\frac{1}{2} = a$
H $14\frac{3}{16} - 7\frac{5}{8} = e$
I $11\frac{1}{3} - 2\frac{4}{5} = g$
J $13\frac{3}{4} - 6\frac{2}{5} = h$
K $37\frac{1}{2} - 32\frac{1}{5} = d$
L $50 - 18\frac{7}{12} = n$
A $6\frac{1}{4} - 5\frac{1}{4} = z$
B $7\frac{5}{12} - 2\frac{1}{6} = f$
C $4\frac{1}{8} - 3\frac{5}{12} = d$
D $9\frac{1}{6} - 3\frac{2}{3} = a$
E $5\frac{3}{4} - 2\frac{13}{16} = b$
F $19\frac{1}{8} - 11\frac{3}{4} = m$
G $31\frac{7}{12} - 20 = r$
H $52\frac{13}{16} - 43\frac{1}{16} = n$
I $20 - 15\frac{5}{8} = s$
J $42\frac{1}{4} - 17\frac{5}{8} = x$

Now you should know how to subtract mixed numbers.





# 96

1. Discuss what is being done with the stick that was cut into fourths. Ask: "How would you describe the amount of butter now? How is this shown by the numerals?"
2. Ask: "What is the hand doing? How much of this part of the butter will be left?"
3. Ask: "What has been done in the computation that is like what is happening in Picture D? To what part of the picture does $\frac{2}{4}$ belong?"
4. Have someone explain what is happening in Picture E and relate it to the computation. Be sure the pupils can explain why there is no whole number in the answer.

# 97

1. Assign Examples A to H as written work. Tell the pupils to do the work without looking at the book, then to verify their work by comparing it with that shown in the book. Help individual pupils as the need arises.
2. When the need arises, explain why it is unnecessary to subtract from the whole number.
3. When the need arises, explain why it is unnecessary to subtract from the fraction.
4. If necessary, explain what is done to the answer.
5. Assign these exercises as independent work. Adjust the assignment to the needs and abilities of the pupils.

# 98 Using arithmetic

Expanded Notes for this lesson are on pages 316-317.

**Objectives**

The pupil solves problems that require the subtraction of numbers expressed as fraction numerals and mixed numerals.

**Vocabulary**

New words page 98 Ed*, gained*, furniture*, almost*, Pat's*, vacation*

**Comments**

The pupils have previously reviewed the five problem types presented in this lesson when they worked with whole numbers (see the notes in this *Teaching Guide* that apply to page 25 of the pupils' book). Now they are to solve these same types of problems when the numbers involved are expressed as fraction numerals and mixed numerals.

If the pupils understand these familiar problem situations and their solutions, they should have no trouble when the same problem situations involve fractions.

**Answers**

A More; $\frac{3}{4}-\frac{2}{3}=[\frac{1}{12}]$

B $3\frac{2}{3}-1\frac{1}{2}=[2\frac{1}{6}]$

C $2\frac{1}{2}-[\frac{1}{2}]=1\frac{5}{6}$

D $1\frac{5}{6}-(1\frac{5}{12})=[\frac{5}{12}]$

E $5\frac{1}{4}-3\frac{2}{3}=[1\frac{7}{12}]$

F $68\frac{1}{2}+[2\frac{1}{2}]=71$
or $71-68\frac{1}{2}=[2\frac{1}{2}]$

G $3\frac{1}{2}-\frac{3}{4}=[2\frac{3}{4}]$

H $[\frac{1}{2}]+1\frac{3}{4}=2\frac{1}{4}$
or $1\frac{3}{4}+[\frac{1}{2}]=2\frac{1}{4}$

I $\frac{5}{8}-[\frac{3}{8}]=\frac{1}{4}$

J $\frac{1}{2}-\frac{3}{8}=[\frac{1}{8}]$

K $97+[8\frac{1}{2}]=105\frac{1}{2}$
or $105\frac{1}{2}-97=[8\frac{1}{2}]$

L $8056\frac{6}{10}+[288\frac{7}{10}]=8345\frac{3}{10}$ or
$8345\frac{3}{10}-8056\frac{6}{10}=[288\frac{7}{10}]$

M $1\frac{2}{3}-\frac{7}{8}=[\frac{19}{24}]$

## Using arithmetic 1

Mrs. May used $\frac{3}{4}$ c. sugar to make lemon drops and $\frac{5}{6}$ c. sugar to make spice bars. Did she use more or less sugar for the lemon drops than for the spice bars? How much more sugar or less sugar did she use?

Mrs. May made $3\frac{5}{8}$ doz. lemon drops and $1\frac{1}{2}$ doz. spice bars. She made how many more dozen lemon drops than spice bars?

Jane bought $2\frac{1}{4}$ yd. of cloth. After she had made an apron, she had $1\frac{5}{8}$ yd. left. How much cloth had she used to make the apron?

Then Jane made aprons for her two sisters. She used $\frac{3}{4}$ yd. of the cloth for one of the aprons and $\frac{2}{3}$ yd. of the cloth for the other. How much of the cloth was left then?

When Ed made a shelf, he used $\frac{5}{8}$ ft. from a board that was $5\frac{1}{4}$ ft. long. How long was the leftover piece of board?

Before her vacation, Ann weighed $81\frac{1}{2}$ lb. After the vacation, she weighed 1 lb. How much had she gained?

Mr. Row had $\frac{3}{4}$ gal. of green paint and $3\frac{1}{2}$ gal. of white paint. He had how much more white paint than green paint?

H Mr. Row mixed $2\frac{1}{4}$ gal. of paint by putting some green paint in a pail and adding $1\frac{3}{4}$ gal. of white paint. Later he wanted to match this color. How much green paint should he add to $1\frac{3}{4}$ gal. of white paint?

I Mrs. Row put $\frac{5}{8}$ gal. of paint in a pail. After she had painted some furniture, she still had $\frac{1}{4}$ gal. of paint in the pail. How much paint had she used on the furniture?

J Mrs. Row used $\frac{1}{2}$ gal. of paint to paint a fence. How much less paint did she use on the furniture than on the fence?

K Jim's cat would not stay on the scales when Jim tried to weigh him. Jim knew that his own weight was 97 lb. He weighed himself holding the cat and found the total weight was $105\frac{1}{2}$ lb. How much did the cat weigh?

L When Pat's father filled the almost empty tank of his car with gasoline, he noticed that he had driven a total of $8056\frac{6}{10}$ mi. By the time this gasoline was used up, Pat's father had driven $8345\frac{3}{10}$ mi. in all. How many miles had he driven on the tank of gasoline?

M Sally bought $1\frac{2}{3}$ yd. of ribbon. She used $\frac{7}{8}$ yd. How much did she have left? 2

Subtraction of fractions in problem-solving situations

## 98

1 Assign these problems as written work. Tell the pupils to write the equation for each problem, do the computation, and then rewrite the equation with the answer inserted. When everyone has finished, supply answers and let the pupils verify their own work.

2 Discuss those problems that caused the most trouble.

# 99 Checking up; Keeping skillful

Exponded Notes are not considered necessory for these lessons.

**Objectives**

The pupil tests his knowledge of addition and subtraction of numbers expressed by fraction numerals and mixed numerals.

**Vocabulary**

There are no new words.

**Comments**

The "Checking up" tests should be used for diagnosis of difficulties; therefore sufficient time should be allowed so that most pupils finish the tests. Test 1 covers addition of proper fractions; Test 2, addition of mixed numbers; Test 3, subtraction of proper fractions; and Test 4, subtraction of mixed numbers.

If the pupils have considerable difficulty, reteaching is necessary. In reteaching, use pages 74-79 of the pupils' book for Test 1, pages 80-85 for Test 2, pages 89-92 for Test 3, and pages 87-88 and 93-97 for Test 4.

An activity for able pupils that might be used in connection with this lesson is described in Activity 4, page 374.

**Answers**

Checking up:

| Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|
| A $1\frac{2}{3}$ | A $9\frac{1}{4}$ | A $\frac{9}{20}$ | A $3\frac{2}{3}$ |
| B $\frac{7}{8}$ | B $6\frac{3}{10}$ | B $\frac{3}{16}$ | B $7\frac{3}{8}$ |
| C $1\frac{1}{2}$ | C $3\frac{5}{12}$ | C $\frac{3}{10}$ | C 6 |
| D $2\frac{11}{24}$ | D $21\frac{1}{20}$ | D $\frac{1}{2}$ | D $\frac{5}{16}$ |
| E $1\frac{8}{15}$ | E $10\frac{1}{2}$ | E $\frac{11}{24}$ | E $4\frac{7}{15}$ |
| F $2\frac{1}{16}$ | F $40\frac{3}{4}$ | F $\frac{1}{16}$ | F $3\frac{13}{24}$ |
| G $1\frac{13}{24}$ | G $25\frac{13}{20}$ | G $\frac{11}{20}$ | G $10\frac{4}{5}$ |
| H 2 | H $20\frac{3}{8}$ | H $\frac{1}{8}$ | H $8\frac{3}{16}$ |
| I $1\frac{11}{16}$ | I 12 | I $\frac{11}{24}$ | I $\frac{1}{4}$ |

Keeping skillful:

(block 1)

| | | |
|---|---|---|
| A $18\frac{25}{86}$ | G 4410 | M 167,007 |
| B 22,819 | H 94 | N 47,081 |
| C 5944 | I 14,316 | O $134\frac{17}{352}$ |
| D 15,588 | J 16,138 | P 25,974 |
| E 77,868 | K 40,408 | Q 484 |
| F 1376 | L 4,117,296 | |

(block 2)

A 12 tbs.
B 5 yd.
C 68 pk.
D 10 yr.
E 15,840 ft.
F 60 min.
G 56 fl. oz.
H 9 gal.
I 33 yd.
J 1460 da.
K 8 da.
L 12 pt.
M 16 bu.
N 216 in.
O 14,080 yd.
P 12 lb.
Q 18,000 lb.

(block 3)

A $2\frac{1}{2}$
B $\frac{7}{12}$
C $2\frac{2}{5}$
D $7\frac{7}{16}$
E $5\frac{1}{4}$
F $7\frac{15}{16}$
G 18
H $\frac{1}{2}$
I $15\frac{1}{6}$
J $7\frac{3}{8}$
K $3\frac{4}{15}$
L $11\frac{23}{24}$
M $4\frac{7}{16}$
N $20\frac{9}{20}$

## Checking up 1

| Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|
| $\frac{5}{8}+\frac{5}{8}=n$ | A $\frac{7}{16}+8\frac{13}{16}=b$ | A $\frac{7}{10}-\frac{1}{4}=s$ | A $7\frac{1}{6}-3\frac{1}{2}=z$ |
| $\frac{3}{4}+\frac{1}{2}=r$ | B $3\frac{5}{8}+2\frac{1}{2}=d$ | B $\frac{15}{16}-\frac{3}{4}=b$ | B $12-4\frac{5}{8}=t$ |
| $\frac{1}{2}+\frac{3}{8}+\frac{5}{8}=s$ | C $1\frac{5}{8}+\frac{11}{12}+\frac{2}{3}=x$ | C $\frac{4}{5}-\frac{1}{2}=n$ | C $6\frac{2}{3}-\frac{2}{3}=b$ |
| $\frac{3}{4}+\frac{5}{8}+\frac{7}{8}=a$ | D $14\frac{1}{4}+6\frac{4}{5}=t$ | D $\frac{7}{8}-\frac{3}{8}=a$ | D $18\frac{7}{16}-17\frac{7}{8}=a$ |
| $\frac{7}{10}+\frac{5}{8}=d$ | E $9\frac{7}{10}+\frac{4}{5}=a$ | E $\frac{5}{8}-\frac{1}{8}=d$ | E $20\frac{4}{5}-16\frac{1}{3}=k$ |
| $\frac{5}{8}+\frac{3}{4}+\frac{11}{16}=r$ | F $26\frac{1}{2}+14\frac{1}{4}=g$ | F $\frac{11}{16}-\frac{5}{8}=h$ | F $9\frac{1}{12}-6\frac{5}{8}=n$ |
| $\frac{5}{12}+\frac{7}{8}+\frac{1}{4}=m$ | G $18\frac{9}{10}+6\frac{3}{4}=k$ | G $\frac{3}{4}-\frac{1}{2}=c$ | G $15\frac{3}{10}-4\frac{1}{2}=d$ |
| $\frac{9}{10}+\frac{3}{8}+\frac{1}{2}=t$ | H $11\frac{7}{8}+\frac{2}{3}+7\frac{5}{8}=z$ | H $\frac{15}{16}-\frac{13}{16}=m$ | H $31\frac{13}{18}-23\frac{5}{8}=m$ |
| $\frac{7}{16}+\frac{3}{4}+\frac{1}{2}=b$ | I $\frac{1}{12}+6\frac{3}{4}+4\frac{1}{3}=m$ | I $\frac{5}{6}-\frac{3}{8}=r$ | I $11\frac{1}{6}-10\frac{11}{12}=r$ 2 |

## Keeping skillful 3

| | | |
|---|---|---|
| 1573 ÷ 86 = m | A 6 fl. oz. = ▒ tbs. | A $s+1\frac{1}{2}=4$ |
| c + 38214 = 61033 | B 180 in. = ▒ yd. | B $\frac{3}{4}+m=1\frac{1}{3}$ |
| 74163 − 68219 = a | C 17 bu. = ▒ pk. | C $14\frac{7}{10}-12\frac{3}{10}=t$ |
| r − 4286 = 11302 | D 120 mo. = ▒ yr. | D $6\frac{5}{16}+\frac{7}{8}+\frac{1}{4}=d$ |
| 927 × 84 = t | E 3 mi. = ▒ ft. | E $5\frac{11}{12}-a=\frac{2}{3}$ |
| s + 38624 = 40000 | F 3600 sec. = ▒ min. | F $r-4\frac{9}{16}=3\frac{3}{8}$ |
| 53036 − y = 48626 | G 7 c. = ▒ fl. oz. | G $7\frac{4}{5}+10\frac{1}{5}=x$ |
| 81310 ÷ z = 865 | H 72 pt. = ▒ gal. | H $20\frac{1}{3}-19\frac{5}{6}=s$ |
| 17432 + k = 31748 | I 99 ft. = ▒ yd. | I $a+15\frac{3}{4}=30\frac{1}{12}$ |
| 96004 − 79866 = n | J 4 yr. = ▒ da. | J $14\frac{1}{8}-n=6\frac{3}{4}$ |
| s + 42617 = 83025 | K 192 hr. = ▒ da. | K $16\frac{6}{8}+d=20\frac{1}{10}$ |
| 496 × 8301 = r | L 24 c. = ▒ pt. | L $m-11\frac{3}{8}=\frac{7}{12}$ |
| w − 65632 = 101375 | M 64 pk. = ▒ bu. | M $34-29\frac{5}{6}=y$ |
| b + 48520 = 95601 | N 18 ft. = ▒ in. | N $g-8\frac{3}{4}=11\frac{7}{10}$ |
| 94370 ÷ 704 = n | O 8 mi. = ▒ yd. | |
| 33406 − z = 7432 | P 192 oz. = ▒ lb. | |
| 30492 ÷ r = 63 | Q 9 T. = ▒ lb. | |

End-of-block tests on addition and subtraction of fractions and mixed numbers 99

# 99

1 Have the pupils work on these tests independently. Tell them to give their answers in lowest terms.

2 After the pupils have corrected their work, do any reteaching that is necessary.

3 Assign the exercises in "Keeping skillful" as time permits.

# 100-101 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil solves a variety of verbal problems, some of which require him to get information from diagrams.

**Vocabulary**

New words **page 100** cabin*, including*, log*, attached*, against*; **page 101** ridge*, stove*, across*, slipcovers*, curtains*, pillow*, building*

**Comments**

Before assigning the problems, you will want to be sure that the pupils know how to interpret the diagrams at the top of page 100. Discuss with the class how the labels and arrows are used, what dimensions are shown on the diagrams, etc.

**Answers**

Using arithmetic:

1 $209-(23\frac{1}{2})=[185\frac{1}{2}]$

2 $(96)+209=[305]$

C $185-11\frac{3}{4}=[173\frac{1}{4}]$

D $11\frac{3}{4}-(5\frac{3}{4})=[6]$

E $5\frac{3}{4}+[2\frac{3}{4}]=8\frac{1}{2}$

F $22\frac{2}{3}-[2\frac{11}{12}]=19\frac{3}{4}$ or $22\frac{2}{3}-19\frac{3}{4}=[2\frac{11}{12}]$

G $60\frac{8}{10}+[8\frac{8}{10}]=69\frac{6}{10}$; temperature is often expressed in tenths of a degree.

H $3\frac{1}{2}-[1\frac{3}{4}]=1\frac{3}{4}$

I $5\frac{7}{10}+6\frac{1}{4}=[11\frac{19}{20}]$

J $36\frac{3}{4}+24=[60\frac{3}{4}]$

K $(70)\times\$1.19=[\$83.30]$

L $25\div[4\frac{1}{6}]=6$

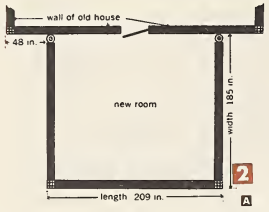
M $[\frac{2}{3}]+2\frac{1}{2}=3\frac{1}{6}$

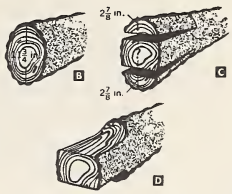
N $475\times\$2.69=[\$1277.75]$

Keeping skillful:

| (block 1) | | (block 2) |
|---|---|---|
| A 7277 | F 349 | A $601\frac{93}{149}$ |
| B 19,398 | G 8417 | B $228\frac{11}{123}$ |
| C 24,510 | H 109,391 | C 75 |
| D 5907 | I 101,400 | D $58\frac{43}{827}$ |
| E 31,278 | J 25,109 | E 403 |

## Using arithmetic




**1** A John Price and his father built a new room on the family summer cabin. Picture A shows the floor plan of this room. The outside length of the room, including the end walls, was 209 inches. Picture B shows a log like those the walls were made of. The diameter of each log was $11\frac{3}{4}$ in. What was the inside length of the room from wall to wall?

**3** B At the point where each side wall of the new room was to be attached to the old room, an upright log was placed against the outside wall of the old cabin. The outer side of each of these logs was 48 inches in from the corner of the old wall. How long was the old wall on the outside?

C What was the inside width of the new room in inches? Study the plan before you answer this question.

**4** D Picture C shows how John's father planned to cut a peg at the end of each log so the logs would fit together at the corners of the room. Picture D shows the finished peg. After the $2\frac{7}{8}$-inch pieces had been cut from opposite sides of the log, how wide was the center piece that was left?

**5** E The walls of the new room were to be $8\frac{1}{2}$ ft. high. John measured the walls one day when he and his father were working. The walls were then $5\frac{3}{4}$ ft. high. How much higher did they need to make the walls?

100

**1** F Mr. Price said the ridge pole should be $19\frac{3}{4}$ ft. long. He chose a log $22\frac{5}{8}$ ft. long. How many feet should he have cut from this log to make the ridge pole?

G When John and his father began working one morning, the temperature was $60\frac{8}{10}°$. By noon the temperature was $69\frac{6}{10}°$. The temperature had gone up how many degrees?

THINK Why do you not need to change the $\frac{8}{10}$ in your answer to $\frac{4}{5}$?

H One Monday there were $3\frac{1}{2}$ gal. of white gasoline for the stove. By Sunday night $1\frac{3}{4}$ gal. were left. How much gasoline had been used that week?

I There were two ways to get from the cabin to the nearest town. By boat across the lake, the distance was $5\frac{7}{10}$ mi. By car on the road around the lake, the distance was $6\frac{1}{4}$ mi. longer. The distance by road from the cabin to the town was how many miles?

J Mrs. Price needed $36\frac{3}{4}$ yd. of cloth for slipcovers and 24 yd. for curtains for the new room. How many yards of cloth did she need in all for curtains and slipcovers?

K When Mrs. Price bought the cloth, she also bought $9\frac{1}{4}$ yd. of another color for pillow covers. She paid $1.19 a yard for all the cloth. How much did she spend for all the cloth?

L Mrs. Price brought 25 lb. of flour to the cabin. She used all of this flour in six weeks. She had used an average of how many pounds of flour a week?

M One day Mrs. Price bought $2\frac{1}{2}$ doz. eggs from a farmer and put them with the eggs she had. Then she found that she had $3\frac{1}{6}$ doz. eggs. What fraction of a dozen eggs did Mrs. Price have before she bought the $2\frac{1}{2}$ doz.?

**2** N John's father worked 475 hours in building the new room. A man from the town would have charged $2.69 per hour to do this work. How much money would Mr. Price have had to pay this man for 475 hours of work?

### Keeping skillful **3**

| | | |
|---|---|---|
| A $6302+975=a$ | F $9526+d=9875$ | A $89642\div 149=c$ |
| B $s-8245=11153$ | G $56400-47983=s$ | B $56110\div r=246$ |
| C $m+25490=50000$ | H $m-48391=61000$ | C $9900\div 132=x$ |
| D $74812-68905=r$ | I $42737+58663=n$ | D $48009\div t=827$ |
| E $40306+t=71584$ | J $W+74891=100000$ | E $391313\div a=971$ |

101

# 100

1. Ask someone to read Problem A aloud. Then have the pupils locate the part of Diagram A that identifies the outside length of the new room. Make sure they realize that the arrows show that this length includes the thickness of the two end walls.
2. Have the pupils find the part of the diagram that gives the outside width of the new room.
3. Call on a pupil to read this problem. Have the pupils identify the upright logs in the diagram and explain how the diagram shows that these logs are each 48 inches from the corners of the old wall.
4. Have a pupil read this problem. Then have the class look at Pictures B to D. Have them identify the diameter of the logs and the pieces of log which are cut off to make a peg.
5. Assign Problems A to E as written work, also Problems F to N on the next page.

# 101

1. Assign Problems F to N as written work.
2. Supply answers (including equations) so that pupils can verify their work. Discuss those problems that caused trouble for more than a few pupils.
3. Assign these exercises as a separate lesson.

# 102-103 Checking up

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil tests his understanding of all problem types studied thus far. He also tests his knowledge of all the computational processes he has learned up to this point. He tests his ability to change units of measure.

**Vocabulary**

New words page 102 drum*, clarinets, trombone, instruments*, jackets*

**Comments**

Test 1 on page 102 is an achievement test on problem solving. At least one example of each type of problem taught so far is included in this test. (A complete list of problem types is given on pages 408-409 of this *Teaching Guide.*)

The tests on page 103 are power tests. Allow sufficient time for most of the pupils to finish the work. Use as many tests in one day as you think wise. All of them will certainly be too much work for one day.

When the pupils have completed their work on the tests on pages 102-103, you will be able to see what their individual weaknesses are. Refer to the Reteaching Chart that follows and reteach the problem-solving and computational techniques that have caused difficulty.

The authors of this book feel that topics originally taught in Grades 3 and 4, such as the addition, subtraction, and multiplication of whole numbers, should not be reviewed in detail in a book for Grade 6. Pupils who still have difficulty with computation or with the solution of the familiar problem types that were studied in earlier grades should be provided with copies of *Seeing Through Arithmetic 4* for review.

Activity 4, page 374, may be used with the able pupils in connection with this lesson.

**Reteaching chart**

This chart will help you locate the lessons in the pupils' book that may be used for any reteaching that is necessary.

Page 102:

| Problem | Pages |
|---|---|
| A | Book 4: 86-87 or Book 5: 21, 207-208 |
| B | Book 4: 92-94 or Book 5: 20 |
| C | Book 6: 26-30 |
| D | Book 6: 26-30 |
| E | Book 4: 134-137 or Book 5: 40-42 |
| F | Book 4: 52-53, 167 or Book 5: 25, 61-64 |
| G | Book 4: 88-90 or Book 5: 196 |
| H | Book 4: 130-133 or Book 5: 43-45 |
| I | Book 6: 80-85 |
| J | Book 6: 26-30 |
| K | Book 5: 101-104 or Book 6: 26-30 |
| L | Book 4: 52-53, 167 or Book 5: 25, 61-64 |

Page 103:

| Test | Pages |
|---|---|
| 2 | Book 4: 20-23, 24-27 |
| 3 | Book 4: 36-41, 45, 54 |
| 4 | Book 4: 142-153, 161-164 or Book 5: 61-64, 77 |
| 5 | Book 4: 200-205 or Book 6: 20-23 |
| 6 | Book 6: 74-79, 80-85 |
| 7 | Book 6: 87-88, 89-92, 93-97 |
| 8 | Book 6: 70-71 |
| 9 | Book 6: 87 |
| 10 | Book 4: 10, 91, 226-233 or Book 5: 92-95 |

**Answers**

In the answers for Test 1, the brackets indicate the place where the pupils would write a letter in the equation. The numeral within the brackets is the answer for the problem. In the equations for multiple-step problems, the parentheses and the numeral within them indicate the preliminary computation that must be done before the final answer can be found.

Test 1

A $\$129.50-\$54.95=[\$74.55]$

B $[\$13.67]+\$8.83=\$22.50$

C $(\$550)-\$495=[\$55]$

D $(\$109.90)+\$129.50=[\$239.40]$

E $[\$25.00]-\$19.95=\$5.05$

F $57\times\$3.79=[\$216.03]$

G $3\frac{3}{4}+[3\frac{1}{4}]=7$

H $\$300-[\$285]=\$15$

I $18\frac{2}{3}+7\frac{1}{2}=[26\frac{1}{6}]$

J $\$1900\div(\$50)=[38]$

K $\$43.75\div[\$1.25]=35$

L $22\times\$1.75=[\$38.50]$

**Test 2**

A 1180
B 184
C 2215
D 7070
E 164,000
F 2229
G 1,004,030
H 1675
I 540,001
J 767,430

**Test 3**

A 786
B 58
C 718
D 68
E 849
F 5036
G 10,146
H 11,279
I 5809
J 43,788

**Test 4**

A 6885
B 688
C 2961
D 36,328
E 24,128
F 538,407
G 678,114
H 66,040
I 564,184
J 212,678

**Test 5**

A 84
B $17\frac{35}{76}$
C 158
D $776\frac{13}{62}$
E 64
F $95\frac{1}{2}$
G $723\frac{1}{4}$
H $75\frac{11}{523}$
I 169
J $271\frac{4}{33}$

**Test 6**

A $1\frac{9}{20}$
B $7\frac{1}{8}$
C $3\frac{1}{5}$
D $9\frac{4}{5}$
E 16
F $31\frac{7}{15}$
G $15\frac{5}{24}$
H $9\frac{5}{16}$

**Test 7**

A $\frac{3}{4}$
B $\frac{2}{3}$
C $8\frac{1}{10}$
D $14\frac{23}{24}$
E 19
F $\frac{13}{16}$
G $6\frac{1}{2}$
H $3\frac{7}{15}$

**Test 8**

A $3\frac{5}{8}$
B 6
C $4\frac{1}{2}$
D $2\frac{5}{12}$
E $2\frac{1}{4}$
F $11\frac{1}{2}$
G $6\frac{1}{2}$
H 9
I $2\frac{5}{8}$
J $8\frac{1}{6}$
K $6\frac{3}{4}$
L $4\frac{2}{3}$
M $2\frac{1}{4}$
N $2\frac{13}{15}$
O $1\frac{5}{6}$
P $2\frac{6}{25}$
Q 4
R $7\frac{1}{5}$

**Test 9**

A $\frac{15}{4}$
B $\frac{11}{2}$
C $\frac{47}{6}$
D $\frac{113}{12}$
E $\frac{17}{4}$
F $\frac{69}{10}$
G $\frac{39}{8}$
H $\frac{103}{12}$
I $\frac{29}{4}$
J $\frac{73}{12}$
K $\frac{28}{3}$
L $\frac{47}{10}$
M $\frac{41}{3}$
N $\frac{37}{2}$
O $\frac{245}{8}$
P $\frac{54}{5}$
Q $\frac{177}{16}$
R $\frac{343}{8}$

**Test 10**

A 84 mo.
B 10 gal.
C 5 da.
D 5 c.
E 68 qt.
F 420 min.
G 7 ft.
H 1825 da.
I 5280 yd.

## Checking up

Test 1 **1**

A John wants to buy a cornet so that he can play in the school band. He looked at two cornets. One was priced at $54.95, and the other at $129.50. What was the difference in price of these two cornets?

B Bill is saving money to buy a drum that costs $22.50. He needs $8.83 more before he can buy the drum. How much money has he saved?

C The school bought 5 clarinets for a total price of $495. If the clarinets had been bought separately, each one would have cost $110. How much money was saved by buying 5 of them at one time?

D The school also bought two cornets at $54.95 each and one trombone for $129.50. What was the total cost of these three instruments?

E Sue bought an instrument case that cost $19.95. Then she had $5.05 left. How much money did she have before she bought the instrument case?

F The school bought 57 music stands at $3.79 each. How much did all the music stands cost?

G Alice is supposed to practice on her trombone for 7 hours each week. This week, from Sunday to Wednesday, she has practiced a total of $3\frac{3}{4}$ hr. How many more hours should she practice this week?

H The Band Mothers' Club raised $300. They used most of this money to buy jackets for the band. Then they had $15 left. How much did they spend for the jackets?

I Philip plays two instruments. He weighed both instruments in their cases to see how many pounds he had to carry when he took both instruments home. One instrument weighed $18\frac{3}{8}$ lb., and the other weighed $7\frac{1}{2}$ lb. How much did Philip's two instruments weigh together?

J In one 10-month school term, the school collected $1900 from renting instruments. Each pupil who rented an instrument paid $5 per month for the term. How many pupils had rented instruments that term?

K 35 of the pupils bought music that cost $43.75 altogether. What was the average cost per pupil for this music?

L 22 other pupils paid $1.75 apiece for music that they needed. How much did the music for these 22 pupils cost? **2**





# 102

**1** Allow enough time for most of your pupils to finish this test. Tell them that for each problem they are to make an equation and to use a letter to hold a place for the numeral that will express the answer. For those problems that require preliminary work, tell the pupils to use letters in the equation to hold places for the numerals that must be found. (See pages 49-51 of the pupils' book.) When they have finished, they should rewrite the equation with the numerals replacing the letters. Have them show the computation needed to obtain the answer.

**2** Supply the pupils with correct answers, and let them verify their own answers. Determine which problems have given the class the most trouble and discuss them. If you feel that reteaching is necessary, the Reteaching Chart on page 109 will be helpful.

1

**Test 2**

A $483+697=t$
B $69+42+18+55=r$
C $862+444+909=a$
D $6347+523+200=x$
E $70059+93941=b$
F $758+346+1125=s$
G $156386+847644=d$
H $114+968+593=w$
I $460095+79906=g$
J $87355+680075=f$

**Test 3**

A $900-114=n$
B $743-685=k$
C $814-96=c$
D $501-433=a$
E $4768-3919=x$
F $6310-1274=t$
G $18742-8596=b$
H $40036-28757=m$
I $92147-86338=d$
J $47086-3298=r$

**Test 4**

A $9\times765=w$
B $86\times8=d$
C $47\times63=t$
D $956\times38=r$
E $58\times416=g$
F $69\times7803=b$
G $746\times909=z$
H $508\times130=c$
I $872\times647=h$
J $2473\times86=s$

**Test 5**

A $2436\div29=d$
B $1327\div c=76$
C $10586\div m=67$
D $96250\div124=s$
E $8256\div t=129$
F $38964\div408=n$
G $49181\div r=68$
H $39236\div523=w$
I $13858\div d=82$
J $53682\div198=e$

**Test 6**

A $\frac{3}{4}+\frac{7}{10}=a$
B $1\frac{1}{2}+\frac{3}{8}+5\frac{1}{4}=x$
C $\frac{5}{6}+\frac{2}{3}+1\frac{7}{10}=t$
D $8\frac{1}{5}+\frac{8}{10}+\frac{7}{10}=d$
E $6\frac{1}{4}+5+4\frac{3}{4}=w$
F $21\frac{2}{3}+9\frac{5}{6}=b$
G $5\frac{7}{12}+\frac{3}{8}+9\frac{1}{4}=k$
H $6\frac{15}{16}+1\frac{3}{4}+\frac{5}{8}=m$

**Test 7**

A $\frac{7}{8}-\frac{1}{8}=c$
B $4\frac{1}{2}-3\frac{5}{8}=x$
C $11-2\frac{9}{10}=a$
D $15\frac{7}{12}-\frac{5}{8}=r$
E $40\frac{5}{6}-21\frac{2}{3}=w$
F $20\frac{3}{4}-19\frac{15}{16}=b$
G $17\frac{9}{16}-11\frac{1}{16}=n$
H $4\frac{3}{10}-\frac{5}{8}=t$

**Test 8**

Change to mixed numbers in lowest terms.

A $\frac{29}{8}$ G $\frac{65}{10}$ M $\frac{63}{28}$
B $\frac{30}{9}$ H $\frac{27}{3}$ N $\frac{43}{16}$
C $\frac{72}{16}$ I $\frac{84}{32}$ O $\frac{55}{30}$
D $\frac{58}{24}$ J $\frac{49}{6}$ P $\frac{56}{25}$
E $\frac{18}{8}$ K $\frac{81}{12}$ Q $\frac{69}{12}$
F $\frac{46}{4}$ L $\frac{28}{6}$ R $\frac{72}{10}$

**Test 9**

Change to improper fractions.

A $3\frac{3}{4}$ G $4\frac{7}{8}$ M $13\frac{3}{5}$
B $5\frac{1}{2}$ H $8\frac{7}{12}$ N $18\frac{1}{2}$
C $7\frac{5}{6}$ I $7\frac{1}{4}$ O $30\frac{5}{8}$
D $9\frac{5}{12}$ J $6\frac{1}{12}$ P $10\frac{4}{5}$
E $4\frac{1}{4}$ K $9\frac{3}{4}$ Q $11\frac{1}{16}$
F $6\frac{9}{10}$ L $4\frac{7}{10}$ R $42\frac{7}{8}$

**Test 10**

A 7 yr. = ▨ mo.
B 80 pt. = ▨ gal.
C 120 hr. = ▨ da.
D 40 fl. oz. = ▨ c.
E 17 gal. = ▨ qt.
F 7 hr. = ▨ min.
G 84 in. = ▨ ft.
H 5 yr. = ▨ da.
I 3 mi. = ▨ yd.

103

# 103

1 Tell the pupils to write the numerals in the proper form for computing. When most of the class has finished, provide answers for verifying the work. Do not impose any rigid time limit for these tests. Provide whatever reteaching is necessary. You will find the Reteaching Chart helpful.

# 104-106 Thinking straight

Expanded Notes for this lesson are on pages 317-319.

**Objectives**

The pupil finds out that there are mathematical sentences other than the equation. He learns how to make statements from the sentences and how to find solution sets for the sentences.

**Vocabulary**

New words page 104 true*, false*, none*; page 105 solution, description

**Comments**

The purpose of this lesson is to make the pupils aware that there are sentences in mathematics as well as in English. As the pupil progresses further in mathematics, he will have use for these sentences. The pupil has had previous experience with the equation, which is one type of sentence. In this lesson the pupil is introduced to sentences that express inequalities. The term sentence, as it is used in mathematics, refers to both equations and inequalities. For example, $x+4=8$, $x+4>8$, and $x+4<8$ are all sentences. A sentence of this kind is said to have no "truth value" (that is, it is neither true nor false). When $x$ has been replaced by a numeral, the sentence is a statement, which is either true or false. Consider the sentence $x+5=8$. If $x$ is replaced with 2, the resulting statement $2+5=8$ is false. If $x$ is replaced with 3, the resulting statement $3+5=8$ is true.

The numbers that make true statements when $x$ is replaced, are called the solution set for the sentence. For example, if you replace $x$ with names of whole numbers, many statements can be made from the sentence $x+1<5$. But the only whole numbers that will make true statements are 0, 1, 2, and 3. These numbers are the solution set for this sentence, and the numerals for them are often enclosed by braces {0, 1, 2, 3}.

It is important to point out to the pupil that up to this point in arithmetic he has been led to think only in terms of the replacements in a sentence that make

true statements. Now the pupil should realize that many replacements are possible, but that only certain ones make true statements and are, therefore, names of members of the solution set. Encourage the pupils to make many statements for a sentence and to pick true statements that determine the solution set. Some pupils may arrive at the solution sets immediately. Other pupils may need guidance.

It is important to note that for some sentences it is possible to list all of the members of the solution set. This is impossible for other sentences. For example, for the sentence $x+3>6$, it is impossible to list the entire solution set, because this set includes all numbers greater than 3.

Refer to the Expanded Notes, page 317, for further information and examples pertaining to this lesson.

**Answers**

**Pages 104-105:**

A The signs point in opposite directions. The point of the sign is toward the smaller quantity.

B $=$ C $=$ D $>$ E $>$

F $<$ G $<$ H $<$

I No; yes, yes, yes; 5, 4, 3, 2, 1, 0; yes

J 0, 1, 2, 3, 4, 5, 6, 7, 8

K Yes; yes; yes; no; 3, 4, 5

L 0, 1, 2, 3, 4, 5 M No; no; yes

N 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0 O 12; no

P 12, 13, 14, 20, 100, 1000, 1,000,000, or any whole number greater than 11

(block 2)

A All whole numbers greater than 14

B 0, 1, 2, and 3

C Any whole number greater than 2

D Any whole number greater than 12

E Any whole number from 0 to 5

F Any whole number from 0 to 13

G Any whole number greater than 7

H Any whole number from 9 to 16

I Any whole number from 7 to 12

J 16 K Any whole number greater than 6 L 7

(block 3)

A $>$ B $<$ C $>$ D $<$ E $=$

F $>$ G $<$ H $>$ I $=$ J $<$

K $>$ L $<$ M 0 N Yes; yes

**Page 106:**

O Yes; no

P No; no, $1\frac{1}{200}$ is not in the solution set for $5>n+4$.

Q 3 R Yes; yes; yes

S No, but the mixed numbers must be between 2 and 3

(block 2)

All the answers given assume that the pupils cannot operate with negative numbers.

A Any number greater than $1\frac{1}{2}$

B Any number less than $5\frac{1}{3}$

C Any number less than $1\frac{3}{4}$

D Any fraction greater than $\frac{11}{16}$, but not greater than $\frac{15}{16}$

E Any number greater than $\frac{7}{10}$

F Any number less than $1\frac{7}{10}$, but not less than $\frac{7}{10}$

G Any number less than $9\frac{2}{3}$, but not less than $1\frac{2}{3}$

H Any number greater than $1\frac{1}{6}$, but not greater than 17

I Any number greater than $3\frac{1}{12}$

J Any number greater than $4\frac{1}{2}$

K Any number greater than $12\frac{4}{5}$

L Any number greater than 1

M Any number less than $2\frac{5}{8}$, but not less than $\frac{5}{8}$

N Any number greater than $\frac{3}{4}$

O Any number greater than $18\frac{1}{3}$

P Any number greater than $38\frac{1}{4}$, but not greater than 41

Q Any number greater than $2\frac{1}{8}$

R Any number greater than $19\frac{4}{5}$

S Any number less than $3\frac{2}{3}$

T Any number less than $10\frac{3}{4}$

U Any number greater than $7\frac{3}{10}$

V Any number

**Keeping skillful:**

(block 1)

A $2\frac{5}{8}$

B $5\frac{11}{30}$

C $32\frac{1}{16}$

D $6\frac{8}{15}$

E $87\frac{23}{24}$

F $22\frac{5}{9}$

G $2\frac{1}{2}$

H $279\frac{7}{15}$

(block 2)

A $8\frac{5}{6}$

B $15\frac{2}{3}$

C $\frac{3}{4}$

D $8\frac{1}{10}$

E $\frac{5}{9}$

F $3\frac{1}{3}$

G $8\frac{2}{3}$

H $11\frac{5}{6}$

I $19\frac{1}{10}$

(block 3)

A 99 ft.

B 14,000 lb.

C 192 qt.

D 9 da.

E 3 yr.

F 684 in.

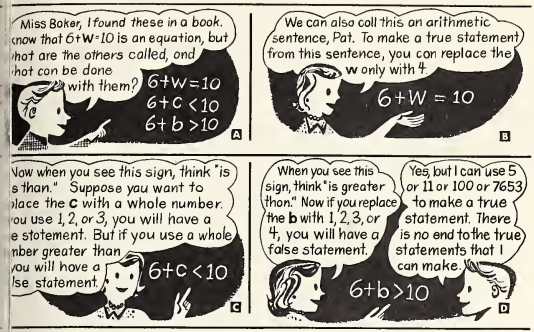
G 80 min.

H 30 fl. oz.

I 4 yr.

J 72 fl. oz.

## Thinking straight


Miss Baker, I found these in a book. I know that 6+w=10 is an equation, but what are the others called, and what can be done with them?
6+w=10
6+c<10
6+b>10
A
We can also call this an arithmetic sentence, Pat. To make a true statement from this sentence, you can replace the w only with 4
6+W = 10
B
Now when you see this sign, think "is less than." Suppose you want to replace the c with a whole number. If you use 1, 2, or 3, you will have a true statement. But if you use a whole number greater than 3, you will have a false statement.
6+c<10
C
When you see this sign, think "is greater than." Now if you replace the b with 1, 2, 3, or 4, you will have a false statement.
Yes, but I can use 5 or 11 or 100 or 7653 to make a true statement. There is no end to the true statements that I can make.
6+b>10
D

In Picture A look at the signs for "is less than" and "is greater than." How are the signs different?

For each exercise below, make a true statement by putting the sign for "is less than," "is greater than," or "is equal to" where it belongs.

3

16 + 5 ⟶ 21
4 × 15 ⟶ 12 × 5
13 + 10 ⟶ 9 + 12
17 ⟶ 56 ÷ 4
71 + 4 ⟶ 4 × 19
21 − 8 ⟶ 12 + 2
10 ÷ 5 ⟶ 10 ÷ 2

Introduction to signs of inequality

**I** Jim said, "I have 9 records. I have more records than you have, Don." Either of the sentences shown below expresses what Jim said.

4

$9 > k$ — Holds a place for the numeral that tells how many records Don has

$k < 9$

Could Don have 9 records? Could he have 8? 7? 6? What other numbers of records could he have? Could he have none?

In the sentences $9 > k$ and $k < 9$, you can replace $k$ with any numeral, but with some numbers you will get false statements. For example, if you write $k > 11$, you have a false statement. If you want to make true statements, here is a set of nine whole numbers that you can use. What are the nine whole numbers in the set?

Now study the arithmetic sentence $6 + c < 11$. Can you use 1 to replace $c$ and make a true statement? Can you use 2? Can you use 0? Can you use 6? What other whole numbers can you use and make true statements?

There is a set of six whole numbers you can use to make true statements for the sentence $5 + c < 11$. Each number in this set is a solution. The set of six whole numbers is a solution set for $5 + c < 11$. Write this solution set.

Is 13 in the solution set for the sentence $13 - d > 1$? Is 12 in the solution set? Is 0 in the solution set?

Write the solution set of twelve whole numbers for $13 - d > 1$.

What is the least whole number in the solution set for $4 + 7 < b$? Is there a greatest whole number in the solution set?

**P** Write ten of the whole numbers in the solution set for $4 + 7 < b$.

For each sentence below, write a short description of the solution set of whole numbers. For example, for Sentence A you might write, "All whole numbers greater than 14." For Sentence B you might write, "0, 1, 2, and 3."

5

A $8 + s > 22$
B $t + 1 < 5$
C $r + 5 > 7$
D $9 < d - 3$
E $10 + b < 16$
F $18 - k > 4$
G $14 < a + 7$
H $8 > 16 - n$
I $12 - f < 6$
J $b - 6 = 10$
K $13 < g + 7$
L $a + 19 = 26$

Make a true statement for each exercise below, using the sign $<$, $>$, or $=$.

6

A $\frac{1}{3} + \frac{1}{3} \longrightarrow \frac{1}{2}$
B $1\frac{1}{4} - \frac{2}{3} \longrightarrow 2$
C $7 \longrightarrow \frac{3}{4} + \frac{9}{10}$
D $\frac{1}{2} - \frac{1}{2} \longrightarrow 1$
E $\frac{15}{16} \longrightarrow \frac{1}{2} + \frac{7}{16}$
F $\frac{3}{4} + \frac{3}{4} \longrightarrow 1$
G $13\frac{1}{4} - 1\frac{3}{4} \longrightarrow 12$
H $119 \longrightarrow 2 \times 59$
I $36 - 35\frac{1}{2} \longrightarrow \frac{2}{3}$
J $45 \div 3 \longrightarrow 15\frac{1}{2}$
K $29\frac{5}{8} - 11 \longrightarrow 15$
L $17 + 2\frac{1}{10} \longrightarrow 20$

**M** There is only one whole number in the solution set for $5 > n + 4$. What is this whole number?

7

**N** Now see if there are any proper fractions in the solution set for $5 > n + 4$. If you use $\frac{11}{12}$, will you have a true statement? If you use $\frac{99}{100}$, will you have a true statement?



# 104

1. Let pupils take turns reading the dialogue in Pictures A to D. Then call on pupils to read the arithmetic sentences in Pictures C and D and replace the place holders with the numerals suggested in the dialogue. These replacements will make true statements.
2. Have the pupils note that the sign points to numerals representing the smaller number.
3. Assign Exercises B to H as written work. Tell the pupils that first they should do the adding, subtracting, etc., indicated on both sides of the missing sign in each statement. Use the answer or answers to determine which sign will make a true statement.
4. Treat this exercise orally. Let the pupils use the suggested numerals to replace the letters in both sentences. Stress that it is possible that Don has no records.

# 105

1. Have the pupils replace the $k$ in the two arithmetic sentences in Exercise J with the names of the numbers they select. Let them decide whether the statements formed are true or false. Show the pupils that the solution set can be written in this way: {0, 1, 2, 3, 4, 5, 6, 7, 8}.
2. Treat Exercises K and L in the same way as J.
3. In Exercises M and N, tell the pupils to think of the sentence as an equation and find the numeral that makes a true sentence when it replaces $d$. When this numeral is known, they can find the numbers in the solution set.
4. In connection with Exercises O and P, bring out that there is no greatest whole number that satisfies the requirements.
5. Assign Exercises A to L as written work.
6. Assign Exercises A to L as written work.
7. Use Exercises M and N to point out that although there is only one whole number in the solution set, there are many fractions.

**1**

O Does the solution set for $5 > n + 4$ contain proper fractions? Is there any end to the number of proper fractions in the solution set?

P Does the solution set for $5 > n + 4$ contain any mixed numbers? For example, can you replace $n$ with $1\frac{1}{200}$?

**2**

Q There is only one whole number in the solution set for $3 - d < 1$. What is this whole number?

R Now see if there are any mixed numbers in the solution set for $3 - d < 1$. Will you have a true statement if you use $2\frac{1}{100}$? If you use $2\frac{24}{25}$? Are all mixed numbers greater than 2 but less than 3 in the solution set?

S Is there any end to the number of mixed numbers in the solution set for $3 - d < 1$?

**3**

For each sentence below, name three solutions from the solution set of whole numbers, mixed numbers, and proper fractions. For the first sentence you could use any number greater than $1\frac{1}{2}$. For example, you could use $1\frac{3}{4}$, 2, $3\frac{1}{10}$. Each of these numbers is a solution for the sentence.

A $8\frac{1}{2} + t > 10$

B $r - \frac{1}{3} < 5$

C $6 > a + 4\frac{1}{4}$

D $\frac{15}{16} - g < \frac{1}{4}$

E $\frac{4}{5} + m > 1\frac{1}{2}$

F $k - \frac{7}{10} < 1$

G $b - 1\frac{2}{3} < 8$

H $17 - c < 15\frac{5}{6}$

I $d + 1\frac{11}{12} > 5$

J $21 < m + 16\frac{1}{2}$

K $w - 8\frac{4}{5} > 4$

L $25 + r > 26$

M $d - \frac{5}{8} < 2$

N $18\frac{1}{4} + m > 19$

O $16 + 2\frac{1}{3} < p$

P $41 - k < 2\frac{3}{4}$

Q $z + 4\frac{3}{8} > 6\frac{1}{2}$

R $n - 9\frac{4}{5} > 10$

S $26\frac{1}{2} + e < 30$

T $40\frac{1}{2} - 29\frac{3}{4} > y$

U $2\frac{7}{10} + 4\frac{3}{5} < f$

V $n + 5\frac{9}{16} > 5\frac{1}{4}$

## Keeping skillful **4**

Find the sum.

A $\frac{1}{2}, \frac{3}{8}, \frac{5}{8}, \frac{11}{12}$

B $1\frac{3}{8}, \frac{7}{10}, 3\frac{1}{5}$

C $9\frac{7}{16}, \frac{3}{4}, 21\frac{3}{8}, \frac{1}{2}$

D $\frac{9}{10}, 2\frac{7}{15}, 3\frac{1}{6}$

E $60\frac{5}{6}, \frac{3}{8}, 11\frac{3}{5}, 15\frac{1}{12}$

F $4\frac{1}{3}, 17\frac{5}{8}, \frac{2}{3}$

G $\frac{7}{12}, \frac{19}{24}, \frac{3}{8}, \frac{1}{4}, \frac{1}{2}$

H $84\frac{3}{5}, 119\frac{2}{3}, 75\frac{1}{5}$

A $s - 5\frac{1}{4} = 3\frac{7}{12}$

B $16\frac{1}{3} - r = \frac{5}{6}$

C $d + 10\frac{3}{4} = 11\frac{1}{2}$

D $16\frac{7}{10} - 8\frac{3}{5} = m$

E $8\frac{7}{9} + a = 9\frac{1}{3}$

F $20\frac{1}{3} - 17 = s$

G $b + 9\frac{7}{12} = 18\frac{1}{4}$

H $c - \frac{5}{6} = 11$

I $32 - w = 12\frac{9}{10}$

A 1188 in. = ▩ ft.

B 7 T. = ▩ lb.

C 48 gal. = ▩ qt.

D 216 hr. = ▩ da.

E 1095 da. = ▩ yr.

F 19 yd. = ▩ in.

G 4800 sec. = ▩ min.

H 60 tbs. = ▩ fl. oz.

I 208 wk. = ▩ yr.

J 9 c. = ▩ fl. oz.

106

# 106

1 Guide the pupils so that they see that there is an unlimited number of proper fractions in the solution set, but that a mixed number would make a false statement.

2 Use Exercises Q, R, and S to point out that although there is only one whole number in the solution set there is an unlimited number of mixed numbers which are greater than 2, but less than 3.

3 Assign Exercises A to V as written work. Remind the pupils that if they cannot find the solution set by inspection or trial and error, they can treat each sentence as if it were an equation and find the numeral that replaces the letter and makes a true statement. Then they can use the number as a starting point to decide what numbers are in the solution set for the original sentence.

4 Assign these exercises as a separate lesson.

# 107-109 Thinking straight

Expanded Notes for this lesson are on pages 320-321.

**Objectives**

The pupil learns ways of regrouping when more than one operation is indicated. He learns that multiplication and division can be indicated in other ways besides using the multiplication and the division signs.

**Vocabulary**

New words page 107 product, examine*, whenever*; page 108 matters*, quotient, earlier*, expressions*

**Comments**

In their work with multiple-step problems (see pages 26-30 and 49-51 of the pupils' book), the pupils have had to deal with equations that involved a grouping symbol (parentheses). The pupils learned to do the work inside the parentheses first. Starting with similar equations in this lesson (see Example A on page 107) they are led to see that there are other ways of grouping when several operations are indicated.

On page 107 the pupils learn how to group numbers to handle the combined operations of multiplication and addition. They learn that when there are two addends that are to be multiplied by the same number, they can either add first and then multiply or multiply each addend by the number and then add. The answer will be the same. On page 108 they learn how to handle subtraction and multiplication. Again, either process can be treated first, and the answer will be the same. And on page 109 they learn to handle addition and subtraction when they appear with division. What they are learning, of course, is an application of the fundamental *distributive law*, but this term should not be used with the pupils. They will need to understand such applications when they study the lessons on mental computation that follow (pages 110-111).

In this lesson pupils learn that they do not need to use the multiplication sign when parentheses are used. When an expression within parentheses is to be multiplied by a number, the number may be written before

or after the expression. The pupils learn that a pair of numerals written in the same way as a fraction numeral or ratio ($\frac{50}{15}$, for example) may indicate division, the top numeral representing the dividend and the bottom one, the divisor. They should be cautioned again to read the text very carefully when they encounter numerals in this form, so that they know what interpretation to put on them. There will be more work later with this way of writing division.

This lesson presents many new ideas to the pupils. They need to become comfortable with all these different ways of recording mathematical ideas, for as they progress in their study of mathematics, they will find fewer uses for operational signs and will make more use of grouping.

Along with the ideas of commutative and associative laws that the pupils have previously studied, the distributive law should help pupils understand arithmetic better.

**Answers**

**Page 107:**

A Add 9 and 30, then multiply 39 by 13; add 9 and 30, then multiply 13 by 39; yes

B $5(8+16)=n$

C The addition

D $13\times39=d$

E 507 F Yes G No

NOTE: For Exercises H to O the pupils are to indicate whether they add first or multiply first. They may show multiplication by using either parentheses or the times sign.

H $15(25)=[375]$; $15(6)+15(19)=[375]$

I $24(71)=[1704]$; $24(62)+24(9)=[1704]$

J $(31)7=[217]$; $(16)7+(15)7=[217]$

K $22(47)=[1034]$; $22(31)+22(16)=[1034]$

L $9(159)=[1431]$; $9(106)+9(53)=[1431]$

M $125(15)=[1875]$; $125(5)+125(10)=[1875]$

N $200(12)=[2400]$; $200(3)+200(9)=[2400]$

O $550(43)=[23,650]$; $550(12)+550(31)=[23,650]$

**Page 108:**

(block 1)

A Subtract 17 from 45 and multiply the answer by 11.

B No, the answer will be the same.

C $11\times28=s$; 308

D $495-187=308$; yes

E No

F $33(9)=[297]$; $33(37)-33(28)=[297]$

G $(6)12=[72]$; $(13)12-(7)12=[72]$

H $25(26)=[650]$; $25(38)-25(12)=[650]$

I $8(12)=[96]$; $8(29)-8(17)=[96]$

J $5(14)=[70]$; $5(26)-5(12)=[70]$

K $30(2)=[60]$; $30(18)-30(16)=[60]$

L $200(25)=[5000]$; $200(50)-200(25)=[5000]$

M $400(40)=[16,000]$; $400(100)-400(60)=[16,000]$

(block 2)

A $50\div5=[10]$; yes; by adding 15 and 35; 10

B Below the line

C The addition

D 50 divided by 5; 10

**Page 109:**

E $3+7=10$; yes

F Add them.

G Subtract 36 from 68, then divide the answer by 4.

H $\frac{68-36}{4}=t$

I The subtraction has been done, and 32 is written over 4 to show that we are to divide. 8

J $17-9=8$; The answer is the same as for Examples N and O.

K $\frac{68}{4}-\frac{36}{4}=t$

L No

M $\frac{156}{12}=[13]$; $\frac{48}{12}+\frac{108}{12}=[13]$

N $\frac{16}{4}=[4]$; $\frac{92}{4}-\frac{76}{4}=[4]$

O $\frac{133}{7}=[19]$; $\frac{49}{7}+\frac{84}{7}=[19]$

P $\frac{45}{15}=[3]$; $\frac{75}{15}-\frac{30}{15}=[3]$

Q $\frac{189}{27}=[7]$; $\frac{54}{27}+\frac{135}{27}=[7]$

R $\frac{253}{23}=[11]$; $\frac{184}{23}+\frac{69}{23}=[11]$

S $\frac{48}{12}=[4]$; $\frac{132}{12}-\frac{84}{12}=[4]$

T $\frac{212}{53}=[4]$; $\frac{318}{53}-\frac{106}{53}=[4]$

U $\frac{902}{41}=[22]$; $\frac{410}{41}+\frac{492}{41}=[22]$

## Thinking straight

A $13\times(9+30)=d$

B $13(9+30)=d$

C $13(39)=d$

D $13(9)+13(30)=d$

A In Example A above, how would you find the numeral that replaces $d$? If the equation were $(9+30)\times 13=d$, how would you find the numeral that replaces $d$? Would the answer be the same as the answer in Example A?

B Now look at Example B. This equation shows one way to rewrite the equation in Example A. Notice that when you use parentheses, you can leave out the multiplication sign. Now rewrite $5\times(8+16)=n$ without using the multiplication sign. 1

C Example C shows another way to rewrite the equation in Example A. In Example C, part of the computation called for in Examples A and B has been done. What part has been done? 2

D Rewrite the equation in Example C, using a multiplication sign. When you do this, you may leave out the parentheses if you want to.

E Whenever you multiply, the answer is called the product. What is the product of 13 times 39? 3

F Now find the numeral that replaces $d$ in Example D. Is the answer the same as the answer to the equation in Example C? When you find the numeral that replaces the letter in an equation, you have solved the equation. 4 5

G Examine the equations in Examples A and D again. Notice that you can add 30 and 9 and then multiply the sum by 13. Or you can multiply 9 by 13 and 30 by 13 and then add the products. Does the way you solve the equation make a difference in the answer that you get? 6

Solve each equation below in two ways. For each way, rewrite the equation in the way that shows how you solved it.

H $15(6+19)=q$
I $24(62+9)=c$ 7
J $(16+15)7=k$
K $22(31+16)=n$
L $9(106+53)=d$
M $125(5+10)=t$
N $200(3+9)=m$
O $550(12+31)=b$



E $(45-17)\times 11=s$

F $11(45-17)=s$

G $11(28)=s$

H $11(45)-11(17)=s$

A How would you solve the equation in Example E above? 1

B The equation in Example F shows the equation in Example E rewritten without the multiplication sign. Do you think it matters whether the equation is rewritten as $11(45-17)=s$ or as $(45-17)11=s$? Why? 2

C Now rewrite the equation in Example G, using the multiplication sign. Leave out the parentheses if you want to. What is the product of 11 and 28? 3

D Solve the equation in Example H. Is the answer the same as the product you found in Example G? 4

E Examine the equation in Example E again. Does it make any difference in the answer whether you subtract before you multiply or multiply before you subtract? 5

Solve each equation below in two ways. For each way, rewrite the equation in the way that shows how you solved it.

F $33(37-28)=f$
G $(13-7)12=n$ 6
H $25(38-12)=m$
I $8(29-17)=a$
J $5(26-12)=s$
K $30(18-16)=z$
L $200(50-25)=c$
M $400(100-60)=e$

A Solve the equation in Example I at the top of the next page. Did you have 50 before you divided? How did you get it? Whenever you divide, the answer is called the quotient. What is the quotient when you divide 50 by 5? 7

B Look at Example J. This equation shows one way to rewrite the equation in Example I. Notice that the division sign (÷) is not used. Where is the divisor, 5, written? 8

C The equation shown in Example I is rewritten again in Example K. What part of the computation called for in Example I has been done now? 9

D Earlier in your work, you learned that expressions like 50 over 5 may be fraction numerals or ratios. In Example K, what does the expression 50 over 5 mean? What is the quotient?



# 107

1 Tell pupils that when the multiplication sign is left out, as in Example B, the work is done exactly as it is in Example A. The work within parentheses is done first.
2 The pupils should see that the addition indicated in Examples A and B has been done.
3 Emphasize that the name for the answer to a multiplication example is the product.
4 Pupils should see that they must first do all the multiplying and then do the addition.
5 Get them to see that "solving the equation" means finding the numeral that replaces the letter.
6 Get pupils to see that either multiplication or addition may be done first in this type of example.
7 Assign Exercises H to O as written work.

# 108

1 Pupils should see that Example E is solved by subtracting first, then multiplying.
2 Explain that 11 may be written before or after the expression in parentheses here, but a problem situation could determine its placement.
3 Have the pupils determine where the 28 comes from. Then let them solve the equation.
4 Help pupils see that they must first do all the multiplication, then the subtraction.
5 Get the pupils to generalize that in examples like these, multiplication or subtraction may be done first.
6 Assign these exercises as written work.
7 Discuss the solution of the equation and the meaning of *quotient*.
8 Be sure pupils understand how division is indicated and which number is the divisor.
9 Pupils should see that addition has been done.

D $(15+35)\div 5=n$

J $\frac{15+35}{5}=n$

K $\frac{50}{5}=n$

L $(15\div 5)+(35\div 5)=n$

M $\frac{15}{5}+\frac{35}{5}=n$

N $(68-36)\div 4=t$

O $\frac{32}{4}=t$

P $(68\div 4)-(36\div 4)=t$

E Solve the equation in Example L. Is your answer the same as the answer you found for Example K?

F The equation in Example M shows another way to write the equation in Example L. Here, too, you first divide 15 by 5 and 35 by 5. Then what do you do with the two quotients?

G How would you solve the equation in Example N?

H Rewrite the equation in Example N, using the form shown in Example J.

I Explain how Example O shows another way to write the equation in Example N. What is the quotient?

J Solve the equation in Example P. What do you notice about the answer?

K Rewrite the equation in Example P, using the form shown in Example M.

L Now think about the equations that you have been solving. Sometimes you have first added or subtracted and then divided. Sometimes you have divided first and then added or subtracted. Has the way you solved these equations made any difference in the answer? 5

Rewrite each equation below in two ways without using a division sign (÷). Then solve your equations.

M $(48+108)\div 12=n$ 6

N $(92-76)\div 4=p$

O $(49+84)\div 7=b$

P $(75-30)\div 15=v$

Q $(54+135)\div 27=h$

R $(184+69)\div 23=x$

S $(132-84)\div 12=d$

T $(318-106)\div 53=z$

U $(410+492)\div 41=a$

109

## 109

1 After the pupils have answered the questions in Exercises E and F, have them compare the actual work involved in both equations. Help pupils understand that Examples J, K, L, and M are different ways of writing Example I.

2 Have the pupils compare Examples N and I.

3 Use Exercises H and I to make comparisons between Examples N and O and Examples J and K.

4 For Exercise J, bring out that although pupils divide first in Example P and then subtract, the answer is the same as that obtained by subtraction first and then division (as in Examples N and O).

5 Help the pupils see that in examples like these it makes no difference whether they add or subtract the numbers and then divide, or whether they divide the numbers first and then add or subtract.

6 Assign Exercises M to U as written work.

# 110 Thinking straight

Expanded Notes for this lesson are on pages 321-322.

**Objectives**

The pupil learns to use the distributive law.

**Vocabulary**

New words page 110 Ted*

**Comments**

The class should be guided in using the easiest manageable groups, which will not always be the same for each pupil. For example, to multiply 33 by 8, the pupils can think of 33 as 30+3 and multiply both 30 and 3 by 8 and add. Or they can think of 33 as 40−7, multiply both 40 and 7 by 8, and then subtract. The majority of pupils will find numerals ending in zero most convenient. Some pupils will not need to regroup, but will be able to compute directly.

In working with Exercise H, $(5\times 9)+(3\times 9)$, the pupils apply principles from the preceding lesson and regroup the numerals as 9(5+3) and then 9(8). In an example such as Exercise K, $(4\times 6)+(4\times 9)$, they regroup first as 4(6+9), then as 4(15). But since $4\times 15$ may still be difficult to handle mentally, they can be taught to regroup even further. 4(15) can be thought of as 4 (10+5), and then as 4(10)+4(5), or 60.

**Answers**

(block 1)

A He wanted to find the total cost of both kinds of seeds.

B The bean seeds and the carrot seeds were the same price (15¢), and there were 9 packages in all.

C 5($.15+4($.15)=[$1.35]; 9($.15)=[$1.35]

D Add them; 264

E 40; 7; subtract them

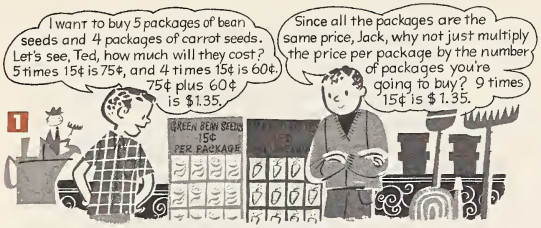
F —

G 50; 40; add them; 270

(block 2)

| | | | | |
|---|---|---|---|---|
| A 196 | D 144 | G 315 | J 424 | M 119 |
| B 294 | E 140 | H 72 | K 60 | N 99 |
| C 426 | F 176 | I 80 | L 231 | |

## Thinking straight


I want to buy 5 packages of bean seeds and 4 packages of carrot seeds. Let's see, Ted, how much will they cost? 5 times 15¢ is 75¢, and 4 times 15¢ is 60¢.
75¢ plus 60¢ is $1.35.
Since all the packages are the same price, Jack, why not just multiply the price per package by the number of packages you're going to buy? 9 times 15¢ is $1.35.
GREEN BEAN SEEDS 15¢ PER PACKAGE

1

2 A In the picture, why did Jack add 60¢ and 75¢?

B Why did Ted multiply 15¢ by 9?

C Write an equation to show how each boy solved the problem. Which of these two ways would you use to solve the problem? Why?

3 D To find the product of 33 and 8 in your head, you can think of 33 as 30 + 3. Then multiply 30 by 8 and 3 by 8. Now what do you do with these two products? What is the answer?

4 E Or you can multiply 33 by 8 by thinking of 33 as 40 − 7. You then multiply ■ by 8 and ■ by 8. What do you do with these two products?

5 F Which way of multiplying 33 by 8 is easier for you, the way shown in Exercise D or that shown in Exercise E?

6 G Suppose you want to multiply 45 by 6. You can think of 45 as ■ − 5 or as ■ + 5. If you think of 45 as 40 + 5, what do you do with the product of 40 and 6 and the product of 5 and 6? What is the answer?

7 Multiply in your head. Use the way that is easier for you.

| | | | |
|---|---|---|---|
| A | 7 × 28 = t | H | (5 × 9) + (3 × 9) = s |
| B | 3 × 98 = p | I | (8 × 7) + (8 × 3) = c |
| C | 6 × 71 = n | J | 8 × 53 = n |
| D | 9 × 16 = c | K | (4 × 6) + (4 × 9) = a |
| E | 4 × 35 = m | L | 3 × 77 = g |
| F | 8 × 22 = b | M | (7 × 8) + (7 × 9) = r |
| G | 5 × 63 = e | N | (9 × 3) + (9 × 8) = f |

110 Mental computation; multiplication

# 110

1 Have the dialogue read and bring out that Jack and Ted are using mental arithmetic.

2 Use Exercises A to C to evaluate the two methods of mental computation. Point out that Jack's method is like Example D on page 107 and Ted's method is like Example B.

3 Point out that the method used in Exercise D is like the method Ted used. 8(33) and 9($.15) can both be thought of in this way: 8(30 + 3) and $.15(4 + 5). Compare $.15(4 + 5) with Example B on page 107.

4 Compare the method described in Exercise E with that used in Example H on page 108.

5 Let the pupils choose whichever method they like. Bring out that sometimes one method is easier than the other.

6 Discuss the answers to the questions in Exercise G.

7 Treat Exercises A to N orally.

# 111 Thinking straight

Expanded Notes for this lesson are on page 322.

**Objectives**

The pupil learns how to divide by the short-division method.

**Vocabulary**

New words page 111 yet*, front*

**Comments**

All division is a combination of written and mental arithmetic, but short division increases the demand for doing more work mentally. The pupil keeps in mind the number that he must subtract and subtracts mentally. Short division is a highly useful technique, and the pupils, especially the able ones, should be encouraged to make use of this technique.

Because of their understanding of long division as it is taught in this program, pupils will be quite aware of the true meaning of each numeral as it appears in the quotient; that is, that it represents hundreds, tens, or ones, as the case may be. When the pupil has made the first division, he knows how many places there will be in the answer. In "long" division he has been taught to think in terms of the entire number. The partial quotients are estimations and usually end in a zero (or zeros). After the final addition of partial quotients, the zeros have been replaced by other numerals.

In the short-division method presented here, the pupil is again taught to work with the entire number. He also makes use of the estimation technique used in "long" division; that is, he rounds down the dividend to get a number that forms a basic fact with the divisor. In an example such as 1578 divided by 6, he rounds the dividend to 1200. 1200 divided by 6 equals 200. But instead of writing 200, he writes only the first figure in the place for the quotient (at the side, as in "long" division). He knows this 2 stands for 200 and that there will be three places in the quotient. He keeps in his mind what the product of 6 times 200 is. He can be guided to "picture" the 1200 written directly under

the dividend to help in subtracting mentally. The only mental subtraction that needs to be done is to subtract 2 from 5 to get 3. Since there are two zeros in 1200, the 78 above them can be temporarily ignored. To help the pupil "see" the next number he has to divide, point out that the last two digits of 378 are in the original dividend, and that by writing the hundreds' figure before them, he can see the entire numeral. (See Cartoons C and D.) Then, rounding 378 to 360 and dividing it by 6 gives the tens' figure in the quotient. Only the first figure of 60 is written. The method used for mental subtraction described above can be repeated here, and the new remainder can be indicated in the original dividend. Dividing 18 by 6 gives the ones' figure.

Zeros in answers with short division may cause difficulty, but if the pupils know beforehand how many places there should be in the answer, there will be no serious problem. The zero situation occurs in Example B below the cartoon. Because 721 divided by 3 rounds down to 600 divided by 3, the pupils know in advance that there must be three places in the answer.

When they divide the next number, 120, by 3, they know that the answer is 40, but if they write only the 4, as is customary in short division, there will be only two places in the answer. Therefore, they must write the zero to get an answer with three places.

**Answers**

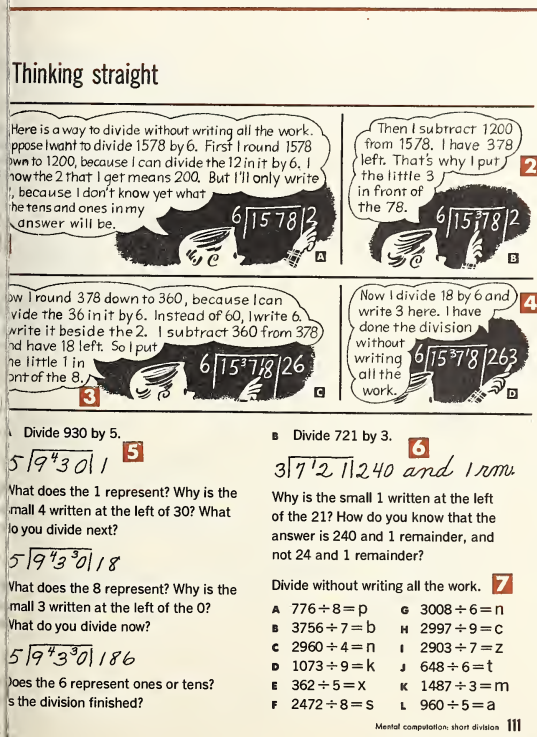
(block 1)

A One hundred; to show the next number to be divided; 430; 8 tens; to show the number to be divided next; 30; ones; yes

B To show that 121 is to be divided. You know that there must be three places in the answer because 600 divided by 3 shows the answer is at least 200.

(block 2)

A 97
B 536 and 4 rm. or $536\frac{4}{7}$
C 740
D 119 and 2 rm. or $119\frac{2}{9}$
E 72 and 2 rm. or $72\frac{2}{5}$
F 309
G 501 and 2 rm. or $501\frac{1}{3}$
H 333
I 414 and 5 rm. or $414\frac{5}{7}$
J 108
K 495 and 2 rm. or $495\frac{2}{3}$
L 192


Thinking straight
Here is a way to divide without writing all the work. ppose I want to divide 1578 by 6. First I round 1578 own to 1200, because I can divide the 12 in it by 6. I now the 2 that I get means 200. But I'll only write , because I don't know yet what he tens and ones in my answer will be.
6|1578|2
A
Then I subtract 1200 from 1578. I have 378 left. That's why I put the little 3 in front of the 78.
2
6|1578|2
B
ow I round 378 down to 360, because I can vide the 36 in it by 6. Instead of 60, I write it beside the 2. I subtract 360 from 378 nd have 18 left. So I put he little 1 in ont of the 8.
3
6|1578|26
C
Now I divide 18 by 6 and write 3 here. I have done the division without writing all the work.
4
6|1578|263
D
Divide 930 by 5.
5
5|9430|1
What does the 1 represent? Why is the small 4 written at the left of 30? What do you divide next?
5|9430|18
What does the 8 represent? Why is the small 3 written at the left of the 0? What do you divide now?
5|9430|186
Does the 6 represent ones or tens? Is the division finished?
B Divide 721 by 3.
6
3|712|1240 and 1 rm.
Why is the small 1 written at the left of the 21? How do you know that the answer is 240 and 1 remainder, and not 24 and 1 remainder?
Divide without writing all the work.
7
A 776÷8=p
B 3756÷7=b
C 2960÷4=n
D 1073÷9=k
E 362÷5=x
F 2472÷8=s
G 3008÷6=n
H 2997÷9=c
I 2903÷7=z
J 648÷6=t
K 1487÷3=m
L 960÷5=a
Mental computation: short division
111

# 111

1 Make sure the pupils understand that the answer will be a 3-figure numeral because 1200÷6=200. Be sure they see that only the first digit is recorded.

2 Suggest to the pupils that they "picture" 1200 written under 1578 to help them subtract mentally. Point out that the remainder can be recorded by writing a 3 in front of the 78.

3 Emphasize that the pupils are finding the tens' figure in the answer.

4 Pupils should understand that they are finding the ones' figure. There is no remainder.

5 Work through Example A step by step.

6 Example B introduces a zero in the answer. Point out that 600÷3 gives an answer in the hundreds. 120÷3=40, but if only the first figure were written, there would not be three places in the answer. So 40 must be written.

7 Before assigning Exercises A to L, discuss Exercise F, which has an internal 0 in the answer.

# 112-116 Exploring problems

Exponded Notes for this lesson are on poges 323-325.

**Objectives**

The pupil applies his knowledge of rate to problem solving.

**Vocabulary**

New words **page 112** rosebushes*; **page 115** grass*, lawn*, weed*, killer*; **page 116** actual*, typing*

**Comments**

All problems about buying items at a stated price, traveling at so many miles per hour, etc., are based on the concept of rate. Traditionally, at this grade level, the pupil has already been taught to handle two types of rate problems: the special and the general. In the special case, the rate is given for one item (10¢ for 1 candy bar, 50 miles per hour, etc.) and the problem is to find the total for several of these items. In solving problems of this kind, the pupils were taught to use multiplication (in the examples above, for 6 candy bars, $6\times10¢$; for 3 hours, $3\times50$ miles). For the general case, in which the rate may be given in such forms as 10¢ for 3 bars or 150 miles in 4 hours, the pupils were taught to use two computational operations. Actually, as shown in this lesson (and as taught in *Seeing Through Arithmetic 5,* pages 215-220), two ratios (both expressing the same rate) are involved in both the special and general cases.

In both cases (10¢ for 3 bars or 10¢ per bar) 3 of the four terms in the two ratios are known. To solve the problem, it is convenient to make an equation of the two ratios and then compute to find the missing numeral. Suppose you want to buy 9 cans of soup, and the soup is priced at 3 cans for 43¢. How much money will you need for 9 cans? Two expressions of the same rate are evident in this problem: 3 cans per 43¢ and 9 cans per *n*¢. Three numbers are known: 3, 43, 9. The missing number for the cost of 9 cans is represented by *n*. The rate at which soup is sold can be expressed by the ratio 3/43. It can also be expressed by the ratio 9/n. The equation $3/43=9/n$ then gives a convenient way to find the numeral to replace *n*. The pupils have already had experience in writing ratios to express rates (see pages 32-37 in the pupils' book) and have learned how to find a numeral to complete one of a pair of equal ratios (see pages 56-58 in the pupils' book). In this lesson they review how to make the proper equation.

Stress the idea that it is not important which numeral becomes the first term in the first ratio. But once the pupil has selected the numeral that is to be the first term of that ratio, he must be consistent when he selects the first term in the other ratio. The first terms of the two ratios must represent the same thing—dollars, miles, items, etc. In the above example, if he chooses to write the price for 3 cans of soup as the first term of the first ratio, he must write *n* to hold a place for the price of 9 cans of soup as the first term of the equal ratio. Thus, instead of the equation above, his equation would be $43/3=n/9$. Point out to the pupils that all the illustrated examples in the text could have been written in another order; the equation for the problem on pages 112-113 could have been $28/12=x/3$; the equation for the problem on pages 113-114 could have been $25/230=150/m$, etc.

**Answers**

**Pages 115-116:**

A $\frac{500}{1}=\frac{3000}{[6]}$; Each first term stands for number of square feet of lawn. Each second term stands for number of pounds of grass seed. Because 500 can be multiplied by 6 to get 3000

B $\frac{15}{4500}=\frac{5}{[1500]}$; Each first term stands for number of pounds of grass seed. Each second term stands for number of square feet of lawn. Because 15 can be divided by 3 to get 5

C $\frac{1785}{15}=\frac{[119]}{1}$; \$1.19; 1785 and c stand for the number of cents paid. 15 and 1 stand for the number of pounds.

D $\frac{8}{1500}=\frac{[16]}{3000}$ or $\frac{1500}{8}=\frac{3000}{[16]}$; no; Since *b* stands for number of fluid ounces of weed killer, it must occupy the same position in its ratio as does 8, which also stands for the number of ounces of weed killer.

E 1 qt. = 32 fl. oz.; $\frac{8}{1500}=\frac{32}{[6000]}$ or $\frac{1500}{8}=\frac{[6000]}{32}$

F $\frac{3}{4}=\frac{[9]}{12}$ or $\frac{4}{3}=\frac{12}{[9]}$

$\frac{3}{10} = \frac{[15]}{50}$ or $\frac{10}{3} = \frac{50}{[15]}$

$\frac{5}{170} = \frac{1}{[34]}$ or $\frac{170}{5} = \frac{[34]}{1}$; $.34

$\frac{75}{4} = \frac{[300]}{16}$ or $\frac{4}{75} = \frac{16}{[300]}$; $3.00

$\frac{40}{6} = \frac{80}{[12]}$ or $\frac{6}{40} = \frac{[12]}{80}$

$\frac{6}{99} = \frac{[2]}{33}$ or $\frac{99}{6} = \frac{33}{[2]}$

$\frac{1}{4} = \frac{6}{[24]}$ or $\frac{4}{1} = \frac{[24]}{6}$

$\frac{2}{79} = \frac{[6]}{237}$ or $\frac{79}{2} = \frac{237}{[6]}$

$\frac{294}{15} = \frac{[98]}{5}$ or $\frac{15}{294} = \frac{5}{[98]}$

$\frac{765}{15} = \frac{[51]}{1}$ or $\frac{15}{765} = \frac{1}{[51]}$

$\frac{735}{15} = \frac{[2940]}{60}$ or $\frac{15}{735} = \frac{60}{[2940]}$

**Keeping skillful:**

| | | |
|---|---|---|
| A 240 oz. | A 1346 | A 30,558 |
| B 20 bu. | B 4557 | B $20\frac{4}{5}$ |
| C 26 yd. | C 9196 | C $48\frac{12}{31}$ |
| D 180 sec. | D 5815 | D 15,503 |
| E 6 fl. oz. | E 18,215 | E 322,072 |
| F 31,680 ft. | F 73,662 | F $35\frac{5}{59}$ |
| G 25 qt. | G 16,799 | G 270,315 |
| H 13 wk. | H 19,359 | H $53\frac{107}{160}$ |
| I 16,000 lb. | I 103,259 | I $24\frac{1}{4}$ |
| J 324 in. | J 8714 | J 441,303 |
| K 38 tbs. | K 33,643 | K 585,168 |

## xploring problems

**see**

A



B

Mr. Castle bought 12 rosebushes for $28. At this rate, he paid how much for 3 rosebushes? **1**

$\frac{12}{28}$ **2** — This ratio shows that the rosebushes were bought at the rate of 12 for $28.

You do not know how much 3 rosebushes cost. Look at Picture B.

Mr. Castle paid for 3 bushes at the same rate as for 12 bushes, but now the rate is expressed by a different ratio. **3**

$\frac{3}{x}$ **4** — Number of rosebushes; Number of dollars 3 rosebushes cost

$\frac{12}{28} = \frac{3}{x}$ **5** — The first term in each ratio stands for the number of rosebushes. The second term in each ratio stands for the number of dollars. You must find the numeral that replaces $x$.

The ratio 3 per $x$ expresses the same rate as the ratio 12 per 28.

Applying knowledge of rate to problem solving (reteaching)

# 112

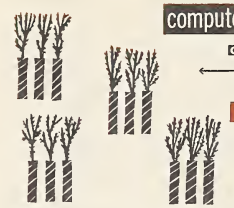
**1** Help the pupils see that a rate, expressed as 12 per 28, is involved in this problem, and that the first task is to find another ratio to express this same rate, using 3 as a first term in the other ratio.

**2** Discuss how the rate is expressed in written form as a ratio. Make sure the pupils understand what the numerals represent.

**3** Point out that they already know one of the numerals for the new ratio [3], but that they must find the other numeral, which will represent the cost of 3 rosebushes.

**4** Discuss this ratio. Point out that $x$, which stands for the cost of 3 rosebushes, is written below the line, as was the cost in the original ratio, and that the numeral for the number of bushes is placed above the line, as it was in the original ratio.

**5** Point out that the numeral that replaces $x$ must make the equation true; that is, the second ratio must equal the first ratio.



**compute**

C You can think of the 12 rosebushes as divided into groups of 3. There are 4 groups.

1 $\frac{12}{28}=\frac{3}{x}$ — 3 can be found by dividing 12 by 4.

D It takes $28 to pay for 4 groups of 3 bushes. So divide $28 into 4 equal groups to find how much a group of 3 bushes cost.

You can compute to find the numeral that replaces $x$.

2 $\frac{12}{28}=\frac{3}{x}$ 3 can be found by dividing 12 by 4. So the numeral that replaces $x$ can be found by dividing 28 by 4. 7 replaces $x$.

3 rosebushes cost $7.

**see**

A Mr. Castle also bought 150 flower bulbs priced at $2.30 for 25 bulbs. How much did he pay for the 150 bulbs?

4 $\frac{230}{25}$ This ratio shows that the bulbs were sold at the rate of $2.30, or 230¢, per 25 bulbs.

Now turn the page.

You do not know how much 150 bulbs cost.

B The rate for 150 bulbs is the same as the rate for 25 bulbs, but now the rate is expressed by a different ratio.

1 $\frac{m}{150}$ ← Number of cents 150 bulbs cost ← Number of bulbs

2 $\frac{230}{25}=\frac{m}{150}$ Do these terms stand for number of cents or for number of bulbs? You must find the numeral that replaces $m$. What do these terms stand for?

**compute**

It takes 6 groups of 25 bulbs to make the group of 150 bulbs.

C $\frac{230}{25}=\frac{m}{150}$ 3 150 can be found by multiplying 25 by 6.

It takes $2.30 to pay for each group of 25 bulbs. It takes 6 groups of $2.30 to pay for 6 groups of 25 bulbs.

D $\frac{230}{25}=\frac{m}{150}$ 4 150 can be found by multiplying 25 by 6. How do you find the numeral that replaces $m$? 1380 replaces $m$.

The 150 bulbs cost 1380¢, or $13.80.





# 113

1 Remind the pupils that they have already studied how to change a ratio to an equal ratio by multiplying or dividing both terms by the same number. Relate the text to Picture C. Point out that the pupils can divide 12 by 4 to get 3.

2 Have the pupils note in Picture D that for each group of 3 bushes, there is a group of $7. They can see that 4 groups of $7 make $28 and that in the equation they must divide 28 by 4.

3 After someone has read the text to the right of the equation, write the completed equation, $12/28=3/7$, on the board. Be sure the pupils understand that the equation is correct because both 12 and 28 can be divided by the same number, 4, to get 3 and 7. Discuss the fact that 7 means $7. Have the pupils read 3/7 as a rate (3 per 7).

4 Have pupils relate the picture to the problem and to the ratio that expresses the rate involved.

# 114

1 Remind pupils that because the numeral for the cost was used as the first term of the first ratio, a numeral for cost must also be used as the first term of the second ratio. Let a pupil explain what $m$ and 150 stand for.

2 Discuss the fact that the two ratios are equal because they express the same rate.

3 Ask: "How did we discover that there are 6 twenty-fives in 150? [150 can be found by multiplying 25 by 6.] If we multiply one term of a ratio, must we also multiply the other term by the same number?"

4 Have pupils read the text and relate it to the picture. Let someone check the computation by multiplying 230 by 6 on the chalkboard. Let another pupil rewrite the equation, putting in the numeral that replaces $m$. Be sure pupils can explain each of the terms in the two ratios.

Mr. Castle knows that he uses 1 lb. grass seed for 500 sq. ft. of lawn. w many pounds of grass seed will he ed for 3000 sq. ft. of lawn?

$$\frac{0}{} = \frac{3000}{k}$$

Each first term stands for ——.

Each second term stands for ——.

y must you multiply 1 by 6 to find numeral that replaces $k$?

Mr. Bills used 15 lb. of grass seed make 4500 sq. ft. of new lawn. He ed grass seed at a rate of 5 lb. per w many square feet of lawn?

$$\frac{15}{500} = \frac{5}{d}$$

What does each first term stand for?

What does each second term stand for?

y must you divide 4500 by 3 to find numeral that replaces $d$?

Mr. Bills paid $17.85 for the 15 lb. grass seed. He bought the seed at a e of how much per pound?

$$\frac{85}{.5} = \frac{c}{1}$$

What do 1785 and $c$ stand for?

What do 15 and 1 stand for?

D Mr. Castle knows that he should use 8 fl. oz. of weed killer for 1500 sq. ft. of lawn. How many fluid ounces of weed killer should he use for his 3000 sq. ft. of lawn?

$\frac{8}{1500} = \frac{b}{3000}$ or $\frac{1500}{8} = \frac{3000}{b}$ **4**

Does it make any difference which equation you use? Why is $b$ in a different place in the two equations?

E How many square feet of lawn will 1 qt. of weed killer cover?

THINK There are how many fluid ounces in 1 qt.? **5**

$\frac{8}{1500} = \frac{32}{n}$ or $\frac{1500}{8} = \frac{n}{32}$

F Mrs. Castle buys about 3 lb. of chicken to serve 4 persons. About how many pounds of chicken should she buy to serve 12 persons?

$\frac{3}{4} = \frac{x}{12}$ or $\frac{4}{3} = \frac{12}{x}$

G A meat loaf made with 3 lb. of ground meat will serve 10 persons. How many pounds of ground meat would be needed to make meat loaf to serve 50 persons?

H Alice paid $1.70 for 5 lb. of seed to feed her parakeet. She bought the seed at a rate of how many cents per pound?



lice has saved 75¢ in 4 weeks. **1** is rate, how much money can she ct to save in 16 weeks?

Irs. Castle paid 80¢ for buttons were priced at 40¢ for 6 buttons. many buttons did she buy?

Irs. Castle bought 6 dishcloths 9¢. The dishcloths were selling e rate of how many for 33¢?

hilip measured the distance een two towns on a road map and d it was 6 inches. 1 inch on the represented a distance of 4 miles. t was the actual distance between wo towns?

M Philip's father drove 237 mi. going from Half Day to Silver City. In the first 2 hr. he drove 79 mi. If he traveled at the same rate for the whole distance, how long did it take him to drive from Half Day to Silver City?

N Mr. Castle drove 294 mi. on 15 gal. of gasoline. At this rate, how far did he drive on 5 gal. of gasoline?

O In a typing test, Betty typed 765 words in 15 minutes. Betty typed at a rate of how many words per minute?

P Bill typed 735 words in 15 minutes. At this rate, how many words would he type in 1 hour? **2**

Now you should be able to make ratios that express rates and to use these ratios to solve problems.

## (eeping skillful **3**

5 lb. = ▩ oz.
0 pk. = ▩ bu.
8 ft. = ▩ yd.
min. = ▩ sec.
2 tbs. = ▩ fl. oz.
mi. = ▩ ft.
0 pt. = ▩ qt.
1 da. = ▩ wk.
T. = ▩ lb.
yd. = ▩ in.
9 fl. oz. = ▩ tbs.

A $8314 + z = 9660$
B $7565 - a = 3008$
C $n - 5067 = 4129$
D $c + 4898 = 10713$
E $36710 - 18495 = w$
F $64450 + 9212 = b$
G $80040 - d = 63241$
H $f + 7983 = 27342$
I $t - 57248 = 46011$
J $28050 - 19336 = x$
K $36450 + r = 70093$

A $6 \times 5093 = m$
B $728 \div 35 = x$
C $3000 \div r = 62$
D $37 \times 419 = v$
E $508 \times 634 = y$
F $2070 \div a = 59$
G $45 \times 6007 = n$
H $8587 \div 160 = c$
I $9021 \div k = 372$
J $867 \times 509 = w$
K $73 \times 8016 = b$

# 115

1 Ask: "How many 500's are in 3000? Why do we want to know? How do we find out?" Then let pupils answer the questions in the text.
2 Discuss the problem and the equation.
3 Make sure pupils know they are dividing 1785 by 15 to find the numeral that replaces c because 1 can be found by dividing 15 by 15. They are *not* dividing 1785 by the 15 in the second term.
4 Pupils should understand that if a numeral that represents ounces is used in the first term of the first ratio, then the numeral in the first term of the equal ratio must also represent ounces; but that if a numeral representing square feet is used in the first term of the first ratio, then the numeral in the first term of the equal ratio must also represent square feet.
5 Discuss Problems E to H and the ratio equations. Let pupils explain how they get the answers.

# 116

1 Assign Problems I to P as written work. Remind the pupils to express the amount of money as cents in the ratio equations, then change the answers to dollars if necessary.
2 Write the completed equations on the board and let the pupils verify their work. For each problem, discuss how the answer was found. Give individual help as necessary.
3 Assign these exercises as written work.

# 117-121 Exploring problems

Expanded Notes for this lesson are on pages 325-327.

**Objectives**

The pupil reviews how to use ratios to solve problems that involve comparisons.

**Vocabulary**

New words page 118 weaving*, wove; page 120 rink, fell*; page 121 job*, streets*

**Comments**

The pupils have just reviewed using ratios to make equations for rate problems (see page 120 of this *Teaching Guide*). They should have no trouble working problems involving comparisons, since both types of problem are solved in the same way. A comparison is expressed by a ratio, and an equal ratio is to be made, one term of which is known and the other of which is to be found. Again, stress the fact that it is not important which numeral is used as the first term of the first ratio, but that the first term of the second ratio must represent the same kind of thing. For example, if the first term of one ratio refers to Bob's money, the first term of the equal ratio must also refer to Bob's money. The pupils should understand that equations for the problems in the text could have been written in different ways; thus, the equation for the problem on pages 117-118 could have been 30/50=a/250.

The equations in this lesson are restricted to ones that can be solved by multiplying or dividing by a whole number. In a subsequent lesson (pages 143-145 of the pupils' book) the pupils will learn a more general procedure for solving equations of ratios. Later, after they have had more work with fractions, they will learn how to handle comparisons in which fractions and mixed numbers will be represented by terms of ratios.

**Answers**

Pages 120-121:

A 25/15=[125]/75; $1.25
B 45/30=9/[6]
C 300/250=600/[500] or 250/300=[500]/600
D 15/12=5/[4] or 12/15=[4]/5
E 24/16=3/[2] or 16/24=[2]/3
F 20/120=[120]/720 or 120/20=720/[120]
G 300/250=[30]/25 or 250/300=25/[30]
H 3/2=150/[100] or 2/3=[100]/150
I 4/3=[80]/60 or 3/4=60/[80]
J 40/30=8/[6] or 30/40=[6]/8
K 150/100=[1800]/1200 or 100/150=1200/[1800]
L 216/192=54/[48] or 192/216=[48]/54
M 240/160=[30]/20 or 160/240=20/[30]
N 6/5=[456]/380 or 5/6=380/[456]

Keeping skillful:

A $13\frac{19}{24}$
B $5\frac{11}{20}$
C $12\frac{1}{3}$
D $9\frac{2}{15}$
E $2\frac{5}{8}$
F $3\frac{31}{36}$
G $50\frac{5}{21}$
H $7\frac{1}{2}$
I $9\frac{7}{10}$
J 13
K $13\frac{1}{12}$
L $22\frac{5}{7}$
M $25\frac{4}{5}$
N $16\frac{11}{24}$
O $\frac{13}{20}$
P $22\frac{2}{3}$
Q $15\frac{1}{9}$
R $16\frac{5}{8}$

**see**

A

Philip and Paul go to different schools. Philip spends 50¢ for round-trip bus fare each day, and Paul spends 30¢ a day. When Philip has spent $2.50 for bus fare, how much will Paul have spent? 1

$\frac{50}{30}$ 2

This ratio compares the number of cents Philip spends in a day with the number of cents Paul spends in a day.

B

Imagine that Philip has spent the $2.50. You can make a ratio that compares what he has spent with the amount that Paul will have spent.

$\frac{250}{a}$ 3

Number of cents Philip has spent

Number of cents Paul will have spent



This new ratio is equal to the ratio 50 to 30.

$\frac{50}{30} = \frac{250}{a}$ 4

Amount Philip spends in cents

Amount Paul spends in cents. You are to find the numeral that replaces *a*.

Now turn the page.



**compute**

C

Every time Philip pays 50¢ for bus fare, Paul pays 30¢. Philip has to pay 50¢ five times before he has spent $2.50 or 250¢. 1 So when Philip has spent 5 groups of 50¢, Paul will have spent 5 groups of 30¢, or 150¢.

You can compute to find the numeral that replaces *a*.

$\frac{50}{30} = \frac{250}{a}$ 2

You can multiply 50 by 5 to get 250. So you multiply 30 by 5 to find the numeral that replaces *a*. 150 replaces *a*.

When Philip has spent $2.50 for bus fare, Paul will have spent $1.50.

**see**

A

May and Ann have been weaving squares. May wove 16 while Ann wove 12. How many squares did May weave for each 3 that Ann wove? 3

$\frac{16}{12}$

This ratio compares the number of squares that May wove with the number that Ann wove.

# 117

1 Ask the pupils to read the problem and find which group of coins in the picture represents Philip's fare and which represents Paul's. Get them to see that a comparison can be made between the money Philip spends and the money Paul spends.

2 Discuss the ratio that expresses the comparison. Point out that the numeral for Philip's fare is the first term and that the numeral for Paul's is the second term.

3 Be sure the pupils understand that we are now going to express the comparison with a ratio using different numerals. We know one of the numerals. Ask: "The numeral 250 above the line represents whose fare? The letter a holds a place for whose fare?" Relate the ratio to Picture B.

4 Help the pupils see that these two ratios are equal because the relationship between Philip's fare and Paul's fare is always the same.

# 118

1 Have the pupils relate the text to the picture, and guide them in seeing that for each 50¢ fare of Philip's there is a matching 30¢ fare for Paul. Point out that there are 5 groups of 50¢, or 250¢, and 5 groups of 30¢, or 150¢.

2 Draw attention to the equation. Remind the pupils that to find a ratio equal to 50/30, they must multiply or divide the numbers represented in both terms by the same number. Discuss what can be done to 50 to get 250. Then discuss the fact that 30 also must be multiplied by 5. Point out that the completed equation 50/30=250/150 is correct because when both 50 and 30 are multiplied by 5 the result is 250 and 150.

3 Have the pupils read the new problem and relate Picture A to it and to the ratio. Point out that the number of squares Ann wove is represented by the second term.

B

You do not know how many squares May wove for each group of 3 that Ann wove.

But you can make a ratio that compares the unknown number of May's squares with Ann's 3 squares.

1 $\frac{n}{3}$ ← Number of squares May wove

← Number of squares Ann wove

This ratio, $n$ to 3, expresses the same comparison as the ratio 16 to 12.

Number of May's squares

$\frac{16}{12} = \frac{n}{3}$ 2

Number of Ann's squares

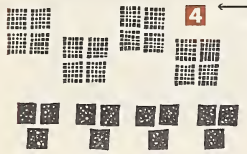
compute

C

Think of Ann's 12 squares as divided into groups of 3. There are 4 groups.

$\frac{16}{12} = \frac{n}{3}$ ← 3 can be found by dividing 12 by 4.

3



D

Think of May's squares also as divided into 4 groups. 4 squares are in each group.

4 ←

3 can be found by dividing 12 by 4. How do you find the numeral that replaces $n$?

$\frac{16}{12} = \frac{n}{3}$ 5

May wove 4 squares for each 3 squares that Ann wove.

119

# 119

1 Have someone explain the new ratio and wh it is n/3 instead of 3/n.

2 Explain that the two ratios are equal becaus the relationship must be the same both time the comparison is expressed.

3 Get the pupils to see that Ann has 4 groups o 3 squares.

4 Be sure the pupils understand that May's 1 squares must be thought of as divided into equal groups, just as Ann's are.

5 Ask: "By what number can we divide 12 to ge the 3? Which picture shows this? By wha number do we divide 16 to find the numera that replaces $n$? Which picture shows this?"

A Don saves 25¢ a week, and George saves 15¢ a week. How much will Don have saved when George has saved 75¢?

1 $\frac{25}{15} = \frac{a}{75}$

25 and $a$ stand for the number of cents Don saves.

What do 15 and 75 stand for?

Why is 75 in the second term of the new ratio? Why do you multiply 25 by 5 to find the numeral that replaces $a$?

B Ann wrote 45 invitations while Sue wrote 30. How many invitations did Sue write for each 9 that Ann wrote?

2 $\frac{45}{30} = \frac{9}{d}$

What do 45 and 9 stand for?

What do 30 and $d$ stand for?

Why is $d$ in the second term of the new ratio? Why do you divide 30 by 5?

C Two planes left Dickens at the same time to go to Redwood, 600 mi. away. One plane flew at 300 mi. per hour. The other plane flew at 250 mi. per hour. How far had the slower plane gone when the faster plane had arrived at Redwood?

3 $\frac{300}{250} = \frac{600}{n}$ or $\frac{250}{300} = \frac{n}{600}$

THINK Each ratio in the first equation compares the distance traveled by the faster plane with the distance traveled by the slower plane. What does each ratio in the second equation compare? Why is $n$ in a different place in this equation?

D Sally skated 15 times around the rink while Jane skated 12 times. When Sally had gone around 5 times, how many times had Jane gone around?

4 $\frac{15}{12} = \frac{5}{t}$ or $\frac{12}{15} = \frac{t}{5}$

THINK What does $t$ stand for in the first equation? Does $t$ stand for the same thing in the second equation? Why is $t$ in a different place in the second equation?

E 24 in. of snow fell in Redwood on Wednesday. During the same time, 16 in. of snow fell in Hometown. How much snow fell in Hometown for each 3 in. of snow that fell in Redwood?

$\frac{24}{16} = \frac{3}{s}$ or $\frac{16}{24} = \frac{s}{3}$

F A large wheel on a machine turns 20 times while a small wheel on the same machine turns 120 times. How many times has the large wheel turned when the small wheel has turned 720 times?

$\frac{20}{120} = \frac{m}{720}$ or $\frac{120}{20} = \frac{720}{m}$

120

# 120

1 Discuss the reasons for writing the second ratio as a/75 instead of 75/a. Ask: "By what do we multiply 15 to get 75? How do we find the number that a represents?"

2 Discuss the equation for this problem.

3 Discuss the two equations for this problem, and be sure that the pupils understand that either way of writing the ratios is correct. Emphasize the fact that both the numerals above the line must represent numbers that belong to the same idea—for example, the number of miles the faster plane flew. Discuss the computational procedure for finding the number represented by the letter.

4 Have the pupils read Problems D to F and explain why both equations for each problem are correct.

One man made 300 machine parts ile another man made 250. How any parts did the first man make for ch 25 parts the second man made?

Mr. Philips said, "I have 150 bu. corn. I have 3 bu. for every 2 bu. at Mr. Price has." How many bushels corn does Mr. Price have?

Bob delivers morning papers, and ck delivers afternoon papers. Bob ys that he delivers 4 papers for ery 3 papers that Jack delivers. Jack livers 60 papers. How many papers es Bob deliver?

Mrs. Farmer says she spends about 0 a week for groceries. Mrs. Bell ys she spends about $30. How much es Mrs. Bell spend for every $8 that rs. Farmer spends?

K Mrs. Johnson says that each month she spends $150 for groceries and $100 for rent. How much money has she spent for groceries when she has spent $1200 for rent?

L One month Mr. Cook worked 216 hr. on his job, and Mr. Hill worked 192 hr. How many hours did Mr. Hill work for each 54 hr. that Mr. Cook worked?

M Park City built 240 blocks of new streets one summer. That summer Deepwater built 160 blocks of new streets. How many blocks of new streets did Park City build for each 20 blocks that Deepwater built?

N Jim said, "During vacation, I traveled 6 mi. to your 5 mi., Bob." Bob had traveled 380 mi. How many miles had Jim traveled?

Now you should know how to use ratios that express comparisons in solving problems.

## Keeping skillful 2

| | | |
|---|---|---|
| $8\frac{2}{3}+5\frac{1}{8}=t$ | G $25\frac{1}{7}+6\frac{2}{3}+18\frac{2}{7}=n$ | M $n-9\frac{3}{10}=16\frac{1}{2}$ |
| $7\frac{3}{4}-a=2\frac{1}{8}$ | H $14\frac{3}{4}-7\frac{1}{4}=b$ | N $33\frac{1}{3}-16\frac{7}{8}=e$ |
| $16\frac{5}{8}-4\frac{1}{2}=x$ | I $s-6\frac{1}{2}=3\frac{1}{8}$ | O $1\frac{3}{8}+w=2\frac{1}{4}$ |
| $g-3\frac{1}{5}=5\frac{4}{5}$ | J $22\frac{1}{10}+g=35\frac{1}{10}$ | P $5\frac{11}{12}+13\frac{1}{6}+3\frac{7}{12}=m$ |
| $9\frac{3}{8}+n=12$ | K $5\frac{2}{3}+7\frac{5}{12}=p$ | Q $h+2\frac{5}{9}=17\frac{2}{3}$ |
| $6\frac{1}{12}-c=2\frac{2}{3}$ | L $d+8\frac{2}{7}=31$ | R $4\frac{3}{4}+5\frac{1}{4}+6\frac{3}{8}=n$ |

121

# 121

1 Assign these problems as written work. Tell pupils they may write the equation for each problem whichever way they wish, but they must be consistent. Afterwards, have the pupils discuss why they multiplied or divided in each equation.

2 Assign these "Keeping skillful" exercises as a separate lesson.

# 122-124 Exploring problems

Expanded Notes for this lesson are on pages 327-328.

**Objectives**

The pupil reviews solving problems in which the result of a comparison of two groups of objects is expressed in terms of "times as many as."

**Vocabulary**

New words page 123 ducklings*

**Comments**

Problems involving "times as many as" have traditionally been taught by a short-cut method. Thus, Problem A on page 122 would be solved by dividing 48 by 16, but the reason for this method of solution was never made clear in traditional arithmetic programs.

In *Seeing Through Arithmetic 5*, the pupils studied how to make an equation of ratios to solve certain kinds of problems involving comparisons, after which they were ready to understand the real meaning of the "times as many as" situation. A lesson presenting the solution of the "times as many as" problem by means of an equation of ratios appeared on pages 227-230. Now, in *Seeing Through Arithmetic 6*, the pupils review the "times as many as" problem.

The pupils must be helped to understand that the expression "times as many as" is a way of expressing something compared with 1. Problem A on page 122 is really the same type as the problem discussed on pages 118-119, except that the expressed comparison 48 to 16, or 48/16, is to be changed to an equal ratio in which 1 takes the place of 16. The real problem is to find how many cards Don printed for every 1 that Pat printed. An equation that describes the problem can be written in the form $48/16=x/1$.

The fact that the pupils must be consistent in making ratios needs continued emphasis. If the numeral that represents Don's cards is the first term of the first ratio of the equation, then the other numeral that also represents Don's cards must be the first term of the second ratio of the equation.

Problem B on page 123 is the type of problem in which the pupils are told that one group is a certain number of times as large as another group. They know the number of things in one group and must find the number of things in the other group. The pupils are helped to see that "2 times as many as" means 2 chicks for 1 duckling. This fact is represented by the ratio 2/1. Since they know that Sue has 19 ducklings, the equation for the problem situation is 2/1=m/19.

Problem C concerns a situation in which the 70 lambs Mr. Peters has are known to be 5 times as many as Mr. Cook has. The second term of the equal ratio is to be found, and the equation is 5/1=70/d.

**Answers**

**Pages 123-124:**

D 2/1=[26]/13
E 45/15=[3]/1
F 2/1=[70]/35
G 4/1=56/[14]
H 2/1=48/[24]
I 96/24=[4]/1
J 2/1=[480]/240
K 6/1=[4560]/760; $45.60
L 80/16=[5]/1
M 10/1=80/[8]
N 300/75=[4]/1

**Keeping skillful:**

(block 1)

A $10\frac{5}{16}$
B $30\frac{19}{24}$
C $13\frac{9}{10}$
D $17\frac{5}{6}$
E $\frac{1}{2}$
F $19\frac{1}{2}$
G $27\frac{19}{30}$
H $31\frac{1}{12}$
I $\frac{5}{24}$
J $6\frac{13}{20}$
K $12\frac{9}{10}$
L 11
M $15\frac{3}{10}$
N $\frac{23}{24}$
O $5\frac{3}{16}$
P $1\frac{7}{8}$
Q 43
R $3\frac{1}{2}$

(block 2)

A $1\frac{7}{12}$
B $1\frac{17}{20}$
C $7\frac{5}{16}$
D $6\frac{13}{15}$
E $9\frac{23}{24}$

## Exploring problems

A Don and Pat have a small printing press. One day Don printed 48 cards on the press, and Pat printed 16 cards. Don printed how many times as many cards as Pat that day?

To answer the question, you must find how many cards Don printed for each card that Pat printed.

The picture shows that Don printed 3 cards for each card that Pat printed.

You can also compute to find the answer. First you make a ratio that compares the total number of cards Don printed with the total number of cards Pat printed.

$\frac{48}{16}$ ← Number Don printed / ← Number Pat printed

Next you make a ratio that compares the number of cards Don printed with one card that Pat printed. Look at the ratio below.

$\frac{x}{1}$ ← Number of cards that Don printed / ← One card that Pat printed

The ratios 48 over 16 and $x$ over 1 express the same comparison. So you can write the equation shown below.

$\frac{48}{16}=\frac{x}{1}$

1 can be found by dividing 16 by ■. So to find the numeral that replaces $x$, you divide ■ by 16.

$\frac{48}{16}=\frac{3}{1}$

3 replaces $x$. Don printed 3 cards for each card that Pat printed.

Don printed 3 times as many cards as Pat printed.





# 122

1 Have the pupils read the problem and the nex five lines of text. They should understand tha this is a comparison situation. Help them se that in the picture Don's 48 cards have bee divided into 16 groups, each group matchin one of Pat's cards. There are 3 of Don's card to each 1 of Pat's.

2 Point out that a ratio is made to show th comparison between the total number of Don' cards and Pat's cards.

3 A ratio that is equal to the first one is made t compare the number of cards Don printed fo 1 card that Pat printed. Point out that since th number of cards Pat printed is written as th second term of the first ratio, 1, which repre sents 1 card that Pat printed, must also be writ ten as the second term of the other ratio.

4 Discuss why 48 must be divided by 16.

D Sue has 19 ducklings. She also has 2 times as many chicks as ducklings. How many chicks does she have?

You are told that Sue has 2 times as many chicks as ducklings. So you know that she has 2 chicks for each duckling.

You can use the ratio 2 over 1 to compare chicks with ducklings. Can you also use the ratio $m$ over 19 to compare chicks with ducklings?

Now you can make an equation of these equal ratios and compute.

$\frac{2}{1}=\frac{m}{19}$ 19 can be found by multiplying ▩ by ▩. How do you find the numeral that replaces $m$?

$\frac{2}{1}=\frac{38}{19}$ ← 38 replaces $m$. Sue has ▩ chicks.

E Mr. Peters said, "I have 70 lambs this spring. I have 5 times as many lambs as Mr. Cook." How many lambs did Mr. Cook have?

You know that Mr. Peters had 5 lambs for each lamb that Mr. Cook had.

You can use the ratio 5 over 1 to compare Mr. Peters' lambs with Mr. Cook's lambs. Does the ratio 70 over $d$ also express the comparison of Mr. Peters' lambs with Mr. Cook's?

When you solve the equation made from these equal ratios, you will know how many lambs Mr. Cook had. 5

$\frac{5}{1}=\frac{70}{d}$ How do you find the numeral that replaces $d$?

$\frac{5}{1}=\frac{70}{14}$ 14 replaces $d$. ← Mr. Cook had ▩ lambs.

F Jack is raising 13 pigs for a 4-H Club project. Paul is raising twice as many pigs as Jack. How many pigs is Paul raising? 6

"Twice as many" means "2 times as many." So Paul is raising 2 pigs for each pig that Jack is raising.

$\frac{2}{1}=\frac{a}{13}$ a: Number of pigs Paul is raising; 13: Number of pigs Jack is raising

What numeral replaces $a$?

G On a trip, Ann sent 45 postcards, and May sent 15. Ann sent how many times as many postcards as May?

$\frac{45}{15}=\frac{x}{1}$

123

1

Patty sold 35 cups of lemonade at e school fair. Kathy sold 2 times much lemonade as Patty did. How any cups of lemonade did Kathy sell?

$=\frac{g}{35}$

Jim has a collection of 56 science ctures. He has 4 times as many ience pictures as Henry has. How any science pictures has Henry?

$=\frac{56}{t}$

Peter brought 48 lb. of paper for e paper drive. He said, "I brought ice as much paper as George did." ow much paper did George bring?

Jim brought 96 lb. of paper. He ought how many times as much per as George did?

J Mr. Parker's garden is 240 sq. ft. in size. Mr. Cook's garden is twice as large as Mr. Parker's. Mr. Cook's garden is how many square feet in size?

K In one summer Jane saved $7.60. Her sister saved 6 times as much as Jane did. How much money did Jane's sister save that summer?

L Jane weighs 80 lb. Her baby brother weighs 16 lb. Jane weighs how many times as much as her baby brother?

M Jane's father said, "You weigh 10 times as much as the cat, Jane." How much did the cat weigh?

N George skated 300 yd., and Henry skated 75 yd. George skated how many times as far as Henry?

Now you should be able to use ratios in solving problems like those in this lesson.

## Keeping skillful 3

$\frac{1}{4}+\frac{3}{8}+9\frac{11}{16}=x$

$a-14\frac{2}{3}=16\frac{1}{3}$

$20\frac{8}{9}-6\frac{9}{10}=t$

$s+42\frac{1}{6}=60$

$11\frac{1}{8}-r=10\frac{5}{8}$

$80\frac{1}{2}+b=100$

G $17\frac{4}{5}+9\frac{6}{8}=c$

H $w-8\frac{11}{12}=22\frac{1}{6}$

I $49\frac{1}{8}-48\frac{11}{12}=s$

J $x+14\frac{3}{5}=21\frac{1}{4}$

K $33\frac{9}{10}-d=21$

L $19\frac{3}{4}-m=8\frac{3}{4}$

M $r-6\frac{4}{5}=8\frac{1}{2}$

N $21\frac{11}{12}-20\frac{23}{24}=f$

O $10\frac{9}{16}-n=5\frac{3}{8}$

P $d+14\frac{11}{24}=16\frac{1}{3}$

Q $25\frac{3}{10}+17\frac{7}{10}=x$

R $\frac{7}{8}+\frac{11}{12}+1\frac{17}{24}=n$

Find the sum.

A $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$

B $\frac{1}{2}, \frac{4}{5}, \frac{1}{4}, \frac{3}{10}$

C $5\frac{7}{16}, 1\frac{1}{2}, \frac{3}{8}$

D $2\frac{1}{10}, \frac{3}{5}, 4\frac{1}{6}$

E $\frac{1}{3}, 6\frac{2}{6}, 2\frac{3}{4}$

# 123

1 When the pupils have read this problem, ask how it differs from Problem A. [Here they are told how many "times as many as."]

2 Discuss the phrase "2 times as many as." Solve this problem by drawing x's for ducklings and circles for chicks on the board, using the idea of 2 chicks for 1 duckling.

3 Get the pupils to see that the ratio is 2/1, and that the equal ratio must be written as m/19. Discuss what $m$ and 19 represent. The pupils should see that 19 can be obtained by multiplying 1 by 19. Ask: "Why should we multiply 2 by 19?"

4 Pupils should realize that "5 times as many as" means that Mr. Peters has 5 lambs for each one that Mr. Cook has.

5 Have the pupils look at the equation and explain why 70 has to be the first term of the second ratio and why $d$ must be the second term. Ask: "Why do we multiply 1 by 14?"

6 Discuss Problems D and E.

# 124

1 For Problems F and G, for which equations are shown in the book, have the pupils explain the position of the numerals and letters. Then have them solve each equation.

2 Assign the remaining problems as written work. Tell the pupils that they are to write each equation first with a letter in the proper place, and then to rewrite the equation replacing the letter with the answer. Supply answers and discuss those problems that caused trouble for more than a few pupils.

3 Assign the "Keeping skillful" exercises as a separate lesson when time permits.

# 125 Thinking straight

Exponded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns the distinction between comparison of a part of a group with the whole group and the comparison of two groups. He also becomes aware of various expressions of comparison.

**Vocabulary**

New words page 125 roses*

**Comments**

The pupils learn that while there are many ways in which comparisons can be expressed in words, it is the *situation* expressed by the words that is important. They learn to distinguish whether a part of a group is being compared with the whole group, or two separate groups are being compared.

**Answers**

(block 1)

A 2/10 B 2/8

C Comparing the blue cards with all the cards

D Comparing part of the group with the whole group; 5/6

E Comparing the white cards with the green cards

F Comparing two groups; 3/7

G Comparing two groups

H Comparing part of the group with the whole group

(block 2)

A Part of a group with the whole group

B Part of a group with the whole group

C Two groups

D Part of a group with the whole group

E Part of a group with the whole group

F Part of a group with the whole group

G Two groups H Two groups

I Part of a group with the whole group

J Two groups K Two groups

L Part of a group with the whole group

M Two groups

N Part of a group with the whole group

O Two groups

P Part of a group with the whole group

## Thinking straight

**1** A Carol has picked 10 roses. 8 are white, and 2 are red. To compare the red roses with all the roses, you can say, "2 out of 10 are red." You are comparing part of the group with the whole group. Write a ratio that shows this comparison.

B But if you want to compare the red roses with the white roses, you can say, "2 red roses to 8 white roses." Now you are comparing one group with another group. Write a ratio that shows this comparison.

**2** C Imagine that you have a pile of cards. Some of them are blue, and some are yellow. Suppose you say, "5 in each 6 are blue." Are you comparing the blue cards with all the cards, or are you comparing the blue cards with the yellow cards?

D In Exercise C, are you comparing part of the group with the whole group, or are you comparing two groups? Write a ratio that shows the comparison made in Exercise C.

**3** E Now imagine that you have another pile of cards. Some of them are white, and some of them are green. Suppose you say, "I have 3 white cards for each 7 green cards." Are you comparing the white cards with all the cards, or are you comparing the white cards with the green cards?

F In Exercise E, are you comparing part of the group with the whole group, or are you comparing two groups? Write a ratio showing the comparison made in Exercise E.

G Bob said, "I have 2 science books to 5 story books." Was he comparing two groups, or was he comparing part of one group with the whole group? **4**

H Suppose Bob had said, "2 of these 7 books are science books." In this case, would he be comparing part of a group with the whole group, or would he be comparing two groups?

Each exercise below expresses a comparison. For each one, decide what is meant, the comparison of a part of a group with the whole group or the comparison of two groups. **5**

A 5 in each 7
B 3 of the 4
C 11 to 15
D 2 in each 10
E 6 of the 9
F 14 out of 16
G 9 for each 12
H 3 to 8
I 6 out of 11
J 25 to 15
K 13 for each 26
L 14 of the 15
M 37 to 100
N 19 of the 40
O 80 to 45
P 18 of the 50





## 125

**1** Use Exercises A and B to bring out how a comparison situation between a part of a group and a whole group and one between two separate groups differ. Have the pupils note the use of the expression "out of" for comparing part of the group with the whole group.

**2** In Exercises C and D help the pupils see that they are comparing part of the group with the whole group when the expression "5 in each 6 are blue" is used. Use cards or other objects to illustrate the situation.

**3** Use Exercises E and F to point out that the expression "3 for 7" indicates a comparison of two groups.

**4** Demonstrate the meaning of the two situations described in Exercises G and H with objects.

**5** Before assigning Exercises A to P as written work, write headings for the two comparison situations on the board. Have the pupils write each phrase in the first four or five exercises under the proper heading.

# 126-129 Moving forward

Expanded Notes for this lesson are on pages 329-331.

**Objectives**

The pupil learns that per cent is a way of expressing a comparison by a ratio that has 100 as the second term. Per cents greater than 100% are included.

**Vocabulary**

New words page 126 goal*; page 127 convenient*; page 128 failed*, crop*, storm*, mistakes*; page 129 finally*

**Comments**

Some of the difficulties that pupils have had with per cent have arisen because per cents were thought of as whole numbers or as fractions instead of as ratios that express comparison. The experiences provided in this lesson show pupils that when the second term of a ratio is 100, it may often be expressed in the language of per cent and by using the sign "%." Thus, the ratio 5/100, representing the comparison 5 for 100, can also be expressed as 5 per cent and by the symbol 5%.

Groups can always be compared by using ratios with common second terms, as is shown in Problems C and D on page 126. Here the pupils see that the accomplishments of the Baker and of the Weston schools can easily be compared by using ratios that have 50 as a common second term. However, custom dictates that 100 be used as a common second term, and as a result, direct comparison via per cents becomes possible.

The preceding lesson (on page 125) helped the pupils to distinguish among different types of comparison situations. In this lesson pupils learn to express the comparison as a per cent. They also compare a group larger than the original one to the whole original group and learn the meaning of per cents greater than 100%.

Per cent is introduced before decimals because the basic idea is that of ratio, and decimals are not needed. Later the pupils are shown that decimals sometimes are useful for computing in per cent problems because they furnish a computational short cut.

**Answers**

Pages 126-128:

Greenfield, $50; Baker, $23; Cook, $18; Weston, $8; Burns, $63; Fisher, $37

B Burns and Fisher; Baker, Cook, and Weston

C $8; $10; yes

D Yes; 40. They have the same second term. Raising $46 toward a goal of $50 is better than raising $40 toward a goal of $50.

E Yes; 25. The second terms are not the same.

F Yes; yes. Raising $24 more than $50 is a better achievement than raising $13 more than $50.

G Only the first terms need to be compared.

H 23/25 and 18/20. The second term of the ratio for the Cook School cannot be easily changed to 50.

I Yes; 90; Baker School. Raising $92 toward a goal of $100 is better than raising $90 toward a goal of $100.

J The second term of each ratio in Column A was multiplied by whatever number necessary to get 100, and then the first term was multiplied by the same number.

K Weston, Cook, Baker, Greenfield, Burns, Fisher

L 100, 100, 100

M Greenfield exactly reached its goal. The per cents are over 100% for Burns and Fisher. The per cents are under 100% for Baker, Cook, and Weston.

(block 2)

| | | | |
|---|---|---|---|
| A 15% | B 99% | C 134% | D 6% |
| E 24% | F 2% | G 43% | H 312% |
| I 44/100 | J 11/100 | K 82/100 | L 101/100 |
| M 19/100 | N 6/100 | O 23/100 | P 140/100 |

(block 3)

Red letters A to L: Innumerable ratios can be written.

(block 4)

A No; no

B Yes; yes

C Yes; yes

D Yes

E No; no

Page 129:

F No. 100% of the examples means all of them.

G 18/20, 90/100, 90% 27/50, 54/100, 54%
11/25, 44/100, 44% 14/10, 140/100, 140%
9/10, 90/100, 90%

H George, Don; George, Peter and Jack, Pat, Don

(block 2)

A 4/5, 80/100, 80% B 1/10, 10/100, 10%

C 6/5, 120/100, 120%
D 8/20, 40/100, 40%
E 11/50, 22/100, 22%
F 19/25, 76/100, 76%
G 40/20, 200/100, 200%
H 7/50, 14/100, 14%
I 15/5, 300/100, 300%
J 13/25, 52/100, 52%
K 9/50, 18/100, 18%
L 7/20, 35/100, 35%
M 66/50, 132/100, 132%
N 14/25, 56/100, 56%
O 6/10, 60/100, 60%
P 8/50, 16/100, 16%

**Keeping skillful:**

(block 1)

A $8\frac{5}{16}$
B $11\frac{13}{18}$
C $7\frac{1}{6}$
D $81\frac{3}{8}$
E $27\frac{2}{3}$
F $1\frac{19}{28}$
G $2\frac{7}{24}$

(block 2)

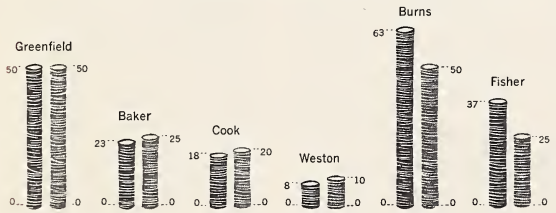
A 54
B 16
C 3
D 3
E 24
F 36

(block 3)

A 36,960 ft.
B 7 min.
C 152 fl. oz.
D 8 lb.
E 14 bu.
F 15 wk.
G 600 hr.
H 16 fl. oz.

## Moving forward

Now you will study a new way to express some comparisons.




A The Lake County schools raised money to buy some new equipment for a park playground. Look at the picture above. For each school the pile of silver dollars on the right shows the goal, which is the total amount the school wanted to raise. The pile of silver dollars on the left shows how much money the school really raised. How much money did each of the schools raise?

B You can tell from the picture that the Greenfield School exactly reached its goal. Which schools passed their goals? Which schools did not reach their goals? You cannot tell from the picture which school did the best.

C To compare some of these schools to see how well they did, you can start with the Baker and Weston schools. The Baker School raised $23 of its goal of $25. The Weston School raised ▩ of its goal of ▩. Do the ratios 23 over 25 and 8 over 10 show the comparisons? You still cannot easily tell which school did better.

D Now express each comparison by a new ratio with 50 in its second term. Can 46 over 50 be a ratio for the Baker School? A ratio for the Weston School can be ▩ over 50. How do these new ratios make it easy to tell that the Baker School did better than the Weston School?





# 126

1 Discuss the picture in the way suggested i Exercise A. Be sure pupils understand that th numeral near the top of each pile of coi stands for the number of silver dollars.

2 Get the pupils to explain how the pictu shows that the Greenfield School raised th $50 it wanted. Have the pupils determine th amount raised and the goal for each of th other schools. Help them see that for the pre ent they cannot tell which school did the be job because the amounts to be raised and th goals were not the same for all the schoo

3 Use Exercise C to bring out that ratios can b used to compare the amount raised with th goal for the Baker School and for the Westo School.

4 Discuss the fact that if each ratio is change so that these ratios have common secon terms, it is easy to tell which school did bett

E To decide which did better, the Burns School or the Fisher School, look at Column A below. Does the ratio 63 over 50 express the comparison for the Burns School? The comparison for the Fisher School is expressed by the ratio 37 over ■. You cannot easily tell which school did better. Why?

F Can a ratio for the Fisher School be expressed with 50 in its second term? Does the ratio 74 over 50 express the same comparison as the ratio 37 over 25? Now how can you tell that the Fisher School did better than the Burns School?

G It is easy to decide which school did better when both ratios have the same second term. Why?

H Now suppose you want to find which school did better, the Baker School or the Cook School. What ratios express the comparisons for the two schools? Why is it not convenient to replace both of these ratios with new ratios that have 50 in the second term?

I People often make comparisons by using ratios that have 100 in the second term. Look at Column B in the table. Can the comparison expressed by the ratio 23 over 25 for the Baker School also be expressed as 92 over 100? The ratio 18 over 20 for the Cook School can be replaced by ■ over 100. Which of these two schools did better? How do you know? 3

J Notice that Column B shows, for each school, a ratio with 100 in its second term. Each ratio compares the money raised with the school's goal. How was each ratio in Column B found? 4

| | A | B | C | D |
|---|---|---|---|---|
| Greenfield | $\frac{50}{50}$ | $\frac{100}{100}$ | 100 per cent | 100% |
| Baker | $\frac{23}{25}$ | $\frac{92}{100}$ | 92 per cent | 92% |
| Cook | $\frac{18}{20}$ | $\frac{90}{100}$ | 90 per cent | 90% |
| Weston | $\frac{8}{10}$ | $\frac{80}{100}$ | 80 per cent | 80% |
| Burns | $\frac{63}{50}$ | $\frac{126}{100}$ | 126 per cent | 126% |
| Fisher | $\frac{37}{25}$ | $\frac{148}{100}$ | 148 per cent | 148% |



Now use the ratios in Column B to make a table listing the schools in the order of how well they did. If you plan to end with the Fisher School, with which school should you begin?

When a ratio that expresses a comparison has 100 in its second term, you can read the ratio as a per cent. Look at Column C. When you say "92 per cent," you mean that the ratio you are using has 100 in its second term. What is the second term of a ratio that is expressed as 90 per cent? As 100 per cent? As 126 per cent?

The sign at the right is called the per cent sign. You read it "per cent." Column D on page 127 shows how to write the per cents in Column C with the per cent sign. From the per cents in Column D decide which school exactly reached its goal. How do the per cents show that two schools passed their goals? How do the per cents show that three schools failed to reach their goals? %

Each of the ratios below expresses a comparison. Write each ratio as a per cent. Use the per cent sign.

A $\frac{15}{100}$ B $\frac{99}{100}$ C $\frac{134}{100}$ D $\frac{6}{100}$

E $\frac{24}{100}$ F $\frac{2}{100}$ G $\frac{43}{100}$ H $\frac{312}{100}$

Write each of the following as a ratio with 100 in the second term. 4

I 44% J 11% K 82% L 101%
M 19% N 6% O 23% P 140%

For each per cent given below, write as many equal ratios as you can without using the per cent sign. The work for 120% might look like this. 5

A $120\% = \frac{120}{100} = \frac{60}{50} = \frac{6}{5}$

A 120% B 50% C 48% D 31%
E 110% F 80% G 75% H 90%
I 250% J 5% K 12% L 36%

A A farmer said that 50% of his corn crop was lost because of a storm. Do you know how many acres of corn were lost? Do you know how many acres of corn he had planted? 6

B Could he have planted 100 acres? Could he have planted 10 acres?

C Can you say that he lost 50 acres out of each 100 acres he planted? Can you say that he lost 5 acres out of every 10 acres?

D Carol worked all the examples in a test on arithmetic. Her grade was 75%. Did she make any mistakes in her work?

E Is there any way to tell how many examples Carol worked? Is there any way to tell how many examples she worked incorrectly?

# 127

1 With Exercises E to G, help the children see that when the ratios have identical second terms, it is much easier to tell which school did better. In connection with Exercise F, discuss why 50 is selected as the second term and how the ratio 74/50 is obtained. Have pupils explain how each of the ratios in Column A is obtained.
2 Exercise H shows that in this comparison only one of the ratios can easily be changed to an equal ratio with 50 as a second term. Since the ratios do not have a common second term, it is hard to compare them.
3 Explain that the use of 100 as the second term of a ratio is conventional when this kind of comparison is made.
4 Point out that each ratio in Column A can be expressed as a ratio with 100 as the second term. Let pupils explain how the changes are made.

# 128

1 Help pupils see why they should start with the Weston School if they end the list with the Fisher School.
2 Be sure the pupils understand that each per cent listed in Column C, page 127, expresses a ratio that has 100 for its second term.
3 Have the pupils note that the only difference between the expressions in Column D, page 127, and those in Column C is the use of the symbol for "per cent." Explain why the failure of a school to reach its goal can be expressed as a per cent less than 100%, why the achievement of the Greenfield School can be expressed as 100%, and why the achievement of a school that exceeded its goal can be expressed as a per cent greater than 100%.
4 Use Exercises A to P as written work.
5 Assign Exercises A to L as written work.
6 Use Exercises A to E to show that while a per cent does not tell the actual number in the comparison, it does indicate a relationship.

F Could Carol have worked more than 100% of the examples correctly? Why or why not?

1

| | Goal | Money saved |
|---|---|---|
| Peter | $20 | $18 |
| Don | $25 | $11 |
| Jack | $10 | $9 |
| Pat | $50 | $27 |
| George | $10 | $14 |

2 G The table above shows how much money five boys planned to save and how much money each of them really did save. For each boy, first write a ratio that compares the amount he saved with his goal. Next write an equal ratio with 100 as its second term. Then write each ratio as a per cent.

H Which boy did best in reaching or passing his goal? Which boy saved the smallest per cent of his goal? Write the names of the boys in an order that shows how well they did. Begin with the boy who did the best.

For each statement below, first write a ratio to show the comparison. Then write an equal ratio with 100 in its second term. Finally, express the ratio as a per cent.

3

A 4 out of 5
B 1 to 10
C 6 for each 5
D 8 in each 20
E 11 in each 50
F 19 in each 25
G 40 to 20
H 7 out of 50
I 15 to 5
J 13 of the 25
K 9 out of 50
L 7 of the 20
M 66 for each 50
N 14 out of 25
O 6 of the 10
P 8 in each 50

Now you can read and write per cents and can express a comparison by a per cent.

## Keeping skillful 4

A $17\frac{7}{8} - 9\frac{9}{16} = f$
B $\frac{11}{18} + 11\frac{1}{3} = h$
C $d + 14\frac{2}{3} = 21\frac{5}{6}$
D $k - 23\frac{3}{4} = 57\frac{5}{8}$
E $34\frac{1}{6} - h = 6\frac{1}{2}$
F $\frac{3}{7} + \frac{3}{4} + \frac{1}{2} = w$
G $7\frac{1}{12} + d = 9\frac{3}{8}$

Solve each equation below.

A $\frac{18}{7} = \frac{f}{21}$
B $\frac{44}{64} = \frac{11}{h}$
C $\frac{9}{27} = \frac{1}{t}$
D $\frac{15}{90} = \frac{k}{18}$
E $\frac{12}{8} = \frac{36}{t}$
F $\frac{17}{9} = \frac{68}{s}$

A 7 mi. = ▒ ft.
B 420 sec. = ▒ min.
C 19 c. = ▒ fl. oz.
D 128 oz. = ▒ lb.
E 56 pk. = ▒ bu.
F 105 da. = ▒ wk.
G 25 da. = ▒ hr.
H 32 tbs. = ▒ fl. oz.

129

# 129

1 Point out, if necessary, that 100% of the examples would be all the examples. Therefore, Carol could not have worked more than 100% of the examples correctly.

2 Assign Exercises G and H as written work. Tell the pupils that in order to answer the questions in Exercise H, they must use the information they found in Exercise G.

3 For Exercises A to P, you may wish to suggest that pupils write three column headings on their papers labeled "Ratio," "Equal ratio," and "Per cent." Their work can then be arranged under these three headings.

4 Treat the "Keeping skillful" exercises as a separate lesson.

# 130-133 Exploring problems

Exponded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to use per cents to express comparison.

**Vocabulary**

New words page 131 auditorium; page 132 won*, Cal's, November*; page 133 Jean*

**Comments**

Traditionally, three "cases" of per cent have been taught in problem solving, and a great deal of time has been devoted to each "case." When per cent is thought of as a ratio, special study of each of the three "cases" is no longer necessary because all three "cases" can be represented as ratio equations, thereby providing a unified idea and, consequently, greater mastery.

Problems using per cent should not be difficult for the pupils. They have already reviewed on pages 117-121, the solution of comparison problems by the use of equations of ratios. The same type of solution is used here. For each problem situation, a ratio can be made from information given. Then, an equal ratio, in which a letter holds a place for the missing numeral, is made. The pupils learn that 100 is always one of the terms.

In this lesson the problems involve whole numbers only. Later on, situations that involve fractions and mixed numbers will be considered.

**Answers**

Pages 131-133:

D 525
E 25
F 84
G $13
H 65%; cents
I 50
J 46/100=[23]/50
K 364/700=[52]/100; 52%
L 30/100=15/[50]
M 60/100=[60]/100; $60
N 88/100=22/[25]
O 23/25=[92]/100; 92%
P 40/100=[280]/700
Q 140/700=[20]/100; 20%
R 40/50=[80]/100; 80%

Keeping skillful:

(block 1)

A 9875
B 39,252
C 7882
D 973
E $229\frac{10}{19}$
F 62,776
G $48\frac{200}{263}$
H 17,299
I 146,030
J 163,566
K 340,516
L $776\frac{12}{67}$
M 778,191
N 91,147
O 37,138
P $380\frac{1}{8}$

(block 2)

A 137
B 11,100
C 1469
D 1564
E 2024
F 10,494
G 10,104
H 84,702
I $515\frac{3}{80}$
J 586,880
K $76\frac{41}{335}$
L 32,556
M 436,150
N $35\frac{45}{52}$
O 21,308
P 440,363

(block 3)

A $2\frac{2}{5}$
B $1\frac{1}{2}$
C $13\frac{3}{10}$
D $21\frac{17}{20}$
E $32\frac{1}{2}$
F $5\frac{19}{24}$
G $13\frac{5}{16}$
H $5\frac{1}{15}$
I $8\frac{7}{10}$
J $2\frac{7}{12}$
K 8
L $5\frac{7}{20}$
M $38\frac{17}{30}$

## xploring problems

700 pupils are enrolled in the ckson School. 420 of the pupils ually walk to school. The pupils who ually walk to school are what per nt of all the pupils? 1

u can use an equation of ratios to d the answer. Start with what the oblem tells you and make a ratio that mpares the number of pupils who lk to school with the total number pupils enrolled.

$\frac{20}{00}$ ← Number of pupils who walk to school
← Number of pupils enrolled

find the per cent who walk to school, u need another ratio. This ratio must press a comparison for 100 pupils at is the same as the comparison of 0 pupils with 700 pupils. You do not ow how many pupils walk for each 0 pupils enrolled. But you can use a ter in your ratio to hold a place for e numeral that tells how many.

$\frac{f}{00}$ ← Number of pupils who walk to school
← Number of pupils enrolled

olving problems involving per cents

The two ratios are equal because they make the same comparison.

$\frac{420}{700} = \frac{f}{100}$

You can divide 700 by 7 to get 100. So you divide 420 by 7 to find the numeral to replace $f$. 60 replaces $f$. 3

The comparison of 420 pupils to 700 pupils is the same as the comparison of 60 pupils to 100 pupils. 60 to 100 can be expressed as 60%. The 420 pupils who walk to school are 60% of the 700 pupils enrolled.

B One day 14% of the 50 sixth graders enrolled in the Jackson School were absent. How many of the sixth graders were absent that day? 4

To find this answer, you again can use an equation of ratios. You are told that 14% of the sixth graders were absent. You know that 14% can mean 14 out of 100. So you can make a ratio that compares 14 absent sixth graders with 100 sixth graders enrolled.

$\frac{14}{100}$ ← Number of sixth graders absent
← Number of sixth graders enrolled

# 130

1 Have the problem read. Then discuss the ratio 420/700, with particular emphasis on the meaning of each term.

2 Be sure the pupils understand that since the problem asks them to find a per cent, the second term of the new ratio must be 100.

3 Discuss how to change 420/700 to an equal ratio with 100 as the second term. Get the pupils to explain why the resulting comparison, 60 to 100, can be expressed as 60%. 420 to 700 is the same as 60 to 100. Consequently, the 420 pupils are 60% of the total number enrolled.

4 The pupils should notice that in this problem a per cent and a number are given. The per cent establishes the ratio 14/100.

The other ratio you make compares the number of sixth graders who really were absent with the 50 sixth graders who were really enrolled.

1

$$\frac{t}{50}$$

$t$ ← Number of sixth graders absent

50 ← Number of sixth graders enrolled

The comparison for the 50 sixth graders is the same as for 100 sixth graders. So the two ratios are equal.

$$\frac{14}{100} = \frac{t}{50}$$

2

You can divide 100 by 2 to get 50. So you divide 14 by 2 to find the numeral to replace $t$. 7 replaces $t$.

7 sixth graders were absent that day.

C One day 95% of the pupils in the fourth, fifth, and sixth grades went to a movie in the auditorium. 190 pupils went to the movie. How many pupils are enrolled in these grades?

3

You are told that 95% of the pupils in the three grades went to the movie. So you know that you can make a ratio that compares 95 pupils who went to the movie with 100 pupils enrolled.

$$\frac{95}{100}$$

95 ← Number of pupils who went to the movie

100 ← Number of pupils enrolled

You know that 190 pupils went to the movie. So the other ratio will compare these 190 pupils with the number of pupils really enrolled.

$$\frac{190}{d}$$

190 ← Number of pupils who went to the movie

4

$d$ ← Number of pupils enrolled

The two ratios are equal because they make the same comparison.

$$\frac{95}{100} = \frac{190}{d}$$

You can multiply 95 by 2 to get 190. So you multiply 100 by 2 to find the numeral to replace $d$. 200 replaces $d$.

200 pupils are enrolled in the three grades.

D About 75% of the 700 pupils at the school eat lunch in the school lunchroom. About how many pupils eat lunch in the school lunchroom?

5

$$\frac{75}{100} = \frac{s}{700}$$

Number of pupils who eat in the school lunchroom

Number of pupils enrolled

Why is 100 in the second term of the ratio 75 over 100? How do you find the numeral to replace $s$?



# 131

1 Point out that if 14%, or 14 per (or out of) 100, were absent, it is possible to discover how many per (or out of) 50 were absent. Have the pupils explain the terms in the equal ratio $t/50$.

2 Discuss the equation and the computation that is done to find the numeral that replaces $t$. Be sure the pupils understand that the replacement for $t$ (which is 7) represents the number of sixth graders who were absent.

3 Point out that in this problem the total number of pupils is not given. The pupils are told only that the 190 pupils are 95% of the total number of pupils. Discuss the ratio 95/100.

4 Discuss why the letter appears as the second term of the equal ratio. Then have a pupil explain how the replacement for $d$ is found. Discuss why the answer represents number of pupils.

5 Point out that this problem is similar to Problem B. Then let pupils explain the equation and how to find the numeral that replaces $s$.

E The Jackson School basketball team has won 64% of its games. The team has won 16 games. How many games has the basketball team played?

1

$$\frac{64}{100} = \frac{16}{p}$$

What do 64 and 16 stand for?

What do 100 and $p$ stand for?

Why is 100 in the second term of the ratio 64 over 100? Why is $p$ in the second term of the ratio 16 over $p$? Why do you divide to find the numeral to replace $p$?

F 42 of the 50 sixth graders have pets. What per cent of the sixth graders have pets?

2

$$\frac{42}{50} = \frac{b}{100}$$

42 and $b$ stand for ~~~.

50 and 100 stand for ~~~.

What numeral will replace $b$? Why can you express the second ratio as 84%?

3 G The sixth-grade class collected $20 for a class party. They spent 65% of the money for food for the party. How many dollars did they spend for food?

$$\frac{65}{100} = \frac{m}{20}$$

H Cal's allowance is $2.00 per week. He spends $1.30 per week of his allowance for lunch at school. What per cent of his allowance does Cal spend for lunch at school?

4

$$\frac{130}{200} = \frac{h}{100}$$

THINK Do the 130, 200, and 100 represent dollars or cents?

I 70% of the pupils in the school band rent their instruments. 35 pupils rent their instruments. How many pupils are there in the school band?

$$\frac{70}{100} = \frac{35}{a}$$

J 46% of the 50 sixth graders are boys. How many of the sixth graders are boys?

K 364 of the 700 pupils in the Jackson School are girls. What per cent of the pupils are girls?

L 15 sixth graders are in the Jackson School Science Club. This is 30% of the total membership of the club. How many pupils in all belong to the Science Club?

M The Science Club had $100 to spend for supplies in one year. By the end of November, the club had used 60% of the money. How many dollars had the club used?





# 132

1 Let the pupils decide which of the preceding problems Problem E resembles [C]. Then let a pupil explain the meaning of each of the terms of the ratios in the equation. Another pupil can explain the computation necessary to find the numeral to replace $p$.

2 Have the pupils find a preceding problem that resembles Problem F [A]. Let them explain how to find the numeral that replaces $b$. Be sure they understand why 84/100 can be expressed as 84%.

3 Discuss Problems G to I.

4 Assign Problems J to M and Problems N to R (page 133) as written work. Remind the pupils that they are to use the information given in each problem to make an equation of equal ratios. Remind them that some answers should be expressed as per cents.

N One day Ted worked 88% of the exercises on an arithmetic test correctly. He worked 22 exercises correctly. How many exercises were there on the arithmetic test?

O On the same test, Jean worked 23 of the exercises correctly. What per cent of the exercises did she work correctly?

P 40% of the 700 pupils at the Jackson School have younger brothers and sisters who are not yet enrolled in school. How many of the pupils have brothers and sisters not yet in school?

Q 140 of the pupils have older brothers and sisters who have already finished high school. What per cent of the pupils have brothers and sisters who have finished high school?

R 40 of the 50 sixth graders are 11 years old. What per cent of the sixth graders are 11 years old?

Now you should be able to solve problems about per cents.

## Keeping skillful 2

| | | | |
|---|---|---|---|
| A | $n+1870=11745$ | A | $863+n=1000$ |
| B | $32497+6755=a$ | B | $w-6305=4795$ |
| C | $563\times14=t$ | C | $424+963+82=d$ |
| D | $7841-6868=b$ | D | $e+437=2001$ |
| E | $4361\div19=c$ | E | $6198+n=8222$ |
| F | $72563-n=9787$ | F | $13404-k=2910$ |
| G | $25648\div w=526$ | G | $c-5230=4874$ |
| H | $62710+z=80009$ | H | $m+24630=109332$ |
| I | $k-53987=92043$ | I | $41203\div80=z$ |
| J | $27\times6058=g$ | J | $640\times917=h$ |
| K | $436\times781=e$ | K | $25501\div d=335$ |
| L | $52004\div67=y$ | L | $82346-49790=v$ |
| M | $833120-54929=a$ | M | $7930\times55=e$ |
| N | $d+9263=100410$ | N | $1865\div52=s$ |
| O | $57637-f=20499$ | O | $r+38692=60000$ |
| P | $18246\div m=48$ | P | $473\times931=w$ |

| | |
|---|---|
| A | $11\frac{9}{10}-n=9\frac{1}{2}$ |
| B | $\frac{5}{12}+\frac{3}{4}+\frac{1}{3}=a$ |
| C | $72\frac{5}{8}+n=86\frac{1}{10}$ |
| D | $y-18\frac{1}{4}=3\frac{3}{5}$ |
| E | $33\frac{1}{3}-\frac{5}{6}=f$ |
| F | $g+4\frac{7}{8}=10\frac{5}{8}$ |
| G | $5\frac{9}{16}+7\frac{3}{4}=w$ |
| H | $9\frac{2}{3}-4\frac{3}{5}=h$, |
| I | $17-8\frac{3}{10}=c$ |
| J | $14\frac{5}{6}-d=12\frac{1}{4}$ |
| K | $e+9\frac{7}{8}=17\frac{7}{8}$ |
| L | $12\frac{1}{4}-m=6\frac{9}{10}$ |
| M | $20\frac{5}{8}+18\frac{1}{8}=v$ |



# 133

1 Have the pupils continue solving the problems. Point out that some of the problems on this page require information obtained from other problems. When all have finished, discuss the equations and solutions that caused difficulty.

2 Assign these exercises as a separate lesson.

# 134-135 Exploring problems

Exponded Notes for this lesson ore on page 331.

**Objectives**

The pupil learns to recognize and use expressions about per cent.

**Vocabulary**

New words page 134 performance, expenses; page 135 refreshments*, scenery

**Comments**

When pupils learn to solve problems that involve per cents by using equations, they do not need to be able to distinguish among the three "cases" of per cent. However, they will encounter statements and questions that are customarily used to express the three cases: "20 is what per cent of 25?, 16% of 25 is what?, 75 is 150% of what number?" This lesson is planned to help pupils become aware of such sentences that express the three cases. They learn how to deal with these abstract statements using the language of per cent.

**Answers**

Pages 134-135:

A 300%; The per cent is more than 100% because 150 is being compared with a smaller number, 50.

B 150

C $50; 50

D 250%; $50 is being compared with $20. 50/20=[250]/100; Sentence 1

E 60/100=30/[50]; Sentence 2

F Fewer than 100%, or all, of the 50 sixth graders were in the class play. 40/100=[20]/50; Sentence 1

G 175/100=[35]/20; Sentence 1

H 24%; 12/50=[24]/100; Sentence 1

I 125/100=5/[4]; Sentence 2

(block 2)

A 84/100=42/n; 50

B 65/100=p/20; 13

C 100/25=r/100; 400%

D 180/100=36/b; 20

E 14/200=a/100; 7%

F 210/100=b/100; 210

G 92/50=x/100; 184%

H 38/100=s/300; 114

I 54/100=27/t; 50

J 16/100=c/25; 4

K 150/100=75/p; 50

L 5/10=r/100; 50%

## Exploring problems

A The sixth-grade class gave a play at the Cook School Friday afternoon and Friday evening. 50 tickets were sold for the afternoon performance, and 150 tickets were sold for the evening performance. The number of tickets that were sold for the evening performance was what per cent of the number of tickets that were sold for the afternoon performance? 1

THINK To solve the problem, you must find what per cent 150 is of 50. Will the per cent be more than 100% or less than 100%? How do you know?

$$\frac{150}{50} = \frac{b}{100}$$ ← Number of evening tickets to 100 afternoon tickets

150 is ■% of 50.

B Of the 200 tickets sold for the play, the sixth graders sold 75% before the first performance. How many tickets did the sixth graders sell before the first performance? 2

THINK You must find 75% of 200.

$$\frac{75}{100} = \frac{m}{200}$$ ← Number of tickets sold by the sixth graders

■ is 75% of 200.

C 40% of the money that the sixth graders raised by selling tickets was used for expenses. $20 was used for expenses. How much money did the sixth graders raise selling tickets for the class play? 3

THINK 20 is 40% of what number?

$$\frac{40}{100} = \frac{20}{t}$$ ← Number of dollars raised in all

20 is 40% of ■.

D The $50 that the sixth graders raised selling the tickets was what per cent of the $20 that they used for expenses? 4

THINK How do you know that the per cent will be greater than 100%? What equation will you make to find what per cent 50 is of 20?

Which of these two sentences is correct for Problem D?

1 50 is ■% of 20.
2 20 is ■% of 50.

E The sixth graders wanted to buy some classroom equipment. The $30 they made from the play was 60% of the money they needed for equipment. How much money did they need for equipment? 5



THINK 30 is 60% of what number? What equation will you make for Problem E? 1

Which of these two sentences is correct for Problem E?

1 60% of 30 is ■.
2 30 is 60% of ■.

F There are 50 sixth graders in the Cook School. 40% of the sixth graders were in the class play. How many of the sixth graders were in the play?

THINK To solve this problem, you must find the number that is 40% of 50. How do you know that the number will be smaller than 50? What equation will you make? 2

Which of these two sentences is correct for Problem F?

1 40% of 50 is ■.
2 50 is 40% of ■.

G The sixth graders sold refreshments at the play. The money they raised by selling refreshments was 175% of the $20 still needed for the equipment. How much money did they raise selling refreshments? 3

THINK Which of these two sentences is correct for Problem G?

1 175% of 20 is ■.
2 20 is 175% of ■.

H 12 of the 50 sixth graders helped make the scenery for the play. What per cent of the sixth graders helped make the scenery? 4

THINK Which of these two sentences is correct for Problem H?

1 12 is ■% of 50.
2 50 is ■% of 12.

I The number of tickets Sue sold for the play was 125% of the number of tickets Ann sold. Sue sold 5 tickets. How many tickets did Ann sell?

THINK Which of these two sentences is correct for Problem I?

1 125% of 5 is ■.
2 5 is 125% of ■.

For each exercise below, first make an equation. Then find the answer. 5

A 42 is 84% of ■.
B 65% of 20 is ■.
C 100 is ■% of 25.
D 36 is 180% of ■.
E 14 is ■% of 200.
F 210% of 100 is ■.
G 92 is ■% of 50.
H 38% of 300 is ■.
I 27 is 54% of ■.
J 16% of 25 is ■.
K 75 is 150% of ■.
L 5 is ■% of 10.

Now you should be able to make equations for problems and sentences that use per cents.





# 134

1 Bring out that since the number of tickets sold for the evening performance is being compared with the number of tickets sold for the afternoon performance and since more evening tickets were sold, the per cent of evening tickets sold is more than 100%. Get the pupils to explain the equation and read the abstract sentence as "150 is *what* per cent of 50?" first; then solve the equation and use the answer in the abstract statement.
2 Discuss the problem and the equation. Have the abstract statement read and the problem solved in the way Problem A was handled.
3 The problem states that $20 is 40% of the total raised. Discuss the equation and proceed as for the first two problems.
4 Have the pupils establish that $50 is more than 100% of $20. Then have them make the equation, choose the correct abstract statement, and solve the equation.
5 Discuss Problem E and continue on page 135.

# 135

1 Discuss "30 is 60% of what number?" in relation to the problem situation. Have the pupils make an equation, then proceed as for Problem D.
2 When the pupils determine why the number they are to find will be smaller than 50, they should choose the correct abstract statement, make an equation, and solve the equation.
3 Have the pupils read the problem and decide whether the amount to be found will be more or less than the amount given. Have them make an equation and proceed as for the other problems.
4 Help the pupils see that since fewer than the total number of sixth graders made scenery, the per cent will be less than 100. Proceed as for the preceding problems.
5 Have the pupils make the equations and find the answers to Exercises A to L. Then discuss the exercises and have pupils replace the screens with the answers.

# 136-137 Thinking straight

Expanded Notes for this lesson are an page 332.

**Objectives**

The pupil discovers short cuts in multiplying and dividing numbers whose numerals end in one or more zeros.

**Vocabulary**

There are no new words.

**Comments**

Because 10 and powers of 10 are the basis of our numeral system, it is important that pupils be able to use the short cuts that are taught in this lesson. Careful attention is given to explanations concerning annexing zeros in multiplication and crossing out zeros in division because it is important that the pupil be able to explain mathematically what he is doing when he annexes zeros or when he crosses out zeros.

Pupils have already multiplied and divided by using multiples of 10. Those who used *Seeing Through Arithmetics 4* and *5* were introduced to multipliers of 10, 20, 100, 200, etc. (pages 159-160 of Book 4 and page 60 of Book 5). In this lesson the work is extended so that both multiplier *and* multiplicand end in one or more zeros. The pupil should observe that the number of zeros in the product is equal to the total number of final zeros in both multiplier and multiplicand. This generalization should result from an understanding of work like that explained in Exercises A to D on page 136.

Division involving numbers that are multiples of 10 was reviewed in connection with estimating quotients on page 19 of the pupils' book. Pupils will now apply this knowledge to the short cuts for finding exact answers. The division examples are written in the form of dividend over divisor, a form with which the pupil became familiar on pages 108-109. The step-by-step process in Row A (page 137) shows the pupil exactly what is happening to the dividend and the divisor. Row B shows the same short cuts with numerals crossed out instead of rewritten. When discussing the exercises on this page, be sure the pupils can explain what the short cuts mean.

**Answers**

Page 136:

(block 1)

A ——  B 16  C 24; 100

D 2; 2; 4

E ——

F 10. A product 10 times 168 is needed.

G 10. A product 10 times 1680 is needed.

H Because 70 and 240 each end in zero; $70=7\times10$ and $240=24\times10$

I 8. Because 8000 ends in 3 zeros and 850 ends in 1 zero; $8000=8\times1000$ and $850=85\times10$

J Because 27000 ends in 3 zeros and 90 ends in 1 zero; $27000=27\times1000$ and $90=9\times10$

(block 2)

| | |
|---|---|
| A 33,000 | H 7,200,000 |
| B 32,000 | I 1,656,000 |
| C 45,000 | J 1,950,000 |
| D 52,800 | K 4,018,000 |
| E 436,600 | L 750,000 |
| F 126,700 | M 5,500,000 |
| G 435,000 | N 3,010,000 |

Page 137:

(block 1)

A Below the line

B $108000=100\times1080$ and $1800=100\times18$; yes

C Yes; yes; yes

D 100

E $1080\div18=60$; $18\div18=1$

F Each is divided by 10; 5; yes

(block 2)

| | |
|---|---|
| A 60 | F 21 |
| B 150 | G 435 |
| C 40 | H 3 |
| D 270 | I 30 |
| E 840 | J 700 |

## Thinking straight

**1** $2400 \times 1600 = r$

| A | B | C |
|---|---|---|
| 1600 | 1600 | 1600 |
| 2400 | 2400 | 2400 |
| 384 | 38400 | 3840000 |

**1** A You can use a short cut to find the product of 1600 and 2400. Study the three steps shown above. Look at Step A. First multiply 16 and 24.

**2** B Look at Step B. You have found the product of 16 and 24. But you should have multiplied 1600 instead of 16. 1600 is 100 times ▪. So write two zeros after 384 to make a new product that is 100 times the product in Step A.

C Look at Step C. The 38400 in Step B is the product of 1600 and 24. But you should have multiplied 1600 by 2400. 2400 is 100 times ▪. You write two more zeros after 38400 to make a new product that is ▪ times the product in Step B.

**3** D 1600 ends in ▪ zeros. 2400 ends in ▪ zeros. Since the product of 16 and 24 is 384, you can put ▪ zeros after 384 to get the product of 1600 and 2400.

**4** E To find the product of 240 and 70, first multiply 24 and 7.

F 240 is ▪ times 24. Why do you now write a zero after 168?

G 70 is ▪ times 7. Why do you write another zero after 1680?

H How do you know that there will be two zeros at the end of the product of 70 and 240?

I To multiply 8000 by 850, first multiply 85 by ▪. Why do you write four zeros after 680?

J When you multiply 27000 by 90, why do you write four zeros after 243?

**5** Solve these equations. Use the short cut to make your work easier.

| | | | |
|---|---|---|---|
| A | $300 \times 110 = r$ | H | $400 \times 18000 = m$ |
| B | $320 \times 100 = g$ | I | $920 \times 1800 = t$ |
| C | $4500 \times 10 = a$ | J | $6500 \times 300 = b$ |
| D | $160 \times 330 = d$ | K | $9800 \times 410 = r$ |
| E | $59 \times 7400 = s$ | L | $750 \times 1000 = h$ |
| F | $1810 \times 70 = n$ | M | $1100 \times 5000 = c$ |
| G | $870 \times 500 = w$ | N | $430 \times 7000 = v$ |



# 136

1. Have the pupils examine the equation and read Exercise A. Ask: "What does 384 in Step A really represent?"
2. Discuss Exercises B and C. Ask: "Is 38,400 one hundred times 384? How do you know?"
3. Read Exercise D. Ask: "Is 3,840,000 one hundred times 38,400? How do you know? Is the product of $2400 \times 1600$ ten thousand times 384, the product of $24 \times 16$? When you put four zeros at the right of 384, do you have the name of the number that is the product of 10,000 times 384?"
4. Have the pupils answer the questions in Exercises E to J orally. Let them verify their answers by multiplying.
5. Assign Exercises A to N as written work. Have the pupils verify their answers by comparing them with a list supplied by you. Then call on several pupils to explain the short cuts they used in various exercises.

$108000 \div 1800 = s$

**1** A $\frac{108000}{1800} = \frac{108000 \div 100}{1800 \div 100} = \frac{1080}{18} = \frac{1080 \div 18}{18 \div 18} = \frac{60}{1} = 60$

**3** B $\frac{108000}{1800} = \frac{108000}{1800} = \frac{1080}{18} = \frac{60}{\frac{18}{1}} = \frac{60}{1} = 60$

**1** A You learned on pages 108 and 109 that you can write division exercises in the way shown above. Where do you write the divisor?

**2** B Look at the work in Row A. How do you know that both 108000 and 1800 can be divided by 100? Will 1080 divided by 18 give the same quotient as 108000 divided by 1800?

C Is 60 the quotient for 108000 divided by 1800? Is 60 also the quotient for 1080 divided by 18? For 60 divided by 1?

D Row B shows another way to do the work. Notice that two zeros have been crossed out in 108000 and in 1800. This is done to show that both 108000 and 1800 have been divided by ▪.

E Now look at the fourth example in Row B. Both 1080 and 18 are divided by 18. Why is 60 written above 1080? Why is 1 written below 18? 1080 and 18 are crossed out to show that they are replaced by 60 and 1.

**4** F $1950 \div 250 = a$

$\frac{1950}{250} = \frac{\overset{39}{\cancel{195}\,0}}{\underset{5}{\cancel{25}\,0}} = \frac{39}{5} = 7\frac{4}{5}$

Why was one zero crossed out in 1950 and in 250? What divisor was used when 195 and 25 were crossed out? Is $7\frac{4}{5}$ the correct quotient for $1950 \div 250$?

**5** Solve these equations. Write your division in the way shown in Row B.

| | |
|---|---|
| A | $42000 \div 700 = x$ |
| B | $9000 \div 60 = d$ |
| C | $180000 \div 4500 = a$ |
| D | $67500 \div 250 = w$ |
| E | $1596000 \div 1900 = p$ |
| F | $1323000 \div 63000 = t$ |
| G | $7612500 \div 17500 = r$ |
| H | $570000 \div 190000 = g$ |
| I | $1050000 \div 35000 = z$ |
| J | $5880000 \div 8400 = e$ |





# 137

1. Have the pupils observe carefully how the division in this equation is written in Row A.
2. Discuss the work in Row A as outlined in Exercises B and C. Have a pupil explain what is being done in each step. Establish the idea that both dividend and divisor can be divided by the same number without affecting the quotient.
3. Have the pupils study Row B and relate the crossing off of two zeros to the division by 100 in Row A. Point out that 1080 and 18 are crossed out and replaced by 60 and 1. Ask: "What does this mean?" Discuss why 60 over 1 equals 60.
4. Let a pupil explain each step in Example F.
5. Assign Exercises A to J as written work. Supply answers and let the pupils verify their own work. Then have them explain the short cuts they used.

# 138-139 Using arithmetic

Expanded Notes for this lesson ore on pages 332-333.

**Objectives**

The pupil solves problems using ratio equations and short cuts for multiples of 10.

**Vocabulary**

New words page 138 Neptune, earth*, equator, Mars, moon*, sound*

**Comments**

Before assigning Problems A to M as written work, review the procedure for setting up ratio equations. Remind the pupils that the corresponding terms of the ratio equation must represent the same kind of thing. Have the pupils write the ratio equation for each problem and then solve it. Remind them to use the short cuts for multiples of 10.

The "Keeping skillful" exercises may be assigned as separate lessons as time permits. Again, remind the pupils to use the short cuts learned on pages 136 and 137.

**Answers**

Pages 138-139:

A 10/1=[31,000]/3100 or 1/10=3100/[31,000]

B 30/1=[108,000]/(3600) or 1/30=(3600)/[108,000]. It represents the number of seconds in one hour.

C 19/1=[68,400]/(3600) or 1/19=(3600)/[68,400]

D 500/1=25,000/[50] or, 1/500=[50]/25,000. 1, or 1 hour, represents time needed to travel 500 mi.

E 500/1=13,000/[26] or 1/500=[26]/13,000; yes

F 12/1=[4380]/365 or 1/12=365/[4380]. 1 represents the earth's year; therefore, 12 represents Jupiter's year which is 12 times as long as the earth's year.

G 1/12=25/[300] or 12/1=[300]/25. 1 represents the 1 trip the earth makes while the moon is making 12 trips.

H 1/1100=[5]/5500 or 1100/1=5500/[5]; 11

I 1100/1=[3,960,000]/(3600) or 1/1100=(3600)/[3,960,000]; to change 1 hr. to seconds

J 5/1=(3600)/[720] or 1/5=[720]/(3600)

K 1100/1=[11,000]/10 or 1/1100=10/[11,000]; yes

L 1200/1=240,000/[200] or 1/1200=[200]/240,000

M 1200/1=24,000/[20] or 1/1200=[20]/24,000

Keeping skillful:

(block 1)

A 1,020,000
B 648,000
C 3,255,000
D 612,000,000
E 46,425,600
F 55,800,000
G 477,260,000
H 404,700,000
I 72,540,000
J 84,280,000
K 738,000,000
L 247,380,000
M 18,312,000
N 52,480,000
O 36,000,000
P 477,260,000
Q 478,560,000
R 166,970,000

(block 2)

A 3
B 40
C $19\frac{9}{65}$
D $5\frac{2}{11}$
E $13\frac{11}{70}$
F $263\frac{5}{16}$
G 20
H $224\frac{2}{7}$
I $100\frac{10}{11}$
J 3
K 300
L $5\frac{5}{18}$
M $21\frac{1}{10}$
N 4
O 300
P $15\frac{5}{31}$
Q $1\frac{3}{10}$
R 6

(block 3)

A 1560
B 400
C 70
D 7
E 300
F 3
G 270
H 9

## Using arithmetic 1

Use two ratios in your equation for each of the following problems.

A The diameter of the planet Neptune is about 10 times as great as the diameter of the planet Mercury. Mercury has a diameter of about 3100 mi. Neptune has a diameter of about how many miles?

B Mercury travels around the sun at a speed of about 30 mi. per second. About how far does Mercury travel in 1 hour?

THINK Why will the second term of the second ratio in your equation be $60 \times 60$?

$$\frac{30}{1} = \frac{a}{60 \times 60}$$

C The earth travels around the sun at a speed of about 19 mi. per second. About how far does the earth travel in 1 hour?

D The distance around the earth at the equator is about 25,000 mi. If an airplane traveled at an average speed of 500 mi. per hour, in how many hours could it travel around the earth at the equator?

THINK Why do you use 1 in one ratio?

$$\frac{500}{1} = \frac{25000}{n}$$

E The distance around the planet Mars at its equator is about 13,000 mi. About how many hours would it take the airplane in Problem D to travel around Mars at the equator?

THINK Is this problem like Problem D?

F The earth travels once around the sun in 365 da. The planet Jupiter takes 12 times as many days to travel once around the sun. How many days does it take Jupiter to travel once around the sun?

THINK One ratio can be 12 over 1. What does the 1 represent?

G For each trip that the earth makes around the sun, the moon makes 12 trips around the earth. When the earth has made 25 trips around the sun, how many trips will the moon have made around the earth?

THINK One ratio can be 1 over 12. What does the 1 represent?

H Sound travels through the air at a speed of about 1100 ft. per second. In about how many seconds will sound travel 1 mi.?

THINK To make your work easier, use 5500 as a round number for the number of feet in 1 mi. You can get the answer by dividing 55 by ▩.



I About how many feet does sound travel in 1 hour?

THINK You will use $60 \times 60$ in one ratio. Why?

J The speed of sound through the air is about how many miles per hour?

K Sound traveled from one point to another point in 10 seconds. The two points were about how many feet apart?

THINK Is it correct to say that the two points were about 2 mi. apart?

L An airplane traveled at a speed of 1200 mi. per hour. About how many hours would it take this airplane to travel the 240,000 mi. from the earth to the moon?

M About how many hours would it take the airplane in Problem L to travel around the earth at the equator?

THINK To make your work easier, use 24,000 mi. as a round number for the distance around the earth at the equator.

## Keeping skillful 2

| | | |
|---|---|---|
| A $600 \times 1700 = b$ | A $18000 \div 6000 = t$ | A $\frac{7800}{2750} = \frac{x}{550}$ |
| B $1800 \times 360 = r$ | B $48000 \div 1200 = a$ | B $\frac{3000}{1200} = \frac{1000}{m}$ |
| C $1550 \times 2100 = m$ | C $49760 \div 2600 = z$ | C $\frac{6300}{4590} = \frac{d}{51}$ |
| D $800 \times 765000 = d$ | D $57000 \div 11000 = r$ | D $\frac{16000}{2800} = \frac{40}{t}$ |
| E $960 \times 48360 = t$ | E $92100 \div 7000 = b$ | E $\frac{5400}{180} = \frac{r}{10}$ |
| F $31000 \times 1800 = n$ | F $42130 \div 160 = m$ | F $\frac{2500}{300} = \frac{25}{n}$ |
| G $4870 \times 98000 = a$ | G $86000 \div 4300 = d$ | G $\frac{81000}{2700} = \frac{c}{9}$ |
| H $57000 \times 7100 = c$ | H $78500 \div 350 = c$ | H $\frac{90000}{1800} = \frac{450}{a}$ |
| I $7800 \times 9300 = f$ | I $111000 \div 1100 = n$ | |
| J $1960 \times 43000 = d$ | J $24000 \div 8000 = p$ | |
| K $9000 \times 82000 = p$ | K $39000 \div 130 = f$ | |
| L $434000 \times 570 = a$ | L $4750 \div 900 = y$ | |
| M $2400 \times 7630 = n$ | M $63300 \div 3000 = h$ | |
| N $82000 \times 640 = b$ | N $80000 \div 20000 = r$ | |
| O $3000 \times 12000 = y$ | O $57000 \div 190 = n$ | |
| P $9800 \times 48700 = d$ | P $94000 \div 6200 = k$ | |
| Q $4800 \times 99700 = g$ | Q $6500 \div 5000 = m$ | |
| R $5900 \times 28300 = n$ | R $78000 \div 13000 = w$ | |





# 138

1 Review ratio equations, using Problem A to illustrate the procedure.

2 Assign Problems A to M as written work. (Problems I to M are on page 139.) Be sure to have the pupils make a ratio equation for each problem. Have them work independently, paying special attention to the helps labeled "Think."

# 139

1 Supply answers for the problems. Discuss the ratio equations for each problem that caused trouble. Be sure the pupils can justify each equation they have used.

2 Assign these exercises as separate lessons as time permits. Remind pupils to use the short cuts learned on pages 136-137.

# 140 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil discovers different ways to write and solve equations involving more than one operation.

**Vocabulary**

There are no new words.

**Comments**

The pupils have studied multiple-step equations on pages 108-109. In this lesson they get further practice in handling equations of this type that involve division. They also learn to cross out numerals to show division. The pupils should examine the equations at the top of the page and notice the different ways in which they are written and how they are solved. It is important that the pupils observe that if the dividend involves multiplication, only one number (factor) is divided; if the dividend involves addition or subtraction, both numbers must be divided. Let the pupils illustrate this by using the examples in the book.

**Answers**

(block 1)

A In Example C the parentheses and the division sign are omitted. The division is indicated by writing the dividend above the line, and the divisor below.

B 4

C Yes

D 8 and 32 are to be added instead of multiplied.

E The sum of 2 and 8 is 10. We still must think of 10 as divided by 1, but dividing a number by 1 always results in the number divided.

F Because both 78 and 36 were divided by 6

G Yes

(block 2)

| | |
|---|---|
| A 1 | G 1 |
| B 42 | H 588 |
| C 5 | I 27 |
| D 11 | J 57 |
| E 168 | K 1 |
| F 2 | L 46 |

## Thinking straight

A $(8 \times 32) \div 4 = w$

B $(8 \times 32) \div 4 = 256 \div 4 = 64$

C $\frac{8 \times 32}{4} = \frac{256}{4} = 64$

D $\frac{\overset{2}{\cancel{8}} \times 32}{\underset{1}{\cancel{4}}} = \frac{2 \times 32}{1} = \frac{64}{1} = 64$

E $(8 + 32) \div 4 = w$ 3

F $(8 + 32) \div 4 = 40 \div 4 = 10$

G $\frac{8 + 32}{4} = \frac{40}{4} = 10$

H $\frac{\overset{2}{\cancel{8}} + \overset{8}{\cancel{32}}}{\underset{1}{\cancel{4}}} = \frac{2 + 8}{1} = \frac{10}{1} = 10$

A How does the work in Example C differ from that in Example B?

B In Example D, 8 and 4 are crossed out to show they are divided by ■

C Another way to do the work is to divide both 32 and 4 by 4. Do you still get 64 as the quotient?

D How does Equation E differ from Equation A?

E In Example H, when you divide 4 by 4, you must also divide both 8 and 32 by 4. How do you get 10?

F $(78 - 36) \div 6 = t$

$\frac{\overset{13}{\cancel{78}} - \overset{6}{\cancel{36}}}{\underset{1}{\cancel{6}}} = \frac{13 - 6}{1} = \frac{7}{1} = 7$

Why were both 78 and 36 crossed out?

G $(11 \times 30) \div 10 = m$

$\frac{11 \times \overset{3}{\cancel{30}}}{\underset{1}{\cancel{10}}} = \frac{11 \times 3}{1} = \frac{33}{1} = 33$

Is the division correct?

When you solve the equations below, cross out numerals to show division. 5

A $(45 - 30) \div 15 = d$

B $(7 \times 24) \div 4 = r$

C $(117 + 78) \div 39 = s$

D $(50 + 60) \div 10 = a$

E $(21 \times 56) \div 7 = n$

F $(91 - 65) \div 13 = w$

G $(125 - 100) \div 25 = t$

H $(49 \times 84) \div 7 = y$

I $(65 + 70) \div 5 = b$

J $(57 \times 19) \div 19 = m$

K $(111 - 74) \div 37 = k$

L $(1400 + 900) \div 50 = f$



# 140

1 Have the pupils observe that Example C is another way of writing the division expressed in Equation B.

2 Be sure the pupils understand that only one numeral is crossed out in the dividend of Example D. Point out that dividing *both* 8 and 32 by 4 would be the same as dividing 256 by 16.

3 Have the pupils point out the difference between Equation E and Equation A. Be sure they notice that both numerals are crossed out in the dividend in Example H. Have a pupil explain this by using Examples F and G.

4 Have Exercises F and G put on the chalkboard and explained by the pupils.

5 Assign Examples A to L as written work. Let the pupils verify their work and then discuss any problems that have caused difficulty.

# 141-142 Thinking straight

Expanded Notes for this lesson are on page 334.

**Objectives**

The pupil learns to apply a test (called the "ratio test") to determine the equality of ratios.

**Vocabulary**

There are no new words.

**Comments**

Until now the pupils have found a ratio equal to a given ratio by multiplying or dividing both terms of the given ratio by the same number. This same principle was used in finding the missing term in a pair of equal ratios, but the examples studied were confined to those in which the known term of one ratio was a multiple or a factor of the corresponding term in the other ratio. With this background, the pupil can benefit from learning that two ratios are equal if the products of the opposite terms are equal. The techniques learned in this lesson will also be useful later when the pupil finds it necessary to use fraction numerals as terms of ratios. Then the "ratio test" becomes a convenient way of solving equations of ratios.

**Answers**

Page 142:

A 24; 8 and 84. The product of 21 and 24 is 504. The product of 8 and 84 is 672. The products are not the same; so the ratios are not equal.

B $\frac{21}{8} \neq \frac{84}{24}$

C The product of 75 and 6 is 450. The product of 18 and 25 is 450. The ratios are equal.

D $\frac{13}{26} = \frac{10}{20}$

E $\frac{16}{42} \neq \frac{4}{12}$

F $\frac{47}{92} \neq \frac{94}{174}$

G $\frac{30}{50} = \frac{12}{20}$

H $\frac{99}{115} = \frac{198}{230}$

I $\frac{650}{125} = \frac{26}{5}$

J $\frac{29}{51} \neq \frac{9}{17}$

K $\frac{100}{75} = \frac{120}{90}$

L $\frac{625}{500} = \frac{125}{100}$

M $\frac{41}{65} \neq \frac{200}{325}$

N $\frac{70}{35} = \frac{100}{50}$

O $\frac{98}{24} \neq \frac{588}{168}$

P $\frac{99}{77} = \frac{198}{154}$

Q $\frac{24}{6} = \frac{80}{20}$

R $\frac{240}{80} \neq \frac{10}{3}$

S $\frac{77}{11} \neq \frac{1}{7}$

**Keeping skillful:**

(block 1)

A $\frac{20}{5} = \frac{[400]}{100}$

B $\frac{18}{100} = \frac{[54]}{300}$

C $\frac{9}{100} = \frac{36}{[400]}$

D $\frac{160}{100} = \frac{80}{[50]}$

E $\frac{500}{50} = \frac{[1000]}{100}$

F $\frac{48}{100} = \frac{[336]}{700}$

G $\frac{21}{300} = \frac{[7]}{100}$

H $\frac{152}{100} = \frac{[38]}{25}$

I $\frac{80}{500} = \frac{[16]}{100}$

J $\frac{5}{100} = \frac{30}{[600]}$

K $\frac{60}{100} = \frac{[30]}{50}$

L $\frac{900}{400} = \frac{[225]}{100}$

M $\frac{32}{100} = \frac{64}{[200]}$

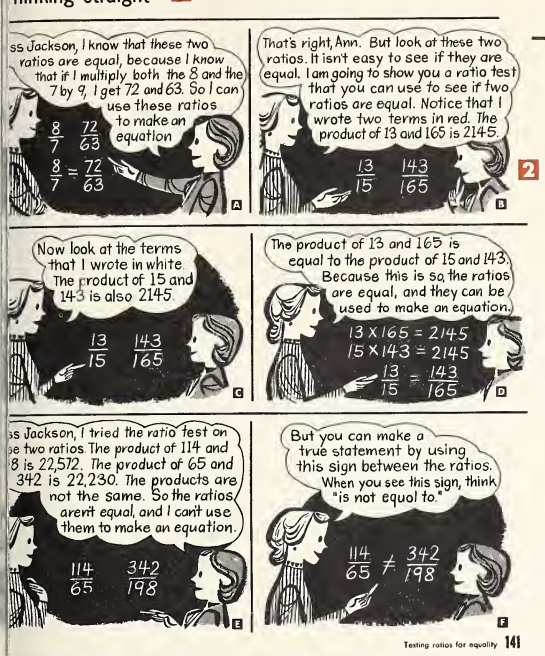
(block 2)

A 14,080 yd.
B 80 hr.
C 10 yd.
D 96 pt.
E 18,000 lb.
F 18 da.
G 1120 oz.
H 12 c.
I 5110 da.
J 24 fl. oz.
K 2 mi.
L 152 pt.
M 12 lb.

(block 3)

A 938,262
B 131,421
C $121\frac{3}{10}$
D $606\frac{209}{297}$
E 850,782,000
F $28\frac{1}{3}$
G 537,900
H $410\frac{19}{30}$
I 9038
J 8351
K $71\frac{2}{5}$

## Thinking straight 1


ss Jackson, I know that these two ratios are equal, because I know that if I multiply both the 8 and the 7 by 9, I get 72 and 63. So I can use these ratios to make an equation
8/7 = 72/63
A
That's right, Ann. But look at these two ratios. It isn't easy to see if they are equal. I am going to show you a ratio test that you can use to see if two ratios are equal. Notice that I wrote two terms in red. The product of 13 and 165 is 2145.
13/15 143/165
B
Now look at the terms that I wrote in white. The product of 15 and 143 is also 2145.
13/15 143/165
C
The product of 13 and 165 is equal to the product of 15 and 143. Because this is so, the ratios are equal, and they can be used to make an equation.
13 × 165 = 2145
15 × 143 = 2145
13/15 = 143/165
D
ss Jackson, I tried the ratio test on e two ratios. The product of 114 and 8 is 22,572. The product of 65 and 342 is 22,230. The products are not the same. So the ratios aren't equal, and I can't use them to make an equation.
114/65 342/198
E
But you can make a true statement by using this sign between the ratios. When you see this sign, think "is not equal to."
114/65 ≠ 342/198
F

Testing ratios for equality 141

To use the ratio test r the ratios at the right, $\frac{21}{8}$ $\frac{84}{24}$ st find the product of and ■. Then find the product of id ■. Make a true statement about e products.

Now write the ratios with the sign at makes a true statement.

Use the ratio test to e whether or not the tios at the right are $\frac{75}{18}$ $\frac{25}{6}$ ual. Then write a true statement out these two ratios. 3

Exercises D to S, use the ratio test see whether or not the ratios are ual. Write each exercise so that it makes a true statement. Use the sign = or the sign ≠. 4

D $\frac{13}{26}$, $\frac{10}{20}$

E $\frac{16}{42}$, $\frac{4}{12}$

F $\frac{47}{92}$, $\frac{94}{174}$

G $\frac{30}{50}$, $\frac{12}{20}$

H $\frac{99}{115}$, $\frac{198}{230}$

I $\frac{650}{125}$, $\frac{26}{5}$

J $\frac{29}{51}$, $\frac{9}{17}$

K $\frac{100}{75}$, $\frac{120}{90}$

L $\frac{625}{500}$, $\frac{125}{100}$

M $\frac{41}{65}$, $\frac{200}{325}$

N $\frac{70}{35}$, $\frac{100}{50}$

O $\frac{98}{24}$, $\frac{588}{168}$

P $\frac{99}{77}$, $\frac{198}{154}$

Q $\frac{24}{6}$, $\frac{80}{20}$

R $\frac{240}{80}$, $\frac{10}{3}$

S $\frac{77}{11}$, $\frac{1}{7}$

## Keeping skillful 5

20 is ■% of 5.
18% of 300 is ■.
36 is 9% of ■.
80 is 160% of ■.
500 is ■% of 50.
48% of 700 is ■.
21 is ■% of 300.
152% of 25 is ■.
80 is ■% of 500.
30 is 5% of ■.
60% of 50 is ■.
900 is ■% of 400.
64 is 32% of ■.

A 8 mi. = ■ yd.
B 4800 min. = ■ hr.
C 360 in. = ■ yd.
D 12 gal. = ■ pt.
E 9 T. = ■ lb.
F 432 hr. = ■ da.
G 70 lb. = ■ oz.
H 96 fl. oz. = ■ c.
I 14 yr. = ■ da.
J 48 tbs. = ■ fl. oz.
K 3520 yd. = ■ mi.
L 19 gal. = ■ pt.
M 192 oz. = ■ lb.

A $897 \times 1046 = d$
B $94872 + 36549 = a$
C $96\frac{3}{8} + 24\frac{7}{10} = r$
D $360382 \div 594 = w$
E $57000 \times 14926 = g$
F $65\frac{1}{4} - 36\frac{11}{12} = c$
G $1100 \times 489 = r$
H $403\frac{5}{8} + 6\frac{4}{5} = t$
I $724 + 99 + 8215 = m$
J $242179 \div 29 = b$
K $118\frac{1}{3} - 46\frac{14}{15} = f$

# 141

1 Let the pupils read the dialogue on this page. Be sure the class understands that when both numbers represented by the terms of a ratio are multiplied or divided by the same number, the new ratio expresses the same relationship as the old. Point out that when one ratio is equal to another ratio, this may be shown by putting an equals sign between the two ratios.

2 Discuss Pictures B and C. Bring out that if the product of two diagonally opposite terms is equal to the product of the other terms, the ratios are equal. Point out that a true statement has been made about the ratios when the equals sign is inserted in Picture D.

3 Pupils may wish to verify the products found in Picture E. Point out that even though the ratios are not equal, a true statement can be made about them by using the special sign ($\neq$), which means "is not equal to."

# 142

1 Have a pupil supply the missing numbers in Exercise A. Be sure the pupils understand why the phrase "is not equal to" must be used in the true statement.

2 A pupil may write the true statement (using the proper sign) on the chalkboard.

3 Have pupils read Exercise C, compute on scratch paper, and write a true statement. Verify the results.

4 Assign Exercises D to S for individual written work. Allow time for discussion.

5 Assign the "Keeping skillful" exercises as written work.

# 143-145 Moving forward

Expanded Notes for this lesson are on pages 334-335.

**Objectives**

The pupil learns how to use the ratio test to solve equations of ratios.

**Vocabulary**

New words page 143 unwise*

**Comments**

The ratio test is presented here as a general mathematical procedure for solving equations of ratios.

The fact that the cross-products are equal follows directly from the mathematical definition of equal ratios, which states that $a/b=c/d$ if and only if $ad=bc$. (You should not discuss this mathematical definition of equal ratios with your pupils. They will arrive at this principle by considering special instances of equal ratios—for example, $2/3=4/6$.) Since the ratio test is a direct application of this definition, it becomes a valuable means of solving ratio equations, especially when the numbers involved are fractions or mixed numbers.

One important property of the multiplication operation—the commutative law—is applied in the process of solving ratio equations by the ratio test. The term "commutative" should not be used in talking with the pupils, but the principle, which says that $18\times2$ and $2\times18$ both represent the same number, is of fundamental importance in mathematics. The four ways of writing the ratio test for Example A on page 143 are permissible because of the commutative law and because the two members, or "sides," of an equation are interchangeable.

The authors have avoided using the word "cancel" in the pupils' text because it is commonly used for several different operations. Note that in Example C on page 143, the pupils are not dividing 18 by 3 or "canceling" 3 into 18. They are using the principle of dividing dividend and divisor by the same number—3, in this case. This principle has a sound mathematical basis, and the pupils already know that equal fraction numerals or ratios may be found by dividing both terms by the same number. Also, the pupils saw, on page 137, that when a division example is written in the form "dividend over divisor," the number above the line (dividend) and the number below the line (divisor) may be divided by the same number with no change in the quotient. And they saw this principle used again on page 140. It is this principle that is used on page 144. The teacher may use the term *cancel*, but should make certain that its meaning in this situation is understood. The use of the principle makes the computation easier in solving the equations in this lesson. Crossing out the numerals provides a record of what has been done.

**Answers**

Pages 143-144:

A ——

B Because it would look like 182

C 3; the product of 6 and 2 is found, and it then is divided by 1; 12

D Yes; divide the product of 36 and 100 by 12; 12; the product of 3 and 100 is divided by 1; 300

E 20; 2; 2; by crossing out 16 and 2; yes

F 100 and 340 are each divided by 10, and then 10 and 34 are each divided by 2; the product of 85 and 5 is 425, and 17 is the result of dividing 340 by 10 and then by 2; 25; 25; yes; by crossing out 85 and 17 and dividing each by 17, then placing 5 above 85 and 1 below 17

Page 145:

(block 1)

A 75 B 70 C 300 D 4 E 55 F 13

(block 2)

A 32
B 30
C 32
D 60
E 252
F 30
G 28
H 135
I 30

## Moving forward

Now you will learn to use the ratio test when you solve certain kinds of equations.

$\frac{8}{3}=\frac{n}{2}$

$3\times n=18\times 2$

$n\times 3=2\times 18$

$8\times 2=3\times n$

$2\times 18=n\times 3$

**1**

Look at Example A. To find the numeral that replaces $n$, you can use the ratio test. The work in green shows different ways of writing the ratio test. All of these ways are correct.

---

$3n=18\times 2$

$3n=36$ — To find the numeral that replaces $n$, divide 36 by 3.

$\frac{36}{3}=$ ▪

$n$ is replaced with 12. $3\times 12=36$

**2**

You can often show, without using the multiplication sign, that you are to multiply. Look at Example B. $3n$ means $3\times n$. Why would it be unwise to write $18\times 2$ in the same way?

Now look at the work in brown. How can you find the numeral that replaces $n$? What is this numeral?

---

$3n=18\times 2$ — To find the numeral that replaces $n$, you can divide $18\times 2$ by 3.

$\frac{18\times 2}{3}=\frac{6\times 2}{1}=\frac{12}{1}=12$

$n$ is replaced with 12. $3\times 12=18\times 2$

**3**

Example C and the work in red show another way to find the numeral that replaces $n$. 18 and 3 are crossed out to show that each number is divided by ▪. What is done next? What numeral replaces $n$?



---

$\frac{12}{5}=\frac{100}{k}$

$12\times k=36\times 100$

$12k=36\times 100$

**1**

In Example D, does $12k$ mean the same as $12\times k$? What must you do to find the numeral that replaces $k$? 36 and 12 are crossed out to show that each is divided by ▪. What is done next? What is the numeral that replaces $k$?

$\frac{36\times 100}{12}=\frac{3\times 100}{1}=\frac{300}{1}=300$

---

$\frac{d}{16}=\frac{20}{16}$

$16d=16\times 30$

**2**

Look at the work for Example E. To find the numeral that replaces $d$, you divide $16\times 30$ by ▪. 30 and 20 are crossed out to show that each is divided by 10. Now you have $16\times 3$ above the line and ▪ below the line.

Then both 16 and 2 are divided by ▪. How is this shown?

A shorter way to write the work is shown in red. Is 24 the numeral that replaces $d$?

$\frac{16\times 30}{20}=\frac{16\times 3}{1}=\frac{24}{1}=24$

$\frac{16\times 30}{20}=\frac{24}{1}=24$

---

$\frac{85}{40}=\frac{t}{100}$

$40t=85\times 100$

**3**

Look at the work for Example F. Explain the division shown by the crossed-out numerals. How do you get 425 over 17? What do you get when you divide 425 by 17? What numeral replaces $t$?

Could 85 and 17 have been divided by the same number? How would you show this?

$\frac{85\times 100}{340}=\frac{85\times 5}{17}=\frac{425}{17}=25$

# 143

1. For Example A, discuss the different ways of writing the ratio test for this equation and emphasize that each is correct.
2. Have Example B read and ask pupils to relate it to the steps that follow. Make sure that pupils understand that the multiplication sign may be omitted between a numeral and a letter. Be sure pupils know that $\frac{36}{3}$ is a way of indicating division.
3. Point out that $\frac{18\times 2}{3}$ is the same as $\frac{36}{3}$ in Example B, except that the product has not been found. Be sure the pupils understand that the division is performed by dividing only one number of the dividend and one of the divisor by the same number.

# 144

1. Bring out that $12k$ means the same as $12\times k$ and $k\times 12$. Then to find a replacement for $k$, it is necessary to divide the product of 36 and 100 by 12. Have pupils observe how the work is simplified by first dividing 36 and 12 by 12.
2. In Example E the pupils should see that $16\times 30$ must be divided by 20 to find a replacement for $d$. Ask pupils to explain why 30 and 20 have been crossed out and what number was used as a divisor. Then ask what was done to $\frac{16\times 3}{2}$. Point out that the work in red is a shorter way to find the numeral that replaces $d$.
3. Help the pupils observe that 340 and 100 are first divided by 10; then 34 and 10 are divided by 2. Assist them in seeing that 425 is the product of 85 and 5. Let a pupil explain how 25, the numeral that replaces $t$, is found. The pupils should see that a final simplifying step is possible (85 and 17 could each have been divided by 17).

Use two or more ways to find the answer. **1**

A $\frac{15 \times 45}{9} = \square$

B $\frac{30 \times 7}{3} = \square$

C $\frac{100 \times 60}{20} = \square$

D $\frac{28 \times 3}{21} = \square$

E $\frac{25 \times 44}{20} = \square$

F $\frac{78 \times 3}{18} = \square$

In each exercise below, find the numeral to replace the letter. Use the ratio test. **2**

A $\frac{48}{15} = \frac{a}{10}$

B $\frac{c}{9} = \frac{40}{12}$

C $\frac{12}{m} = \frac{15}{40}$

D $\frac{15}{18} = \frac{50}{w}$

E $\frac{k}{63} = \frac{68}{17}$

F $\frac{9}{15} = \frac{n}{50}$

G $\frac{c}{16} = \frac{35}{20}$

H $\frac{b}{90} = \frac{21}{14}$

I $\frac{12}{z} = \frac{16}{40}$

Now you can solve equations of ratios by using the ratio test.

## Using arithmetic

A Mr. Anderson and Tom put a picket fence around the yard. The perimeter of the yard is about 600 ft. They used 25 pickets to make 6 ft. of fence. About how many pickets did they use for the 600-ft. fence?

$\frac{25}{6} = \frac{t}{600}$

How do you know you can find the numeral to replace $t$ by multiplying 25 by 100?

THINK You can also use the ratio test to find the numeral to replace $t$. You get the equation $6t = 25 \times 600$. How do you solve this equation?

B It took Mr. Anderson and Tom 2 hr. to put up 80 ft. of the picket fence. At this rate, how long did it take them to put up the entire fence?

$\frac{80}{2} = \frac{600}{b}$

THINK Can 80 be multiplied by a whole number to get 600? Use the ratio test to find the numeral to replace $b$.

C For every 10 ft. of fence that Mr. Anderson put up, Tom put up 5 ft. of fence. When Mr. Anderson had put up 30 ft. of fence, how many feet of fence had Tom put up?

$\frac{10}{5} = \frac{30}{y}$

THINK When you use the ratio test, what equation do you get? How do you solve this equation?

Using the ratio test in problem solving 145

# 145

1 Encourage pupils to use as many ways to c Exercises A to F as their understanding ar ingenuity permit. The computation for ea exercise may be placed on the chalkboard s that the class can see and discuss the differe ways of finding the answer. Discuss any exe cise that seems troublesome.

2 Assign the exercises in Block 2 as written wor Provide answers so pupils can verify their wor

# 145-146 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns how to apply the ratio test when he solves problems.

**Vocabulary**

New words page 145 Anderson, picket, entire*; page 146 ceilings*, border*

**Comments**

When the pupils have completed this lesson, they will have a means of solving a wide variety of rate and comparison problems by applying the ratio test.

The pupils have solved only ratio equations such as $2/3=4/n$. They noted that since 2 is multiplied by 2 to get 4, therefore, 3 must also be multiplied by 2 to find the numeral that replaces $n$.

Before the pupils studied ratio, they solved problems by direct multiplication or division. For example, the problem "If 1 apple costs 6¢, how much do 5 apples cost?" was solved by using the multiplicative equatio $5 \times 6 = n$. Note, however, that a ratio equation $6/1 = n/5$ can be made for this problem. When the ratio te is applied, the equation that results is $n = 5 \times 6$. Sim larly, for the problem "If you have 2 apples and I hav three times as many, how many apples do I have? the ratio equation is $3/1 = x/2$. When the ratio test applied, the equation $x = 3 \times 2$ results. In this case als a direct multiplicative equation could have been wri ten, since the comparison statement "three times c many" is a way of expressing "three for each one," c 3/1. It is possible to use a short-cut equation only whe the rate or comparison is expressed as "so much per 1 or "so much to 1," so that one of the ratios has 1 for second term. This is a special case, and the pupi should be taught the general case expressed by a rati equation.

The pupils are encouraged to write the equation c ratios because this equation indicates that the pup understands the problem situation. The authors fee

trongly that a short-cut, or special case, equation hould not be stressed. One of the reasons "times as nany" problems have been difficult in the past is that nly the short cut was taught.

Some able pupils may discover, after solving problems using the ratio test, that a shorter way may be used. These pupils will usually be able to recognize when the short cut can be used and will also be able to make the ratios.

**Answers**

**Pages 145-146:**

A 2500; because 6 multiplied by 100 is 600; by dividing the product of 25 and 600 by 6

B 15 hr.; no; $80b = 2 \times 600$

C 15 ft.; $10y = 5 \times 30$; by dividing the product of 5 and 30 by 10

D $90

E Because dividing by 1 will not affect the product of 35 and 5; 175 min.

F Because you are to find how many spice cookies were made for each raisin cooky; 2; 2 times as many

G $\frac{[69]}{3} = \frac{2300}{100}$; cents

H \$27; $\frac{81}{3} = \frac{[2700]}{100}$

I \$1.15; $\frac{[115]}{5} = \frac{2300}{100}$

J 2 times as many; $\frac{14}{7} = \frac{[2]}{1}$

K 30¢; $\frac{600}{20} = \frac{[30]}{1}$; cents

L 15%; $\frac{6}{40} = \frac{[15]}{100}$

se two or more ways to find the nswer.

$\frac{15 \times 45}{9} =$ ▒ D $\frac{28 \times 3}{21} =$ ▒

$\frac{30 \times 7}{3} =$ ▒ E $\frac{25 \times 44}{20} =$ ▒

$\frac{100 \times 60}{20} =$ ▒ F $\frac{78 \times 3}{18} =$ ▒

In each exercise below, find the numeral to replace the letter. Use the ratio test.

A $\frac{48}{15} = \frac{a}{10}$ D $\frac{15}{18} = \frac{50}{w}$ G $\frac{c}{16} = \frac{35}{20}$

B $\frac{c}{9} = \frac{40}{12}$ E $\frac{k}{63} = \frac{68}{17}$ H $\frac{b}{90} = \frac{21}{14}$

C $\frac{12}{m} = \frac{15}{40}$ F $\frac{9}{15} = \frac{n}{50}$ I $\frac{12}{z} = \frac{16}{40}$

Now you can solve equations of ratios by using the ratio test.

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J Mr. Anderson and Tom put a picket ence around the yard. The perimeter f the yard is about 600 ft. They used 5 pickets to make 6 ft. of fence. bout how many pickets did they use or the 600-ft. fence?

$\frac{25}{6} = \frac{t}{600}$

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J It took Mr. Anderson and Tom 2 hr. o put up 80 ft. of the picket fence. At this rate, how long did it take them to put up the entire fence?

$\frac{80}{2} = \frac{600}{b}$

THINK Can 80 be multiplied by a whole number to get 600? Use the ratio test to find the numeral to replace $b$.

3

K For every 10 ft. of fence that Mr. Anderson put up, Tom put up 5 ft. of fence. When Mr. Anderson had put up 30 ft. of fence, how many feet of fence had Tom put up?

$\frac{10}{5} = \frac{30}{y}$

THINK When you use the ratio test, what equation do you get? How do you solve this equation?



# 145

1 Have Problem A read and discuss the question relating to the equation. Ask a pupil to work the problem, using the ratio test. Be sure he can explain how he solved it.

2 Point out that to solve Problem B, information from Problem A must be used. Point out that it is not easy to see what number 80 is multiplied by to get 600. Have a pupil solve the problem by using the ratio test, and let the class verify his work.

3 Have Problem C read and let a pupil write the equation on the chalkboard. Let other pupils solve the equation and verify the results.

D Mr. Anderson bought the fence pickets on sale for $72. The price he paid was 80% of the usual price. How much would he have paid for the pickets at the usual price?

1

$\frac{80}{100}=\frac{72}{d}$ ← Use the ratio test to find the numeral to replace $d$.

E Mrs. Anderson cooked a 5-lb. roast for dinner. The roast had to be cooked 35 minutes per pound. How many minutes did it take to cook the roast?

$\frac{35}{1}=\frac{f}{5}$

2

THINK When you use the ratio test to solve Problem E, why do you not have to divide to solve the equation?

F Kathy Anderson and her mother made 56 raisin cookies and 112 spice cookies. They made how many times as many spice cookies as raisin cookies?

$\frac{112}{56}=\frac{n}{1}$

3

THINK Why do you use $n$ over 1 as one of the equal ratios? When you use the ratio test, you get $56n=112\times 1$. What numeral replaces $n$? How do you read it as the answer to Problem F?

4 G Mr. Anderson paid $23 for a 100-ft. roll of screening. He paid how much per yard for the screening?

THINK Problem G will be easier if you think of $23 as 2300¢. You can use $n$ over 3 for one of the ratios in your equation. What will you use for the other ratio? Will the numeral that replaces $n$ represent dollars or cents?

H If the screening had been bought by the yard, it would have cost 81¢ per yard. How much would Mr. Anderson have had to pay for 100 ft. of the screening if he had bought it by the yard?

5

I Mr. Anderson used 5 ft. of screening for one screen. How much did he pay for the screening for one screen?

J When the Andersons decided to put wallpaper on two rooms, they found that they needed 14 rolls of paper for the walls and 7 rolls for the ceilings. They needed how many times as many rolls of paper for the walls as for the ceilings?

K Mr. Anderson paid $6 for 20 yards of border paper. What was the price of the border paper per yard?

THINK Problem K will be easier if you think of $6 as 600¢. Will your answer represent dollars or cents?

L All the paper for the two rooms cost $40. The amount paid for the border paper was what per cent of the total cost?



# 146

1 Have pupils read Problem D. Get someone to explain what the 100 means in the first ratio. Have a pupil write the equation that results when the ratio test is used. Let pupils solve the equation and verify their work.

2 Have pupils determine where the 1 comes from. Help them understand that in an equation such as $1f=35\times 5$, they do not need to divide by 1 to solve the equation. The numeral that replaces $f$ is found immediately by multiplying the 35 and 5.

3 In Problem F bring out that to find how many times as many spice cookies as raisin cookies there were, they must find how many spice cookies there were for 1 raisin cooky.

4 Discuss Problem G. Help the pupils see where the 3 comes from. Also help them make the ratio equation.

5 Assign Problems H to L as individual work. Provide answers so the pupils can verify their solutions.

# 147 Checking up; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil tests his ability to use the ratio test in solving equations. He also uses ratios to solve equations involving per cents.

**Vocabulary**

There are no new words.

**Comments**

Give each pupil plenty of time to complete all these tests. Provide time and opportunity so the computation can be verified. If reteaching is necessary, materials (for Test 1) can be found on pages 143-145. Equations for per cent problems are taught on pages 130-135.

**Answers**

Test 1

| | | |
|---|---|---|
| A 32 | D 68 | G 82 |
| B 1500 | E 16 | H 15 |
| C 26 | F 7 | I 102 |

Test 2

A $\frac{20}{100}=\frac{[90]}{450}$

B $\frac{25}{100}=\frac{[125]}{500}$

C $\frac{175}{100}=\frac{[84]}{48}$

D $\frac{42}{100}=\frac{[294]}{700}$

E $\frac{225}{100}=\frac{[360]}{160}$

F $\frac{50}{100}=\frac{[48]}{96}$

G $\frac{125}{100}=\frac{[80]}{64}$

H $\frac{90}{100}=\frac{[162]}{180}$

I $\frac{124}{100}=\frac{[217]}{175}$

Test 3

A $\frac{40}{200}=\frac{[20]}{100}$

B $\frac{130}{65}=\frac{[200]}{100}$

C $\frac{24}{96}=\frac{[25]}{100}$

D $\frac{234}{52}=\frac{[450]}{100}$

E $\frac{468}{72}=\frac{[650]}{100}$

F $\frac{180}{450}=\frac{[40]}{100}$

G $\frac{405}{900}=\frac{[45]}{100}$

H $\frac{792}{2200}=\frac{[36]}{100}$

I $\frac{96}{480}=\frac{[20]}{100}$

Test 4

A $\frac{4}{100}=\frac{160}{[4000]}$

B $\frac{36}{100}=\frac{81}{[225]}$

C $\frac{5}{100}=\frac{80}{[1600]}$

D $\frac{75}{100}=\frac{150}{[200]}$

E $\frac{110}{100}=\frac{11}{[10]}$

F $\frac{1}{100}=\frac{6}{[600]}$

G $\frac{80}{100}=\frac{16}{[20]}$

H $\frac{104}{100}=\frac{52}{[50]}$

I $\frac{24}{100}=\frac{96}{[400]}$

Test 5

A $\frac{20}{100} = \frac{[14]}{70}$

B $\frac{80}{160} = \frac{[50]}{100}$

C $\frac{25}{100} = \frac{54}{[216]}$

D $\frac{150}{100} = \frac{75}{[50]}$

E $\frac{184}{100} = \frac{[46]}{25}$

F $\frac{600}{40} = \frac{[1500]}{100}$

G $\frac{130}{100} = \frac{65}{[50]}$

H $\frac{130}{650} = \frac{[20]}{100}$

I $\frac{32}{100} = \frac{[200]}{625}$

**Keeping skillful:**

| (block 1) | (block 2) | |
|---|---|---|
| A 41,359 | A 31,725 | H 18,269 |
| B 3683 | B 107,708 | I 4719 |
| C 208,541 | C 24,678 | J 88,275 |
| D 38,690 | D 88,185 | K 8887 |
| E 69,800 | E 201,877 | L 23,709 |
| F 8775 | F 126,946 | M 20,031 |
| | G 32,988 | N 72,580 |

## Checking up 1

Test 1

Use the ratio test when you solve these equations.

A $\frac{64}{a} = \frac{10}{5}$

B $\frac{100}{3} = \frac{w}{45}$

C $\frac{r}{91} = \frac{6}{21}$

D $\frac{3}{4} = \frac{51}{m}$

E $\frac{b}{40} = \frac{200}{500}$

F $\frac{35}{55} = \frac{x}{11}$

G $\frac{123}{t} = \frac{3}{2}$

H $\frac{16}{10} = \frac{24}{c}$

I $\frac{d}{85} = \frac{6}{5}$

Test 2

A 20% of 450 is ▒.
B ▒ is 25% of 500.
C 175% of 48 is ▒.
D 42% of 700 is ▒.
E ▒ is 225% of 160.
F 50% of 96 is ▒.
G ▒ is 125% of 64.
H 90% of 180 is ▒.
I ▒ is 124% of 175.

Test 3

A 40 is ▒% of 200.
B 130 is ▒% of 65.
C ▒% of 96 is 24.
D 234 is ▒% of 52.
E ▒% of 72 is 468.
F 180 is ▒% of 450.
G 405 is ▒% of 900.
H ▒% of 2200 is 792.
I 96 is ▒% of 480.

Test 4

A 160 is 4% of ▒.
B 36% of ▒ is 81.
C 80 is 5% of ▒.
D 75% of ▒ is 150.
E 110% of ▒ is 11.
F 6 is 1% of ▒.
G 16 is 80% of ▒.
H 104% of ▒ is 52.
I 96 is 24% of ▒.

Test 5

A 20% of 70 is ▒.
B 80 is ▒% of 160.
C 54 is 25% of ▒.
D 75 is 150% of ▒.
E 184% of 25 is ▒.
F 600 is ▒% of 40.
G 130% of ▒ is 65.
H 130 is ▒% of 650. 2
I 32% of 625 is ▒.

## Keeping skillful 3

Find the sum.

A 2074, 69, 39216
B 895, 636, 2152
C 113862, 94679
D 562, 36988, 1140
E 64862, 938, 4000
F 7213, 649, 866, 47

A $s + 38276 = 70001$
B $114003 - 6295 = a$
C $24836 - t = 158$
D $m - 84531 = 3654$
E $198746 + k = 400623$
F $30705 + 96241 = d$
G $n + 81062 = 114050$
H $27383 - r = 9114$
I $9608 + f = 14327$
J $y - 79836 = 8439$
K $g + 16434 = 25321$
L $41628 - z = 17919$
M $30065 + d = 50096$
N $t - 63824 = 8756$

End-of-block tests on per cents and equations of ratios 147

# 147

1 Before the pupils start to work, be sure that they understand what they are to do. Assign no time limit, since all pupils should have an opportunity to complete each test.

2 Verify each pupil's equations and computation so that reteaching can be done on the basis of need.

3 Assign the blocks of work in the "Keeping skillful" section as individual work. Let pupils verify their answers.

# 148-149 Thinking straight

Exponded Notes for this lesson are on pages 336-338.

**Objectives**

The pupil generalizes further the idea of perimeter and area. He learns to recognize a parallelogram.

**Vocabulary**

New words page 148 parallelograms, special*

**Comments**

In this lesson geometric shapes are introduced informally. In later grades, the definitions of such shapes as parallelograms, rectangles, etc., will be made explicit. Here the pupils learn to recognize different members of the parallelogram family. They discover that all rectangles belong to the class of parallelograms.

The pupils discover that parallelograms with bases of the same length and altitudes of different lengths have different areas. They also discover that parallelograms with equal perimeters may have different areas.

The pupils review parallel lines, perpendicular lines, and the ideas of base and altitude for polygons.

The purpose of this lesson is to find differences and likenesses among parallelograms and to *compare* their areas by counting square units.

**Answers**

Pages 148-149:

A 4 B A and B; C C Yes; no; yes

D The dimensions are shown on the diagrams. Also, many pupils will remember that the opposite sides of a rectangle are equal.

E All its angles are right angles; A and B

F All its sides are equal; B; yes

G Yes; The perimeter is the sum of the lengths of the sides.

H 9; r and t; yes; 5 I 9; yes; yes; 3

J No; The altitude must be perpendicular to the base.

K No; F L 9; F; yes

M Yes; yes; F and G are examples of this.

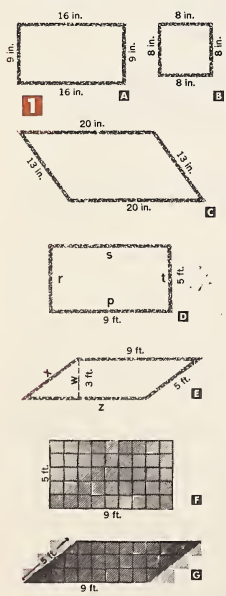
N Yes; Compare F and G.

O A, D, E, G, and I; A, E, and I; A

P [A] t or v; [D] g; [E] v or t; [G] v; [I] g or s

Q B, C, F, and H R [C] x; [H] z S Diameter

## Thinking straight




A Look at Polygons A, B, and C. Each of these polygons has ■ straight sides. 2

B Which of the three polygons have 4 right angles? Which polygon has no right angles?

C Are any of the sides of Polygon A perpendicular to each other? Are any of the sides of Polygon C perpendicular to each other? Do sides that are perpendicular also form right angles?

D In each of the three polygons, the opposite sides are parallel. How do you know that the opposite sides are also equal in length? 3

E Polygons A, B, and C are parallelograms because their opposite sides are parallel. The rectangle is a special kind of parallelogram. In what way is it special? Which of the three polygons are rectangles? 4

F The square is a special kind of rectangle. In what way is it special? Which polygon is a square? Is a square also a parallelogram?

G Polygons D and E are parallelograms. Do they have equal perimeters? 5

THINK How do you find the perimeter of any polygon?

H Look at Polygon D. Line p represents the base. The length of the base is ■ ft. Which lines represent the altitude? Is the altitude perpendicular to the base? The length of the altitude is ■ ft.





# 148

1 Have the pupils observe the polygons pictured here and discuss their likenesses and differences.

2 As the pupils discuss Exercises A to C, review the ideas of right angles, perpendicular lines, and parallel lines.

3 Have the pupils point out the parallel sides of Polygons A, B, and C. Be sure they note that the parallel sides are also equal in length.

4 Ask the pupils to study Polygons A and B, and let them tell why these are special kinds of parallelograms.

5 Before the pupils discuss Exercises G and H, review the meaning of the terms *perimeter*, *base*, and *altitude*.

In Polygon E, Line z represents the ase. The length of the base is ■ ft. Line w perpendicular to the base? Line w the altitude? The length of e altitude is ■ ft.

In Polygon E, can Line x also present the altitude? Why or why not?

Picture F shows a rectangle the me size as Rectangle D. Picture G ows a parallelogram the same size as ue one in Picture E. The small squares vering Parallelograms F and G present square feet. Are the areas Parallelograms F and G equal? Which arallelogram, F or G, has the greater rea?

Each parallelogram has a base ft. long. Which parallelogram has the nger altitude? Is the parallelogram ith the longer altitude also the arallelogram with the greater area?

M Do Parallelograms F and G have equal perimeters? Is it possible for two parallelograms to have equal perimeters and areas that are not equal? How do you know? 3

N If two parallelograms have bases that are of equal length and altitudes that are not equal, will they have areas that are not equal? How do you know?

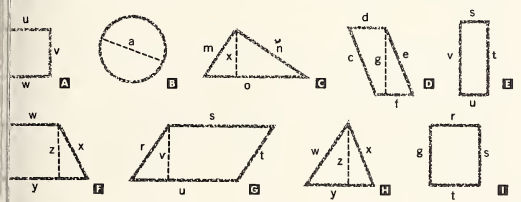
O Which pictures below show parallelograms? Which of these parallelograms are rectangles? Which rectangle is also a square? 4

P For each parallelogram, name a line that represents the altitude.

Q Which pictures show shapes that are not parallelograms?

R For each triangle, name the line that represents the altitude.

S What do you call Line a on Circle B?




149

# 149

1 Have the pupils relate the questions in Exercises I and J to Polygon E on page 148. Be sure they understand that an altitude is always perpendicular to the base. Have a pupil explain why a side of a rectangle is an altitude, but a side of Polygon E is not.

2 To compare the areas of Parallelograms F and G, let the pupils count the superimposed squares. (45 squares on F and 27 squares on G) See if they can make a generalization concerning parallelograms with equal bases but different altitudes after they have answered Exercise L.

3 In connection with Exercises M and N, let the pupils experiment with parallelograms that have equal perimeters, equal bases, and unequal altitudes. Let them make generalizations about the areas. These polygons may be drawn on squared paper to make it easier to determine the areas.

4 Discuss Exercises O to S.

# 150-151 Moving forward

Expanded Notes for this lesson are on pages 338-339.

**Objectives**

The pupil learns that a parallelogram that is not a rectangle has the same area as a rectangle if both have the same number of units in base and altitude.

**Vocabulary**

There are no new words.

**Comments**

This lesson demonstrates that a parallelogram that is not a rectangle has the same area as a rectangle if the dimensions (base and altitude) are the same. The pupils can verify this themselves by placing 1-inch squares on suitable parallelograms, and also by drawing various parallelograms on squared paper. No computing should be done in this lesson. In the lesson that follows this one, the pupils will compute to find areas.

Some pupils may have difficulty in seeing that the number of square inches in one whole row along the base of a parallelogram is the same as the number of units in the length of the base. This idea is carefully developed in Exercises C, E, and I.

**Answers**

Page 151:

A The part of the 1-inch square that is cut off at the left fits the uncovered part at the right.

B One C Yes D 8; 8

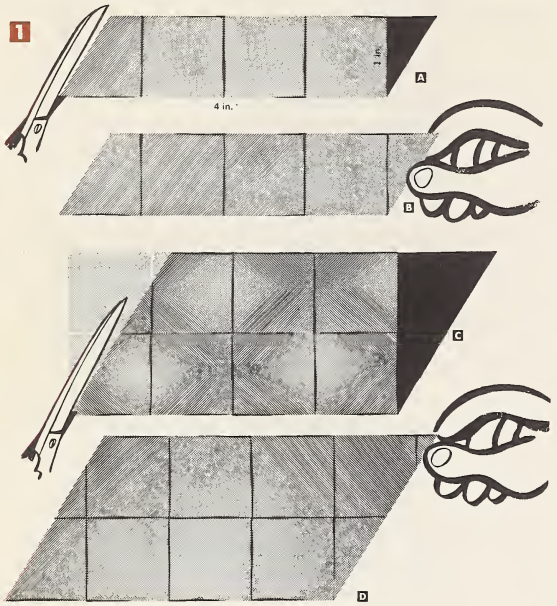
E 2; The base of the parallelogram is 4 in. long; yes

F Yes G 4 sq. in.; yes H Yes; 8 sq. in.

I [E] base, 8 in.; altitude, 3 in.
[F] base, 3 in.; altitude, 4 in.
[G] base, 5 in.; altitude, 4 in.
[H] base, 4 in.; altitude, 4 in.

J [E] 24 sq. in.; [F] 12 sq. in.; [G] 20 sq. in; [H] 16 sq. in.

## Moving forward Now you will learn more about parallelograms.






**1** A In Pictures A and B you can see a parallelogram that is not a rectangle. It has a base 4 in. long and an altitude 1 in. long. How do the 1-inch squares placed on this parallelogram show that its area is 4 square inches?

B How many rows of 1-inch squares are there in the parallelogram shown in Pictures A and B?

C Are there as many square inches in the row as there are inches in the length of the base?

**2** D Now look at Pictures C and D. They show a parallelogram with a base 4 in. long and an altitude 2 in. long. How many 1-inch squares does it take to cover this parallelogram? The area of this parallelogram is ▭ sq. in.

E How many rows of square inches are there in the parallelogram in Pictures C and D? How do you know that it takes four 1-inch squares to cover each row? Are there as many square inches per row as there are inches in the length of the base?

F Are there as many rows as there are inches in the length of the altitude?

**3** G Now look at Picture A again. You can see that the 1-inch squares form a rectangle with a base of 4 in. and an altitude of 1 in. What is its area? Is the area of this rectangle the same as the area of the parallelogram that is not a rectangle?

H In Picture C, the 1-inch squares also form a rectangle. Does this rectangle have the same base and altitude as the parallelogram that is not a rectangle? What is the area of the rectangle?

**4** I Imagine that each small square placed on the parallelograms below is a square inch. How long are the base and altitude of each parallelogram?

J Use the small squares to find the area of each parallelogram.

Now you know that if a rectangle and a parallelogram that is not a rectangle have equal bases and altitudes, they also have equal areas.



# 150

**1** Explain to the pupils that they are going to use what they know about rectangles to study other parallelograms. Prepare large rectangles and parallelograms similar to those shown here, so that the pupils can see the demonstrations shown by the pictures. For the parallelograms, use a scale of 6 inches for the large figure for each 1 inch shown in the picture.

# 151

**1** After the pupils have discussed Exercises A, B, and C and studied the pictures, ask someone to demonstrate Pictures A and B by cutting a triangular piece from one end of a large rectangle and transferring it to the other end. (See also Expanded Notes.)

**2** Use the same procedure to demonstrate Pictures C and D after the class has discussed Exercises D to F.

**3** For Exercises G and H, have pupils find the areas by counting the squares. Emphasize that Pictures A and B and Pictures C and D have the same area and that the bases and altitudes have the same length.

**4** For Exercises I and J have pupils find answers by counting the squares. If possible, prepare duplicates of drawings of rectangles and other parallelograms for the pupils to use to get practice in finding areas by counting the number of 1-inch squares they can place on each.

# 152-156 Exploring problems

Expanded Notes for this lesson are on pages 339-341.

**Objectives**

The pupil learns to compute the area of a parallelogram.

**Vocabulary**

New words page 152 plywood*; page 156 mushrooms*, tile

**Comments**

By now the pupils know that the measurement of area requires finding the number of square units that cover a given surface. They know what a square inch is, and they know what the altitude of a parallelogram is. They are now ready to find the area of the parallelogram by computation.

The usual way of introducing the rule for finding the area of a parallelogram by multiplying the number of units in the base by the number of units in the altitude does not give the pupil a clear idea of what the numbers really represent. The pupil may not understand why he can multiply two "lengths" to find the area or why his answer means square units. The traditional rule is a short cut, and, like most short cuts, suppresses meaning.

Actually, finding an area involves a rate expressed by two ratios. One ratio expresses the number of square units per row. The other ratio expresses the relationship of the total number of square units to the total number of rows. Because both ratios express the same rate, an equation can be written. In this lesson, once the equation is made, it is solved just as the pupil first solved ratio equations, and also by the "ratio test" method.

By using the ratio test, the traditional short-cut procedure is developed, although the pupil should not be required to use it.

If a pupil shows that he clearly understands why he can use the short cut, allow him to do so. Note that the second answer for each exercise in the answer key shows the computational short cut. Note also that the short cut may be written in several acceptable ways. For example, the short cut for Exercise A may be written as $14\times6=[84]$, as $6\times14=[84]$, as $[84]=14\times6$, or as $[84]=6\times14$.

**Answers**

Page 156:

(block 1)

A $\frac{14}{1}=\frac{[84]}{6}$; $14\times6=[84]$; 84 sq. in.

B $\frac{13}{1}=\frac{[65]}{5}$; $13\times5=[65]$; 65 sq. in.

C $\frac{8}{1}=\frac{[104]}{13}$; $8\times13=[104]$; 104 sq. in.

D $\frac{5}{1}=\frac{[40]}{8}$; $5\times8=[40]$; 40 sq. in.

E $\frac{8}{1}=\frac{[64]}{8}$; $8\times8=[64]$; 64 sq. in.

F $\frac{12}{1}=\frac{[72]}{6}$; $12\times6=[72]$; 72 sq. in.

G $\frac{7}{1}=\frac{[77]}{11}$; $7\times11=[77]$; 77 sq. in.

H $\frac{9}{1}=\frac{[63]}{7}$; $9\times7=[63]$; 63 sq. in.

(block 2)

A $\frac{4}{1}=\frac{[12]}{3}$; $4\times3=[12]$; 12 sq. in.; 3 in.

B $\frac{15}{1}=\frac{[90]}{6}$; $15\times6=[90]$; 90 sq. in.

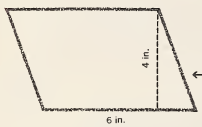
C $\frac{17}{1}=\frac{[255]}{15}$; $17\times15=[255]$; 255 sq. in.

D $\frac{60}{1}=\frac{[2520]}{42}$; $60\times42=[2520]$; 2520 sq. in.

E $\frac{6}{1}=\frac{[36]}{6}$; $6\times6=[36]$; 36 sq. in.

F $\frac{9}{1}=\frac{[81]}{9}$; $9\times9=[81]$; 81 sq. in.; $(81)-36=[45]$; 45 sq. in.

G $\frac{22}{1}=\frac{[374]}{17}$; $22\times17=[374]$; 374 tiles



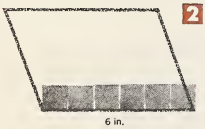

A

**1** A piece of plywood is in the shape of a parallelogram with a base 6 in. long and an altitude 4 in. long. What is its area in square inches?

← Look at the diagram.

Imagine that 1-inch squares are being laid in rows to cover the parallelogram. There will be 6 square inches in each row because the base of the parallelogram is 6 in.

← the base of the parallelogram is 6 in.




**2** B This is a rate of 6 square inches per 1 row. Express this rate as a ratio.

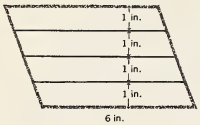
$$\frac{6}{1}$$

6 ← Number of square inches in each row

1 ← Number of rows

Now you must find how many rows of square inches there will be. The altitude is 4 in., and each row will use 1 in. of the altitude.

← There will be 4 rows.




C

**3** You do not know how many square inches will be used for all 4 rows, but you can express the rate as $x$ square inches per 4 rows.

$$\frac{x}{4}$$

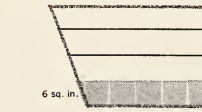
x ← Total number of square inches

4 ← Total number of rows



← The rate at which square inches will be used will be the same for the 4 rows as for 1 row. So you can make an equation of the ratios.

D




**1**

$$\frac{6}{1} = \frac{x}{4}$$

Do these terms stand for number of square inches or number of rows? You must find the numeral that replaces $x$.

What do these terms stand for?

E

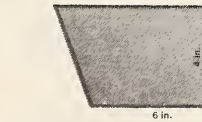
← 6 square inches are used for 1 row. You can see that 24 square inches are used for all 4 rows. You can also compute to find how many square inches there are in the parallelogram.

**2**

$$\frac{6}{1} = \frac{x}{4}$$

4 can be found by multiplying 1 by 4. So the numeral that replaces $x$ can be found by multiplying 6 by 4.

F




**3** If you use the ratio test when you solve this equation, you get $1x = 6 \times 4$. What do you do next?

$$\frac{6}{1} = \frac{24}{4}$$

24 ← 24 replaces $x$.

**4** The area of the piece of plywood is 24 sq. in.





# 152

1 Have the pupils read the problem and point out the base and the altitude of the parallelogram. Ask them how they would go about finding the area.

2 Have the pupils read the text in connection with Picture B. Be sure they understand that "6 square inches per 1 row" is a rate, just as 35 miles per 1 hour is a rate. The entire row of red square units along the base of the parallelogram is represented by 1.

3 Be sure the pupils see that, since the altitude is 4 inches and since each row of units takes up 1 inch of the altitude, as indicated by the horizontal red lines, there are 4 rows of units in the whole parallelogram. Now the rate is expressed as x square inches per 4 rows. Help the pupils observe that the first term in both ratios means number of square inches, and the second term in both ratios means number of rows.

# 153

1 Be sure the pupils understand that the two ratios express the same rate and, therefore, can be written as an equation. Have pupils relate Picture D to the equation to show the 6 square inches per row and the 4 rows. Have them note, too, that part of the square extending to the right in Picture B is now shown inserted at the left.

2 Have the pupils read the text and study Picture E. Ask: "How can we find the numeral to replace $x$? Why do we multiply $4 \times 6$?" Let the pupils count the squares to verify the answer.

3 Have the pupils explain how to use the ratio test for this equation. If necessary, review what the ratio test is.

4 Picture F suggests the surface enclosed by the parallelogram. Have the pupils compare it with Picture A, in which just the outline and not the interior region or "surface" is indicated.

G A piece of plywood is in the shape of a parallelogram with a base 4 in. long and an altitude 5 in. long. What is its area in square inches?

1

Look at the diagram. This parallelogram has right angles. What special name do you know for this kind of parallelogram?

H The base of this rectangle is 4 in. long. So in 1 row there will be ▩ sq. in. The rate is ▩ square inches per 1 row. Express this rate as a ratio.

2

$\frac{4}{1}$ ← What does this term stand for?

$\frac{\blacksquare}{1}$ ← What does this term stand for?

3

I The altitude of the rectangle is 5 in. long. There will be ▩ rows of square inches. Why can the rate also be expressed as $b$ square inches per 5 rows?

$\frac{b}{5}$ ← What does this term stand for?

4

$\frac{\blacksquare}{5}$ ← What does this term stand for?

J Square inches will be used at the same rate for 5 rows as for 1 row.

1

$\frac{4}{1} = \frac{b}{5}$

What do the 4 and the $b$ stand for?

What do the 1 and the 5 stand for?

K

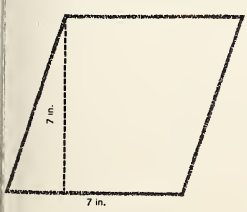
2

$\frac{4}{1} = \frac{b}{5}$

Why do you multiply 4 by 5 to find the numeral that replaces $b$? ▩ replaces $b$.

What equation do you get when you use the ratio test?

The area of this parallelogram is ▩ sq. in.




3

L The base of Parallelogram L is 7 in. long. The altitude is 7 in. long. Find the area in square inches.

$\frac{7}{1} = \frac{k}{7}$

This ratio means ▩ sq. in. per 1 row.

When you have found the numeral that replaces $k$, you will know how many square inches there are in ▩ rows.

Why can you also use the equation $1k = 7 \times 7$?

The area of Parallelogram L is ▩ sq. in.



# 154

1 Have the pupils relate the problem to the picture. Let them establish that the altitude of the rectangle is the same as one vertical side.

2 Have the pupils study Picture H and relate it to the ratio. Let them discuss what the 4 and the 1 represent.

3 Have the pupils read the text in connection with Picture I. Be sure they understand why there will be 5 rows of squares.

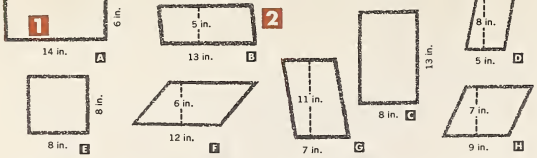
4 Be sure the pupils understand that this ratio expresses the same rate as 4/1. They should observe that the first term in both ratios represents number of square inches and the second term represents the number of rows.

# 155

1 Be sure the pupils understand how the equation is formed and can answer the questions about it. Relate the equation to the illustration to show that they both give the same information.

2 Have the pupils answer the question. Be sure they know what 20 represents. Have them tell how they know 20 is the number they want. Let them verify it by counting the squares in Picture J and by using the ratio test. Have them state the equation for the ratio test and tell how to solve it. Relate this to the illustration, which shows the interior as a whole.

3 Have the pupils read the problem. Let one pupil draw a parallelogram like Picture L on the chalkboard. He should include 1-inch squares. Have him explain his drawing in relation to the equation. Have another pupil explain the ratio test. Then let them supply the missing numerals.




Each diagram above represents a piece of cardboard that is in the shape of a parallelogram. Find the area of each piece of cardboard in square inches.

**3**

A Henry is cutting a piece of glass in the shape of a rectangle 4 in. long and 3 in. wide. What is the area of the piece of glass in square inches?

THINK If you use 4 inches for the base, what do you use for the altitude?

B Don has a piece of plywood in the shape of a parallelogram with a base 15 in. long and an altitude 6 in. long. Find the area of the piece of plywood in square inches.

C Mr. Beach grows mushrooms in rectangular trays that are 17 in. long and 15 in. wide. What is the area in square inches of the bottom of one of these trays?

D Mr. May plans to put a tile floor in a hall. The hall floor has the shape of a rectangle. It is 60 in. long and 42 in. wide. What is the area of the floor in square inches?

E The tiles Mr. May will use are 6 in. square. Find the area of one tile in square inches.

F The tiles also come in 9-inch squares. A 9-inch tile will cover how many more square inches of floor than a 6-inch tile?

THINK When you have found the area of a 9-inch tile, what should you do next?

G Mr. Henry is setting tiles in a table top. The tiles are about 1 inch square. The space to be filled with tiles is a rectangle 22 inches by 17 inches. About how many 1-inch tiles will Mr. Henry need for the table top?

Now you should know how to solve problems about the areas of parallelograms.

156

# 156

1 Discuss the area of Polygon A with the class. Let a pupil write the equation "14/1=n/6" on the chalkboard and have someone explain what each numeral represents. Let another pupil rewrite the equation using the ratio test and explain how to find the numeral to replace *n*.

2 Let the pupils work independently to find the areas of Parallelograms B to H. Have them write a ratio equation for each, do the necessary computation, and rewrite the equation with the correct numeral in place of *n*. Then have them use the ratio test with the original ratio equation and solve the new equation. Have them express their answers in the form, "65 square inches."

3 Follow the procedure suggested in Note 2 for Problems A to G. When the pupils have finished, supply answers and let them verify their work.

# 157-160 Looking back

Expanded Notes for this lesson are on pages 341-343.

**Objectives**

The pupil reviews finding areas measured in square feet and square yards. He also changes units of square measure.

**Vocabulary**

New words page 158 rug*; page 159 mirror*, carpet

**Comments**

In the previous lesson on computing area, only the square inch was used as the measuring unit for areas.

Now the computation of area can be extended to include the use of the square foot and the square yard. And, since it is frequently necessary to change from one unit of square measure to another in real-life situations, such changes of units of square measures are included in this lesson. Since such changes depend upon a knowledge of square measure equivalents, the idea of the square foot is introduced in terms of its area in square inches. The square yard is introduced in terms of its area in square feet.

The rate method provides a means of finding equivalents of square measure. The pupil does not have to rely on his memory. He has a method to use that has been developed in many different problem situations.

**Answers**

(Answers to area problems are given in terms of the unit of measure asked for in the problem. When dimensions are given in different units, the answers are given in both units unless mixed numbers are involved. Exceptions are noted in the answer key.)

Pages 158-159:

A 12/1=[108]/9; 108 sq. ft.

B There are 9 sq. ft. per 1 sq. yd.; number of square yards in 108 sq. ft.; by dividing 108 by 9; 12

C 3/1=[6]/2; 6 sq. yd.

D 6; Diagram E shows 1 sq. yd. marked off into 9 sq. ft.; yes; number of square feet in the area; multiply 9 and 6; 54

48/1=[1584]/33; 1584 sq. in.

11 sq. ft.; 144 sq. in. equal 1 sq. ft.; 1584 represents area in square inches, as does 144 in the first ratio; number of square feet in 1584 sq. in.

144/1=[1008]/7; 1008 sq. in.; number of square inches in 1 sq. ft.; number of square inches in 7 sq. ft.

40 sq. yd.; 20/1=[360]/18; 360 sq. ft.; 9/1=(360)/[40]; 9 per 1 expresses number of square feet in 1 sq. yd.; 360 represents square feet as does 9 in the first ratio; number of square yards in 360 sq. ft.

5 sq. ft.; 24/1=[720]/30; 720 sq. in.; 144/1=720/[5]; 720 represents square inches as does 144 in the first ratio; number of square feet in the area

135 sq. ft.; 3/1=[15]/5; 15 sq. yd.; 9/1=[135]/15; number of square feet in 15 sq. yd., which is the area of the carpet; It represents square yards, as does 1 in the first ratio.

(block 2)

A 2448 sq. in.
B 171 sq. ft.
C 8 sq. yd.
D 6 sq. ft.
E 13 sq. yd.
F 2160 sq. in.
G 207 sq. ft.
H 10 sq. ft.
I 1296 sq. in.
J 70 sq. yd.
K 21 sq. ft.
L 567 sq. ft.
M 3744 sq. in.

Page 160:

A (48)/1=[1728]/36; 1728 sq. in.

B 144/1=1728/[12] or 4/1=[12]/(3); 12 sq. ft.

C [B] 26/1=[390]/15; 390 sq. in.; [C] 4/1=[4]/(1); 4 sq. yd. or (12)/1=[36]/3; 36 sq. ft.; [D] 11/1=[99]/(9); 99 sq. ft. or 9/1=(99)/[11]; 11 sq. yd.; [E] (3)/1=[24]/8; 24 sq. ft.; [F] 5/1=[10]/(2); 10 sq. ft. or (60)/1=[1440]/24; 1440 sq. in.; [G] 2/1=[2]/(1); 2 sq. yd. or (6)/1=[18]/3; 18 sq. ft.

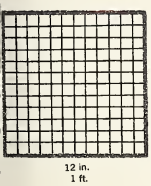
Keeping skillful:

A 1237
B 12,976
C 11,657
D 3428
E 13,521
F 32,688
G 126,793
H 37,162
I 157,550
J 242,827
K 55,231
L 465,759

## Looking back

In this lesson you will learn more about the measurement of area.

**1** Diagram A represents a 1-foot square of paper marked off in square inches. The area of the paper square is 1 square foot. How can you find the number of square inches in the square foot?




**2** You can use the equation below to find the area of 1 square foot in square inches.

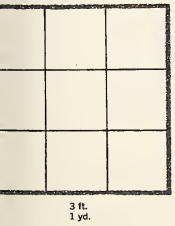
**3** $\frac{12}{1}=\frac{b}{12}$

This ratio expresses a rate of 12 sq. in. per 1 row.

What does this ratio express? How do you find the numeral that replaces $b$?

1 square foot = 144 square inches

**4** Diagram B represents a 1-yard square of paper marked off in square feet. The area of the paper square is 1 square yard. You can see that the area is also 9 square feet. But you can also use the equation below to find the number of square feet in 1 square yard.




**5** $\frac{3}{1}=\frac{x}{3}$

This ratio expresses a rate of 3 sq. ft. per 1 row.

What does this ratio express? How do you find the numeral that replaces $x$?

1 square yard = 9 square feet



# 157

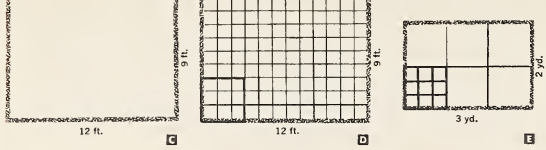
1 On oaktag or ordinary paper prepare a full scale drawing of a square foot marked off in square inches. Let pupils read the text and experiment with the drawing to answer the question [count the 1-inch squares].

2 Have a pupil read the sentence. Explain to the class how an equation can help them find the number of square inches without their having to count square inches.

3 Discuss the two ratios in the equation, and have the pupils tell what the terms represent. Be sure the pupils understand why the two ratios are equal. Have them solve the equation. Point out that they now know the number of square inches in a square foot.

4 Follow the same procedure suggested in Notes 1 and 2, using a full-scale drawing of a square yard marked off in square feet.

5 Follow the procedure suggested in Note 3.




**A** Diagram C represents a rectangular rug 12 ft. long and 9 ft. wide. Find the area of the rug in square feet. **1**

**B** You can also find the area in square yards. One way to do this is to change 108 sq. ft. to square yards.

The rate of square feet per square yards is always the same. Why does 9 per 1 express this rate? Look at Diagram D. In Problem B, the rate of square feet per square yards can also be expressed as 108 per $t$. What does $t$ represent? **2**

$$\frac{9}{1}=\frac{108}{t}$$ **3**

The numeral that replaces $t$ will tell you how many square yards there are in 108 sq. ft.

If you use the ratio test to solve this equation, you get $9t = 1 \times 108$. How do you find the numeral that replaces $t$?

108 sq. ft. = ▩ sq. yd.

**C** Diagram E represents a rectangular tablecloth 3 yd. long and 2 yd. wide. There are how many square yards of cloth in the tablecloth? **4**

**D** There are how many square feet of cloth in the tablecloth? You can change ▩ sq. yd. to square feet.

How does Diagram E show that 9 per 1 expresses the rate of square feet per square yards? In Problem D, does $n$ per 6 also express the rate of square feet per square yards? What does $n$ represent? **5**

$$\frac{9}{1}=\frac{n}{6}$$ **6**

When you find the numeral that replaces $n$, you will know how many square feet there are in 6 sq. yd.

If you use the ratio test when you solve this equation, you get $1n = 9 \times 6$. What do you do now?

6 sq. yd. = ▩ sq. ft.

158

**E** A table top is in the shape of a rectangle 48 in. long and 33 in. wide. What is its area in square inches? **1**

**F** What is the area of the table top in square feet? **2**

The rate of square inches per square feet can be expressed as 144 per 1. Why?

$$\frac{144}{1}=\frac{1584}{b}$$

Why is 1584 in the first term of the second ratio? What does $b$ represent?

**G** A piece of plywood has an area of 7 sq. ft. Find its area in square inches. **3**

$$\frac{144}{1}=\frac{w}{7}$$

What does 144 represent? What does $w$ represent?

**H** A rectangular floor is 20 ft. long and 18 ft. wide. What is its area in square yards? **4**

The dimensions are given in feet. So first find the area in square feet. Then make an equation that you can use to change square feet to square yards.

$$\frac{9}{1}=\frac{360}{x}$$

Why must you use 9 over 1 as one of the ratios in the equation? Why is 360 in the first term of the second ratio? What does $x$ represent?

**I** The dimensions of a rectangular mirror are 24 in. and 30 in. Find the area of the mirror in square feet. **5**

First find the area in square inches. Then make an equation that you can use to change square inches to square feet.

$$\frac{144}{1}=\frac{720}{k}$$

Why is 720 in the first term of the second ratio? What does $k$ represent?

**J** Mrs. Jackson bought a rectangular piece of carpet 3 yd. wide and 5 yd. long. How many square feet of carpet did she buy?

$$\frac{9}{1}=\frac{m}{15}$$ **5**

What does $m$ represent? Why is 15 in the second term of the second ratio?

**A** 17 sq. ft. = ▩ sq. in. **6**
**B** 19 sq. yd. = ▩ sq. ft.
**C** 72 sq. ft. = ▩ sq. yd.
**D** 864 sq. in. = ▩ sq. ft.
**E** 117 sq. ft. = ▩ sq. yd.
**F** 15 sq. ft. = ▩ sq. in.
**G** 23 sq. yd. = ▩ sq. ft.
**H** 1440 sq. in. = ▩ sq. ft.
**I** 9 sq. ft. = ▩ sq. in.
**J** 630 sq. ft. = ▩ sq. yd.
**K** 3024 sq. in. = ▩ sq. ft.
**L** 63 sq. yd. = ▩ sq. ft.
**M** 26 sq. ft. = ▩ sq. in.

159

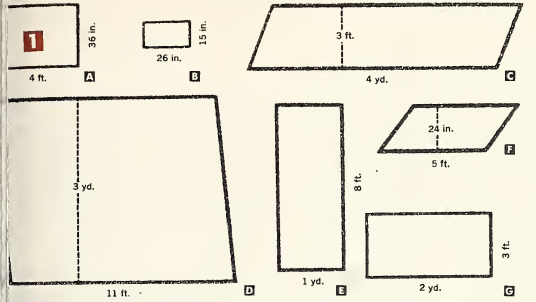


# 158

1 Let a pupil write and solve the equation o the chalkboard.
2 Have the pupils study Diagram D. Explai that the area outlined in black shows ther are 9 square feet in one square yard.
3 Help pupils understand why $t$ holds a plac for the numeral representing the number o square yards in 108 sq. ft., and that it mus be the second term (as the 1 stands for 1 sq yd. in the first ratio). Have someone find the numeral that replaces $t$. Have pupils observe that when they express the area in terms of a larger unit of measure, the number of units is smaller.
4 Have a pupil write and solve the equation.
5 Have pupils observe they are expressing the area in terms of a smaller unit of measure. Ask if the answer will be larger or smaller than 6, and why.
6 Have pupils explain each term in the equation and find the numeral that replaces $n$.

# 159

1 Have the pupils write the equation and find the numeral that replaces the letter they use.
2 Have the pupils observe that they are expressing the area in terms of a larger unit of measure. Have them answer the questions and find the numeral that replaces $b$.
3 Ask the pupils if the answer should be more or less than 7 and why. Make sure they understand what 144 and $w$ represent.
4 This exercise involves two ratio equations. Have the pupils write the equation to find the number of square feet [20/1 = n/18]. Then ask them what ratios will be in the second equation. Be sure they can explain what each numeral in the ratios represents.
5 Follow the procedure suggested in Note 4.
6 Assign these exercises as written work. Have the pupils work independently, but continue to give help wherever it is needed.




1 In Diagram A, the length of the ase is given in feet, and the length of ne altitude is given in inches. To find ne area in square inches, first change ft. to inches. Then what do you do?

Suppose you want to find the area f Rectangle A in square feet. You can change 1728 sq. in. to square feet. Or you can first change 36 in. to feet. Then what do you do?

G Find the area of each of the other parallelograms above. You may have to change one dimension before you make your equation. 3

Now you know how to find areas in square inches, square feet, and square yards. You can also change square measures when you need to.

## Keeping skillful 4

| | | |
|---|---|---|
| 3765 + d = 5002 | E 52430 − w = 38909 | I 351426 + n = 508976 |
| n − 8309 = 4667 | F a + 67312 = 100000 | J 723905 − c = 481078 |
| 20745 − 9088 = v | G m − 49325 = 77468 | K 150013 − 94782 = g |
| f + 87123 = 90551 | H 200714 − 163552 = z | L e − 63579 = 402180 |

# 160

1 Diagram A illustrates that dimensions can be given in different units of measure, one of which must be changed before the area can be computed in standard units. The pupils should write the equation and find the area in square inches.

2 Help the pupils understand the two ways of using the data to find the area of Rectangle A in square feet. Have them write the equations for each way and observe the results. They should note that it is much simpler first to change 36 in. to feet.

3 Have the pupils find these areas. Assist anyone who needs help.

4 Assign the "Keeping skillful" exercises as time permits.

# 161 Checking up; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil tests his knowledge of how to find areas and change units of square measure.

**Vocabulary**

There are no new words.

**Comments**

Use the "Checking up" tests to discover how well the pupils understand the terminology used in giving dimensions, finding areas, and changing from one unit to another in expressing square measures. If Tests 1 and 2 reveal weaknesses, review the lessons on pages 150-156. If Test 3 causes trouble, review pages 157-160.

**Answers**

Test 1

A 345 sq. in.
B 16 sq. ft.
C 1152 sq. in. or 8 sq. ft.
D 117 sq. yd.
E 504 sq. ft. or 56 sq. yd.

Test 2

A 625 sq. ft.
B 1920 sq. in.
C 867 sq. ft.
D 420 sq. yd.
E 21 sq. ft. or 3024 sq. in.
F 88 sq. yd. or 792 sq. ft.

Test 3

A 864 sq. in.
B 5 sq. yd.
C 135 sq. ft.
D 3 sq. ft.
E 324 sq. ft.
F 2592 sq. in.
G 15 sq. yd.
H 13 sq. ft.
I 3024 sq. in.
J 20 sq. yd.
K 20 sq. ft.

**Keeping skillful:**

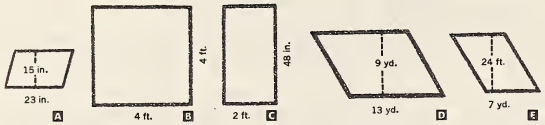
(block 1)

A $7\frac{1}{2}$
B $6\frac{3}{4}$
C $2\frac{11}{12}$
D $4\frac{4}{5}$
E $10\frac{3}{4}$
F $16\frac{4}{9}$
G $6\frac{9}{10}$
H $27\frac{3}{16}$
I $21\frac{7}{24}$
J $4\frac{11}{30}$
K $27\frac{5}{16}$
L $1\frac{14}{15}$
M $20\frac{1}{3}$
N 7
O $94\frac{1}{20}$
P $3\frac{4}{9}$

(block 2)

A 5
B 8
C 50
D 5
E 72
F 60
G 90
H 80
I 46

## Checking up




Test 1 **1**
Find the area of each parallelogram shown above.

Test 2
Find the area of each parallelogram whose dimensions are given below. **2**

A 25 ft. square
B base, 5 ft.; altitude, 32 in.
C base, 17 ft.; altitude, 51 ft.
D base, 30 yd.; altitude, 14 yd.
E base, 36 in.; altitude, 7 ft.
F base, 24 ft.; altitude, 11 yd.

Test 3
A 6 sq. ft. = ▩ sq. in.
B 45 sq. ft. = ▩ sq. yd. **3**
C 15 sq. yd. = ▩ sq. ft.
D 432 sq. in. = ▩ sq. ft.
E 36 sq. yd. = ▩ sq. ft.
F 18 sq. ft. = ▩ sq. in.
G 135 sq. ft. = ▩ sq. yd.
H 1872 sq. in. = ▩ sq. ft.
I 21 sq. ft. = ▩ sq. in.
J 180 sq. ft. = ▩ sq. yd.
K 2880 sq. in. = ▩ sq. ft.

## Keeping skillful **4**

A $4\frac{5}{8}+2\frac{7}{8}=z$
B $t+1\frac{5}{8}=8\frac{7}{12}$
C $46\frac{3}{4}+n=49\frac{2}{3}$
D $12-a=7\frac{1}{5}$
E $\frac{1}{4}+3\frac{9}{10}+6\frac{3}{5}=f$
F $p-11\frac{1}{3}=5\frac{5}{9}$
G $10\frac{2}{5}-3\frac{1}{2}=b$
H $w+24\frac{3}{16}=52$
I $5\frac{1}{3}+7\frac{1}{8}+8\frac{5}{8}=c$
J $11\frac{1}{6}-6\frac{2}{3}=m$
K $13\frac{5}{8}+4\frac{1}{2}+9\frac{3}{16}=x$
L $8\frac{2}{3}+d=10\frac{1}{3}$
M $g-6\frac{5}{12}=13\frac{1}{12}$
N $n+14\frac{5}{8}=21\frac{5}{8}$
O $70\frac{3}{4}+23\frac{9}{10}=v$
P $11\frac{5}{8}+e=15$

A $3500\div700=n$
B $64000\div8000=p$
C $60000\div1200=r$
D $16000\div3200=c$
E $10080\div140=t$
F $16200\div270=a$
G $450000\div5000=h$
H $720000\div9000=k$
I $322000\div7000=b$

End-of-block tests on square measure **161**

# 161

1 Have the pupils write an equation to find the area of each parallelogram. Remind them that when two dimensions are given in different units, they should change one of the dimensions before computing, and then be sure to label the answer in terms of the square unit in which the area was found. Discuss the two ways in which the areas for C and E can be found.

2 Use Note 1.

3 Have the pupils write equations to find the answers to Test 3.

4 The "Keeping skillful" exercises may be assigned as a separate lesson. Remind the pupils that they learned a short cut that may be used for the division exercises in Block 2.

# 162-164 Side trip; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil writes long division in the traditional way.

**Vocabulary**

There are no new words.

**Comments**

The purpose of this lesson is to acquaint the pupil with the traditional method of long division. The traditional method will be better understood if it is compared with the method that is taught in the pupils' book and with the previous work on short division (page 111). This lesson is not an essential part of the content of the program, and its use is optional.

On pages 162-163, the traditional method has been developed by a sequence of steps. The method shown at the top of page 162 may be referred to for clarification of various points (for example, to show why the 6 in Step C represents 600).

The pupils should not be required to work many exercises by the traditional method. This applies especially to the slow learners, who may become confused if forced to learn the traditional method.

**Answers**

Page 164:

A $481\frac{6}{11}$
B $1360\frac{5}{62}$
C 31
D $87\frac{3}{5}$
E 692
F $218\frac{3}{8}$
G $3807\frac{42}{89}$
H $1655\frac{548}{583}$
I $186\frac{101}{206}$
J $24{,}591\frac{2}{19}$
K $44\frac{39}{112}$
L $30\frac{10}{333}$
M $110\frac{37}{144}$
N $293\frac{314}{651}$
O $267\frac{80}{99}$
P $212\frac{23}{28}$
Q $42\frac{11}{96}$
R $230\frac{149}{575}$

Keeping skillful:

A $1\frac{23}{24}$
B $13\frac{1}{5}$
C $7\frac{2}{3}$
D $44\frac{1}{2}$
E $\frac{11}{12}$
F $14\frac{1}{8}$
G $19\frac{7}{15}$
H $24\frac{7}{8}$
I $25\frac{31}{36}$
J $13\frac{5}{6}$
K $43\frac{3}{16}$
L $16\frac{1}{8}$
M $8\frac{1}{4}$
N $28\frac{1}{10}$
O $67\frac{6}{35}$
P $\frac{17}{18}$
Q $22\frac{1}{3}$
R $41\frac{1}{6}$
S $13\frac{21}{40}$
T $63\frac{7}{20}$
U $23\frac{7}{10}$

## Side trip

ook at the division example below. Is
ne computation correct?

```
34)21098 | 600
 20400   |
   698   |  20
 1 680   |
    18   | 620
```

ere is another way to do division.
this way you work with parts of
umbers in each step of the
mputation.

```
1 34)21098
```

Divide the 21 in 21098 by the 3 in 34. Try 7 in the quotient.

2

Write 7 here. It stands for 700. Multiply the divisor 34 by 7.

```
      7 ←
1 34)21098
     238 ←
```

Write the product of 34 and 7 here. 238 is really 23,800. 238 is greater than 210. So 7 is too large.

```
      6 ←
1 34)21098
     204
```

Try 6 in the quotient. 6 stands for 600. Multiply the divisor 34 by 6. 204 is really 20,400.

Writing division examples in the traditional way

```
      6      3
D 34)21098
     204 ←
```

204 is smaller than 210. So subtract.

```
       6 ←
D 34)21098
     204
```

This number is smaller than the divisor. So you know 6 is the correct number to use in the quotient.

```
       6
E 34)21098
     204      4
```

```
       6
E 34)21098
     204
      69 ←
```

Bring down the 9. 69 is really 698.

```
       6
F 34)21098
     204
      69 ←
```

Divide the 6 in 69 by the 3 in 34. Try 2 in the quotient. 5

```
      62 ←
G 34)21098
     204
      69
      68 ←
```

Write 2 here. It stands for 20. Multiply the divisor 34 by 2.

Write the product here. 68 is really 680. 68 is smaller than 69. So 2 in the quotient is not too large. 6

```
      62
J 34)21098
     204
      69
      68 ←
       1 ←
```

Now subtract.

This number is smaller than the divisor. So you know that 2 is the correct number in the quotient.

1

```
      62
I 34)21098
     204
      69
      68
       18 ←
```

Bring down the 8.

2

```
     620 ←
I 34)21098
     204
      69
      68
       18
```

You do not divide because 18 is less than the divisor 34. But you must have a numeral in ones' place in the quotient. So write 0 in the quotient.

The quotient is 620 with 18 remainder, or $620\frac{18}{34}$, or $620\frac{9}{17}$.

Divide 29,568 by 96.

```
       3 ←
A 96)29568
     288 ←
       7 ←
```

Why is 3 a good number to try in the quotient? Is this really 3 or 300?

3

How do you get this number?

How does this number show that 3 is the correct number to use in the quotient? What do you do next?

```
      30 ←
B 96)29568
     288
      76
```

You do not divide 76 by 96. But you must have a numeral in tens' place in the quotient. What is this numeral? Now what do you do?

4

```
      308 ←
C 96)29568
     288
      768 ←
      768 ←
```

Why is 8 the correct number to use in the quotient?

How do you get this number?

5 How do you get this number? Is there any more work to do?

The quotient is 308.



# 162

1 Have pupils observe that the computation is correct.

2 Discuss Steps A to C to bring out that sometimes numbers tried in the quotient are too large, and smaller ones have to be tried.

3 In Step D point out that two comparisons have to be made to know whether the 6 in the quotient is correct.

4 In Step E point out that only one numeral is brought down at a time.

5 The 2 suggested in the quotient in Step F is the estimate for the number of times 34 is contained in 69. The product of 34 and 2, then, is 68 (Step G), and so 2 can be used because 68 is smaller than 69.

6 Discuss the relationship of the method outlined on this page to the method the pupils were taught in *Seeing Through Arithmetic* (shown at the beginning of this lesson). Note that more steps follow on the next page.

# 163

1 Give the pupils a chance to explain why 68 is subtracted from 69. Get them to see that the 1 remainder is really 10.

2 Emphasize that 8 is brought down because we must use all the numerals in the dividend. Be sure the pupils understand why 0 must be written in ones' place in the quotient.

3 Help the pupils understand that 295 is divided by 96, and that 3 is an estimate of the number of times 9 is contained in 29. Have them observe that the product of 96 and 3 can be subtracted from 295 and that the remainder is less than the divisor, 96.

4 Point out that 6 is brought down and that there is a new number to divide by 96. Discuss why 76 is not divided by 96 and why a zero must be put in the quotient.

5 Discuss Step C, and bring out that 8 is tried because 9 is contained in 76 eight times. Since the product of 8 and 96 is 768, there is no remainder.

A $785623 \div 145 = n$

$$145\overline{)785623} \quad 5418\tfrac{13}{145}$$
725
606
580
262
145
1173
1160
13

B $139734 \div b = 18$

$$18\overline{)139734} \quad 7763$$
126
137
126
113
108
54
54

C $278901 \div 695 = w$

$$695\overline{)278901} \quad 401\tfrac{206}{695}$$
2780
901
695
206

1 2 2

For each exercise below, write your computation in the way shown in this lesson. 3

A $5297 \div 11 = t$
B $84325 \div 62 = n$
C $217 \div 7 = x$
D $438 \div r = 5$
E $17300 \div m = 25$
F $1747 \div 8 = q$
G $338865 \div 89 = a$
H $965413 \div 583 = b$
I $76834 \div 412 = c$
J $934462 \div 38 = p$
K $4967 \div h = 112$
L $10000 \div 333 = y$
M $47631 \div 432 = z$
N $191057 \div d = 651$
O $26513 \div 99 = g$
P $5959 \div w = 28$
Q $8086 \div 192 = s$
R $132399 \div 575 = v$

## Keeping skillful 4

A $\frac{3}{8} + \frac{3}{4} + \frac{5}{8} = m$
B $16\frac{7}{10} - 3\frac{1}{2} = x$
C $a + 17\frac{1}{3} = 25$
D $32\frac{3}{8} + 6\frac{1}{2} + 5\frac{5}{8} = n$
E $11\frac{3}{4} - t = 10\frac{5}{8}$
F $7 + q = 21\frac{1}{8}$
G $f - 15\frac{5}{8} = 3\frac{4}{8}$
H $18\frac{2}{3} + 5\frac{3}{8} + \frac{5}{8} = b$
I $29\frac{5}{12} - 3\frac{5}{6} = e$
J $18\frac{1}{2} + v = 32\frac{1}{3}$
K $47\frac{7}{16} - s = 4\frac{1}{4}$
L $p - 13\frac{5}{8} = 2\frac{1}{2}$
M $c + 6\frac{1}{12} = 14\frac{1}{3}$
N $\frac{3}{8} + 8\frac{4}{5} + 18\frac{7}{10} = r$
O $79\frac{4}{7} - 12\frac{2}{5} = z$
P $1\frac{1}{9} - h = \frac{1}{6}$
Q $y + 11 = 33\frac{1}{3}$
R $27 + 6\frac{1}{2} + 7\frac{2}{3} = d$
S $83\frac{1}{10} + n = 96\frac{5}{8}$
T $46\frac{3}{8} + 16\frac{3}{4} = t$
U $38 - 14\frac{3}{10} = q$

164

# 164

1 Discuss each step to determine what each numeral in the quotient stands for, what each product stands for, and what each remainder stands for. Follow the steps outlined for pages 162-163.

2 Let pupils try working the example without referring to the book. Then let them verify their computation with that in the book. Answer any questions they have concerning the work.

3 Assign Exercises A to R according to the pupils' abilities. Encourage, but do not force, pupils to use the traditional way. Pupils who do not understand the traditional method should use the method originally taught.

4 Assign the "Keeping skillful" exercises according to individual needs and abilities. Provide answers for the pupils to verify their work.

# 165-168 Moving forward

Expanded Notes for this lesson are on pages 343-344.

**Objectives**

The pupil learns that each number may have many names, and that they may be used as names for a point on a number line.

**Vocabulary**

New words page 165 extended*

**Comments**

The main purpose of this lesson is to help the pupils see that different names for a number may be used interchangeably. For example, $\frac{1}{2}$ and $\frac{2}{4}$ are names for the same number. So $\frac{2}{4}$ may be used instead of $\frac{1}{2}$ in "$\frac{1}{2} + \frac{1}{4} = n$." Other understandings to be gained by the pupils are that a point on a line can be named by an infinite number of names, and that the line may be extended to include a point for any number.

The names shown for 0 ($\frac{0}{1}$, $\frac{0}{2}$, $\frac{0}{3}$, etc.) are totally new to the pupils. Special attention should be given to these names. For $\frac{0}{4}$, for example, you may explain that if $\frac{1}{4}$ is subtracted from $\frac{1}{4}$, the answer would be $\frac{0}{4}$, or 0.

It should be pointed out to the pupils that, on a given number line, the distance between any two consecutively named points is the same as the distance between any other pair of such points. When the pupils are asked to extend the sets horizontally for specified points, they should observe that each member in the set has the same denominator. It is important to point out that any number may be represented by a point. For example, it would be possible to represent a point for 1,000,000 by extending the line far enough to the right.

**Answers**

Pages 165-167:

A 5, 6; yes; yes; 7, 8, 9, 10, 11, 12, etc.
B 1
C $\frac{5}{1}$; $\frac{6}{1}$
D Yes; $\frac{7}{1}$, $\frac{8}{1}$, $\frac{9}{1}$, $\frac{10}{1}$, $\frac{11}{1}$, $\frac{12}{1}$, etc.

E 2; $\frac{8}{2}$, $\frac{10}{2}$, $\frac{12}{2}$

F $\frac{14}{2}$, $\frac{16}{2}$, $\frac{18}{2}$, $\frac{20}{2}$, $\frac{22}{2}$, $\frac{24}{2}$, etc.

G 3; $\frac{9}{3}$, $\frac{12}{3}$, $\frac{15}{3}$, $\frac{18}{3}$

H $\frac{21}{3}$, $\frac{24}{3}$, $\frac{27}{3}$, $\frac{30}{3}$, $\frac{33}{3}$, $\frac{36}{3}$, etc.

I 3; $\frac{8}{4}$, $\frac{10}{5}$, $\frac{12}{6}$, $\frac{14}{7}$, etc.

J $0\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$; $4\frac{1}{2}$, 5; $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$, 8, etc.

K No; no; yes, $\frac{5}{1}$

L No

M $\frac{8}{2}$; $\frac{9}{2}$; $\frac{10}{2}$; $\frac{11}{2}$, $\frac{12}{2}$, $\frac{13}{2}$, $\frac{14}{2}$, $\frac{15}{2}$, $\frac{16}{2}$, etc.

N $0\frac{1}{4}$ or $\frac{1}{4}$, $0\frac{1}{3}$ or $\frac{1}{3}$, $0\frac{1}{2}$ or $\frac{1}{2}$, $0\frac{2}{3}$ or $\frac{2}{3}$; $0\frac{3}{4}$ or $\frac{3}{4}$; $2\frac{1}{4}$, $2\frac{1}{3}$, $2\frac{1}{2}$, $2\frac{2}{3}$, $2\frac{3}{4}$, 3

O $3\frac{1}{4}$, $3\frac{1}{3}$, $3\frac{1}{2}$, $3\frac{2}{3}$, $3\frac{3}{4}$, 4, etc.

P Set B: [z] $\frac{3}{1}$; Set C: [w] $\frac{5}{2}$, [z] $\frac{6}{2}$; Set D: [v] $\frac{7}{3}$, [x] $\frac{8}{3}$, [z] $\frac{9}{3}$; Set E: [u] $\frac{9}{4}$, [w] $\frac{10}{4}$, [y] $\frac{11}{4}$, [z] $\frac{12}{4}$; no

Q Set D: $\frac{10}{3}$, $\frac{11}{3}$, $\frac{12}{3}$, $\frac{13}{3}$, $\frac{14}{3}$, $\frac{15}{3}$, etc.; Set E: $\frac{13}{4}$, $\frac{14}{4}$, $\frac{15}{4}$, $\frac{16}{4}$, $\frac{17}{4}$, $\frac{18}{4}$, etc.

R 2; $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, etc.; $\frac{10}{6}$, $\frac{15}{9}$, $\frac{20}{12}$, etc.

S Set D

T No; $\frac{0}{5}$; $\frac{5}{5}$; $\frac{12}{5}$

U $\frac{25}{1}$; $\frac{50}{2}$; $\frac{75}{3}$; $\frac{100}{4}$

V C or E; $\frac{15}{6}$, $\frac{20}{8}$, $\frac{25}{10}$, $\frac{30}{12}$, $\frac{35}{14}$, etc.

W $\frac{2}{4}$; yes; no; no; yes

X $\frac{2}{2}$; yes; no; C or E; because $\frac{1}{2}$ and 1 are in the same set and have the same denominator

Y $\frac{1}{3}$ subtracted from $\frac{1}{3}$ leaves 0 thirds; yes; yes; yes; yes

Z No; yes; yes

(block 2)

| | | | |
|---|---|---|---|
| A > | F < | K > | P > |
| B < | G = | L > | Q > |
| C = | H > | M < | R = |
| D < | I < | N > | S = |
| E > | J < | O = | T < |

**Page 168:**

A $\frac{5}{5}$, $\frac{7}{5}$

B $\frac{5}{6}$, $\frac{7}{6}$

C $\frac{11}{3}$

D $\frac{6}{11}$

E $\frac{0}{8}$, $\frac{2}{8}$

F $\frac{1}{100}$, $\frac{4}{100}$, $\frac{5}{100}$

**Keeping skillful:**

(block 1)

A 39/100=78/[200]

B 16/100 = [4]/25

C 25/20=[125]/100; 125%

D 50/100 = 5/[10]

E 230/100 = [115]/50

F 26/100 = 130/[500]

G 188/400=[47]/100; 47%

H 140/100 = 28/[20]

I 82/100=[205]/250

J 24/80=[30]/100; 30%

K 175/100 = 7/[4]

L 40/100 = [28]/70

M 92/25=[368]/100; 368%

N 88/100 = 66/[75]

O 135/100 = [27]/20

P 92/200=[46]/100; 46%

Q 72/100 = [198]/275

R 45/100 = 144/[320]

S 77/25=[308]/100; 308%

T 3/150=[2]/100; 2%

U 40/100 = 30/[75]

V 160/100 = [8]/5

W 39/100 = 39/[100]

X 15/100=[12]/80

(block 2)

A 160 qt.

B 10 hr.

C 99 sq. ft.

D 2080 oz.

E 46,000 lb.

F 12 c.

G 7040 yd.

H 96 pt.

I 8 sq. ft.

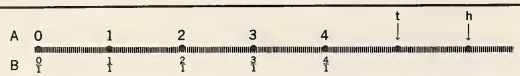
J 126 da.

K 13 bu.

L 8 yd.

## Moving forward

In this lesson you will learn that a number may have many different names.

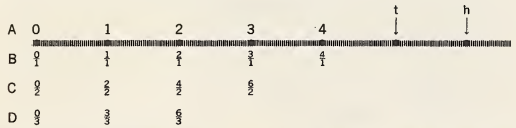



**A** The numerals in Row A above are a set of names for some of the points on the line. What numerals can you use to name Points $t$ and $h$? Can the line be extended so there is a point for 100? For 1,000,000? Give six other numerals that belong in Set A.

**B** Look at the pairs of numerals in Row B below the line. They belong in another set of names for the points on the line. Each pair has ■ in the denominator.

**C** What pair of numerals from this set can you use to name Point $t$? Point $h$?

**D** If you extend the line, will all the other pairs of numerals in Set B have the same denominator? Give six other pairs of numerals that belong in Set B.




**E** The pairs of numerals in Row C belong in another set of names for the points on the line. Each pair has ■ in the denominator. What pair of numerals from this set can you use to name Point 4? Point $t$? Point $h$?

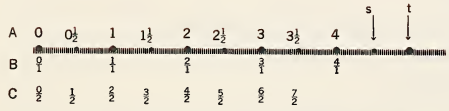
**F** Suppose you extend the line farther to the right. Give six other pairs of numerals that belong in Set C.

**G** The pairs of numerals in Set D have what denominator? Name the pairs of numerals that belong in this set for the other points shown.

**H** Give six other pairs of numerals that belong in Set D.

**I** How many names are shown for Point 2? Give four other names for Point 2.






**J** More points are now named on the line. What numerals are used in Set A to name these new points? What numerals can you use to name Points $s$ and $t$? Give six other numerals that belong in Set A.

**K** Are there pairs of numerals from Set B that can be used to name the new points on the line? Is there a pair of numerals from Set B for Point $s$? For Point $t$?

**L** Is there any end to the pairs of numerals that belong in Set B?

**M** Now look at Set C. What pair of numerals that belongs in this set can you use to name Point 4? Point $s$? Point $t$? Give six other pairs of numerals that belong in Set C.

**N** Notice that other points have been named on the line above. What numerals that belong in Set A can you use to name Points $m$, $n$, $p$, $q$, and $r$? Points $u$, $v$, $w$, $x$, $y$, and $z$?

**O** Give six other numerals that belong in Set A.

**P** What pairs of numerals that belong in Sets B, C, D, and E can you use to name Points $u$, $v$, $w$, $x$, $y$, and $z$? Is there a pair of numerals in each set for each point?

**Q** Give six other pairs of numerals that belong in Sets D and E.





# 165

1. Point out that the names for the points in Set A are those of whole numbers. Help pupils observe that the names for the points labeled "$t$" and "$h$" are 5 and 6 and that the line can be extended to include any whole numbers.
2. Call attention to Set B, which gives other names for the points on the line.
3. Help pupils understand that the names in Set B for the points labeled "$t$" and "$h$" must have "1" as the denominator.
4. For Exercises E and F, discuss the names given in Set C, have the other points named, and have names given for points that can be shown by extending the line.
5. For Exercises G and H, discuss the names in Set D and have the other points named. Have names given for points beyond those shown.
6. Point out in Exercise I that three names are shown for Point 2. Help pupils understand that all names for Point 2 will have numerators that are twice the denominators.

# 166

1. Have pupils observe that between the original points on the line more points are named for Set A. Have them name the points labeled "$s$" and "$t$" and give six other numerals that belong in Set A.
2. Help pupils understand that the points named in red in Set A cannot be named in Set B.
3. Help pupils see that there is no end to the numerals that belong in Set B.
4. The pupils should give new names for Set C that have the denominator "2".
5. Help pupils name the lettered points that belong in Set A. Then help them extend the line, as suggested in Exercise O.
6. Pupils should determine that only one point can be named in Set B, that two points can be named in Set C, three in Set D, and four in Set E. Let them give other names that belong in Sets D and E.

R How many names have been given to Point $p$? Give four other names for Point $p$. Give three other names for Point $1\frac{2}{3}$.

S Suppose $8\frac{3}{3}$ were shown in Set A. Will Set C or Set D have a pair of numerals to name the same point?

T Suppose $2\frac{5}{5}$ were shown in Set A. Could you use a pair of numerals from Sets B, C, D, or E to name the same point? What pair of numerals with denominator 5 can you use to name Point 0? To name Point 1? To name Point $2\frac{5}{5}$?

U If 25 were shown in Set A, what pairs of numerals could you use from Sets B, C, D, and E to name the same point?

V Which set has a pair of numerals to name Point $w$? Write five other names for Point $w$.

W When you add $\frac{1}{2}$ and $\frac{3}{4}$, what name do you use for $\frac{1}{2}$? Does Set C have a pair of numerals to name $\frac{1}{2}$? To name $\frac{3}{4}$? Does Set D have pairs of numerals to name either $\frac{1}{2}$ or $\frac{3}{4}$? Does Set E have pairs of numerals to name both $\frac{1}{2}$ and $\frac{3}{4}$? When you add $\frac{1}{2}$ and $\frac{3}{4}$, you think of $\frac{1}{2}$ as $\frac{2}{4}$, so that both $\frac{2}{4}$ and $\frac{3}{4}$ will be in the same set.

X When you subtract $\frac{1}{2}$ from 1, what name do you use for 1? Does Set B have a pair of numerals to name 1? To name $\frac{1}{2}$? Which set has pairs of numerals to name both $\frac{1}{2}$ and 1? When you subtract $\frac{1}{2}$ from 1, why do you think of 1 as $\frac{2}{2}$?

Y When you subtract $\frac{1}{3}$ from $\frac{1}{3}$, you do not need to change the name. How do you get the answer $\frac{0}{3}$? Is $\frac{0}{3}$ another name for 0? Can you use $\frac{0}{0}$ to name 0? Can you use $\frac{0}{800}$? Can you use $\frac{0}{10,000}$?

Z When you subtract $\frac{1}{12}$ from $\frac{5}{12}$, do you need to use different names for the numbers? Is the answer $\frac{4}{12}$? Is $\frac{4}{12}$ another name for $\frac{1}{3}$?

You may think of all the pairs of numerals in Sets B, C, D, and E on page 166 as one set of names for the points on the line. Use this set of names to decide which sign, >, <, or =, makes a true statement for each exercise below.

2

| | | | |
|---|---|---|---|
| A | $\frac{4}{3} \frac{5}{4}$ | K | $\frac{1}{2} \frac{1}{3}$ |
| B | $\frac{5}{3} \frac{7}{4}$ | L | $\frac{3}{3} \frac{0}{1}$ |
| C | $\frac{4}{4} \frac{1}{1}$ | M | $1\frac{1}{4} \; 1\frac{1}{3}$ |
| D | $1\frac{1}{2} \; 1\frac{2}{3}$ | N | $\frac{2}{1} \frac{5}{3}$ |
| E | $\frac{4}{2} \frac{0}{3}$ | O | $\frac{6}{4} \frac{3}{2}$ |
| F | $\frac{3}{2} \frac{5}{3}$ | P | $\frac{2}{1} \frac{4}{3}$ |
| G | $\frac{1}{2} \frac{2}{4}$ | Q | $\frac{1}{1} \frac{1}{3}$ |
| H | $1\frac{3}{4} \; 1\frac{2}{3}$ | R | $\frac{4}{2} \frac{2}{1}$ |
| I | $\frac{0}{4} \frac{1}{3}$ | S | $\frac{0}{1} \frac{0}{3}$ |
| J | $\frac{4}{3} \frac{3}{2}$ | T | $1\frac{1}{3} \; 1\frac{1}{2}$ |

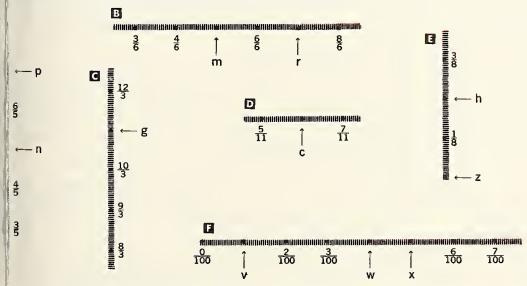


# 167

1 Refer to the second number line on page 166 to discuss Exercises R to Z.

2 Before the exercises are assigned, refer to the lesson on pages 104-106 to review the meanings of the signs. These exercises can be used for individual work. Let the pupils use the second number line on page 166 to help them decide which sign makes a true statement for each exercise. Verify each pupil's work and clear up any misunderstandings through class discussion.

udy each line below. Then replace each letter ith the pair of numerals that belongs in the set. 1




Now you should be able to give different names to fractions, whole numbers, and mixed numbers.

## Keeping skillful 2

78 is 39% of ▪.
16% of 25 is ▪.
25 is ▪% of 20.
50% of ▪ is 5.
230% of 50 is ▪.
26% of ▪ is 130.
188 is ▪% of 400.
140% of ▪ is 28.
82% of 250 is ▪.
24 is ▪% of 80.
175% of ▪ is 7.
40% of 70 is ▪.

M 92 is ▪% of 25.
N 88% of ▪ is 66.
O 135% of 20 is ▪.
P 92 is ▪% of 200.
Q 72% of 275 is ▪.
R 45% of ▪ is 144.
S 77 is ▪% of 25.
T ▪% of 150 is 3.
U 30 is 40% of ▪.
V 160% of 5 is ▪.
W 39% of ▪ is 39.
X 15% of 80 is ▪.

A 5 bu. = ▪ qt.
B 36,000 sec. = ▪ hr.
C 11 sq. yd. = ▪ sq. ft.
D 130 lb. = ▪ oz.
E 23 T. = ▪ lb.
F 192 tbs. = ▪ c.
G 4 mi. = ▪ yd.
H 12 gal. = ▪ pt.
I 1152 sq. in. = ▪ sq. ft.
J 18 wk. = ▪ da.
K 52 pk. = ▪ bu.
L 288 in. = ▪ yd.

# 168

1 Use Exercises A to F for individual work. Supply answers so that pupils may verify their work.

2 Adjust the assignment of the "Keeping skillful" exercises to individual needs and abilities. Supply the pupils with answers so they can verify their work.

# 169-172 Moving forward

Expanded Notes for this lesson are on pages 345-346.

**Objectives**

The pupil learns to read and write decimal fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

It is important for pupils to understand that decimal fraction numerals are a part of the numeral system. The place value of a numeral to the right of the decimal point is determined by its position in the same way that the place value of a numeral to the left of the point is determined. The names used to signify place value are symmetrical with respect to the ones' place. Thus, a numeral one place to the right of ones' place represents tenths just as a numeral one place to the left of ones' place represents tens; a numeral two places to the right of ones' place represents hundredths just as a numeral two places to the left represents hundreds; etc. Give special attention to the use of "and." Be sure the pupils understand that it is used in reading to separate a whole number from a fraction.

**Answers**

Page 171:

E $\frac{718}{1000}$; thousandths

F $\frac{2904}{10000}$; ten-thousandths

G $\frac{6}{100}$; and, hundredths

H $\frac{503}{10000}$; ten-thousandths

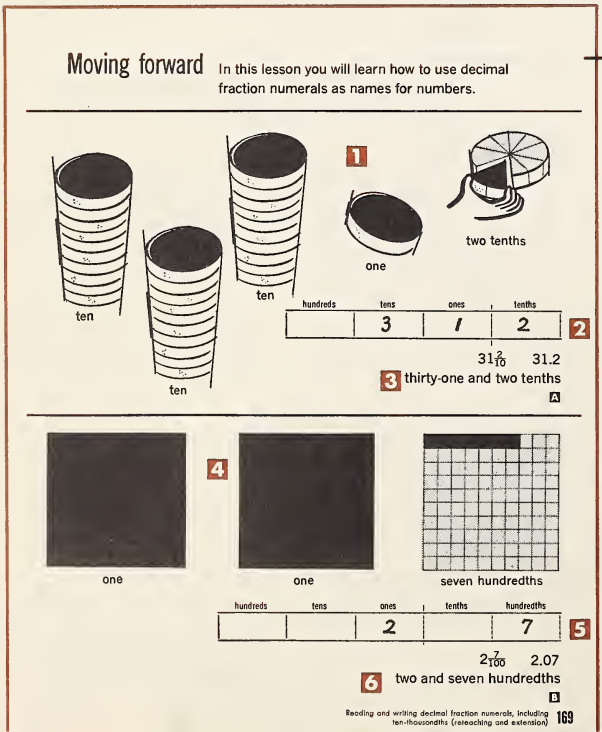
I $\frac{5}{10}$; and, tenths

J $\frac{705}{1000}$; and, thousandths

Page 172:

(block 2)

| | | |
|---|---|---|
| A 36.9 | F 375.53 | K 70.0006 |
| B 518.213 | G 1634.1256 | L 539.041 |
| C 69.25 | H 930.0784 | M 2103.0001 |
| D 7.002 | I 2060.005 | N 6254.07 |
| E 88.4 | J 429.01 | |






# 169

1 Have pupils observe the three groups of disks with ten in each group, the single disk, and the two tenths of a disk.

2 Call attention to the spaces, pointing out the longer dotted line that separates the tenths from the ones. Discuss whether the numerals have been placed in the correct spaces.

3 Call attention to the three ways of representing the number, and point out that each is read in the same way.

4 Guide pupils to see that the picture shows two wholes and seven hundredths of a whole. Be sure they understand that the seven red squares represent seven hundredths of a whole square.

5 Call attention to the shorter dotted line that separates the spaces for numerals representing fractions. Discuss whether the numerals have been placed in the correct spaces.

6 Discuss the three ways of writing the name of the number.

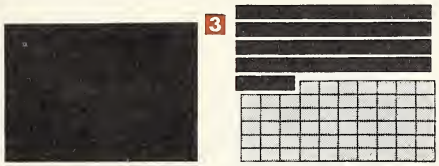
one forty-three hundredths

| hundreds | tens | ones | tenths | hundredths |
|---|---|---|---|---|
| | | 1 | | 43 |

$1\frac{43}{100}$ 1.43

one and forty-three hundredths

C



one forty-three hundredths or four tenths and three hundredths

| hundreds | tens | ones | tenths | hundredths |
|---|---|---|---|---|
| | | 1 | 4 | 3 |

$1\frac{43}{100}$ 1.43

one and forty-three hundredths

# 170

1 Pupils should observe that one whole rectangle and part of another rectangle are dark red. Help pupils understand that the small sections are called hundredths because each section is one of the 100 equal parts in the whole.

2 Relate the position of the numerals in the spaces to the illustration. Have pupils read the three ways in which the number is expressed.

3 Have pupils observe that now the sections showing hundredths have been grouped into tenths so that each group is 1 tenth of the whole. Help them see that there are 1 whole, 4 tenths, and 3 hundredths.

4 Help pupils discover that the numerals in the spaces say the same thing as the numerals (green) in the spaces at the top of the page, but that the grouping is different.

| hundreds | tens | ones | tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|---|---|---|
| | 6 | 7 | | | 9 | |

$67\frac{9}{1000}$ 67.009

sixty-seven and nine thousandths

| hundreds | tens | ones | tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|---|---|---|
| 3 | 5 | | | | | 1 |

$350\frac{1}{10,000}$ 350.0001

three hundred fifty and one ten-thousandth

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| 2 | | | 65 |
| 2 | | 6 | 5 |

C $2\frac{65}{1000}$ 2.065

two and sixty-five thousandths

| tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|
| | | | 411 |
| | 4 | 1 | 1 |

D $\frac{411}{10,000}$ .0411

four hundred eleven ten-thousandths

| ones | tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|---|
| 7 | 1 | 8 | 718 | |

E $\frac{718}{\square}$ .718 718 ~~

| tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|
| 2 | 9 | | 4 |
| | | | 2904 |

F $\frac{2904}{\square}$ .2904 2904 ~~

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| 9 | | 6 | |

G $9\frac{6}{\square}$ 9.06 9 ~~ 6 ~~

| tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|
| | 5 | | 3 |

H $\frac{503}{\square}$ .0503 503 ~~

| tens | ones | tenths | hundredths |
|---|---|---|---|
| 4 | | 5 | |

I $40\frac{5}{\square}$ 40.5 40 ~~ 5 ~~

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| 3 | 7 | | 5 |

J $3\frac{705}{\square}$ 3.705 3 ~~ 705 ~~

171

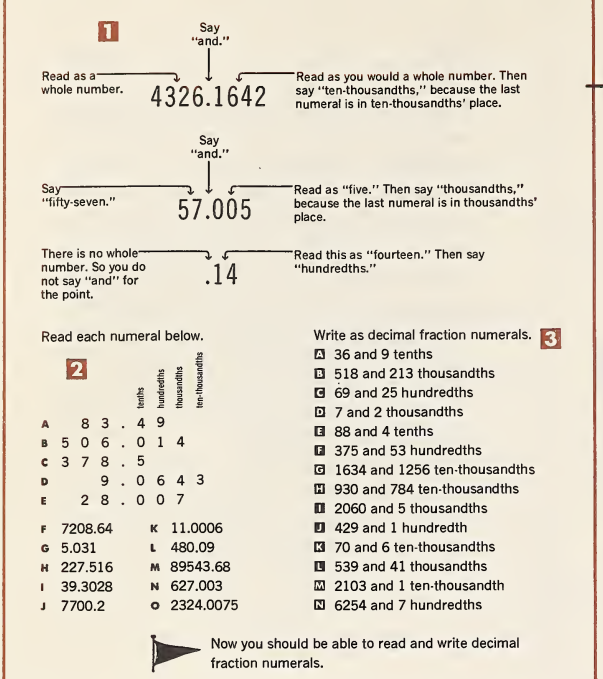
# 171

1 Have pupils compare the positions of the numerals in the spaces with the different ways in which the number is expressed. Point out that zeros are used in the numerals as place holders to establish the place value of the other figures. For example, in 67.009, the two zeros are used to show that the 9 is in thousandths' place. It should also be stressed that the point establishes place value.

2 Help the pupils understand that in Exercises C and D the numeral in the second row (brown) expresses a regrouping of the number represented in the first row. Point out that for both groupings the numeral is read the same way. Let pupils read the three ways in which the numbers are expressed.

3 Discuss the regrouping in Exercises E and F with the class and have them supply the missing numerals and words.

4 Let the pupils work individually on Exercises G to J.


1
Say "and."
Read as a whole number.
4326.1642
Read as you would a whole number. Then say "ten-thousandths," because the last numeral is in ten-thousandths' place.
Say "and."
Say "fifty-seven."
57.005
Read as "five." Then say "thousandths," because the last numeral is in thousandths' place.
There is no whole number. So you do not say "and" for the point.
.14
Read this as "fourteen." Then say "hundredths."
Read each numeral below.
2
tenths
hundredths
thousandths
ten-thousandths
A 8 3 . 4 9
B 5 0 6 . 0 1 4
C 3 7 8 . 5
D 9 . 0 6 4 3
E 2 8 . 0 0 7
F 7208.64
G 5.031
H 227.516
I 39.3028
J 7700.2
K 11.0006
L 480.09
M 89543.68
N 627.003
O 2324.0075
Write as decimal fraction numerals.
3
A 36 and 9 tenths
B 518 and 213 thousandths
C 69 and 25 hundredths
D 7 and 2 thousandths
E 88 and 4 tenths
F 375 and 53 hundredths
G 1634 and 1256 ten-thousandths
H 930 and 784 ten-thousandths
I 2060 and 5 thousandths
J 429 and 1 hundredth
K 70 and 6 ten-thousandths
L 539 and 41 thousandths
M 2103 and 1 ten-thousandth
N 6254 and 7 hundredths
Now you should be able to read and write decimal fraction numerals.
172

# 172

1 Have the pupils read the three numerals at the top of the page by following the instructions. Make sure they understand that they should read the numerals that come before the point as expressing a whole number, then say "and" for the point, and then read the numerals after the point as they would those for a whole number, followed by the name of the place for the last numeral. Stress the function of the zeros in the second example.

2 Use Exercises A to O for class discussion. Have the pupils take turns reading the numerals.

3 Assign Exercises A to N in the last block as written work. Verify the results by having pupils take turns writing the answers on the chalkboard so the class may both see and hear the correct solution. Discuss any points that need clarification.

# 173-174 Moving forward

Expanded Notes for this lesson are on page 346.

**Objectives**

The pupil learns the principles of regrouping needed in computation involving decimal fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

The regrouping taught in this lesson is basic to an understanding of the processes in addition and subtraction of decimal fractions. On page 173 two series of illustrations are presented. The first shows the regrouping used in carrying, and the second shows the regrouping used in borrowing. Practice in regrouping is provided on page 174, so that the pupil will be able to apply these principles to the lessons that follow.

**Answers**

Page 174:

A 4,0 B 2,8 C 4,9 D 4,18 E 4,0 F 4,9,9

(block 2)

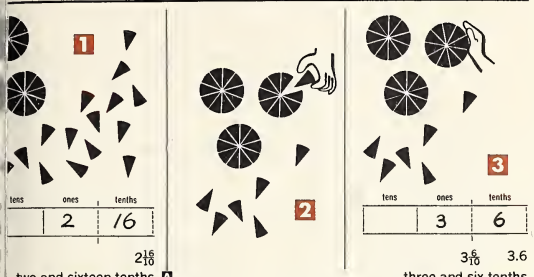
A 1.08
B 2.218
C 2.236
D .214
E 10.10 or 10.1
F 14.527
G .187
H 24.19
I .250 or .25
J 10.000 or 10
K .101
L 4.735

(block 3)

A 0,13
B 7,5,9
C 8,3
D 8,9
E 3,10
F 0,9,9

## Moving forward

In this lesson you will learn how to change decimal fraction numerals as you will when you compute.




$2\frac{16}{10}$

two and sixteen tenths A





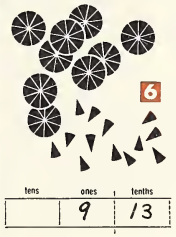


$3\frac{6}{10}$ 3.6

three and six tenths

$10\frac{3}{10}$ 10.3

ten and three tenths B




$9\frac{13}{10}$

nine and thirteen tenths



1

A

| tens | ones | tenths | hundredths |
|---|---|---|---|
| 4 | 3 | 9 | 16 |
| 4 | ? | ? | 6 |

B

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| | 19 | 28 | 3 |
| ? | 1 | ? | 3 |

2

C

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| 6 | 5 | | 2 |
| 6 | ? | ? | 12 |

D

| tens | ones | tenths | hundredths |
|---|---|---|---|
| 5 | 2 | 3 | 8 |
| ? | 11 | 12 | ? |

E

| enths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|
| 3 | 17 | 9 | 11 |
| ? | 8 | ? | 1 |

F

| tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|
| 5 | | | 6 |
| ? | ? | ? | 16 |

4

ite in the usual way each number presented below.

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| | 9 | 18 | |
| 2 | 1 | 9 | 28 |
| | 22 | 3 | 6 |
| | 2 | 0 | 14 |
| 9 | 9 | 20 | |
| 14 | 5 | 0 | 27 |
| | | 18 | 7 |
| 24 | | 19 | |
| | 2 | 3 | 20 |
| 9 | 9 | 9 | 10 |
| | | 9 | 11 |
| 4 | 6 | 9 | 45 |

5 What numeral belongs where each question mark is?

A

| ones | tenths | hundredths | thousandths |
|---|---|---|---|
| 1 | 0 | 3 | |
| ? | 9 | ? | |

B

| | | | |
|---|---|---|---|
| 7 | 6 | 0 | 9 |
| ? | ? | ? | 19 |

C

| | | | |
|---|---|---|---|
| 9 | 1 | 4 | 2 |
| ? | 11 | ? | 12 |

D

| | | | |
|---|---|---|---|
| 4 | 9 | 0 | 0 |
| 4 | ? | ? | 10 |

E

| | | | |
|---|---|---|---|
| | 4 | 3 | 0 |
| | ? | 12 | ? |

F

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 3 |
| ? | ? | ? | 13 |

Now you should know how to change decimal fraction numerals when you need to.

# 173

1 Help pupils relate the first picture in Exercise A to the numerals in the spaces. Call attention to the ways of representing the number.

2 Direct attention to the second picture and bring out that 10 tenths are being regrouped to form another whole.

3 Relate the third picture to the numerals in the spaces. Tell pupils that this kind of regrouping is used in addition. Call attention to the three ways of representing the number.

4 Have pupils relate the first picture in Exercise B to the numerals in the spaces. Call attention to the three ways of representing the number.

5 Help pupils observe that in the second picture one of the wholes is taken from the group of 10 wholes and separated into tenths.

6 Relate the last picture to the numerals in the spaces. Tell pupils that this kind of regrouping is used in subtraction.

# 174

1 Have pupils compare Exercises A and B at the top of the page with Exercise A on page 173. Have them compare the numerals in the top row with those in the bottom row and decide what numerals can replace the question marks. They should see that this is the kind of regrouping that is required for addition.

2 Have pupils compare Exercises C and D with Exercise B on page 173, and bring out that this kind of regrouping is required for subtraction.

3 Discuss Exercise E to bring out that the regrouping involves carrying.

4 Examination of Exercise F will show that the regrouping involves borrowing.

5 Treat Exercises A to L and A to F orally first. Let pupils take turns answering. Then assign these exercises as written work and have pupils verify their answers.

# 175-176 Moving forward

Expanded Notes for this lesson are on pages 346-347.

**Objectives**

The pupil learns that decimal fraction numerals can be used as names for points on a number line.

**Vocabulary**

There are no new words.

**Comments**

In teaching lessons dealing with the number line, stress that there are always two types of sets to consider: (1) The set of different names for a point ($\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{5}{10}$, etc.); (2) The set of fraction numerals that name different points but have the same numeral in the denominator, ($\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$, etc.).

It should be pointed out that decimal fraction numerals can be related to the common fraction numerals whose denominators are multiples of ten.

**Answers**

(block 1)

A 3

B Yes, but those for *y* and *z* are not shown. Yes, they may all be read in the same way. Yes

C —— D "two and zero tenths" or "two"

E Some of the points cannot be expressed as fraction numerals whose denominators are 5 or 2.

F $\frac{21}{10}$; $\frac{22}{10}$ G 2.1; 2.2

H No; yes, $\frac{11}{5}$; *y* and *z* cannot be expressed as halves.

I $\frac{85}{10}$; 8.5; no; $\frac{17}{2}$

J Set A, 15; Set B, $\frac{150}{10}$; Set C, 15.0; Set D, $\frac{75}{5}$; Set E, $\frac{30}{2}$

K Yes; $\frac{2}{4}$, $\frac{4}{4}$, $\frac{8}{4}$ L Yes; $\frac{3}{6}$, $\frac{6}{6}$, $\frac{9}{6}$

M $\frac{0}{10}$ to $\frac{1}{10}$; yes N Yes; yes

O $\frac{11}{100}$; $\frac{12}{100}$; .11; .12

P Yes; *m*; *p*, *r*, *t*, *v*, *x*

Q Yes; *p*, *t*, *x* R Yes; yes

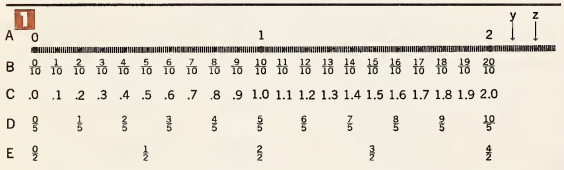
S Yes; yes; yes; .010

(block 2)

| | | | | | | |
|---|---|---|---|---|---|---|
| A < | C > | E = | G < | I > | K = | M > |
| B = | D > | F > | H < | J > | L < | N = |

## Moving forward

In this lesson you will learn more about decimal fraction numerals and the number system.




1 A Five sets of names are shown for points on the line above. For how many points are there names in Set A?

2 B Look at Set B and Set C. Is there a name in each set for each point shown on the line? Do you read the numerals in the two sets in the same way? Are the denominators the same?

3 C Notice how the name for Point 1 is written in Set C. You can read it as "one and zero tenths" or as "one."

D How do you read the name in Set C for Point 2?

4 E Why are names not given in Set D and Set E for all the points?

F What name from Set B would you use for Point *y*? For Point *z*? 5

G What name from Set C would you use for Point *y*? For Point *z*?

H Is there a name in Set D for Point *y*? For Point *z*? How do you know that there is no name in Set E for either Point *y* or Point *z*?

I If Point $8\frac{1}{2}$ were shown on the line, what name could you give for this point from Set B? From Set C? Would there be a name for Point $8\frac{1}{2}$ from Set D? What name could you use from Set E for this point?

J Suppose Point 15 were shown on the line. Give a name from each set for this point.

K Can you use $\frac{0}{4}$ to name Point 0? Which of these pairs of numerals can you use to name points on the line: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{4}{4}$, $\frac{7}{4}$, $\frac{8}{4}$? 6

L Can you use $\frac{0}{6}$ to name Point 0? Which of these pairs of numerals can you use to name points on the line: $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{6}{6}$, $\frac{8}{6}$, $\frac{9}{6}$? 6





# 175

1 Notice that only three points have names in every set. These names correspond to zero and to whole numbers. Have someone answer the question in Exercise A.

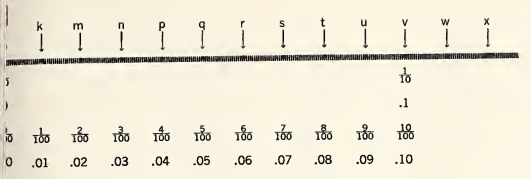
2 In Exercise B, stress that Sets B and C show other names for points on the line. Point out that all the numerals in these sets can be read in the same way because the denominators are the same. Note, however, that 1.1 commonly is read "one *and* one tenth," rather than "eleven tenths."

3 Pupils should notice that the information given in Exercise C can be used for Exercise D.

4 Explain why some of the points on this line are not named in Sets D and E. (.5, for example, is not commonly expressed as $2\frac{1}{2}$ fifths.)

5 Let pupils take turns answering the questions in Exercises F to J.

6 Pupils should note that some pairs of numerals cannot be used to name points on this line.




J The line above shows only part of the line on page 175. What part? Are the points named in Sets G and H on the line above also named on the line on page 175?

J Look at Sets I and J. Is there a name from each set for each point shown on the line? Do you read the numerals in the two sets in the same way?

J What name can you use in Set I for Point $w$? For Point $x$? What names can you use from Set J for these two points?

J Can you use $\frac{0}{50}$ to name Point 0? For which point on the line can you use the name $\frac{1}{50}$? For which point can you use $\frac{2}{50}$? $\frac{3}{50}$? $\frac{4}{50}$? $\frac{5}{50}$? $\frac{6}{50}$?

J Can you use $\frac{0}{25}$ to name Point 0? For which point on the line can you use $\frac{1}{25}$? $\frac{2}{25}$? $\frac{3}{25}$?

R Look at the line on page 175 again. Can you use $\frac{100}{100}$ to name Point 1? Can you also use 1.00 to name Point 1?

S Make a line like the one above to show the thousandths between 0 and $\frac{1}{10}$. Can you use $\frac{0}{1000}$ to name Point 0? Can you also use .000? Can you use $\frac{10}{1000}$ to name $\frac{1}{100}$? What decimal fraction numeral equal to $\frac{10}{1000}$ can you use to name $\frac{1}{100}$? 4

Rewrite each exercise below, using the sign >, =, or <, to make a true statement. 5

A .01 ⟶ .1
B 1.0 ⟶ 1.000
C .10 ⟶ .04
D 1.3 ⟶ .13
E .00 ⟶ .000
F 2.8 ⟶ 2.0
G .09 ⟶ .1
H .005 ⟶ .03
I 1.6 ⟶ 1.08
J .12 ⟶ .012
K .01 ⟶ .010
L .009 ⟶ .07
M 3.60 ⟶ .361
N .2 ⟶ .20

Now you should understand how decimal fraction numerals can be used to name points of a line.

# 176

1 Have pupils notice that this number line is an enlargement of a small part of the number line on page 175.
2 Point out for Exercises N and O that Sets I and J show other names for the points on the line and that both these sets of names are read in the same way.
3 Have pupils answer the questions in Exercises P, Q, and R.
4 Make a number line, as suggested in Exercise S, on a 1 ft. x 6 ft. strip of wrapping paper, allowing 5 inches between points. Use this line to answer the questions in the text.
5 Assign Exercises A to N as individual work. Provide answers so that pupils may verify their results. Have them refer to pages 104 and 105, if they need to refresh their memory of what the signs > and < mean.

# 177-180 Learning how

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns how to add and subtract numbers represented by decimal fraction numerals.

**Vocabulary**

New words page 177 August*, Miller*, steel*, rod*; page 178 odometer

**Comments**

The addition and subtraction of numbers expressed by decimal fraction numerals are based upon the pupils' knowledge and understanding of the addition and subtraction of whole numbers, of decimal fraction notation, and of the regrouping necessary for carrying and borrowing. It is the purpose of this lesson to extend the operations of addition and subtraction to include numbers represented by decimal fraction numerals.

The teacher should point out that the *process* of addition or the *process* of subtraction is no different when applied to decimal fractions than when applied to whole numbers. However, the points in the numerals indicate that fractions, and not whole numbers, are being processed, and the sum or difference must be adjusted accordingly.

The four-step teaching method is used. Examples and explanations help pupils SEE; examples and questions help pupils THINK; exercises with completed examples provide opportunities to TRY; and individual practice material gives them an opportunity to DO exercises without guidance.

Page 180:
(block 1)

| | | | |
|---|---|---|---|
| A .01 | H 11.0 | O 40.7 | V 3.008 |
| B 18.67 | I .3673 | P 1.72 | W 1.7230 |
| C 4.32 | J 20.0 | Q 4.3000 | X .17 |
| D 1.2872 | K 15.471 | R 3.0 | |
| E 86.000 | L 1.333 | S .421 | |
| F 165.1 | M 14.142 | T .05 | |
| G .058 | N .02 | U 5.00 | |

(block 2)

| | | |
|---|---|---|
| A 5.0 | E 2.87 | I 1.106 |
| B .27 | F .0617 | J 16.0 |
| C 1.8 | G 1.14 | K 3.973 |
| D 347.92 | H 52.270 | L 37.59 |

Keeping skillful:

(block 1)

| | | | |
|---|---|---|---|
| A 76 | E 27 | I 33 | M 33 |
| B 18 | F 221 | J 8 | N 70 |
| C 64 | G 20 | K 70 | |
| D 21 | H 154 | L 44 | |

(block 2)

| | |
|---|---|
| A $364\frac{17}{32}$ | I $581\frac{67}{110}$ |
| B 140,244 | J $513\frac{2}{15}$ |
| C $698\frac{14}{15}$ | K 16,384 |
| D 1250 | L 629,800 |
| E 28,519 | M 375 |
| F 34,500 | N 240,714 |
| G $2343\frac{7}{18}$ | O $852\frac{5}{8}$ |
| H 468,628 | P $404\frac{724}{725}$ |

(block 3)

| | | |
|---|---|---|
| A $9\frac{13}{15}$ | F $2\frac{7}{8}$ | K $\frac{3}{4}$ |
| B $13\frac{3}{4}$ | G $7\frac{1}{2}$ | L $6\frac{4}{5}$ |
| C $1\frac{23}{24}$ | H $10\frac{7}{12}$ | M $3\frac{1}{8}$ |
| D $11\frac{9}{14}$ | I $4\frac{11}{12}$ | N $\frac{7}{20}$ |
| E 9 | J $17\frac{2}{5}$ | |

## Learning how

In this lesson you will learn how to add and subtract numbers expressed with decimal fraction numerals.

**see** A Bill read that New York City had 4.34 in. of rainfall in August and 3.67 in. of rainfall in September of that year. New York City had a total of how many inches of rain in these two months?

$4.34 + 3.67 = p$ **1**

You must find the numeral to replace $p$.

**2**

$$\begin{array}{r} 4.34 \\ 3.67 \\ \hline 1 \end{array}$$

First add in the hundredths' column. The sum is 11 hundredths. Think of 11 hundredths as 1 tenth and 1 hundredth.

Remember the 1 tenth. Write 1 hundredth in hundredths' place.

$$\begin{array}{r} {\scriptstyle 1} \\ 4.34 \\ 3.67 \\ \hline 01 \end{array}$$

Write 1 in tenths' column to show there is 1 more tenth. Add in the tenths' column. The sum is 10 tenths. Think of 10 tenths as 1.

Remember the 1. Write 0 in tenths' place.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 4.34 \\ 3.67 \\ \hline .01 \end{array}$$

Write 1 in ones' column to show there is 1 more one.

Put the point here. You have really added $\frac{34}{100}$ and $\frac{67}{100}$. So the fraction part of your sum must be hundredths.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 4.34 \\ 3.67 \\ \hline 8.01 \end{array}$$

Add the ones. The sum is 8.

Write 8 in ones' place.

New York City had a total of 8.01 in. of rain in August and September. **3**

$4.34 + 3.67 = 8.01$

B Mr. Miller has a steel rod that is 3.028 in. long. He needs to cut .175 in. from the rod. How long will the rod be when he has cut it down?

$3.028 - .175 = q$ **4**

You must find the numeral to replace $q$.

Now turn the page.





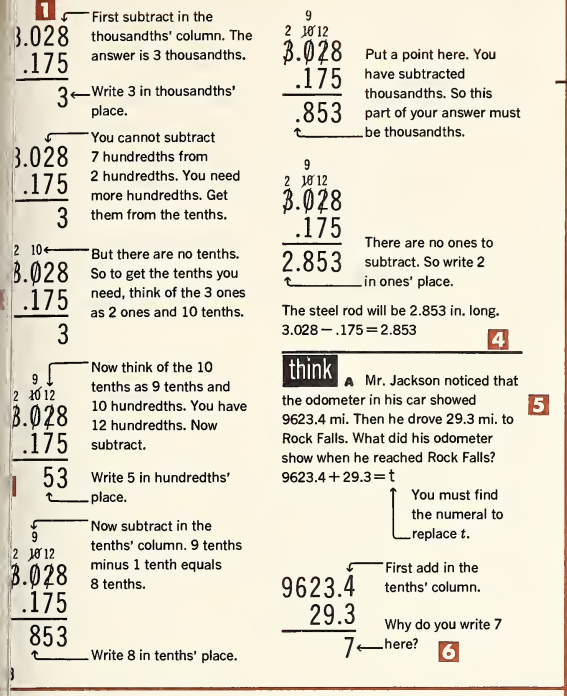
# 177

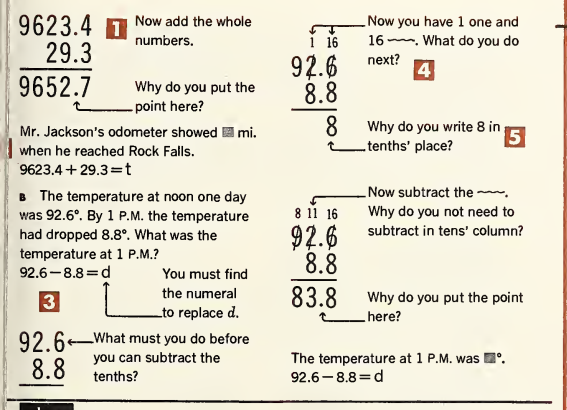
1 Have the problem and the equation read.

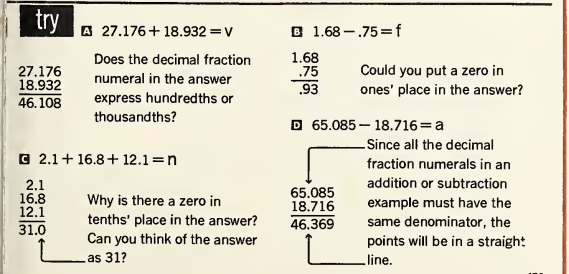
2 The pupils should note that the decimal fraction numerals have been placed in vertical form. Emphasize that hundredths are placed under hundredths and tenths under tenths, and that the decimal points are also in a vertical column. Read and discuss each step as it is developed. Make certain pupils understand the regrouping. If clarification is necessary, refer them to page 173.

3 Discuss the statement and the completed equation.

4 Have the problem read. Let the class determine whether or not the equation is correct.


1
First subtract in the thousandths' column. The answer is 3 thousandths.
3.028
.175
3
Write 3 in thousandths' place.
You cannot subtract 7 hundredths from 2 hundredths. You need more hundredths. Get them from the tenths.
2 10
But there are no tenths. So to get the tenths you need, think of the 3 ones as 2 ones and 10 tenths.
9
2 10 12
Now think of the 10 tenths as 9 tenths and 10 hundredths. You have 12 hundredths. Now subtract.
53
Write 5 in hundredths' place.
Now subtract in the tenths' column. 9 tenths minus 1 tenth equals 8 tenths.
853
Write 8 in tenths' place.
Put a point here. You have subtracted thousandths. So this part of your answer must be thousandths.
.853
There are no ones to subtract. So write 2 in ones' place.
2.853
The steel rod will be 2.853 in. long.
3.028 − .175 = 2.853
4
think
A Mr. Jackson noticed that the odometer in his car showed 9623.4 mi. Then he drove 29.3 mi. to Rock Falls. What did his odometer show when he reached Rock Falls?
5
9623.4 + 29.3 = t
You must find the numeral to replace t.
9623.4
29.3
First add in the tenths' column.
Why do you write 7 here?
7
6


9623.4
29.3
9652.7
1
Now add the whole numbers.
Why do you put the point here?
Mr. Jackson's odometer showed ▩ mi. when he reached Rock Falls.
9623.4 + 29.3 = t
B The temperature at noon one day was 92.6°. By 1 P.M. the temperature had dropped 8.8°. What was the temperature at 1 P.M.?
92.6 − 8.8 = d
3
You must find the numeral to replace d.
92.6
8.8
What must you do before you can subtract the tenths?
1 16
92.6
8.8
8
Now you have 1 one and 16 ~~. What do you do next?
4
Why do you write 8 in tenths' place?
5
8 11 16
Now subtract the ~~. Why do you not need to subtract in tens' column?
83.8
Why do you put the point here?
The temperature at 1 P.M. was ▩°.
92.6 − 8.8 = d


try
A 27.176 + 18.932 = v
27.176
18.932
46.108
Does the decimal fraction numeral in the answer express hundredths or thousandths?
B 1.68 − .75 = f
1.68
.75
.93
Could you put a zero in ones' place in the answer?
C 2.1 + 16.8 + 12.1 = n
2.1
16.8
12.1
31.0
Why is there a zero in tenths' place in the answer? Can you think of the answer as 31?
D 65.085 − 18.716 = a
Since all the decimal fraction numerals in an addition or subtraction example must have the same denominator, the points will be in a straight line.
65.085
18.716
46.369



# 178

1 The pupils should observe that subtraction begins on the right, just as with whole numbers.

2 Help pupils understand that since 7 cannot be subtracted from 2 in the hundredths' column and there are no tenths, it is necessary to take 1 whole (or 10 tenths) from the 3 ones, leaving 2 ones.

3 Relate the borrowing shown here to the way pupils borrowed when they subtracted whole numbers. Refer to the series of pictures at the bottom of page 173, if necessary.

4 Have the pupils read the completed equation and decide whether or not the answer is reasonable.

5 Have the problem and the equation read. Let the pupils determine whether or not the equation has been written correctly.

6 Have the pupils explain how 7 is obtained.

# 179

1 Bring out that the answer shows tenths because tenths were added.

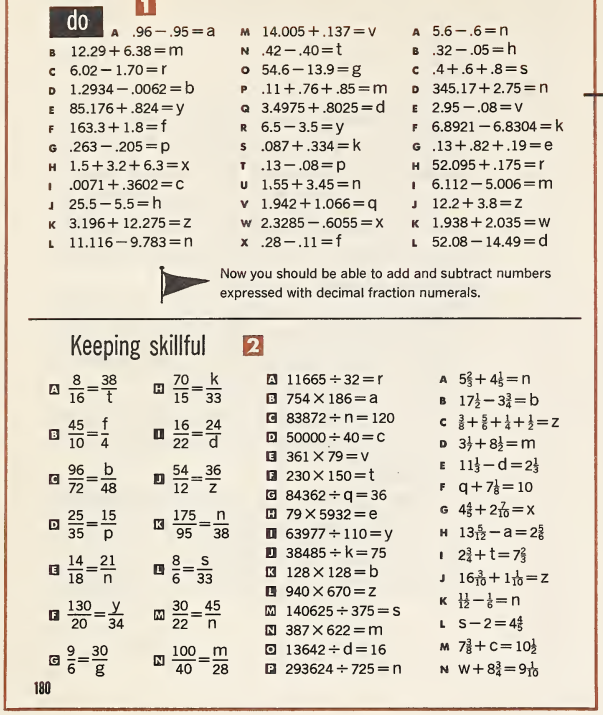
2 Have the pupils complete the statement and the equation. The class should determine whether or not the answer is reasonable.

3 Have the problem and the equation read. Call attention to the black numerals to bring out that more tenths are needed.

4 Pupils should note in this step that 1 whole is thought of as 10 tenths, making 16 tenths in all. They should understand that 8 is written in tenths' place because tenths have been subtracted.

5 Let the pupils answer the remaining questions, then complete the statement and equation.

6 For the TRY step, let each pupil work Examples A to D and compare his answers with those in the book when he has finished. Then discuss the questions.



do 1

A .96 − .95 = a
B 12.29 + 6.38 = m
C 6.02 − 1.70 = r
D 1.2934 − .0062 = b
E 85.176 + .824 = y
F 163.3 + 1.8 = f
G .263 − .205 = p
H 1.5 + 3.2 + 6.3 = x
I .0071 + .3602 = c
J 25.5 − 5.5 = h
K 3.196 + 12.275 = z
L 11.116 − 9.783 = n
M 14.005 + .137 = v
N .42 − .40 = t
O 54.6 − 13.9 = g
P .11 + .76 + .85 = m
Q 3.4975 + .8025 = d
R 6.5 − 3.5 = y
S .087 + .334 = k
T .13 − .08 = p
U 1.55 + 3.45 = n
V 1.942 + 1.066 = q
W 2.3285 − .6055 = x
X .28 − .11 = f
A 5.6 − .6 = n
B .32 − .05 = h
C .4 + .6 + .8 = s
D 345.17 + 2.75 = n
E 2.95 − .08 = v
F 6.8921 − 6.8304 = k
G .13 + .82 + .19 = e
H 52.095 + .175 = r
I 6.112 − 5.006 = m
J 12.2 + 3.8 = z
K 1.938 + 2.035 = w
L 52.08 − 14.49 = d

Now you should be able to add and subtract numbers expressed with decimal fraction numerals.

Keeping skillful 2

A $\frac{8}{16}=\frac{38}{t}$
B $\frac{45}{10}=\frac{f}{4}$
C $\frac{96}{72}=\frac{b}{48}$
D $\frac{25}{35}=\frac{15}{p}$
E $\frac{14}{18}=\frac{21}{n}$
F $\frac{130}{20}=\frac{y}{34}$
G $\frac{9}{6}=\frac{30}{g}$
H $\frac{70}{15}=\frac{k}{33}$
I $\frac{16}{22}=\frac{24}{d}$
J $\frac{54}{12}=\frac{36}{z}$
K $\frac{175}{95}=\frac{n}{38}$
L $\frac{8}{6}=\frac{s}{33}$
M $\frac{30}{22}=\frac{45}{n}$
N $\frac{100}{40}=\frac{m}{28}$

A 11665 ÷ 32 = r
B 754 × 186 = a
C 83872 ÷ n = 120
D 50000 ÷ 40 = c
E 361 × 79 = v
F 230 × 150 = t
G 84362 ÷ q = 36
H 79 × 5932 = e
I 63977 ÷ 110 = y
J 38485 ÷ k = 75
K 128 × 128 = b
L 940 × 670 = z
M 140625 ÷ 375 = s
N 387 × 622 = m
O 13642 ÷ d = 16
P 293624 ÷ 725 = n

A $5\frac{2}{3}+4\frac{1}{6}=n$
B $17\frac{1}{2}-3\frac{3}{4}=b$
C $\frac{3}{8}+\frac{5}{6}+\frac{1}{4}+\frac{1}{2}=z$
D $3\frac{1}{7}+8\frac{1}{2}=m$
E $11\frac{1}{3}-d=2\frac{1}{3}$
F $q+7\frac{1}{8}=10$
G $4\frac{2}{5}+2\frac{7}{10}=x$
H $13\frac{5}{12}-a=2\frac{5}{8}$
I $2\frac{3}{4}+t=7\frac{2}{3}$
J $16\frac{3}{10}+1\frac{1}{10}=z$
K $\frac{11}{12}-\frac{1}{6}=n$
L $s-2=4\frac{4}{5}$
M $7\frac{3}{8}+c=10\frac{1}{2}$
N $w+8\frac{3}{4}=9\frac{1}{10}$

180

# 180

1 Assign the blocks of work in the DO step according to the needs and abilities of the pupils. Verify each pupil's work so that any necessary reteaching may be done.

2 The three blocks of work in the "Keeping skillful" section may be used separately, or they may be used at one time, as individual needs require. Provide answers so pupils may verify their work.

# 181-182 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to solve problems that require addition and subtraction of numbers expressed as decimal fraction numerals.

**Vocabulary**

New words page 181 Thomas*, hatching*, brooder, increased*, Allen's, production, produced*, remaining*, storage*, butterfat*, gauge; page 182 binding, pastures*, cattle*, sown, clover*

**Comments**

This lesson gives experience in solving problems that involve addition and subtraction of fractions expressed in decimal notation. The pupil writes an equation to represent the problem situation and then does the necessary computation using decimal fraction numerals.

The "Checking up" tests on page 182 should help you determine the pupils' understanding of decimal fraction notation and their ability to add and subtract numbers expressed as decimal fraction numerals. For Tests 1 and 2 use pages 169-172 for reteaching. For Tests 3, 4, and 5, use pages 177-180.

An activity for able pupils that might be used with this lesson is described in Activity 4, page 374.

**Answers**

Using arithmetic:

A 102.8 − 94.6 = [8.2]; 8.2°
B 94.6 − 89.6 = [5.0] or 94.6 − [5.0] = 89.6; 5°
C 2.10 + [2.15] = 4.25 or 4.25 − 2.10 = [2.15]
D [.6] + .5 = 1.1
E (.9) + [.2] = 1.1 or 1.1 − (.9) = [.2]
F 23,686.80 − 22,945.35 = [741.45]
G 38.32 + [26.56] = 64.88
H 23,686.80 − 919.47 = [22,767.33]
I .8 + 3.4 + 52.6 = [56.8]
J 172.6 + [27.4] = 200.0
K 45.8 + 3.6 + 45.8 + 3.6 = [98.8]
L 72.5 + [26.3] = 98.8

M 12.9 + 8.3 = [21.2]
N 18.50 − 9.75 = [8.75]
O (11.3) + [9.9] = 21.2

**Checking up:**

Test 1
A thousandths
B hundredths
C tenths
D ten-thousandths
E thousandths
F tenths
G hundredths
H ten-thousandths

Test 2
A 17.03
B .0600
C .807
D 5.2
E 9.011
F 2.16
G 8.017

Test 3
A 9.382
B 26.89
C 1.4083
D 3.0
E 12.00
F 19.119
G 8.1500

Test 4
A .416
B .8
C 2.8450
D 11.08
E 1.0
F .418
G 2.92

Test 5
A 2.47
B .5686
C 8.4
D 6.839
E 4.26
F .612
G 9.5
H 19.529

## Using arithmetic 1

J Mr. Thomas kept the temperature for hatching chicks at 102.8°. When the chicks hatched, he moved them to a brooder, where he kept the temperature at 94.6°. The brooder temperature was how many degrees lower than the hatching temperature?

K When the chicks were one week old, Mr. Thomas lowered the brooder temperature to 89.6°. He lowered the brooder temperature how many degrees at the end of the first week?

L When one chick was 12 weeks old, its weight was 2.10 lb. At 24 weeks, its weight was 4.25 lb. What was the gain in weight during that time?

M When the chickens were 12 weeks old, Mr. Thomas increased the amount of feed for each chicken by .5 lb. per week. Then each chicken was eating 1.1 lb. of feed per week. About how much feed per week was a chicken eating before the feed was increased?

N The feed for a 12-week-old chicken for one week contained .4 lb. of wheat and .5 lb. of corn. How much did the other material in the feed weigh?

O At the fair, one of Mr. Allen's cows won first prize for milk production. This cow had produced 23,686.80 lb. of milk in one year. The cow that won second prize produced 22,945.35 lb. of milk that year. Mr. Allen's cow had produced how many more pounds of milk than the cow that won second prize?

G Mr. Allen's prize cow produced 64.88 lb. of milk one day. The amount produced in the morning milking was about 38.32 lb. About how much milk was produced in the evening milking?

H The 23,686.80 lb. of milk that Mr. Allen's prize cow produced in one year contained 919.47 lb. of butterfat. What would be the weight of the remaining milk if all butterfat were removed?

I The Allens use the milk produced by their cows in three ways. About .8 gal. is used each day by the family. About 3.4 gal. is used each day for feeding animals. About 52.6 gal. is sold per day. About how many gallons of milk do the cows produce per day?

J Mr. Allen's milk storage tank will hold 200.0 gal. of milk. One day the gauge showed that the tank contained 172.6 gal. How many more gallons of milk could have been put in the tank?



# 181

1 Assign these problems and problems K to O on page 182 as written work. Remind the pupils to watch for problems that require information from a previous problem. Tell them to write an equation for each one and use a letter to hold a place for the numeral they are to find. Then they should compute to find the answer and rewrite the equation with the answer in place of the letter.

K Mr. Allen built a feed box for his cows. The bottom of the box is 45.8 ft. long and 3.6 ft. wide. What is the perimeter of the bottom of the box?

L Mr. Allen put a steel binding around the edge of the feed box. He had a piece of steel binding 72.5 ft. long. How many more feet of steel binding did he need for the feed box?

M Mr. Townsend has two pastures for his cattle. The area of one pasture is 12.9 acres, and the area of the other is 8.3 acres. How many acres are there in both pastures?

N If grass seed were sown in the pastures, 18.50 lb. of seed per acre would be needed. If clover were sown, 9.75 lb. per acre would be needed. How much more grass seed than clover seed would be needed per acre?

O Mr. Townsend sowed 5.5 acres of pasture in the morning and 5.8 acres in the afternoon. How many acres of pasture did he have left to sow? 1

## Checking up 2

Test 1

| | |
|---|---|
| A 2.014 | two and fourteen ~~~ |
| B .23 | twenty-three ~~~ |
| C 6.2 | six and two ~~~ |
| D 5.0025 | five and twenty-five ~~~ |
| E .380 | three hundred eighty ~~~ |
| F 100.6 | one hundred and six ~~~ |
| G 3.70 | three and seventy ~~~ |
| H .5000 | five thousand ~~~ |

Test 2

Write as decimal fraction numerals.

A seventeen and three hundredths
B six hundred ten-thousandths
C eight hundred seven thousandths
D five and two tenths
E nine and eleven thousandths
F two and sixteen hundredths
G eight and seventeen thousandths

Test 3

Find the sum.

A 3.178, 6.204
B 12.92, 11.18, 2.79
C .6007, .1752, .6324
D .9, .8, .5, .6, .2
E 5.48, 6.52
F 12.735, 6.384
G 2.3975, 5.7525

Test 4

Find the difference.

A .724, .308
B 1.7, .9
C 16.3025, 13.4575
D 29.38, 18.30
E 4.2, 3.2
F 1.293, .875
G 3.75, .83

Test 5

A $1.79 + .68 = n$
B $.6028 - .0342 = b$
C $18.9 + y = 27.3$
D $p - 2.111 = 4.728$
E $14.92 - c = 10.66$
F $s + 5.116 = 5.728$
G $2.8 + 6.7 = z$
H $h - 4.637 = 14.892$

182 End-of-block tests on reading, writing, adding, and subtracting decimal fractions

# 182

1 After the problems have been solved, verify the answers and discuss any problems that are troublesome.

2 Allow ample time on the "Checking up" tests so that all pupils can finish the work. Verify each pupil's work so that any reteaching may be based upon the needs of the pupils.

# 183 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns that he can use fraction numerals and mixed numerals as terms of ratios.

**Vocabulary**

New words page 183 blouse, revolutions

**Comments**

The purpose of this lesson is to get pupils to realize that fraction numerals can be used in ratios. Up to this time the pupils have been using only numerals that represent whole numbers in the terms of the ratios. On page 166, however, the pupils learned that fraction numerals and mixed numerals can be used as names for points on a number line and represent numbers that are a part of the complete number system. Thus they should understand that fraction numerals and mixed numerals, just as whole numerals, can be used as terms of a ratio. Beginning on page 186, the pupils will learn to multiply fractions and mixed numbers that are expressed as terms of ratios.

**Answers**

A The cost of 2 lb. of bananas

B The number of pounds of bananas; the cost of $1\frac{3}{4}$ lb. of bananas

C The number of yards for the blouse; the number of yards for the skirt

D 1, number of yards for the blouse; $2\frac{1}{4}$, number of yards for the skirt

E 2, number of blocks Mary lives from school for every 1 block Alice lives from school; 1, number of blocks Alice lives from school for every 2 blocks Mary lives from school

F $2\frac{1}{3}$, number of blocks Peggy lives from school for every 1 block Sue lives from school; 1, number of blocks Sue lives from school for every $2\frac{1}{3}$ blocks Peggy lives from school

G $\frac{3}{4}$, fraction of a block June lives from school for every 1 block Tony lives from school; 1, number of blocks

Tony lives from school for every $\frac{3}{4}$ of a block June lives from school

(block 2)

A $100/2\frac{2}{3}$ or $2\frac{2}{3}/100$

B $8\frac{1}{2}/7$ or $7/8\frac{1}{2}$

C $3\frac{7}{10}/1$

D $75/70\frac{5}{8}$ or $70\frac{5}{8}/75$

E $260/3\frac{1}{2}$ or $3\frac{1}{2}/260$

F $4\frac{1}{2}/1$ or $1/4\frac{1}{2}$

G $\frac{4}{5}/1$

H $33\frac{1}{3}/1$ or $1/33\frac{1}{3}$

I $1\frac{3}{4}/2$ or $2/1\frac{3}{4}$

J $16\frac{1}{2}/1$ or $1/16\frac{1}{2}$

K $1\frac{2}{3}/1$

L $2\frac{5}{8}/100$ or $100/2\frac{5}{8}$

M $5\frac{1}{3}/1$ or $1/5\frac{1}{3}$

N $1/5\frac{5}{12}$ or $5\frac{5}{12}/1$

O $\frac{1}{2}/1$

P $5\frac{1}{2}/1$

Q $10/\frac{3}{8}$ or $\frac{3}{8}/10$

## Thinking straight

A Mrs. Parker bought 2 lb. of bananas for 29¢. You can use the ratio at the right to express this rate. In the ratio, 2 stands for the number of pounds. What does 29 stand for? $\frac{2}{29}$

B Mrs. Page bought $1\frac{3}{4}$ lb. of bananas for 29¢. You can use the ratio at the right to express this rate. In the ratio, what does $1\frac{3}{4}$ stand for? What does 29 stand for? $\frac{1\frac{3}{4}}{29}$

C Ann used 1 yd. of material for a blouse and 2 yd. for a skirt. You can use the ratio at the right to express this comparison. In the ratio, what does 1 stand for? What does 2 stand for? $\frac{1}{2}$

D Sue used 1 yd. of material for a blouse and $2\frac{1}{4}$ yd. for a skirt. You can use the ratio at the right to express this comparison. What does each term in the ratio stand for? $\frac{1}{2\frac{1}{4}}$

E Mary lives twice as far from school as Alice. This means that Mary lives 2 blocks from school for each block Alice lives from school. You can use the ratio above to express this comparison. What does each term stand for? $\frac{2}{1}$

F Peggy lives $2\frac{1}{3}$ times as far from school as Sue. This means that Peggy lives $2\frac{1}{3}$ blocks from school for each block Sue lives from school. What does $2\frac{1}{3}$ in the ratio stand for? What does 1 stand for? $\frac{2\frac{1}{3}}{1}$

G June lives $\frac{3}{4}$ as far from school as Tony. This means that June lives $\frac{3}{4}$ of a block from school for each block Tony lives from school. What does each of the terms of the ratio stand for? $\frac{\frac{3}{4}}{1}$

Write a ratio for each exercise below.

A 100 mi. in $2\frac{2}{3}$ hr.

B $8\frac{1}{2}$ pages to 7 pages

C $3\frac{7}{10}$ times as many buttons

D 75 lb. to $70\frac{5}{8}$ lb.

E $2.60 per $3\frac{1}{2}$ lb.

F $4\frac{1}{2}$ mi. per 1 in.

G $\frac{4}{5}$ as many records

H $33\frac{1}{3}$ revolutions per minute

I $1\frac{3}{4}$ cups to 2 cups

J $16\frac{1}{2}$¢ for 1 can

K $1\frac{2}{3}$ times as many hours

L $2\frac{5}{8}$ in. per 100 mi.

M $5\frac{1}{3}$ yd. per window

N 1 in. per $5\frac{5}{12}$ ft.

O $\frac{1}{2}$ as many cookies

P $5\frac{1}{2}$ times as much money

Q 10 mi. per $\frac{3}{8}$ in.

4



# 183

1 Have pupils read Exercises A and B and answer the questions in each. Guide pupils to understand that it is possible to express a rate by a ratio in which one term is a mixed numeral.

2 Compare Exercises C and D. Point out that a comparison also may be expressed by a ratio in which one term is a mixed numeral.

3 Guide the reading and discussion of Exercises E to G to bring out that the expressions "twice as far" and "2 times as far," which involve the idea of "times as many," are ways of comparing something with 1. Direct attention to the use of a mixed numeral in Exercise F and a fraction numeral in Exercise G.

4 Let pupils work individually in writing ratios for Exercises A to Q. Provide answers. Then clarify any misunderstandings.

# 184-185 Thinking straight

Exponded Notes ore not considered necessary for this lesson.

**Objectives**

The pupil is introduced to the multiplication of fractions by using the number line.

**Vocabulary**

New words page 184 labeled; page 185 select

**Comments**

In this lesson the number line is used to introduce pupils to multiplication of fractions. First they are given the equation $3 \times 2 = n$, which they can easily solve. Then, by referring to the number line, they see that fraction numerals can be used as other names for 2 and 3. Two of these names are selected. The pupils are shown that multiplication of the numbers expressed by the two numerators and of the numbers expressed by the two denominators results in a product that is just a different name for $3 \times 2$. Several other examples are demonstrated in this way. The pupils see that this method for multiplication of fractions produces a result that is reasonable and consistent.

On page 185 the pupils are provided with exercises (A to N) in which they must select new names for whole numbers and then perform a multiplication. The pupils either can use their knowledge of the number line or they can make the numerical replacements through computation. For example, for Exercise B, they can think of 12 first as $\frac{12}{1}$, which they then can reduce by multiplication to an equivalent pair of numerals with any second term they want. Thus $\frac{12}{1}$ equals $\frac{24}{2}$, $\frac{36}{3}$, etc.

In discussing these exercises, the pupils may verify their answers by performing the multiplication with the whole numbers as well as with the fractions.

This lesson sets the stage for understanding multiplication of mixed numbers and proper fractions, which is presented in the next lesson, by showing the pupils that the product of two pairs of numbers can be found by finding the products of the numbers in corresponding terms (numerators and denominators).

**Answers**

**Thinking straight:**

A 6

B $\frac{4}{2}, \frac{6}{3}, \frac{24}{12}, \frac{6}{2}, \frac{9}{3}, \frac{36}{12}$

C Yes; yes

D 12 and 6; above the line after the equals sign

E 4 and 3; below the line after the equals sign

F $\frac{72}{12}$; yes

G Yes

H Yes

I Yes

J Yes

K 70

L Yes; $\frac{3150}{45}$

M Yes; yes

N Yes; yes, 135

O Allow the pupils to find the product by using any of the names shown on the number line for Points 7 and 8.

(block 2)

In Exercises A to N, innumerable fractions can be multiplied.

**Keeping skillful:**

(block 1)

A 4

B $6\frac{2}{5}$

C 6

D $2\frac{5}{8}$

E 8

F 8

G $5\frac{5}{6}$

H $4\frac{7}{10}$

I 2

J $7\frac{1}{6}$

K $5\frac{4}{5}$

L 11

(block 2)

A $\frac{79}{12}$

B $\frac{19}{5}$

C $\frac{289}{32}$

D $\frac{125}{6}$

E $\frac{169}{10}$

F $\frac{359}{8}$

G $\frac{73}{16}$

H $\frac{464}{15}$

I $\frac{56}{3}$

J $\frac{103}{2}$

K $\frac{1211}{12}$

L $\frac{643}{10}$

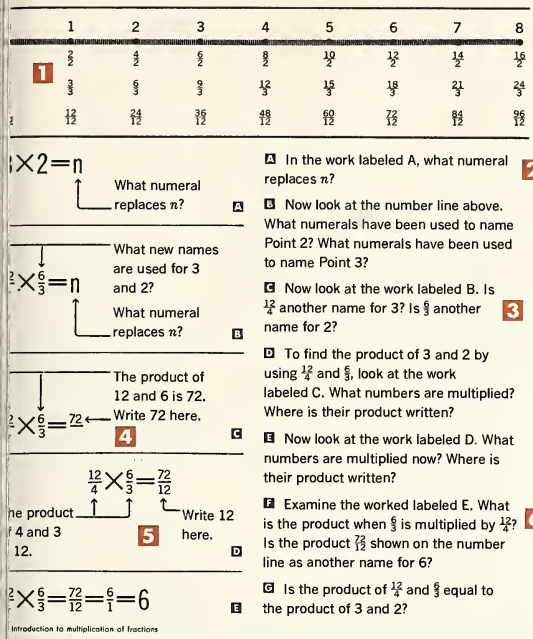
M $\frac{35}{4}$

N $\frac{126}{5}$

O $\frac{75}{8}$

P $\frac{67}{6}$

## Thinking straight




A In the work labeled A, what numeral replaces $n$?

B Now look at the number line above. What numerals have been used to name Point 2? What numerals have been used to name Point 3?

C Now look at the work labeled B. Is $\frac{12}{4}$ another name for 3? Is $\frac{6}{3}$ another name for 2? 3

D To find the product of 3 and 2 by using $\frac{12}{4}$ and $\frac{6}{3}$, look at the work labeled C. What numbers are multiplied? Where is their product written?

E Now look at the work labeled D. What numbers are multiplied now? Where is their product written?

F Examine the worked labeled E. What is the product when $\frac{6}{3}$ is multiplied by $\frac{12}{4}$? 6 Is the product $\frac{72}{12}$ shown on the number line as another name for 6?

G Is the product of $\frac{12}{4}$ and $\frac{6}{3}$ equal to the product of 3 and 2?

H Can $\frac{15}{5}$ and $\frac{16}{8}$ be used as other names for 3 and 2?

$\frac{15}{5} \times \frac{16}{8} = d$ 1

Multiply 16 by 15.

$\frac{15}{5} \times \frac{16}{8} = \frac{240}{\ } \leftarrow$ Write the product here.

$\frac{15}{5} \times \frac{16}{8} = \frac{240}{40}$

Multiply 8 by 5. Write the product here.

I Can $\frac{240}{40}$ be used as another name for 6?

J Is the product of $\frac{15}{5}$ and $\frac{16}{8}$ equal to the product of 3 and 2?

K What is the product of 10 and 7?

L Can $\frac{90}{9}$ and $\frac{35}{5}$ be used as other names for 10 and 7? The product of $\frac{90}{9}$ and $\frac{35}{5}$ is $\frac{\square}{\square}$.

M Can the name of the product in Exercise L be used as another name for 70? Is the product of $\frac{90}{9}$ and $\frac{35}{5}$ equal to the product of 10 and 7?

N Are $\frac{15}{1}$ and $\frac{27}{3}$ other names that can be used for 15 and 9? Is the product of $\frac{15}{1}$ and $\frac{27}{3}$ equal to the product of 15 and 9?

O Find the product of 7 and 8. Now look at the number line on page 184. For each of the points 7 and 8, select another name and find the product by using these names. Is the product equal to the product of 7 and 8? 4

In each exercise below, first select new names that can be used for each number. Then find the product.

5

| | | | |
|---|---|---|---|
| A | $4 \times 8 = t$ | H | $9 \times 5 = g$ |
| B | $12 \times 5 = a$ | I | $13 \times 7 = v$ |
| C | $3 \times 7 = r$ | J | $10 \times 8 = b$ |
| D | $15 \times 2 = s$ | K | $12 \times 3 = m$ |
| E | $6 \times 4 = d$ | L | $6 \times 14 = e$ |
| F | $2 \times 9 = c$ | M | $5 \times 15 = z$ |
| G | $6 \times 11 = w$ | N | $9 \times 11 = k$ |

## Keeping skillful 6

Express each improper fraction below as a mixed number or a whole number.

| | | | | | |
|---|---|---|---|---|---|
| A $\frac{40}{10}$ | B $\frac{32}{5}$ | C $\frac{18}{3}$ | D $\frac{21}{8}$ | E $\frac{16}{2}$ | F $\frac{64}{8}$ |
| G $\frac{70}{12}$ | H $\frac{94}{20}$ | I $\frac{100}{50}$ | J $\frac{86}{12}$ | K $\frac{116}{20}$ | L $\frac{99}{9}$ |

Express each mixed number below as an improper fraction.

| | | | | |
|---|---|---|---|---|
| A $6\frac{7}{12}$ | E $16\frac{9}{10}$ | I $18\frac{2}{3}$ | M $8\frac{3}{4}$ |
| B $3\frac{4}{5}$ | F $44\frac{7}{8}$ | J $51\frac{1}{2}$ | N $25\frac{1}{5}$ |
| C $9\frac{1}{32}$ | G $4\frac{9}{16}$ | K $100\frac{11}{12}$ | O $9\frac{3}{8}$ |
| D $20\frac{5}{6}$ | H $30\frac{14}{15}$ | L $64\frac{3}{10}$ | P $11\frac{1}{6}$ |



# 184

1 Have the pupils study the number line. Discuss Sets B, C, and D.

2 When discussing Exercises A and B, point out that the work labeled A presents a basic fact. Refer to the number line to bring out that other numerals can be used as names for 2 and 3.

3 In Exercise C, if pupils have any difficulty in deciding that $\frac{12}{4}$ is another name for 3, refer them to the number lines on pages 165-166. Be sure they understand that the pairs of numerals shown in the work labeled B are new names for 2 and 3.

4 Study the work labeled C. Help the pupils see that the numbers expressed by the numerators have been multiplied. Discuss where the product is written.

5 Discuss the multiplication of the numbers expressed by the denominators.

6 Use Exercises F and G to bring out that the product of $\frac{12}{4}$ and $\frac{6}{3}$ is 6.

# 185

1 In discussing Exercise H, you may want to have the pupils construct a number line showing sets with 5 and 8 written in the denominators, to show that $\frac{15}{5}$ and $\frac{16}{8}$ are other names for 3 and 2. Discuss the multiplication of the numbers expressed by the numerators and the denominators. Have the pupils observe where the products are written.

2 Point out that $\frac{240}{40}$ is an improper fraction numeral and discuss how it can be reduced to 6. Point out that the answer is the same as for $3 \times 2$.

3 Treat Exercises K to N in the same way.

4 Have the pupils use the number line on page 184 to answer this question.

5 Assign Exercises A to N as written work.

6 Assign the "Keeping skillful" exercises as a separate lesson.

# 186-191 Learning how

Expanded Notes for this lesson are on pages 349-351.

**Objectives**

The pupil learns how to multiply numbers expressed by mixed numerals and proper fraction numerals.

**Vocabulary**

New words page 191 tied*

**Comments**

In the preceding lesson the pupils were introduced to the technique of using improper fraction numerals to express whole numbers and multiplying the numbers in the numerators and the numbers in the denominators to find the product. This was demonstrated by showing that the product of $\frac{6}{2}$ and $\frac{4}{1}$, for example, corresponds to the product of 3 and 4. Thus, $\frac{6}{2}\times\frac{4}{1}=\frac{6\times4}{2\times1}=\frac{24}{2}=\frac{12}{1}$; $\frac{12}{1}=12$. Earlier, the pupils learned that ratios can have mixed numerals and fraction numerals in their terms. The present lesson uses equations of ratios with such terms to show the *need* for multiplication of fractions and mixed numbers. And the present lesson uses the technique of multiplying the numbers in numerators and denominators in teaching the multiplication of fractions and mixed numbers.

Note that the pictures in this lesson are used to verify the computation, rather than to explain the process.

In this lesson the pupils are given examples using various possible combinations of mixed numbers and proper fractions. The pupil learns to multiply any combination of mixed numbers, proper fractions, and whole numbers in one lesson.

"Cancellation" is not introduced in this lesson. The removal of a common factor before multiplying is a short cut. Since the authors believe that most short cuts, including "cancellation," frequently hinder the pupil when he is first developing a new understanding, the short cut will be introduced later. (See page 204 of the pupils' book.)

**Answers**

Page 191:

(block 1)

A $3\frac{7}{16}$ C $7\frac{29}{48}$ E $23\frac{1}{3}$

B 12 D $\frac{28}{45}$ F $1\frac{1}{8}$

G $\frac{11}{40}$ M $39\frac{1}{3}$ S 84

H $14\frac{9}{20}$ N 45 T $\frac{39}{64}$

I $3\frac{7}{15}$ O $12\frac{7}{48}$ U 56

J $3\frac{15}{16}$ P $10\frac{5}{12}$ V $21\frac{13}{24}$

K $48\frac{3}{5}$ Q $\frac{3}{8}$ W $6\frac{21}{40}$

L 16 R $57\frac{1}{27}$

(block 2)

A $1\frac{1}{4}$ D $5\frac{5}{6}$ G $1\frac{4}{45}$

B $3\frac{53}{96}$ E $14\frac{11}{15}$

C $\frac{5}{8}$ F $53\frac{1}{8}$

Keeping skillful:

(block 1)

A 7.071 E .256 I .8804

B .54445 F 507.9 J 7.0440

C 65.1 G 2.6 K 129.610

D 7.61 H .4786

(block 2)

A 16/64=[25]/100; 25%

B 25/100=[15]/60

C 4/100=44/[1100]

D 75/100=[225]/300

E 120/40=[300]/100; 300%

F 200/100=650/[325]

G 144/100=[144]/100; 144%

H 80/100=[40]/50

I 10/100=85/[850]

J 112/140=[80]/100; 80%

K 37/100=[148]/400

(block 3)

A $193\frac{19}{20}$ E $180\frac{7}{27}$ I 314

B 12,285 F 86 J $116\frac{23}{34}$

C 4 G 211,896 K 16

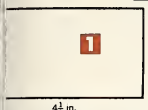
D 21 H $68\frac{104}{165}$

## Learning how

Now you will learn how to find the product when you use mixed numbers and proper fractions.

### see

A




What are the dimensions of the piece of paper in Diagram A? Find its area in square inches.

B

Diagram B shows why you can use the equation below to find the area.

$$\frac{4\frac{1}{3}}{1} = \frac{w}{2\frac{1}{2}}$$

Number of square inches in 1 row

Number of square inches in $2\frac{1}{2}$ rows

When you find the numeral that replaces $w$, you will know the area of the piece of paper.

3

...en you use the ratio test, you get ...s equation.

$$=4\frac{1}{3}\times 2\frac{1}{2}$$

...solve this equation, you must find ... product of $4\frac{1}{3}$ and $2\frac{1}{2}$. To multiply ...se mixed numbers, first find other ...mes for them. Express them with ...ction numerals.

$$=\frac{13}{3}\times\frac{5}{2}$$

$\frac{13}{3}$ is another name for $4\frac{1}{3}$.

$\frac{5}{2}$ is another name for $2\frac{1}{2}$.

Now multiply the numerators.

Multiply 5 by 13.

$$\frac{13}{3}\times\frac{5}{2}=\frac{65}{\ }$$

Write the product here.

Next multiply the denominators.

4

$$\frac{13}{3}\times\frac{5}{2}=\frac{65}{6}$$

Multiply 2 by 3.

Write the product here.

Express $\frac{65}{6}$ as a mixed number.

$$\frac{65}{6}=10\frac{5}{6}$$

$10\frac{5}{6}$ is another name for $\frac{65}{6}$.

5



C

The work you have just done shows that the area is $10\frac{5}{6}$ sq. in. Diagram C also shows that the area is $10\frac{5}{6}$ sq. in.

1

Each square inch is how many one sixths of a square inch?

Each $\frac{1}{2}$ square inch is how many one sixths of a square inch?

Each $\frac{1}{3}$ square inch is how many one sixths of a square inch?

Now use Diagram C to find how many one sixths of a square inch there are in the area. You can either count or add. Do not forget the $\frac{1}{6}$ sq. in. in the upper right corner.

Is the total area $\frac{65}{6}$ sq. in.?

How does the diagram show that the area can also be expressed as $10\frac{5}{6}$ sq. in.? The area of the paper is $10\frac{5}{6}$ sq. in.

3

$4\frac{1}{3}\times 2\frac{1}{2}=10\frac{5}{6}$

### see

4

A

Mr. Jackson's farm is a rectangle $\frac{3}{4}$ mi. long and $\frac{2}{3}$ mi. wide. Find its area in square miles.

You can use the equation below to find the area.

$$\frac{\frac{3}{4}}{1}=\frac{t}{\frac{2}{3}}$$

4

When you find the numeral that replaces $t$, you will know the area.

Now turn the page.



# 186

1 Discuss the fact that the dimensions of the rectangle in Diagram A are represented by mixed numerals.

2 Be sure the pupils see that the green rectangles in Diagram B represent the $4\frac{1}{3}$ square inches in 1 row. The class should also understand that the length of the altitude, $2\frac{1}{2}$ inches, indicates that there will be $2\frac{1}{2}$ rows of 1-inch squares. Ask someone to relate the equation of ratios to Diagram B.

3 Call on someone to explain how the ratio test produces the new equation. Ask another pupil to explain how to replace the mixed numerals with improper fraction numerals.

4 Discuss the multiplication of the numbers in the numerators and denominators. Remind the pupils that they learned how to do this in the preceding lesson.

5 Let someone explain how $10\frac{5}{6}$ is obtained from $\frac{65}{6}$.

# 187

1 The pupils should understand that $10\frac{5}{6}$ replaces $w$ in the original equation on page 186.

2 Explain that Diagram C can be used to verify the result of the multiplication. Point out that the square inch shown below and at the left shows the number of sixths in 1 square inch. Have pupils point out the diagrams that show the number of sixths in $\frac{1}{2}$ square inch and in $\frac{1}{3}$ square inch. Let someone explain where the $\frac{1}{6}$ in the upper right corner comes from. Then have the pupils count or add to find the total number of sixths.

3 Help the pupils count the parts of the rectangle again in terms of the number of whole and fractional square inches, so they can see that there are $10\frac{5}{6}$ square inches.

4 Have the new problem read. Point out that the measures of both dimensions are represented by proper fraction numerals. Then discuss the terms of the ratio equation.

When you use the ratio test, you get this equation.

$t = \frac{3}{4} \times \frac{2}{3}$ 1

Multiply 2 by 3. The product is ■.

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{}$ ← Write 6 here.

2 To solve the equation, you must find the product of $\frac{3}{4}$ and $\frac{2}{3}$.

Look at the work in the next column. Since the numbers are already expressed with fraction numerals, you do not need to use new names for them.

Multiply 3 by 4. The product is ■.

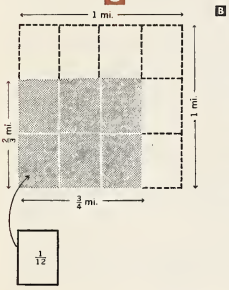
$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Write 12 here.

$\frac{6}{12} = \frac{1}{2}$ 2

$\frac{1}{2}$ is another name for $\frac{6}{12}$.

3 ← Now look at Diagram B. The dotted lines help show the whole square mile of which Mr. Jackson's farm is a part.




One side of the square is marked off in fourths because the farm is $\frac{3}{4}$ mi. long. The other side is marked off in thirds because the farm is $\frac{1}{3}$ mi. wide. Now the whole square is marked off in twelfths.

You can see from the diagram that the area of Mr. Jackson's farm is $\frac{6}{12}$ sq. mi.

How does Diagram B show that you can also think of the area of the farm as $\frac{1}{2}$ sq. mi.? 4

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$, or $\frac{1}{2}$

The area of Mr. Jackson's farm is $\frac{1}{2}$ sq. mi.



think Jane said, "My story is $1\frac{1}{4}$ pages long." Ann said, "My story is 2 times as long as yours, Jane." Ann's story was how many pages long?

1 Is the equation below correct?

$\frac{2}{1} = \frac{d}{1\frac{1}{4}}$

How do you get the equation below?

$d = 2 \times 1\frac{1}{4}$ 1

Before you multiply $1\frac{1}{4}$ by 2, express $1\frac{1}{4}$ and 2 with fraction numerals.

Are $\frac{?}{1}$ and $\frac{5}{4}$ correct names for 2 and $1\frac{1}{4}$? 2

$\frac{2}{1} \times \frac{5}{4} = \frac{10}{4}$

How do you find the 10? The 4?

Now express $\frac{10}{4}$ as a mixed number.

$\frac{10}{4} = 2\frac{?}{2}$

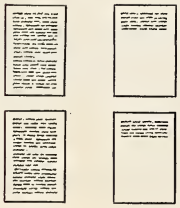
Ann's story was how many pages long?

3

You can also use the diagram to find how many pages Ann used for her story.

For the 1 page that Jane used, Ann used ■ pages.

For the $\frac{1}{4}$ page that Jane used, Ann used ■ fourths of a page.

4 

Count or add Ann's pages by fourths. Do you get $\frac{10}{4}$?

Can you use the diagram to show that Ann used $2\frac{1}{2}$ pages?

$2 \times 1\frac{1}{4} = ■\frac{1}{2}$





# 188

1 Let a pupil explain how this equation was obtained from the ratio equation on page 187.

2 Be sure the pupils understand the computation. Discuss the reduction of $\frac{6}{12}$ to $\frac{1}{2}$.

3 Make sure the pupils understand that the shaded portion of Diagram B shows the size of Mr. Jackson's farm and that it is less than 1 square mile. Have the pupils point out the sides of the square mile that are marked off in fourths and in thirds. Have the pupils count the parts to see that there are 12 of them in the square mile.

4 After the pupils have seen that the farm is $\frac{6}{12}$ sq. mi. in area, they can compare the shaded portion of the diagram with the unshaded portion to see that the farm is also $\frac{1}{2}$ sq. mi. in area.

# 189

1 Have pupils read the new problem and then discuss the terms of the equation of ratios. Let a pupil explain how the second equation is obtained.

2 Call on a pupil to explain how the $\frac{2}{1}$ and the $\frac{5}{4}$ are obtained. Discuss the multiplication of the numbers in the numerators and the denominators. Then discuss how to replace $\frac{10}{4}$ by a mixed numeral. Remind the pupils that the mixed numeral $2\frac{1}{2}$ replaces $d$ in the original equation.

3 Explain to the pupils that Jane's story is on the shaded pages. Have someone explain the relationship between Jane's pages and Ann's pages. When the pupils count or add Ann's pages by fourths, remind them that 1 page is 4 fourths.

4 Help the pupils see that Ann's pages can also be counted as 2 whole pages with 2 fourths, or $\frac{1}{2}$, additional page.

## think

A pipe that is $\frac{7}{8}$ ft. long weighs 1 lb. A longer pipe of the same kind weighs $2\frac{1}{2}$ lb. How long is this pipe?

Is the equation below correct?

$$\frac{\frac{7}{8}}{1}=\frac{n}{2\frac{1}{2}}$$

How is the equation below found?

$$n=\frac{7}{8}\times 2\frac{1}{2}$$

**1**

Before you multiply, for which number must you have a new name?

$$\frac{7}{8}\times\frac{5}{2}=\frac{35}{16}$$

← How is 35 found?
← How is 16 found?

Can $\frac{35}{16}$ be expressed as $2\frac{3}{16}$?

The pipe is how many feet long?

## try

A Sue paid 28¢ for 1 lb. of plums. Jane bought $1\frac{1}{4}$ lb. at the same rate. How much did her plums cost?

$$\frac{28}{1}=\frac{a}{1\frac{1}{4}}$$

$$\frac{28}{1}=\frac{a}{1\frac{1}{4}}$$

$$a=28\times 1\frac{1}{4}$$

$$\frac{28}{1}\times\frac{5}{4}=\frac{140}{4}=35$$

**2**

B Mr. Stone has a truck garden. One day he set out $\frac{1}{4}$ row of tomato plants in 1 hr. At this rate, what part of a row had he planted in $\frac{1}{2}$ hr.?

$$\frac{\frac{1}{4}}{1}=\frac{s}{\frac{1}{2}}$$

$$\frac{1}{4}\times\frac{1}{2}=s$$

← What do you multiply to get 1?

$$\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

C $8\frac{1}{10}\times 1\frac{2}{3}=d$

$$\frac{81}{10}\times\frac{5}{3}=\frac{405}{30}=13\frac{15}{30}\text{ or }13\frac{1}{2}$$

D $w=\frac{5}{16}\times 14$

← Why do you use $\frac{14}{1}$?

$$\frac{5}{16}\times\frac{14}{1}=\frac{70}{16}=4\frac{6}{16}\text{ or }4\frac{3}{8}$$

E $\frac{1}{5}\times 11\frac{1}{3}=r$

$$\frac{1}{5}\times\frac{34}{3}=\frac{34}{15}=2\frac{4}{15}$$

F $b=3\frac{3}{8}\times 4\frac{1}{6}$

$$\frac{27}{8}\times\frac{25}{6}=\frac{675}{48}=14\frac{3}{48}\text{ or }14\frac{1}{16}$$

0

## do

A Mrs. Johnson bought a $2\frac{3}{4}$-lb. roast. Mrs. Price bought a roast that weighed $1\frac{1}{2}$ times as much as Mrs. Johnson's. Mrs. Price's roast weighed how many pounds?

$$\frac{1\frac{1}{2}}{1}=\frac{k}{2\frac{3}{4}}$$

$$1\tfrac{1}{2}\times 2\tfrac{3}{4}=k$$

B Kathy wrapped and tied 16 small packages that were all the same size. She used $\frac{3}{4}$ yd. of ribbon for each package. How many yards of ribbon did she use in all?

$$\frac{\frac{3}{4}}{1}=\frac{v}{16}$$

$$v=\tfrac{3}{4}\times 16$$

**1**

C $9\frac{1}{8}\times\frac{5}{8}=d$

D $m=\frac{14}{15}\times\frac{2}{3}$

E $7\times 3\frac{1}{3}=a$

F $k=9\times\frac{1}{8}$

G $\frac{11}{24}\times\frac{3}{8}=w$

H $8\frac{1}{2}\times 1\frac{7}{10}=t$

I $s=5\frac{1}{3}\times\frac{2}{3}$

J $n=1\frac{5}{8}\times 4\frac{1}{5}$

K $2\frac{7}{10}\times 18=b$

L $c=32\times\frac{1}{2}$

M $r=6\frac{2}{3}\times 5\frac{9}{10}$

N $40\times 1\frac{1}{8}=d$

O $h=13\frac{1}{4}\times\frac{11}{12}$

P $\frac{5}{6}\times 12\frac{1}{2}=t$

Q $\frac{15}{32}\times\frac{4}{5}=a$

R $w=8\frac{5}{8}\times 6\frac{2}{3}$

S $10\times 8\frac{5}{8}=m$

T $b=\frac{13}{16}\times\frac{3}{4}$

U $1\frac{13}{15}\times 30=r$

V $n=7\frac{5}{6}\times 2\frac{3}{4}$

W $k=7\frac{1}{4}\times\frac{8}{10}$

**2**

A $d=5\times\frac{1}{4}$

B $\frac{11}{12}\times 3\frac{6}{8}=t$

C $v=1\frac{5}{16}\times\frac{8}{3}$

D $10\times\frac{7}{12}=r$

E $m=8\frac{1}{2}\times 1\frac{11}{15}$

F $14\frac{1}{8}\times 3\frac{3}{4}=s$

G $c=2\frac{1}{3}\times\frac{7}{15}$

Now you know how to find the products of mixed numbers and of proper fractions.

## Keeping skillful

**3**

A $11.003+b=18.074$
B $.67314-.12869=d$
C $22.7+30.5+11.9=r$
D $5.06+.84+1.22+.49=w$
E $92.001-c=91.745$
F $1387.5-879.6=a$
G $.4+.9+.3+.8+.2=n$
H $1.3725-.8939=b$
I $m+.7498=1.6302$
J $14.0002-6.9562=h$
K $z-52.618=76.992$

A 16 is ■% of 64.
B 25% of 60 is ■.
C 44 is 4% of ■.
D 75% of 300 is ■.
E 120 is ■% of 40.
F 650 is 200% of ■.
G 144 is ■% of 100.
H 80% of 50 is ■.
I 85 is 10% of ■.
J 112 is ■% of 140.
K 37% of 400 is ■.

A $3879\div d=20$
B $65\times 189=c$
C $1740\div m=435$
D $319\times r=6699$
E $4867\div 27=s$
F $k\times 945=81270$
G $648\times 327=t$
H $11324\div 165=w$
I $d\times 472=148208$
J $7934\div 68=y$
K $230\times a=3680$



# 190

1 Discuss the ratio equation, and the equation that results from the use of the ratio test. Point out that one of the numbers is already expressed as a fraction numeral and that the other must be similarly expressed. Discuss the multiplication process for fractions and how the improper fraction numeral for the answer is replaced by a mixed numeral.

2 Assign the problems and examples in the TRY step as written work. Tell the pupils to imagine that Examples C to F came from problem situations and that the numerals in them were written first in ratio equations. Point out that in Examples B, C, and E the letter holding the place for the answer is on the right side of the equals sign. The pupils should know that these equations are solved in the same way as those with the letter on the left.

# 191

1 Assign Problems A and B and Exercises C to W as written work. Discuss them afterward.

2 Assign Exercises A to G as extra work for those who need more practice.

3 Assign these three blocks of exercises as time permits. Tell the pupils to write equations and then use the ratio test in working the per cent exercises in Block 2. In Block 3, note Exercises D, F, I and K. Help the pupils see that these equations are solved by division.

# 192-193 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil solves problems that require multiplication of proper fractions and mixed numbers.

**Vocabulary**

New words page 192 result*; page 193 Beth, winter*

**Comments**

In this lesson the pupils encounter problem situations that require multiplication of fractions and mixed numbers. For some of the problems, ratio equations have been given. For the others, the pupils are to write ratio equations and then rewrite them, using the ratio test. If any of the pupils feel proficient enough and understand the problem situation, they may immediately write the equation they would get from the ratio test. Encourage this procedure *only* if you are sure the pupils understand how to get this equation without first writing an equation of ratios.

After the pupils have worked the problems, supply equations and answers and let them verify their work. Discuss any problems that have caused difficulty.

The "Keeping skillful" exercises on page 193 give the pupils practice in multiplying, adding, and subtracting fractions and mixed numbers. These should be assigned as a separate lesson as time permits.

**Answers**

**Using arithmetic:**

A $3/1 = [15]/5$ or $5 \times 3 = [15]$

B $2\frac{3}{4}/1 = [13\frac{3}{4}]/5$ or $[13\frac{3}{4}] = 2\frac{3}{4} \times 5$

C $\frac{1}{2}/1 = [8\frac{1}{8}]/16\frac{1}{4}$ or $[8\frac{1}{8}] = \frac{1}{2} \times 16\frac{1}{4}$

D $11\frac{1}{2}/1 = [71\frac{7}{8}]/6\frac{1}{4}$ or $11\frac{1}{2} \times 6\frac{1}{4} = [71\frac{7}{8}]$

E $1\frac{1}{2}/1 = [3]/2$ or $[3] = 1\frac{1}{2} \times 2$

F $2\frac{1}{2}/1 = [50]/20$ or $[50] = 2\frac{1}{2} \times 20$

G $\frac{3}{4}/1 = [13\frac{1}{2}]/18$ or $[13\frac{1}{2}] = 18 \times \frac{3}{4}$

H $\frac{2}{3}/1 = [78]/117$ or $[78] = \frac{2}{3} \times 117$

I $\frac{3}{4}/1 = [3]/4$ or $[3] = \frac{3}{4} \times 4$

J $\frac{1}{2}/1 = [\frac{1}{8}]/\frac{1}{4}$ or $[\frac{1}{8}] = \frac{1}{2} \times \frac{1}{4}$

K $\frac{1}{2}/1 = [17]/34$ or $[17] = \frac{1}{2} \times 34$

L $2\frac{1}{2}/1 = [8\frac{3}{4}]/3\frac{1}{2}$ or $[8\frac{3}{4}] = 2\frac{1}{2} \times 3\frac{1}{2}$

M $\frac{1}{2}/1 = [\frac{3}{8}]/\frac{3}{4}$ or $[\frac{3}{8}] = \frac{1}{2} \times \frac{3}{4}$

N $\frac{5}{8}/1 = [6\frac{2}{3}]/10\frac{2}{3}$ or $[6\frac{2}{3}] = \frac{5}{8} \times 10\frac{2}{3}$

**Keeping skillful:**

(block 1)

| | | | |
|---|---|---|---|
| A $2\frac{1}{4}$ | E $11\frac{1}{2}$ | I $4\frac{1}{32}$ | M $11\frac{1}{3}$ |
| B $19\frac{1}{2}$ | F $\frac{35}{96}$ | J $\frac{9}{80}$ | N 70 |
| C $\frac{1}{5}$ | G $36\frac{2}{3}$ | K 1 | O $\frac{6}{25}$ |
| D 22 | H $\frac{16}{27}$ | L $5\frac{37}{40}$ | P $1\frac{5}{12}$ |

(block 2)

| | | | |
|---|---|---|---|
| A $3\frac{1}{6}$ | E $37\frac{1}{3}$ | I $3\frac{1}{5}$ | M $\frac{1}{6}$ |
| B $1\frac{13}{16}$ | F $8\frac{5}{12}$ | J $4\frac{8}{15}$ | N $5\frac{3}{20}$ |
| C $\frac{19}{20}$ | G $5\frac{19}{30}$ | K $1\frac{31}{32}$ | O $10\frac{8}{15}$ |
| D $2\frac{1}{6}$ | H $5\frac{1}{15}$ | L $3\frac{5}{16}$ | P $8\frac{5}{8}$ |

## Using arithmetic

A Pete rides his bicycle to school. He rides 3 mi. a day on each round trip to and from school. How many miles does he ride in 5 days?

You can use an equation of ratios to solve Problem A.

$\frac{3}{1}=\frac{k}{5}$

Does the equation below also show how to solve Problem A?

$5\times3=k$

When you use the ratio test, can you also write your equation as $k=3\times5$ or as $3\times5=k$? Does the same numeral replace $k$ in all these equations? What numeral replaces $k$?

B Jim also rides his bicycle to school. He rides $2\frac{3}{4}$ mi. a day on each round trip to and from school. How many miles does he ride in 5 days?

Does the equation below show how to solve Problem B?

$\frac{2\frac{3}{4}}{1}=\frac{a}{5}$

Does the equation below show the result when you use the ratio test?

$a=2\frac{3}{4}\times5$

What numeral replaces $a$?

C Kathy bought a watermelon that weighed $16\frac{1}{4}$ lb. Ann bought one that weighed $\frac{1}{2}$ as much as Kathy's. How many pounds did Ann's watermelon weigh?

You know that Ann bought $\frac{1}{2}$ lb. of watermelon for each 1 lb. that Kathy bought. Does the equation below show how to solve Problem C? 3

$\frac{\frac{1}{2}}{1}=\frac{w}{16\frac{1}{4}}$

Does the equation below show the result when you use the ratio test?

$W=\frac{1}{2}\times16\frac{1}{4}$

What numeral replaces $w$?

D Mrs. Brown has a flower bed that is $11\frac{1}{2}$ yd. long and $6\frac{1}{4}$ yd. wide. What is the area of the flower bed in square yards? 4

Write an equation of ratios for Problem D. Do you get the equation below by using the ratio test?

$11\frac{1}{2}\times6\frac{1}{4}=d$

What numeral replaces $d$?

E May made 2 lb. of candy. Patty made $1\frac{1}{2}$ times as much candy as May. How much candy did Patty make?

If you are sure that you understand Problem E, write just the equation you would get from using the ratio test. If you are not sure, write an equation of ratios and use the ratio test.



F Ellen weighs $2\frac{1}{2}$ times as much as her little sister Beth. Beth weighs 20 lb. How much does Ellen weigh?

$\frac{1}{2}\times20=n$

G Mr. Cook uses about $\frac{3}{4}$ T. of coal a week during the winter. He burns coal for about 18 weeks. About how much coal does he need for the winter?

$n=18\times\frac{3}{4}$

H Mr. Cook drove 117 mi. one day. Mr. Peters drove $\frac{8}{9}$ as far as Mr. Cook that day. Mr. Peters drove how many miles that day?

I Patty practiced the piano for $\frac{3}{4}$ hr. each day for 4 days. How many hours had she practiced at the end of the fourth day?

J A field is $\frac{1}{2}$ mi. long and $\frac{1}{4}$ mi. wide. The area of the field is what fraction of a square mile?

K There were 34 children at a party. For a game, each child was given $\frac{1}{2}$ of a sheet of paper. How many whole sheets of paper must have been cut in half?

L Bob's home is $3\frac{1}{2}$ blocks from the school. Andy's home is $2\frac{1}{2}$ times as far from school as Bob's. Andy's home is how many blocks from the school?

M Jack has two dogs. The larger one is fed a total of $\frac{3}{4}$ lb. of fresh meat each day. The smaller one is fed $\frac{1}{2}$ as much fresh meat as the larger one. How much fresh meat is the smaller dog fed each day?

N Mrs. Jackson used $10\frac{2}{3}$ yd. of cloth for bedroom curtains. She used about $\frac{5}{8}$ as much cloth for a chair cover. About how much cloth did she use for the chair cover? 2

## Keeping skillful 3

J $6\times\frac{3}{8}=m$

I $2\frac{1}{4}\times8\frac{2}{3}=f$

E $\frac{1}{3}\times\frac{3}{8}=a$

J $4\times5\frac{1}{2}=w$

E $3\frac{5}{8}\times3=b$

A $\frac{7}{8}\times\frac{5}{12}=c$

G $5\frac{1}{2}\times6\frac{2}{3}=n$

H $\frac{5}{8}\times1\frac{1}{3}=z$

I $3\frac{7}{12}\times1\frac{1}{8}=v$

J $\frac{1}{5}\times\frac{9}{16}=c$

K $6\times\frac{1}{6}=d$

L $7\frac{9}{10}\times\frac{3}{4}=x$

M $5\times2\frac{15}{16}=g$

N $20\times3\frac{1}{2}=h$

O $\frac{3}{5}\times\frac{2}{5}=t$

P $2\frac{5}{6}\times\frac{1}{2}=k$

A $n+3\frac{1}{2}=6\frac{2}{3}$

B $1\frac{5}{16}+\frac{7}{8}=d$

C $4\frac{3}{10}+s=5\frac{1}{4}$

D $11-y=8\frac{5}{8}$

E $a-13\frac{1}{3}=24$

F $z+6\frac{7}{12}=15$

G $w-4\frac{2}{10}=1\frac{1}{3}$

H $6\frac{2}{3}-1\frac{3}{8}=b$

I $14\frac{1}{8}-k=11$

J $3\frac{5}{6}+\frac{7}{10}=m$

K $r-3\frac{1}{32}=1$

L $h+\frac{11}{16}=4$

M $7\frac{5}{12}-r=7\frac{1}{4}$

N $n-1\frac{9}{10}=3\frac{1}{4}$

O $8\frac{1}{5}+k=19$

P $14-5\frac{3}{8}=c$



# 192

1 Have the pupils read the problem and note the ratio equation. Then have them answer the questions. They should understand that when the ratio test is used, the equation can be written in different ways. Have them find the numeral that replaces $k$.

2 Have the pupils read Problem B and discuss the equations. Ask the pupils what other name they can use for $2\frac{3}{4}$ and for 5. Then have them solve the equation.

3 Be sure the pupils understand what the ratio $\frac{1}{2}/1$ expresses. Have them give another name for $16\frac{1}{4}$ and solve the equation.

4 Assign Problems D to N as written work. (F to N are on page 193.) Point out that different names are needed for both factors in some problems.

# 193

1 If the pupils understand the problem situation, they may immediately write the equation they get from using the ratio test and solve it. If they are not sure, have them write the ratio equation first, then rewrite it using the ratio test, and solve the equation they get from the ratio test.

2 Supply answers and have the pupils verify their work. Discuss any problems that have caused difficulty.

3 The "Keeping skillful" exercises should be assigned as a separate lesson.

# 194 Side trip

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns different uses for numerals and signs.

**Vocabulary**

New words page 194 slant*

**Comments**

In everyday experiences, numerals and signs are used in many different situations.

Some numerals are used for identification only and have no relation to numerousness. Phone numbers and numerals on an athlete's sweater are examples of such numerals. Numerals on houses and streets usually have some relation to the location of the house or street, but in most cases the position indicated is not so exact as it would be in a counting situation.

Pupils also become acquainted with the slant line used in writing ratios and fraction numerals and with the symbols for *feet* (′) and *inches* (″). The symbol (x) for the word *by* in stating dimensions, which was used on page 17, is reviewed here. Pupils may get extra practice with these symbols by measuring and recording the dimensions of objects in the classroom.

**Answers**

A Ninety-eight and six tenths; degrees

B Fifty-four point oh five, five four point oh five, fifty-four and five hundredths

C Eight and one hundred eigthty-three thousandths, eight point one hundred eighty-three, eight point one eight three

D One three oh five two; to make sure of being understood

E Three two four oh; It is used as a label, not as a counting number. Yes

F To make them easier to say, read, and remember

G Eighteen sixty-seven

H Any fractions whose denominators are powers of ten, as $\frac{7}{10}$, 7/10, .7; $\frac{15}{100}$, 15/100, .15; etc.

I Any ratios

J 4′, .7″, .03″, $6\frac{1}{2}$″, $8\frac{3}{16}$″, 15′, 1.1″, $10\frac{1}{2}$′, $\frac{9}{10}$″

K 6′ x 12″ or 6′ x 1′

## Side trip

A How do you read the decimal fraction numeral at the right? How do you read the sign that follows the numeral? 98.6°

B Another way to read the decimal fraction numeral 98.6 is to say, "Ninety-eight point six." Or you also may say, "Nine eight point six." In what three ways might you read 54.05?

C Read the decimal fraction numeral 8.183 in three ways.

D The numeral at the right represents a telephone number. When giving this number, would you say, "Thirteen thousand fifty-two," or would you say, "One three oh five two"? Why? 13052

E The numeral below represents a house number. Would you read this number as "Three thousand two hundred forty" or as "Three two four oh"? Why? Might you also read it as "Thirty-two forty"? 3240

F Why are house and telephone numbers often read differently from other numbers?

G The numeral below represents a year. You can read it as "Sixteen seven." How would you read the date 1867? 1607

H Three ways to write five tenths are shown here. The first two are common fraction numerals. The other is a decimal fraction numeral. You can read all three in the same way. Now write the numeral for each of five fractions in these three ways. $\frac{5}{10}$ 5/10 .5

I You can also use the slant line to write a ratio, as shown at the right. Write five ratios, using the slant line. 25/60

J At the right are the abbreviations for "inch" or "inches" and "foot" or "feet." 3 in. 3″ $7\frac{1}{2}$ ft. $7\frac{1}{2}$′

Now write each dimension below, using these two abbreviations.

| | | |
|---|---|---|
| 4 feet | $6\frac{1}{2}$ inches | 1.1 inches |
| .7 foot | $8\frac{3}{16}$ inches | $10\frac{1}{2}$ feet |
| .03 inch | 15 feet | $\frac{9}{10}$ inch |

K The dimensions of a box are shown below in two ways. You read both of these in the same way.
18 ft. by 3 ft. by $1\frac{1}{2}$ ft.
18′ x 3′ x $1\frac{1}{2}$′

Now write the dimensions of a board that is 6 ft. long and 12 in. wide. Use the sign that is read "by."



# 194

1 Have a pupil read Exercise A and answer the questions. Perhaps some pupil will know that this numeral represents the normal body temperature of human beings.

2 Discuss Exercises B to E. Have some of the pupils give their own phone numbers or house numbers, and have the class observe how these numerals are read.

3 Discussion of Exercise F should bring out that in these situations numerals are used to label and to identify, not to count. Pupils may think of other examples, such as room "numbers", license "numbers", "numbers" assigned to merchandise in catalogues, etc.

4 Have several pupils write the numerals in Exercises H to K on the chalkboard. Point out that for Exercise H each fraction numeral written must have a denominator of 10, 100, 1000, etc., so that when the numerals are written in three different ways, they can be read in the same way.

# 195 Thinking straight; Checking up

Expanded Notes for these lessons are on page 351.

**Objectives**

The pupil is introduced to the concept of the reciprocal of a number. The pupil's skill in multiplying numbers expressed by proper fraction numerals and mixed numerals is tested.

**Vocabulary**

New words page 195 reciprocal

**Comments**

An understanding of the reciprocal of a number is basic to the study of division of fractions, which will be taken up in the following lesson. If the product of two numbers is 1, each number is the reciprocal of the other. Zero has no reciprocal. Thus, $\frac{3}{1}$ and $\frac{1}{3}$ are reciprocals because $\frac{3}{1} \times \frac{1}{3} = 1$. The reciprocal of a fraction can be found by merely interchanging the numerals in the numerator and denominator in the expression of the original fraction.

On page 195 the pupils discover that the product of two fractions like $\frac{2}{5}$ and $\frac{5}{2}$ is 1. The pupils are then told that when this is true the fractions are called reciprocals. The pupils are also shown that two fractions are mutually reciprocal, that is, $\frac{3}{8}$ is the reciprocal of $\frac{8}{3}$, and $\frac{8}{3}$ is the reciprocal of $\frac{3}{8}$. And the pupils learn that the reciprocal of a whole number can be written as 1 over the numeral representing the whole number.

The "Checking up" at the bottom of the page should be used to evaluate the ability of the pupils to multiply numbers represented by proper fraction numerals and mixed numerals. If pupils have difficulty with any of the tests, review pages 186-191.

**Answers**

Thinking straight:

(block 1)

A Yes; yes

B Yes; yes

C Yes; yes

D The product in each exercise is 1. E $\frac{3}{2}$; $\frac{1}{8}$; $\frac{8}{1}$

(block 2)

A $\frac{15}{16} \times \frac{16}{15} = [1]$

B $\frac{7}{1} \times \frac{1}{7} = [1]$

C $\frac{21}{11} \times \frac{11}{21} = [1]$

D $\frac{5}{16} \times \frac{16}{5} = [1]$

E $\frac{9}{10} \times \frac{10}{9} = [1]$

F $\frac{24}{1} \times \frac{1}{24} = [1]$

Checking up:

| Test 1 | Test 2 |
|---|---|
| A $\frac{5}{12}$ | A 3 |
| B $\frac{7}{72}$ | B 5 |
| C $\frac{3}{10}$ | C $5\frac{1}{4}$ |
| D $\frac{10}{21}$ | D $7\frac{1}{5}$ |
| E $\frac{3}{4}$ | E $4\frac{7}{12}$ |
| F $\frac{31}{40}$ | F $3\frac{1}{2}$ |
| G $\frac{3}{8}$ | G $23\frac{3}{4}$ |

| Test 3 | Test 4 |
|---|---|
| A 21 | A 1 |
| B 51 | B $\frac{21}{50}$ |
| C $2\frac{23}{36}$ | C $9\frac{19}{30}$ |
| D $14\frac{17}{20}$ | D 33 |
| E $13\frac{7}{8}$ | E $\frac{9}{20}$ |
| F $4\frac{19}{24}$ | F 24 |
| G 141 | G $18\frac{4}{7}$ |

## Thinking straight

**1**

A $\frac{2}{3}\times\frac{3}{2}$

B $\frac{15}{12}\times\frac{12}{15}$

C $\frac{6}{5}\times\frac{5}{6}$

D $\frac{100}{59}\times\frac{59}{100}$

E $8\times\frac{1}{8}$

F $\frac{1}{6}\times 6$

A Look at Exercise A. Is the denominator of $\frac{2}{3}$ the same as the numerator of $\frac{3}{2}$? Is the numerator of $\frac{2}{3}$ the same as the denominator of $\frac{3}{2}$?

B Examine Exercises A, B, C, and D. In each exercise, is the numerator of one fraction numeral the same as the denominator of the other? Is the denominator of one fraction numeral the same as the numerator of the other?

C Look at Exercises E and F. Can you use $\frac{8}{1}$ as another name for 8? Can you use $\frac{6}{1}$ as another name for 6?

D Find the product in each exercise. **2**

E When the product of two numbers is 1, one number is the reciprocal of the other. $\frac{8}{1}$ is the reciprocal of $\frac{1}{8}$. What is the reciprocal of $\frac{5}{8}$? What is the reciprocal of 8? Of $\frac{1}{8}$? **3**

Write the reciprocal of each number below. Multiply each number by its reciprocal. Is each product 1? **4**

A $\frac{16}{15}$ B 7 C $\frac{11}{11}$ D $\frac{16}{5}$ E $\frac{10}{9}$ F 24

## 5 Checking up

| Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|
| A $\frac{5}{8}\times\frac{2}{3}=r$ | A $4\times\frac{3}{4}=n$ | A $10\frac{1}{2}\times 2=b$ | A $3\frac{1}{3}\times\frac{3}{10}=f$ |
| B $\frac{1}{6}\times\frac{7}{12}=t$ | B $\frac{1}{2}\times 10=g$ | B $5\frac{5}{8}\times 9=w$ | B $\frac{7}{10}\times\frac{2}{5}=t$ |
| C $\frac{3}{8}\times\frac{1}{2}=a$ | C $\frac{7}{16}\times 12=a$ | C $2\frac{2}{3}\times 1\frac{1}{2}=d$ | C $6\frac{4}{5}\times 1\frac{5}{12}=a$ |
| D $\frac{9}{5}\times\frac{5}{9}=b$ | D $8\times\frac{9}{10}=r$ | D $4\frac{1}{8}\times 3\frac{3}{8}=a$ | D $1\frac{1}{12}\times 36=e$ |
| E $\frac{15}{16}\times\frac{8}{5}=w$ | E $\frac{11}{12}\times 5=k$ | E $12\frac{1}{2}\times 1\frac{1}{8}=v$ | E $\frac{9}{16}\times\frac{8}{5}=k$ |
| F $\frac{31}{32}\times\frac{8}{5}=s$ | F $\frac{1}{10}\times 35=w$ | F $1\frac{1}{4}\times 3\frac{5}{8}=m$ | F $21\times 1\frac{1}{7}=z$ |
| G $\frac{21}{40}\times\frac{9}{7}=f$ | G $\frac{13}{52}\times 40=d$ | G $18\times 7\frac{5}{8}=t$ | G $10\frac{5}{8}\times 1\frac{5}{7}=h$ |



# 195

1 Have the pupils observe the fraction numerals in the exercises at the left to see what similarities they can point out. Then have them answer Exercises A, B, and C.

2 Allow enough time for the pupils to work each exercise. Be sure they observe that the answer for each is 1. Then write equations ($\frac{3}{4}\times n=1$, for example) on the chalkboard. For each equation, have the pupils tell what numeral replaces $n$ [$\frac{4}{3}$, in this example].

3 Point out that the 8 in Exercise E could have been written as $\frac{8}{1}$. Have the pupils observe that the reciprocal of a whole number is written as 1 over the numeral representing that number.

4 When the pupils have finished this block of exercises, discuss their work.

5 Have the pupils work independently on these tests. Tell them to write their answers in simplest form. After the pupils have corrected their work, do any reteaching that is necessary.

# 196-197 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns that division of whole numbers, mixed numbers, and fractions may be indicated in different ways.

**Vocabulary**

New words page 196 sweet*; page 197 knitted*, scarf, period*

**Comments**

This lesson prepares the pupils for division of fractions by showing that division (of whole numbers, mixed numbers, and fractions) can be expressed in different ways. The pupils are given division problems for which they are to write expressions that *indicate* how the problems are to be solved. They are not to *solve* the problems because as yet they have not learned all the concepts and computational skills needed for finding the solutions.

To indicate how Problems E to N are solved, some pupils may wish first to write an equation of ratios, then write the equation that results from the ratio test. Next, the pupils are to show two ways to write the division that is needed to solve the equation.

**Answers**

A $2/1=24/[12]$ B $1\frac{3}{4}/1=21/[12]$

C $\frac{1}{2}/1=1\frac{1}{2}/[3]$ D $3.5/1=14/[4]$

E $\frac{1}{8}/1 = \frac{7}{8}/a;\ \frac{7}{8}\div\frac{1}{8};\ \frac{\frac{7}{8}}{\frac{1}{8}}$

F $\frac{1}{2}/1 = 14/x;\ 14\div\frac{1}{2};\ \frac{14}{\frac{1}{2}}$

G $\frac{3}{4}/1 = 30/t;\ 30\div\frac{3}{4};\ \frac{30}{\frac{3}{4}}$

H $129/1 = 387/r;\ 387\div 129;\ \frac{387}{129}$

I $8.5/13 = w/1;\ 8.5\div 13;\ \frac{8.5}{13}$

J $10540/42.5 = m/1;\ 10540\div 42.5;\ \frac{10540}{42.5}$

K $\frac{3}{4}/1 = 5\frac{1}{4}/d$; $5\frac{1}{4} \div \frac{3}{4}$; $\frac{5\frac{1}{4}}{\frac{3}{4}}$

L $4\frac{1}{2}/1 = 180/b$; $180 \div 4\frac{1}{2}$; $\frac{180}{4\frac{1}{2}}$

M $60.5/1 = 484/r$; $484 \div 60.5$; $\frac{484}{60.5}$

N $\frac{1}{3}/1 = 4/n$; $4 \div \frac{1}{3}$; $\frac{4}{\frac{1}{3}}$

(block 2)

A $\frac{1.5}{5}$

B $\frac{\frac{15}{16}}{\frac{3}{4}}$

C $12\frac{1}{2} \div 1\frac{1}{3}$

D $\frac{20}{6\frac{7}{10}}$

E $17\frac{1}{8} \div 5$

F $\frac{\frac{1}{2}}{\frac{1}{4}}$

G $\frac{1.7}{1.6}$

H $18 \div \frac{1}{2}$

I $\frac{\frac{19}{20}}{\frac{4}{5}}$

J $21.6 \div 4.3$

**Keeping skillful:**

A 45/15=[300]/100; 300%
B 2/100=[26]/1300
C 100/100=59/[59]
D 17/100 = [68]/400
E 50/100=23/[46]
F 81/2700=[3]/100; 3%
G 42/35=[120]/100; 120%
H 14/100 = [140]/1000
I 30/100=72/[240]
J 28/350=[8]/100; 8%
K 1/100=[49]/4900
L 155/100 = [310]/200
M 486/648=[75]/100; 75%
N 5/100=60/[1200]
O 23/100=161/[700]

## Thinking straight

Mrs. Long used 24 cups of grapes
o make grape juice. She used 2 cups
f grapes for each quart of juice. How
nany quarts of juice did she make?

lake an equation of ratios for
roblem A.

Vhen you use the ratio test, do you get
ne equation shown below?
d = 24

o find the numeral that
eplaces *d*, you divide 24
y ■. The expressions at
ne right show two ways
o write the division.

$24 \div 2$

$\frac{24}{2}$

Mrs. Parks used 21 cups of grapes
o make grape juice. She used $1\frac{3}{4}$ cups
f grapes for each quart of juice. How
nany quarts of juice did she make?

Make an equation of ratios for
Problem B.

low do you get the equation below?
$\frac{3}{4}w = 21$

o find the numeral that replaces *w*,
ou divide ■ by $1\frac{3}{4}$.
The expression at the
ight shows one way to
rite the division.

$21 \div 1\frac{3}{4}$

Another way to write the
livision is also shown at
he right.

$\frac{21}{1\frac{3}{4}}$

Ways to indicate division involving whole numbers, mixed numbers, and proper fractions

**C** Mr. Cook had $1\frac{1}{2}$ lb. of sweet corn seed. He planted $\frac{1}{2}$ lb. of the seed every other week until he had used all of it. How many plantings had Mr. Cook made? **4**

Make an equation of ratios for Problem C.

How is the equation below found?
$\frac{1}{2}x = 1\frac{1}{2}$

To find the numeral that replaces *x*, divide $1\frac{1}{2}$ by ■. You may write the division in either of the two ways shown below.

$1\frac{1}{2} \div \frac{1}{2}$ $\quad \frac{1\frac{1}{2}}{\frac{1}{2}}$

**D** Mr. Henry sawed a 14-foot board into 3.5-foot lengths. How many of the shorter boards did he have?

Make an equation of ratios for Problem D.

How is the equation below found?
$3.5a = 14$

To find the numeral that replaces *a*, you divide ■ by 3.5. You may write the division in either of these two ways.

$14 \div 3.5$ $\quad \frac{14}{3.5}$

For each problem that follows, write two expressions to show that you

# 196

1 Have Problem A read. Then have the pupils make an equation of ratios for the problem. Be sure they can explain what each term represents. Then have them study the equation resulting from the ratio test and observe that division is needed to solve it.

2 Have the pupils study the two ways of expressing division. Be sure they observe the position of the dividend and the divisor in each expression.

3 Have the pupils make the ratio equation and explain what each term represents, then study the equation that results from the ratio test. Point out that even though the divisor represents a mixed number, division can be expressed in the two ways shown for Exercise A.

4 Follow the procedure outlined above for Problems C and D. Have the pupils notice that both the dividend and the divisor involve fractions in Problem C and that Problem D involves a decimal fraction numeral.

divide to solve the problem. You do not need to compute. You will learn how to do the computation later.

**1** E A gardener had $\frac{7}{8}$ lb. of prize seed. He put this seed into packages with $\frac{1}{8}$ lb. of seed in each package. How many packages did he make?

F 14 pt. of cream was put into $\frac{1}{2}$-pt. bottles. How many bottles were used?

G When Patty knitted a scarf, she found that she could knit about $\frac{3}{4}$ inch in 1 hr. About how many hours did it take her to knit a scarf 30 in. long?

H Mr. Peterson spent $3.87 for wallpaper that was priced at $1.29 per roll. How many rolls of this wallpaper did Mr. Peterson buy?

I A piece of metal 8.5 in. wide was cut into 13 strips of equal width. How wide was each strip?

J A man earned $105.40 for 42.5 hr. of work. How much was he paid per hour?

K Patty spends $5\frac{1}{4}$ hr. in school each day. Each class period is $\frac{3}{4}$ hr. long. There is time for how many classes in each school day?

L 180 fl. oz. of liquid was put into bottles that held $4\frac{1}{2}$ fl. oz. each. How many bottles were used?

M A piece of land 484 ft. wide was marked off into lots. Each lot was 60.5 ft. wide. How many lots were in the piece of land?

N Mrs. Peterson has a 4-lb. roast. If she allows $\frac{1}{3}$ lb. of meat per serving, how many servings will she get? **2**

For each exercise below, write the division in another way. **3**

A $1.5 \div 5$

B $1\frac{5}{16} \div \frac{3}{4}$

C $\frac{12\frac{1}{2}}{1\frac{1}{3}}$

D $20 \div 6\frac{7}{10}$

E $\frac{17\frac{1}{8}}{5}$

F $\frac{1}{2} \div \frac{1}{4}$

G $1.7 \div 1.6$

H $\frac{18}{\frac{1}{2}}$

I $\frac{18}{20} \div \frac{4}{5}$

J $\frac{21.6}{4.3}$

## Keeping skillful **4**

A 45 is ■% of 15.
B 2% of 1300 is ■.
C 59 is 100% of ■.
D 17% of 400 is ■.
E 23 is 50% of ■.
F 81 is ■% of 2700.
G 42 is ■% of 35.
H 14% of 1000 is ■.
I 72 is 30% of ■.
J 28 is ■% of 350.
K 1% of 4900 is ■.
L 155% of 200 is ■.
M 486 is ■% of 648.
N 60 is 5% of ■.
O 161 is 23% of ■.

197

# 197

1 Assign Problems E to N as written work. Stress that the pupils are not to solve the problems. Suggest to the pupils that they may follow the procedure used for the problems on page 196, in which they write an equation of ratios, use the ratio test, and then express the required division in two ways.
2 When the pupils have finished, let the pupils justify the expressions they wrote for each problem.
3 Assign Exercises A to J as written work.
4 The "Keeping skillful" exercises may be assigned according to the needs of the pupils.

# 198-200 Learning how

Exponded Notes for this lesson are on poges 352-353.

**Objectives**

The pupil learns to divide numbers expressed by fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

In the *Seeing Through Arithmetic* program, division of whole numbers is presented as a process of repeated subtraction. It is possible to use a pictorial representation of the process because the dividend (the total group) is larger than the divisor (the size of the equal groups into which the total group was divided).

However, in division of fractions and mixed numbers, a physical representation of the process is not always possible because the divisor often will be larger than the dividend. Nevertheless, the pupil can use concepts he has already learned to follow a mathematical process *logically*. These concepts are basic to the understanding of division of mixed numbers and fractions:

(1) Every number, except zero, has a reciprocal.
(2) The product of a fraction and its reciprocal is 1.
(3) The dividend and the divisor can be multiplied (or divided) by the same number without changing the quotient.
(4) If the divisor is 1, the quotient is equal to the dividend.

To make the computation easier, a division exercise such as $\frac{2}{3} \div \frac{1}{2}$ is written in the form dividend over divisor. The first step is to get 1 for the divisor. So the divisor is multiplied by its reciprocal. Then the dividend also is multiplied by the reciprocal of the divisor. Now the dividend is $\frac{2}{3} \times \frac{2}{1}$. The product of $\frac{1}{2} \times \frac{2}{1}$ in the divisor is 1, and thus can be disregarded. The computation can be completed by multiplying $\frac{2}{3}$ by $\frac{2}{1}$.

The four-step teaching process is used in this lesson. The TRY step on pages 199-200 demonstrates a variety of examples, including division of one mixed number

by another, division of one fraction by another, division of a whole number by a mixed number or a fraction, and vice versa. Make sure the pupils understand all the examples in the TRY step before they work the exercises in the DO step.

**Answers**

Page 200:

(block 1)

| | |
|---|---|
| A 6 | M $1\frac{1}{3}$ |
| B $17\frac{7}{9}$ | N $12\frac{3}{16}$ |
| C 12 | O $1\frac{1}{4}$ |
| D $2\frac{53}{72}$ | P $\frac{3}{8}$ |
| E $\frac{7}{32}$ | Q $1\frac{2}{9}$ |
| F $1\frac{103}{116}$ | R $2\frac{2}{35}$ |
| G $4\frac{2}{3}$ | S $\frac{18}{31}$ |
| H $1\frac{3}{5}$ | T $1\frac{7}{11}$ |
| I 3 | |
| J $\frac{13}{100}$ | |
| K 35 | |
| L $4\frac{4}{7}$ | |

(block 2)

| | |
|---|---|
| A $34\frac{4}{5}$ | K $2\frac{2}{9}$ |
| B $7\frac{7}{11}$ | L $\frac{69}{130}$ |
| C $2\frac{2}{9}$ | M $\frac{3}{16}$ |
| D $\frac{13}{44}$ | N 2 |
| E $1\frac{2}{9}$ | O $\frac{19}{44}$ |
| F $\frac{1}{8}$ | P $1\frac{51}{224}$ |
| G $3\frac{5}{18}$ | Q 16 |
| H 32 | R $\frac{5}{32}$ |
| I $5\frac{2}{5}$ | S $\frac{21}{40}$ |
| J 9 | T $\frac{2}{3}$ |

## Learning how

Now you will learn how to divide when the numerals represent proper fractions or mixed numbers.

**see**

**A** The equation below was made for a problem.

$\frac{1}{2} \div \frac{3}{4} = w$

The work below shows how to divide $5\frac{1}{2}$ by $\frac{3}{4}$. Use $\frac{11}{2}$ as another name for $5\frac{1}{2}$.

$\frac{\frac{11}{2}}{\frac{3}{4}}$ Express the division in this way.

$\frac{\frac{11}{2}}{\times\frac{4}{3}}$ First you need to get 1 for the divisor. You multiply $\frac{3}{4}$ by $\frac{4}{3}$ to get 1.

$\frac{\frac{11}{2}}{\frac{3}{4}} \times \frac{4}{3}$ $\frac{3}{4}$ has been multiplied by $\frac{4}{3}$. So you must also multiply $\frac{11}{2}$ by $\frac{4}{3}$.

$\frac{\frac{11}{2}}{\frac{3}{4}} \times \frac{4}{3}$ You do not need to write the divisor when it is 1.

$\frac{11}{2} \times \frac{4}{3}$ So now you can write the computation in this way.

$\frac{11}{2} \times \frac{4}{3} = \frac{44}{6} = 7\frac{1}{3}$ You found $7\frac{1}{3}$ by multiplying $\frac{4}{3}$ by $\frac{11}{2}$.

$5\frac{1}{2} \div \frac{3}{4} = 7\frac{1}{3}$

**B** The equation below was also made for a problem.

$\frac{7}{8} \div 4 = d$ **6**

You can find the quotient in the way shown below.

$\frac{\frac{7}{8}}{4}$ Use $\frac{4}{1}$ as another name for 4.

$\frac{\frac{7}{8}}{\frac{4}{1} \times \frac{1}{4}}$ To get 1 for the divisor, multiply $\frac{4}{1}$ by $\frac{1}{4}$.

$\frac{7}{8} \times \frac{1}{4}$ You multiply $\frac{7}{8}$ by $\frac{1}{4}$ because $\frac{4}{1}$ was multiplied by $\frac{1}{4}$.

$\frac{7}{8} \times \frac{1}{4}$ You do not need to write the divisor when it is 1.

$\frac{7}{8} \times \frac{1}{4}$ So now you can write the computation in this way.

$\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$ How is this numeral found?

$\frac{7}{8} \div 4 = \frac{7}{32}$



# 198

1. Have the pupils examine the equation and identify the number to be divided and the divisor.
2. Have the pupils observe the form in which the division is expressed. Explain that this form is more convenient for computation.
3. Have the pupils recall that when the divisor is 1, the quotient will be equal to the dividend. Explain that since we want a divisor of 1, we will have to multiply $\frac{3}{4}$ by some number to get 1. Remind the pupils that they learned that the reciprocal of $\frac{3}{4}$ [$\frac{4}{3}$] can be used.
4. Have the pupils explain why the dividend must also be multiplied by $\frac{4}{3}$. Ask them why they do not need to write the divisor when it is 1.
5. Have the pupils study the multiplication and explain the simplified answer.
6. Adapt the preceding notes. Point out that this time a fraction is to be divided by a whole number. Ask why $\frac{4}{1}$ can be used as another name for 4.

## think

A $7/10 \div 4\frac{1}{2} = a$ **1**

$\frac{7}{10}$ / $\frac{9}{2}$ ← Is $\frac{9}{2}$ another name for $4\frac{1}{2}$?

$\frac{7}{10}$ / $\frac{9}{2} \times \frac{2}{9}$ ← What divisor do you want to get? Why do you multiply $\frac{9}{2}$ by $\frac{2}{9}$?

$\frac{7}{10} \times \frac{2}{9}$ ← Why is $\frac{7}{10}$ also multiplied by $\frac{2}{9}$?

1 ← How is this 1 found? Can you now write the work without showing the 1?

$7/10 \times 2/9 = 14/90$ or $7/45$ ← How is $\frac{7}{45}$ found?

$7/10 \div 4\frac{1}{2} = 7/45$

B $5/6 \div 1/3 = b$

$\frac{5}{6}$ / $\frac{1}{3}$ ← Is it necessary to use another name for either $\frac{5}{6}$ or $\frac{1}{3}$? **2**

$\frac{5}{6}$ / $\frac{1}{3} \times \frac{3}{1}$ ← Why do you multiply $\frac{1}{3}$ by its reciprocal, $\frac{3}{1}$?

$\frac{5}{6} \times \frac{3}{1}$ ← Why is $\frac{5}{6}$ also multiplied by $\frac{3}{1}$?

1 ← You can rewrite the work without the 1.

$5/6 \times 3/1 = 15/6 = 2\frac{1}{2}$ ← How is this numeral found?

$5/6 \div 1/3 = 2\frac{1}{2}$

## try

A $9 \div 3/4 = t$ **3**

$\frac{9}{1} \times \frac{4}{3} = \frac{9 \times 4}{1 \times 3}$

$\frac{9}{1} \times \frac{4}{3} = \frac{36}{3}$ or 12

B $6\frac{1}{4} \div 3\frac{2}{5} = r$

$\frac{25}{4} \times \frac{5}{17}$

$\frac{17}{5} \times \frac{5}{17}$

$\frac{25}{4} \times \frac{5}{17} = \frac{125}{68}$ or $1\frac{57}{68}$

C $5\frac{7}{12} \div 3 = m$

$\frac{67}{12} \times \frac{1}{3}$

$\frac{3}{1} \times \frac{1}{3}$

$\frac{67}{12} \times \frac{1}{3} = \frac{67}{36}$ or $1\frac{31}{36}$

D $27 \div 5\frac{1}{10} = w$

$\frac{27}{1} \times \frac{10}{51}$

$\frac{51}{10} \times \frac{10}{51}$

$\frac{27}{1} \times \frac{10}{51} = \frac{270}{51}$ or $5\frac{15}{51}$ or $5\frac{5}{17}$



E $4/5 \div 8/8 = b$

$\frac{4}{5} \times \frac{8}{8}$

$\frac{5}{8} \times \frac{8}{5}$

$\frac{4}{5} \times \frac{8}{5} = \frac{32}{25}$ or $1\frac{7}{25}$

**1**

F $24/25 \div 8 = d$

$\frac{24}{25} \times \frac{1}{8}$

$\frac{8}{1} \times \frac{1}{8}$

$\frac{24}{25} \times \frac{1}{8} = \frac{24}{200}$ or $\frac{3}{25}$

G $40 \div 5/6 = k$

$\frac{40}{1} \times \frac{6}{5}$

$\frac{5}{6} \times \frac{6}{5}$

$\frac{40}{1} \times \frac{6}{5} = \frac{240}{5}$ or 48

H $14/15 \div 2/5 = a$

$\frac{14}{15} \times \frac{5}{2}$

$\frac{2}{5} \times \frac{5}{2}$

$\frac{14}{15} \times \frac{5}{2} = \frac{70}{30}$ or $2\frac{10}{30}$ or $2\frac{1}{3}$

I $9/16 \div 3 = z$

$\frac{9}{16} \times \frac{1}{3}$

$\frac{3}{1} \times \frac{1}{3}$

$\frac{9}{16} \times \frac{1}{3} = \frac{9}{48}$ or $\frac{3}{16}$

J $8 \div 1/10 = c$

$\frac{8}{1} \times \frac{10}{1}$

$\frac{1}{10} \times \frac{10}{1}$

$\frac{8}{1} \times \frac{10}{1} = \frac{80}{1}$ or 80

**2**

## do

Show your answers in lowest terms.

**3**

A $3/4 \div 1/8 = a$
B $6\frac{3}{8} \div 3/8 = r$
C $10 \div 5/8 = t$
D $12\frac{5}{16} \div 4\frac{1}{2} = k$
E $7/16 \div 2 = m$
F $9\frac{1}{8} \div 4\frac{5}{6} = s$
G $7/32 \div 3/64 = b$
H $8/15 \div 1/3 = e$
I $7 \div 2\frac{1}{3} = z$
J $13/20 \div 5 = f$

K $11\frac{1}{3} \div 1/3 = w$
L $4 \div 7/8 = d$
M $5\frac{1}{3} \div 4 = s$
N $9\frac{3}{8} \div 4/8 = v$
O $16\frac{1}{4} \div 13 = n$
P $2\frac{1}{32} \div 1\frac{3}{4} = a$
Q $1\frac{1}{12} \div 3/8 = r$
R $9 \div 4\frac{3}{8} = h$
S $2\frac{1}{4} \div 3\frac{6}{7} = c$
T $12 \div 7\frac{1}{3} = n$

**4**

A $5\frac{4}{5} \div 1/6 = c$
B $7 \div 1\frac{1}{4} = r$
C $3\frac{1}{3} \div 1\frac{1}{2} = w$
D $1\frac{3}{16} \div 2\frac{3}{4} = a$
E $3\frac{1}{4} \div 3/8 = t$
F $7/8 \div 7 = d$
G $9\frac{5}{8} \div 3 = m$
H $8 \div 1/4 = e$
I $5/10 \div 1/6 = k$
J $15 \div 1\frac{1}{3} = g$

K $10 \div 4\frac{1}{2} = b$
L $6\frac{9}{10} \div 13 = a$
M $9/16 \div 3 = r$
N $7/12 \div 7/24 = m$
O $13/32 \div 1\frac{3}{8} = w$
P $3\frac{7}{16} \div 2\frac{3}{8} = v$
Q $14 \div 6/8 = c$
R $3\frac{3}{8} \div 20 = t$
S $1\frac{5}{16} \div 2\frac{1}{2} = n$
T $4\frac{2}{3} \div 7 = y$



Now you should be able to divide when the numerals represent fractions.



# 199

1 Point out that this equation calls for a fraction to be divided by a mixed number. Have the pupils follow each step and answer the questions.

2 Point out that this equation calls for a fraction to be divided by a fraction, and have the pupils answer the questions.

3 Have the pupils copy and work Examples A to J (E to J are on page 200). Ask them to compare their work with the computation in the book when they have finished.

# 200

1 Have the pupils continue working the examples in the TRY step. (See Note 3 for page 199.)

2 When the pupils have finished all the examples, have various pupils discuss what they did in each example. When they understand each of these examples thoroughly, they may work the DO exercises.

3 Assign the first block of exercises (A to T) as written work. After the pupils have finished, supply answers and let them verify their work. Discuss any exercises that have caused difficulty.

4 The second block of exercises (A to T) may then be assigned to those pupils who need more practice, or to all the pupils if time permits.

# 201 Thinking straight; Keeping skillful

Exponded Notes are not considered necessary for these lessons.

**Objectives**

The pupil makes generalizations about 0 and 1 when addition, subtraction, multiplication, and division of fractions are involved.

**Vocabulary**

There are no new words.

**Comments**

This lesson will give pupils an opportunity to generalize the rôle of 0 and 1 in our number system. They should observe that when 0 is added or subtracted, the sum or difference is the "original" number; and when a number is multiplied or divided by 1, the product or quotient is also the "original" number. It may be brought out also that when a number is subtracted from the same number, the result is 0, and when a number is divided by the same number, the result is 1. The lesson also brings out that multiplying and dividing 0 by any other number will result in 0.

**Answers**

A [A] C [A] E [D] G [E] I [F] K [C]
B [A] D [D] F [D] H [E] J [B] L [G]

Keeping skillful:

(block 1)

A 24/96=[25]/100; 25%
B 15/100=[18]/120
C 2/100=60/[3000]
D 140/100=[35]/25
E 120/100=114/[95]
F 30/100=75/[250]
G 48/16=[300]/100; 300%
H 63/100=[126]/200
I 1/100=[3]/300
J 28/112=[25]/100; 25%
K 20/100=150/[750]
L 40/100=[140]/350
M 19/38=[50]/100; 50%

(block 2)

A 56
B $364\frac{85}{177}$
C $49\frac{18}{43}$
D $2\frac{18}{59}$
E 275
F 63
G $3271\frac{2}{29}$
H $58\frac{24}{73}$
I 332,628
J $111\frac{27}{41}$
K $248\frac{81}{89}$
L 68
M $70\frac{3}{40}$

(block 3)

A $5\frac{5}{48}$
B $26\frac{2}{5}$
C $6\frac{1}{2}$
D $\frac{1}{24}$
E 14
F $\frac{5}{6}$
G $37\frac{1}{2}$
H $\frac{3}{8}$
I $19\frac{4}{5}$
J $52\frac{1}{2}$
K $97\frac{1}{2}$

## Thinking straight

A $4\frac{1}{4}+0=4\frac{1}{4}$
B $0+\frac{2}{3}=\frac{2}{3}$
C $\frac{11}{12}-0=\frac{11}{12}$
D $\frac{7}{8}\times 1=\frac{7}{8}$
E $1\times 4\frac{2}{3}=4\frac{2}{3}$
F $\frac{29}{32}\div 1=\frac{29}{32}$
G $1\frac{5}{18}\times 0=0$
H $0\times\frac{11}{12}=0$
I $0\div\frac{5}{8}=0$
J $\frac{7}{12}-\frac{7}{12}=0$
K $\frac{15}{16}\div\frac{15}{16}=1$
L $\frac{3}{8}\times\frac{8}{3}=1$

For each example at the left, find a statement below that belongs with it. 1

A When 0 is added to, or subtracted from, a number, the answer is the number you started with.
B When a number is subtracted from the same number, the answer is 0.
C When a number other than zero is divided by the same number, the answer is 1.
D When a number is multiplied or divided by 1, the answer is the number you started with.
E The answer is 0 when a number is multiplied by 0.
F When 0 is divided by a number, the quotient is 0.
G When a number is multiplied by its reciprocal, the product is 1. 3

## Keeping skillful 4

A 24 is ■% of 96.
B 15% of 120 is ■.
C 60 is 2% of ■.
D 140% of 25 is ■.
E 114 is 120% of ■.
F 75 is 30% of ■.
G 48 is ■% of 16.
H 63% of 200 is ■.
I 1% of 300 is ■.
J 28 is ■% of 112.
K 150 is 20% of ■.
L 40% of 350 is ■.
M 19 is ■% of 38.

A $w\times 207=11592$
B $64513\div 177=t$
C $86\times k=4250$
D $136\div m=59$
E $25850\div b=94$
F $n\times 402=25326$
G $94861\div 29=r$
H $73\times c=4258$
I $6276\times 53=m$
J $9156\div 82=v$
K $44306\div 178=t$
L $17\times e=1156$
M $28030\div z=400$

A $\frac{7}{16}\times 11\frac{2}{3}=m$
B $4\frac{4}{5}\times 5\frac{1}{2}=d$
C $3\frac{9}{10}\times 1\frac{2}{3}=w$
D $\frac{1}{16}\times\frac{8}{9}=z$
E $20\times\frac{7}{10}=a$
F $4\frac{4}{8}\times\frac{1}{5}=n$
G $1\frac{15}{16}\times 40=v$
H $\frac{8}{9}\times\frac{3}{5}=k$
I $1\frac{4}{5}\times 11=b$
J $14\times 3\frac{3}{4}=g$
K $6\frac{1}{2}\times 15=f$



## 201

1 Discuss each of the generalizations (A to G).
2 Beside the letter for each of these examples, have the pupils write on their papers the letter of the statement that describes it. When they have finished, supply answers and let them verify their work. Discuss any questions they may have.
3 The pupils should now understand that the generalizations about 0 and 1 in addition, multiplication, etc., apply to fractions as well as to whole numbers.
4 Assign the three blocks of exercises in "Keeping skillful" as written work according to the needs of the pupils.

# 202-203 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to solve problems that involve division of fractions.

**Vocabulary**

New words page 203 costumes*, crust*, perfume

**Comments**

The lesson on pages 196-197 gave pupils practice in forming, not solving, equations involving division of fractions. Now they can solve equations of the type they learned to make in that lesson. Thus, in the present lesson, the pupils are required to make ratio equations with fraction numeral and mixed numeral terms, and to solve these equations.

The tests on page 203, involving division of fractions and of mixed numbers, are diagnostic in nature. Allow sufficient time for pupils to complete each test. Use the lesson on pages 198-200 for any reteaching that is necessary.

**Answers**

**Using arithmetic:**

A $2/1 = 16/[8]$

B $\frac{3}{4}/1 = 4\frac{1}{2}/[6]$

C $1\frac{1}{2}/1 = 57/[38]$

D $360/[4] = 90/1$

E $\frac{3}{8}/1 = \frac{3}{4}/[2]$

F $2\frac{1}{2}/1 = 6\frac{2}{3}/[2\frac{2}{3}]$

G $3\frac{1}{2}/3 = [1\frac{1}{6}]/1$

H $2\frac{3}{8}/1 = 14\frac{1}{4}/[6]$

I $\frac{5}{6}/\frac{2}{3} = [1\frac{1}{4}]/1$

J $\frac{3}{4}/1 = 1\frac{1}{3}/[1\frac{7}{9}]$

K $\frac{2}{3}/1 = 2/[3]$

L $\frac{3}{8}/1 = 1\frac{1}{2}/[4]$

M $147/1\frac{1}{2} = [98]/1$; \$.98

N $325/\frac{1}{8} = [2600]/1$; \$26

**Checking up:**

| Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|
| A $1\frac{1}{2}$ | A $2\frac{8}{9}$ | A $\frac{7}{40}$ | A 9 |
| B $\frac{4}{7}$ | B $\frac{10}{33}$ | B $1\frac{2}{3}$ | B $1\frac{3}{17}$ |
| C 1 | C $\frac{1}{7}$ | C $1\frac{17}{18}$ | C $\frac{12}{13}$ |
| D $\frac{11}{12}$ | D 21 | D $\frac{4}{35}$ | D $7\frac{1}{2}$ |
| E $\frac{9}{10}$ | E $5\frac{1}{16}$ | E $\frac{1}{27}$ | E 3 |
| F $1\frac{1}{2}$ | F $\frac{3}{40}$ | F $1\frac{13}{24}$ | F $1\frac{25}{31}$ |
| G 2 | G 8 | G $1\frac{2}{5}$ | G 144 |

## Using arithmetic 1

A Mrs. Peters used 16 yd. of cloth to make curtains. She used 2 yd. of the cloth for each curtain. How many curtains did she make?

You can write an equation of ratios for Problem A. Is the equation below correct?

$$\frac{2}{1}=\frac{16}{n}$$

When you use the ratio test, can you get the equation shown below?

$2n = 16$

To find the numeral that replaces $n$, you divide 16 by ■.
What numeral replaces $n$?

B Mrs. Peters used $4\frac{1}{2}$ yd. of cloth to make kitchen towels. She used $\frac{3}{4}$ yd. of cloth for each towel. How many towels did she make? 2

Can you write the equation of ratios below for Problem B?

$$\frac{\frac{3}{4}}{1}=\frac{4\frac{1}{2}}{w}$$

How do you get the equation below?

$\frac{3}{4}w = 4\frac{1}{2}$

To find the numeral that replaces $w$, you divide $4\frac{1}{2}$ by ■.
What numeral replaces $w$?

3 C George said, "I have read $1\frac{1}{2}$ times as many pages as you have, Jim. I have read 57 pages." How many pages had Jim read?

You know that George read $1\frac{1}{2}$ pages for each page that Jim read. Can you write this equation for Problem C?

$$\frac{1\frac{1}{2}}{1}=\frac{57}{t}$$

Does the equation below show the result when you use the ratio test?

$1\frac{1}{2}t = 57$

To find the numeral that replaces $t$, you divide 57 by ■.
What numeral replaces $t$?

D The area of a rectangular garden is 360 sq. yd. Its length is 90 yd. What is its width?

Can you write the equation below for Problem D?

$$\frac{360}{d}=\frac{90}{1}$$

Does the equation below show the result when you use the ratio test?

$90d = 360$

To find the numeral that replaces $d$, you divide 360 by ■.
What numeral replaces $d$?

When you solve each problem below, first write an equation of ratios.

E Jim gives his ducks about $\frac{3}{8}$ peck of feed each day. One day he noticed 4





# 202

1 Have Problem A read and let the pupils answer the questions. Be sure they understand what each term of the equation represents, how the second equation is obtained, and how the answer is found.

2 Let someone read Problem B. Then explain that this problem is much like Problem A, and that the same type of equation can be made for it. The only difference is that the ratios will have mixed numeral and fraction numeral terms. Discuss the equation and its solution in the same way you discussed the work for Problem A.

3 Continue the oral presentation and discussion for Problems C and D.

4 When you have finished Problem D, ask the pupils to read the statement following it and to begin working Problems E to N independently.

hat he had only about $\frac{3}{4}$ peck of feed eft. This amount would make about low many feedings?

John and Bill planned a $6\frac{2}{3}$-mile ike one Saturday. They expected to valk at a rate of about $2\frac{1}{2}$ mi. per iour. About how long should the hike ave taken?

Mr. Brown needed some poles for is tomato plants. He had some poles $1\frac{1}{2}$ yd. long. He cut each of these poles nto 3 equal shorter poles. How long vas each of these shorter poles?

When Mrs. Wood made costumes or a school play, she used $2\frac{3}{4}$ yd. of cloth for each costume. She used $14\frac{1}{4}$ yd. of cloth in all. How many costumes did she make?

Mr. Walker's pasture fence is $\frac{5}{8}$ mi. ong. Mr. Baker's pasture fence is $\frac{2}{3}$ mi. long. Mr. Walker's fence is how many times as long as Mr. Baker's?

J The area of a rectangular farm is $1\frac{1}{3}$ sq. mi. The width of this farm is $\frac{3}{4}$ mi. What is its length?

K When Mrs. Castle makes an apple pie, she uses $\frac{2}{3}$ cup of lard for the crust. There are 2 cups of lard in 1 pound. She can make crust for how many apple pies with 1 pound of lard?

L Mary wanted to buy $1\frac{1}{2}$ lb. of salted peanuts. At the store, the peanuts came in $\frac{3}{8}$-pound cans. How many cans should Mary have bought?

M Mrs. Wood paid $1.47 for $1\frac{1}{2}$ lb. of fudge. She paid how much per pound for the fudge?

N Mrs. Brown paid $3.25 for $\frac{1}{8}$ oz. of perfume. At the same rate, how much would 1 oz. of this perfume cost? 2

## Checking up 3

| Test 1 | Test 2 | Test 3 | Test 4 |
|---|---|---|---|
| A $\frac{3}{8}\div\frac{1}{4}=x$ | A $4\frac{1}{3}\div1\frac{1}{2}=r$ | A $\frac{7}{20}\div2=s$ | A $6\div\frac{2}{3}=m$ |
| B $\frac{2}{5}\div\frac{7}{10}=t$ | B $\frac{2}{5}\div2\frac{2}{4}=x$ | B $11\frac{2}{3}\div7=a$ | B $5\div4\frac{1}{2}=t$ |
| C $\frac{5}{6}\div\frac{5}{6}=a$ | C $\frac{2}{5}\div4\frac{2}{5}=s$ | C $5\frac{5}{8}\div3=c$ | C $9\div9\frac{3}{4}=d$ |
| D $1\frac{1}{10}\div\frac{3}{4}=d$ | D $10\frac{1}{2}\div\frac{1}{2}=b$ | D $\frac{4}{5}\div7=r$ | D $4\div\frac{8}{15}=f$ |
| E $\frac{9}{20}\div\frac{1}{2}=w$ | E $6\frac{3}{4}\div1\frac{1}{3}=n$ | E $\frac{1}{3}\div9=w$ | E $10\div3\frac{1}{3}=z$ |
| F $\frac{7}{8}\div\frac{7}{12}=b$ | F $\frac{1}{2}\div6\frac{2}{3}=d$ | F $9\frac{1}{2}\div6=k$ | F $14\div7\frac{3}{4}=g$ |
| G $\frac{3}{5}\div\frac{3}{10}=c$ | G $9\frac{3}{5}\div1\frac{1}{5}=m$ | G $5\frac{3}{5}\div4=h$ | G $36\div\frac{1}{4}=x$ |

End-of-block tests on division of proper fractions and of mixed numbers 203

# 203

1 Assign Problems E to N as individual written work.

2 Allow sufficient time so that the majority of pupils have time to complete all problems. To verify the work, let pupils volunteer to read the problems, write their equations on the chalkboard, and explain their work.

3 For the "Checking up" tests, give all pupils an opportunity to work each test. Verify each pupil's work so that reteaching may be done according to individual needs.

# 204 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns that "cancellation" may be used as a short cut in multiplication of fractions.

**Vocabulary**

New words page 204 wish*

**Comments**

On pages 184-191 the pupils learned to multiply fractions. In this lesson cancellation is introduced as a short-cut method for performing the computation. The pupils have previously learned that dividing both the numbers represented by the numerator and denominator by the same number gives them another name for the same fraction. (Operating on both terms of a fraction, whether by multiplication or by division, is called "reduction.") When a pupil uses "cancellation," he is performing a reduction *before* he finds the product. Therefore, he shortens the computation.

**Answers**

(block 1)

A By multiplying 7 by 8; by multiplying 12 by 15; $\frac{14}{45}$

B With the times sign between them below the line

C 12 divided by 4 equals 3; 8 divided by 4 equals 2; 2

D $7\times2=14$; $3\times15=45$

E Because 12 and 8 have both been divided by 4; by multiplying 7 by 2, and 3 by 15; yes

F 10; because 21 and 49 have both been divided by 7

G $\frac{6}{7}$; the way shown in Example D

(block 2)

| | | |
|---|---|---|
| A $\frac{7}{12}$ | E $\frac{9}{40}$ | I $1\frac{2}{5}$ |
| B $3\frac{1}{2}$ | F $5\frac{1}{2}$ | J $\frac{25}{56}$ |
| C $\frac{7}{30}$ | G $\frac{7}{15}$ | |
| D $12\frac{7}{24}$ | H $7\frac{1}{2}$ | |

## Thinking straight

A $\frac{7}{12} \times \frac{8}{15} = \frac{56}{180}$ or $\frac{14}{45}$

B $\frac{7}{12} \times \frac{8}{15} = \frac{7 \times \overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}} \times 15} = \frac{14}{45}$

C $\frac{7}{\underset{3}{\cancel{12}}} \times \frac{\overset{2}{\cancel{8}}}{15} = \frac{14}{45}$

D $2\frac{1}{10} \times \frac{20}{49} = \frac{\overset{3}{\cancel{21}}}{\underset{1}{\cancel{10}}} \times \frac{\overset{2}{\cancel{20}}}{\underset{7}{\cancel{49}}} = \frac{6}{7}$

1 A In Example A, how is the 56 found? How is the 180 found? In lowest terms, $\frac{56}{180}$ is $\frac{\blacksquare}{\blacksquare}$.

2 B In Example B, the 7 and the 8 are written with the times sign between them above the line. How are the 12 and the 15 written?

3 C In the work in red in Example B, both 12 and 8 are divided by 4. Why is 3 written below 12? Why is 2 written above 8? The 12 and the 8 are crossed out to show that they are replaced with 3 and ■.

D How is the 14 in $\frac{14}{45}$ found? How is the 45 in $\frac{14}{45}$ found?

4 E Example C shows that you can use the short cut without rewriting the work. Why are the 12 and the 8 crossed out? How is the product, $\frac{14}{45}$, found? Is $\frac{14}{45}$ in lowest terms?

5 F In Example D, the short cut is used twice. 20 and 10 are crossed out and replaced with 2 and 1 because both 20 and 10 have been divided by ■. Why are 21 and 49 crossed out and replaced with 3 and 7?

6 G Now multiply $\frac{20}{49}$ by $\frac{21}{10}$ in the way shown in Example A. The product, in lowest terms, is $\frac{\blacksquare}{\blacksquare}$. Which way of finding the product is more convenient, the way shown in Example A or the way shown in Example D?

In the exercises below, you may use the short cut if you wish. Notice that sometimes you cannot use the short cut.

7
A $\frac{7}{10} \times \frac{5}{8} = d$
B $4\frac{1}{12} \times \frac{6}{7} = r$
C $\frac{1}{3} \times \frac{7}{10} = w$
D $3\frac{11}{15} \times 3\frac{1}{3} = n$
E $\frac{3}{4} \times \frac{3}{10} = e$
F $\frac{33}{35} \times \frac{5}{3} = a$
G $\frac{14}{15} \times \frac{1}{2} = k$
H $\frac{8}{16} \times 13\frac{1}{3} = m$
I $\frac{21}{5} \times \frac{1}{3} = t$
J $\frac{10}{21} \times \frac{18}{10} = b$



# 204

1 Discuss Exercise A. Stress that 56 is the product of 7 and 8, and 180 is the product of 12 and 15. The pupils should see that both 56 and 180 in $\frac{56}{180}$ are divided by 4 to give the answer in lowest terms.

2 Have the pupils compare Examples A and B.

3 Let pupils volunteer to answer the questions in Exercises C and D. Help them see that dividing 8 and 12 by 4 results in smaller numbers to multiply. Note that the answer is in lowest terms.

4 Guide pupils to see that Example C is also a short cut for Example A.

5 Be sure the pupils understand how the short cut is used twice in Example D.

6 When the pupils have found the answer in lowest terms, make sure they see the difference between this method and the short cut shown in Example D.

7 Assign Exercises A to J as written work. Provide answers.

# 205 Keeping skillful

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns a convenient way to multiply large numbers when one is a whole number and the other number is expressed as a mixed numeral.

**Vocabulary**

There are no new words.

**Comments**

In the lesson on pages 186-191 the pupils learned to multiply fractions. This lesson shows the pupils a procedure that will make computation easier when large numbers are involved.

In the second part of this lesson the pupils learn that when a number is to be divided by a larger number ($6 \div 18 = t$), a short cut can be used to find the answer. First the pupils are shown the division performed in the usual way. (Each number is renamed—6 is thought of as $\frac{6}{1}$; 18 as $\frac{18}{1}$.) The second example shows that after the indicated division has been performed, the resulting fraction numeral has in its numerator the numeral that was the dividend; in its denominator it has the numeral that was the divisor. As a short cut, the symbol 6 over 18 can be thought of as a fraction numeral. Frequently this short cut is introduced without explaining why the division can be expressed by a fraction numeral. Here, however, the work in the first example shows the pupils why this short cut works.

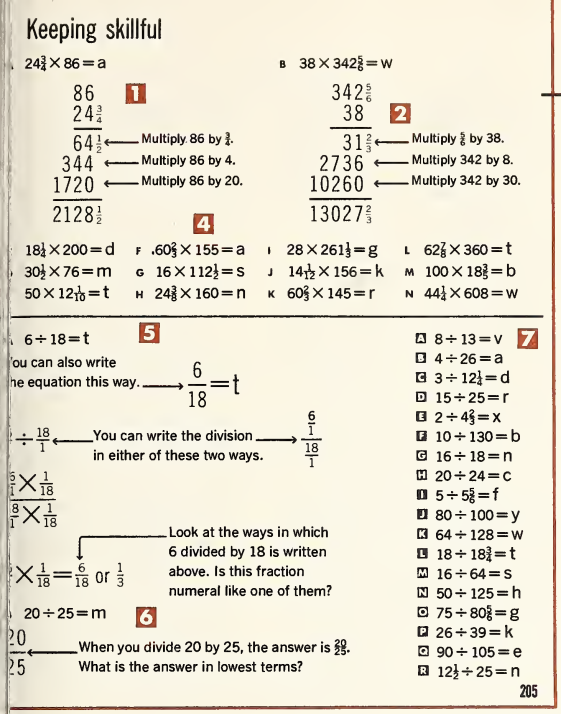
**Answers**

(block 1)

| | | | |
|---|---|---|---|
| C 3650 | F $9403\frac{1}{3}$ | I $7317\frac{1}{3}$ | L 22,635 |
| D 2318 | G 1800 | J 2197 | M 1860 |
| E 605 | H 3900 | K $8796\frac{2}{3}$ | N 26,904 |

(block 2)

| | | | | |
|---|---|---|---|---|
| A $\frac{8}{13}$ | E $\frac{3}{7}$ | I $\frac{6}{7}$ | M $\frac{1}{4}$ | Q $\frac{6}{7}$ |
| B $\frac{2}{13}$ | F $\frac{1}{13}$ | J $\frac{4}{5}$ | N $\frac{2}{5}$ | R $\frac{1}{2}$ |
| C $\frac{12}{49}$ | G $\frac{8}{9}$ | K $\frac{1}{2}$ | O $\frac{90}{97}$ | |
| D $\frac{3}{5}$ | H $\frac{5}{6}$ | L $\frac{24}{25}$ | P $\frac{2}{3}$ | |


Keeping skillful
24¾ × 86 = a
B 38 × 342⅝ = w
86
24¾
64½ ← Multiply 86 by ¾.
344 ← Multiply 86 by 4.
1720 ← Multiply 86 by 20.
2128½
1
342⅝
38
31⅔ ← Multiply ⅝ by 38.
2736 ← Multiply 342 by 8.
10260 ← Multiply 342 by 30.
13027⅔
2
4
18¼ × 200 = d
30½ × 76 = m
50 × 12 1/10 = t
F .60⅔ × 155 = a
G 16 × 112½ = s
H 24⅜ × 160 = n
I 28 × 261⅓ = g
J 14½ × 156 = k
K 60⅔ × 145 = r
L 62⅞ × 360 = t
M 100 × 18⅜ = b
N 44¼ × 608 = w
6 ÷ 18 = t
5
ou can also write
he equation this way.
6/18 = t
÷ 18/1 ← You can write the division in either of these two ways. → 6/1 ÷ 18/1
6/1 × 1/18
8/1 × 1/18
Look at the ways in which 6 divided by 18 is written above. Is this fraction numeral like one of them?
× 1/18 = 6/18 or 1/3
20 ÷ 25 = m
6
20/25 ← When you divide 20 by 25, the answer is 20/25. What is the answer in lowest terms?
A 8 ÷ 13 = v
B 4 ÷ 26 = a
C 3 ÷ 12¼ = d
D 15 ÷ 25 = r
E 2 ÷ 4⅔ = x
F 10 ÷ 130 = b
G 16 ÷ 18 = n
H 20 ÷ 24 = c
I 5 ÷ 5⅝ = f
J 80 ÷ 100 = y
K 64 ÷ 128 = w
L 18 ÷ 18⅜ = t
M 16 ÷ 64 = s
N 50 ÷ 125 = h
O 75 ÷ 80⅝ = g
P 26 ÷ 39 = k
Q 90 ÷ 105 = e
R 12½ ÷ 25 = n
7
205

# 205

1 In Example A, point out that the work for multiplication of a "whole number" and a number expressed as a "mixed number" can be written in the same form as the work for multiplication of whole numbers. Discuss the computation.

2 Discuss the multiplication shown in Example B. Make sure each step of the computation is clear.

3 Exercises C to E may be used for chalkboard demonstrations.

4 Assign Exercises F to N as written work. Provide answers for pupils to verify their work.

5 Discuss Example A with the class. Pupils should understand that the quotient, $\frac{6}{18}$, or $\frac{1}{3}$, is *not* a reduction of the indicated division but develops from the complete division shown immediately above.

6 You may need to work through this example in the way shown for Example A.

7 Use these exercises for independent work.

# 206-208 Checking up

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil tests his ability to solve problems of various types, his ability to make arithmetic generalizations, and his skill in computation.

**Vocabulary**

New words **page 206** Clark, cabbage*, double*, postage; **page 207** tulips, pansy

**Comments**

Test 1 in this series of achievement tests is on problem solving. The pupils are required to write equations for the types they have studied, including rate problems, comparison problems, and per cent problems. Test 2 on page 207 covers generalizations about fractions, ratios, and inequalities. This test should help you evaluate the pupils' understanding of various concepts. By means of the tests on page 208 the pupils' achievement in computation can be measured. There are tests on addition and subtraction of fractions in decimal notation, multiplication and division of whole numbers, all four processes with fraction numerals, and the solving of per cent equations.

**Reteaching chart**

This chart will help locate the pages in the pupils' book that may be used for any reteaching that is necessary. Page references are to Book 6 unless otherwise indicated.

Test 1

| Problem | Pages | Problem | Pages |
|---|---|---|---|
| A | 112-116, 192-193 | J | 122-124, 192-193 |
| B | 112-116 | K | 112-116 |
| C | 26-30, 112-116 | L | 112-116, 202-203 |
| D | 112-116 | M | 130-133, 134-135 |
| E | 152-156, 192-193 | N | 122-124, 202-203 |
| F | 112-116, 192-193 | O | 130-133, 134-135 |
| G | 112-116 | P | 122-124, 202-203 |
| H | 130-133, 134-135 | Q | 112-116, 202-203 |
| I | 112-116, 202-203 | | |

Test 2

Reteaching references are not considered necessary for these exercises on terminology and generalizations.

| Test | Pages | Test | Pages |
|---|---|---|---|
| 3 | 177-180 | 7 | 89-92, 93-97 |
| 4 | 177-180, 181-182 | 8 | 186-191 |
| 5 | 20-23 | 9 | 198-200 |
| | Book 4: 161-164, 167 | 10 | 80-85, 86, 93-97, 98, |
| | Book 5: 61-64, 77 | | 186-191, 198-200 |
| 6 | 74-79, 80-85 | 11 | 130-133, 134-135 |

**Answers**

Test 1

A $110/1 = [55]/\frac{1}{2}$
B $75/1 = 300/[4]$ or $\$3.00 \div \$.75 = [4]$; 4 oz.
C $1680/320 = [5\frac{1}{4}]/1$ or $1680 \div [5\frac{1}{4}] = 320$; $5\frac{1}{4}$ lb.
D $525/7 = [75]/1$ or $\$5.25 \div [\$.75] = 7$; \$.75
E $20\frac{1}{2}/1 = [328]/16$; 328 sq. ft.
F $12/1 = [4]/\frac{1}{3}$; 4 lb.
G $220/1 = 3300/[15]$; 15 lb.
H $80/100 = [48]/60$
I $135/\frac{1}{4} = [540]/1$; \$5.40
J $1\frac{1}{2}/1 = [18]/12$; 18 oz.
K $3250/250 = [6500]/500$; \$65
L $\frac{1}{4}/1 = 1\frac{1}{2}/[6]$
M $10/100 = [260]/2600$; \$2.60
N $8\frac{1}{4}/5\frac{1}{2} = [1\frac{1}{2}]/1$
O $65/100 = 130/[200]$
P $\frac{1}{8}/\frac{1}{16} = [2]/1$
Q $300/\frac{1}{8} = [2400]/1$; \$24

Test 2

A Yes
B Yes
C Yes
D No
E Smallest
F No
G No
H Yes
I No
J No
K Equal to
L $15 < a$
M $<, \neq$
N $1\frac{1}{3}/3$, 12/27, $1/2\frac{1}{4}$

Test 3

A 51.440
B 169.3
C 8.6868
D 210.10
E 2.6209
F 96.983
G 6.0006
H 235.4
I 208.76

Test 4

A 45.2
B .252
C 34.425
D 78.23
E 1.8289
F 4.7
G 6.171
H 105.13
I .9102
J 33.4549

Test 5

A 36
B $48\frac{98}{189}$
C $65\frac{60}{593}$
D $17\frac{19}{23}$
E 1,389,052
F $138\frac{2}{7}$
G $31\frac{35}{88}$
H 36,190
I 804
J $176\frac{40}{49}$

Test 6

A $8\frac{3}{8}$
B 26
C $191\frac{7}{16}$
D 35
E 9
F $11\frac{23}{24}$
G $86\frac{3}{4}$
H $24\frac{41}{48}$

Test 7

A $1\frac{9}{20}$
B $8\frac{1}{3}$
C $\frac{3}{32}$
D $10\frac{1}{6}$
E $15\frac{1}{24}$
F $20\frac{19}{32}$
G $\frac{1}{2}$
H 21
I $\frac{41}{64}$

Test 8

A $\frac{1}{4}$
B $\frac{77}{128}$
C 25
D $43\frac{1}{15}$
E $73\frac{5}{8}$
F $48\frac{29}{80}$
G 171
H $138\frac{5}{6}$
I 170

Test 9

A $\frac{7}{8}$
B $1\frac{1}{5}$
C 24
D $1\frac{1}{2}$
E $18\frac{2}{9}$
F $\frac{1}{6}$
G $\frac{9}{10}$
H $1\frac{1}{3}$

Test 10

A $3\frac{5}{8}$
B $5\frac{1}{6}$
C 39
D $11\frac{9}{16}$
E $14\frac{1}{12}$
F $14\frac{5}{7}$
G $7\frac{1}{10}$
H $1\frac{1}{2}$

Test 11

A $15/20 = [75]/100$; 75%
B $152/100 = [456]/300$
C $11/100 = 22/[200]$
D $180/100 = 90/[50]$
E $40/100 = [250]/625$
F $27/135 = [20]/100$; 20%
G $155/100 = 465/[300]$
H $2/100 = [3]/150$
I $75/15 = [500]/100$; 500%

## Checking up

Test 1

A Mr. Clark bought ½ lb. of pole beans for seed. The package stated that 1 lb. would plant 110 hills of beans. How many hills should Mr. Clark plant with the seeds he bought?

B Mr. Cook paid $3 for cabbage seed priced at $.75 per ounce. How many ounces of cabbage seed did he buy?

C In the fall, Mr. Cook sold 320 heads of cabbage. The total weight of the cabbage was 1680 lb. What was the average weight per head?

D Mrs. Cook paid $5.25 for 7 packages of flower seed. Each package cost the same amount. What was the cost of each package?

E Mrs. Cook had a rectangular flower garden that was 20½ ft. long and 16 ft. wide. What was its area in square feet?

F Mr. Clark planned to plant ½ acre of sweet corn. He read that 12 lb. of seed would plant 1 acre. How many pounds of seed should Mr. Clark have bought?

G Mr. Clark bought lawn seed for his 3300 sq. ft. of lawn. He planned to use it at the rate of 1 lb. for every 220 sq. ft. of lawn. How much lawn seed should he have bought?

H When Mrs. Cook bought some plants, she was told that about 80% of the plants would have double flowers. About how many of her 60 plants could be expected to have double flowers?

I Mr. Cook bought 1 lb. of melon seed priced at ¼ lb. for $1.35. How much did he pay for the 1 lb. of seed?

J Mr. Clark bought 12 oz. of weed killer to use on his 3300 sq. ft. of lawn. His neighbor, Mr. Cook, said, "I have about 1½ times as much lawn as you do. So I will need 1½ times as much weed killer as you do." How much weed killer did Mr. Cook need?

K Mr. Cook bought 500 strawberry plants at $32.50 per 250 plants. How much did he pay for all the plants?

L One kind of flower seed that Mrs. Cook wanted was sold in ¼ ounce packages. She wanted 1½ oz. of the seed. How many packages should she have bought?

M Mr. Clark ordered plants and seeds at a total cost of $26. He had to pay an extra 10% of this amount for packing and postage. How much money did he have to pay for packing and postage?



N Mrs. Cook's tallest sunflower plant was 8½ ft. high. This plant was how many times as tall as the shortest plant, which was 5½ ft. high?

O Mrs. Cook says that last spring she had 130 red tulips. These tulips were about 65% of all her tulips. About how many tulips did Mrs. Cook have?

P Mrs. Cook bought ⅛ oz. of pansy seed. Mrs. Clark bought $\frac{1}{16}$ oz. of the same kind of pansy seed. Mrs. Cook bought how many times as much pansy seed as Mrs. Clark?

Q Mrs. Cook paid $3 for ⅛ oz. of pansy seed. At this rate, how much would 1 oz. of the seed cost? 1

Test 2

A Can the sum of two proper fractions be less than 1?

B Can the sum of two proper fractions be 1?

C Can the sum of two proper fractions be more than 1?

D Can the sum of two proper fractions be 2?

E Suppose you have six different fraction numerals, each of which has 1 for the numerator. Which fraction numeral represents the largest fraction, the one with the largest denominator or the one with the smallest denominator?

F If each of two fractions is greater than ½, can their sum be equal to 1?

G If each of two fractions is greater than ½, can their sum be less than 1?

H If each of two fractions is greater than ½, is their sum greater than 1?

I When you subtract a proper fraction from a whole number, is the answer ever a whole number?

J When a mixed number is subtracted from a whole number, is the answer always a mixed number?

K If you multiply the number in each term of a fraction numeral by the same number, will the new fraction numeral be equal to, greater than, or less than the fraction numeral you began with?

L Which sentence below has more whole numbers in its solution set?

$15 > a$ $15 < a$

M Which sign or signs below can you use to make a true statement of the sentence $13 \times 3 \text{ —— } 2 \times 26$?

$<$ $>$ $=$ $\neq$

N Of the ratios given below, which are equal to 4 over 9? 3

$\frac{8}{32}$ $\frac{1\frac{1}{3}}{3}$ $\frac{6}{11}$ $\frac{12}{27}$ $\frac{1}{2\frac{1}{4}}$ $\frac{16}{27}$



# 206

1 Give the pupils enough time to complete all the problems in Test 1. Instruct the pupils to write an equation of ratios for each problem, using a letter to hold a place for the missing numeral. After the problem has been solved, the pupils should rewrite the equation and replace the letter with the numeral they found by computing.

# 207

1 Verify each pupil's work. Refer to the Reteaching Chart on page 199 of this *Teaching Guide* to find the textbook pages on which reteaching materials may be found.

2 Ask pupils to give examples to illustrate their answers for Exercises A to K in this test. For Exercise E, for example, a pupil might list fraction numerals such as $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{12}$, and $\frac{1}{4}$. He would then point out that $\frac{1}{3}$ represents the largest fraction.

3 Go over the exercises orally. Have pupils give their answers and explain them. The nature of this test makes it impossible to give reteaching references. But any serious difficulties indicate that the pupil needs more work on understanding the meaning of fractions and the related arithmetic terminology.

**Test 3** 1

Find the sum.

A .014, 3.686, 47.740
B 16.7, 43.2, 96.8, 12.6
C .3372, .8657, 7.4839
D 99.99, 4.65, 104.61, .85
E 1.0003, .9797, .6409
F 86.915, .742, 9.326
G 5.7777, .2223, .0006
H 48.3, 92.0, 76.4, 18.7
I 24.76, 30.05, 63.99, 89.96

**Test 4**

A $62.4 - x = 17.2$
B $m + .748 = 1.000$
C $39.051 - 4.626 = w$
D $d - 15.83 = 62.40$
E $.1716 + b = 2.0005$
F $g - 2.8 = 1.9$
G $11.006 - f = 4.835$
H $n + 6.78 = 111.91$
I $.8396 + .0706 = t$
J $42.1034 - k = 8.6485$

**Test 5**

A $m \times 87 = 3132$
B $9170 \div 189 = w$
C $593 \times d = 38605$
D $1640 \div k = 92$
E $746 \times 1862 = b$
F $a \times 49 = 6776$
G $2763 \div n = 88$
H $94 \times 385 = d$
I $r \times 63 = 50652$
J $8664 \div 49 = y$

**Test 6**

Find the sum.

A $6\frac{1}{2}$, $1\frac{7}{8}$
B $8\frac{1}{4}$, $7\frac{3}{12}$, $10\frac{1}{3}$
C $60\frac{1}{16}$, $120\frac{3}{8}$, 11
D $14\frac{5}{6}$, $2\frac{3}{4}$, $9\frac{6}{8}$, $8\frac{1}{4}$
E $\frac{9}{10}$, $\frac{3}{5}$, $\frac{4}{8}$, $2\frac{2}{3}$
F $3\frac{7}{12}$, $2\frac{5}{8}$, $\frac{3}{4}$, $5\frac{1}{2}$
G $28\frac{5}{8}$, $11\frac{9}{20}$, $46\frac{7}{10}$
H $17\frac{5}{16}$, $5\frac{3}{4}$, $11\frac{1}{12}$, $\frac{5}{8}$

**Test 7**

A $10\frac{7}{20} - 8\frac{9}{10} = x$
B $14\frac{1}{3} - 6 = t$
C $\frac{15}{32} - \frac{3}{8} = a$
D $60 - 49\frac{5}{8} = r$
E $15\frac{11}{12} - \frac{5}{6} = b$
F $21 - 13\frac{1}{32} = d$
G $8\frac{5}{16} - 8\frac{1}{16} = c$
H $47\frac{11}{20} - 26\frac{11}{20} = w$
I $\frac{15}{16} - \frac{13}{64} = m$

**Test 8**

A $\frac{5}{6} \times \frac{3}{10} = m$
B $1\frac{3}{8} \times \frac{7}{16} = r$
C $15 \times 1\frac{3}{5} = v$
D $9\frac{1}{2} \times 4\frac{8}{15} = d$
E $\frac{13}{32} \times 124 = b$
F $7\frac{3}{10} \times 6\frac{8}{9} = c$
G $45 \times 3\frac{5}{9} = k$
H $21\frac{1}{4} \times 6\frac{8}{15} = w$
I $100 \times 1\frac{7}{10} = t$

**Test 9**

A $\frac{21}{32} \div \frac{3}{4} = w$
B $8\frac{2}{3} \div 7 = t$
C $40 \div 1\frac{3}{5} = d$
D $9\frac{3}{8} \div 6\frac{1}{4} = m$
E $27\frac{1}{3} \div 1\frac{1}{2} = x$
F $\frac{11}{16} \div 4\frac{1}{8} = n$
G $\frac{15}{32} \div \frac{28}{48} = r$
H $18\frac{2}{3} \div 14 = b$

**Test 10**

A $r + 3\frac{1}{4} = 6\frac{7}{8}$
B $11 - x = 5\frac{2}{6}$
C $\frac{3}{4} \times 52 = w$
D $9\frac{15}{18} + 1\frac{5}{8} = c$
E $v - 12\frac{11}{12} = 1\frac{1}{6}$
F $10\frac{3}{10} \div \frac{7}{10} = b$
G $8\frac{1}{10} - m = 1$
H $14\frac{7}{12} - 13\frac{1}{12} = t$

**Test 11** 2

A 15 is ■% of 20.
B 152% of 300 is ■.
C 11% of ■ is 22.
D 180% of ■ is 90.
E 40% of 625 is ■.
F 27 is ■% of 135.
G 155% of ■ is 465.
H 2% of 150 is ■.
I 75 is ■% of 15.

208

# 208

1 Allow ample time for pupils to complete the tests. Verify each pupil's work and see that he makes the needed corrections. Provide reteaching if necessary. The Reteaching Chart on page 200 lists the pages in the pupils' book that present the work involved in each test.

2 For Test 11 instruct pupils to write an equation of ratios for each exercise. After the missing term has been found, have the pupils rewrite the ratios with the correct numeral replacing the letter. In cases where the answer to be found represents what per cent one number is of another, be sure the pupils understand that the term they find is not the answer. Rather, the entire ratio, or the term with a per cent sign written after it is the answer. Again, verify, make corrections, and reteach if necessary.

# 209 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns a convenient short cut to use in solving ratio equations that have 1 in the second term of one ratio.

**Vocabulary**

There are no new words.

**Comments**

This lesson shows the pupil that he may use a short cut to solve equations of ratios that have the numeral 1 in the second term of one ratio. In each of the three equations shown in Examples A, B, and C, the intermediate step (Step 2) of writing the equation that results from the use of the ratio test can be omitted. The pupil can go directly to the computational step of division or multiplication. Study of the textbook examples and practice with other examples should enable the pupil to recognize when he can use this short cut. The equations given in the examples are types that may be made for familiar problems.

The purpose of this lesson is to help the pupils see that, *after* the equation has been written, a short cut can be used to solve it if 1 is a term of one ratio.

**Answers**

A Yes; that the replacement for *n* is found by dividing 120 by 15
B Yes; 15
C Yes; the equation that results from using the ratio test; that the replacement for *n* is found by multiplying 15 by 8
D Yes; 15 by 8
E Yes; the equation that results from using the ratio test; that the replacement for *n* is found by dividing 120 by 8
F Yes; 120 by 8

(block 2)

| | | | | |
|---|---|---|---|---|
| A 4 | C 5 | E 364 | G 19 | I 5 |
| B 450 | D 5 | F 400 | H 1331 | |

## Thinking straight

| | A | B | C |
|---|---|---|---|
| Step 1 | $\frac{120}{15}=\frac{n}{1}$ | $\frac{8}{1}=\frac{n}{15}$ | $\frac{8}{1}=\frac{120}{n}$ |
| Step 2 | $15n=120$ | $n=8\times15$ | $8n=120$ |
| Step 3 | $120\div15$ | $8\times15$ | $120\div8$ |

A Step 1 in Example A above shows an equation that was made for a problem. Notice that 1 is the second term of one ratio. Does Step 2 show what you write when you use the ratio test? What does Step 3 show?

B Look again at Steps 1 and 3 of Example A. Can you solve the equation without using the ratio test? To find the numeral that will replace $n$, you divide 120 by ■.

C Study the equation in the first step of Example B. Is 1 the second term of one ratio? What does Step 2 show? What does Step 3 show?

D Can you solve the equation in Example B without using the ratio test? To find the numeral that will replace $n$, you multiply ■ by ■.

E Is 1 the second term of one ratio in the equation in Example C? What does Step 2 show? What does Step 3 show? 5

F Can you solve the equation in Example C without using the ratio test? To find the numeral that will replace $n$, you divide ■ by ■. 6

When you have an equation of ratios, and 1 is the second term of one ratio, you do not need to use the ratio test.

You can solve each equation below without using the ratio test. 7

A $\frac{60}{15}=\frac{b}{1}$ B $\frac{9}{1}=\frac{x}{50}$ C $\frac{25}{5}=\frac{n}{1}$ D $\frac{7}{1}=\frac{35}{r}$ E $\frac{14}{1}=\frac{m}{26}$ F $\frac{5}{1}=\frac{b}{80}$ G $\frac{95}{5}=\frac{w}{1}$ H $\frac{11}{1}=\frac{t}{121}$ I $\frac{21}{1}=\frac{105}{f}$

Short cut in solving equations in which the second term of one ratio is 1 209

# 209

1 The pupils should see that Step 2 of Example A shows the application of the ratio test, and that Step 3 shows the computation.
2 Have Exercise B read. The pupils should see that Step 3 shows that the answer can be found by dividing the number represented in the first term of the first ratio by the number represented in the second term.
3 Adapt Note 1, emphasizing the 1 that appears in the second term of the first ratio.
4 Point out that in Example B the answer can also be found without using the ratio test—by multiplying the number represented in the first term of one ratio by the number represented in the second term of the other.
5 Adapt Note 1.
6 Have the pupils note that the number represented in the first term of one ratio is divided by the number represented in the first term of the other.
7 Assign these exercises as written work.

# 210-213 Exploring problems

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to solve comparison problems in which fraction numerals are used in the terms of ratio equations.

**Vocabulary**

New words page 210 honey*; page 213 plowed*

**Comments**

Pupils have already studied comparison problems of the type traditionally known as "times as many as." In this lesson the pupils learn to solve the "fraction as many as" type of problem. The solution of the equations for such problems requires multiplication or division of fractions instead of multiplication or division of whole numbers.

The comparison problem situations studied here cover those commonly described as the "three cases" of fractions. But these distinctions disappear when equations of ratios are used to express the problem situation. The pupil knows that in making an equation of ratios for these situations, one of the ratios will be based on a comparison with 1; the other ratio must be equal to it. He then solves the equation he has made in the same way he has learned to solve other ratio equations.

In each problem, it is helpful for the pupil to be aware of whether the comparison is between two groups or between a part of a group and the whole group. This distinction does not affect the equation that is made for the problem, but it does help in interpreting the answer.

**Answers**

Pages 212-213:

Exploring problems:

G $3\frac{3}{4}/15 = [\frac{1}{4}]/1$
H $\frac{1}{4}/1 = [5]/20$
I $\frac{4}{5}/1 = 8/[10]$; \$10
J $\frac{1}{8}/1 = 7/[56]$; \$56
K $\frac{3}{4}/1 = 36/[48]$
L $36/60 = [\frac{3}{5}]/1$
M $\frac{3}{5}/1 = [3\frac{3}{4}]/6\frac{1}{4}$
N $\frac{1}{3}/1 = 3\frac{3}{4}/[11\frac{1}{4}]$
O $14/21 = [\frac{2}{3}]/1$
P $\frac{5}{6}/1 = [100]/120$

Q $\frac{1}{2}/1 = [10\frac{1}{3}]/20\frac{2}{3}$

R $\frac{1}{2}/1 = [17]/34$; \$17

S $75/250 = [\frac{3}{10}]/1$

**Keeping skillful:**

A $2/1 = [3]/1\frac{1}{2}$ or $24/1 = [432]/18$; 3 sq. ft. or 432 sq. in.

B $1\frac{2}{3}/1 = [2]/1\frac{1}{5}$; 2 sq. mi.

C $15/1 = [300]/20$; 300 sq. yd.

D $3\frac{1}{2}/1 = [4\frac{3}{8}]/1\frac{1}{4}$; $4\frac{3}{8}$ sq. yd.

E $35/1 = [840]/24$ or $2\frac{11}{12}/1 = [5\frac{5}{6}]/2$; 840 sq. in. or $5\frac{5}{6}$ sq. ft.

F $2\frac{3}{10}/1 = [3\frac{91}{100}]/1\frac{7}{10}$; $3\frac{91}{100}$ sq. mi.

G $4\frac{7}{12}/1 = [12\frac{29}{48}]/2\frac{3}{4}$; $12\frac{29}{48}$ sq. in.

H $50/1 = [450]/9$ or $150/1 = [4050]/27$; 450 sq. yd. or 4050 sq. ft.

I $40/1 = [1600]/40$; 1600 sq. ft.

J $4/1 = [14]/3\frac{1}{2}$; 14 sq. mi.

K $60/1 = [4320]/72$ or $1\frac{2}{3}/1 = [3\frac{1}{3}]/2$; 4320 sq. in. or $3\frac{1}{3}$ sq. yd.

L $1\frac{1}{3}/1 = [2\frac{1}{2}]/1\frac{7}{8}$; $2\frac{1}{2}$ sq. in.

M $5\frac{1}{4}/1 = [27\frac{9}{16}]/5\frac{1}{4}$; $27\frac{9}{16}$ sq. ft.

N $3/1 = [22]/7\frac{1}{3}$; 22 sq. mi.

O $\frac{1}{3}/1 = [1\frac{1}{2}]/4\frac{1}{2}$ or $4/1 = [216]/54$; $1\frac{1}{2}$ sq. ft. or 216 sq. in.

P $17\frac{1}{8}/1 = [293\frac{17}{64}]/17\frac{1}{8}$; $293\frac{17}{64}$ sq. ft.

## Exploring problems

A A farmer had $20\frac{2}{3}$ gal. of honey. In one week he sold $15\frac{1}{2}$ gal. What fraction of his total supply did he sell?

When you find what fraction of the total supply the farmer sold, you are really finding what fraction of a gallon he sold for each gallon he had to start with.

First, make an equation of ratios. Each ratio will express the comparison of gallons the farmer sold with the gallons he had to start with.

$$\frac{15\frac{1}{2}}{20\frac{2}{3}} = \frac{d}{1}$$

Ratio expressing the comparison for $20\frac{2}{3}$ gal.

Ratio expressing the comparison for 1 gal.

How is the equation below found?

$20\frac{2}{3}d = 15\frac{1}{2}$

$15\frac{1}{2} \div 20\frac{2}{3} = \frac{31}{2} \div \frac{62}{3} = \frac{31}{2} \times \frac{3}{62} = \frac{3}{4}$

$\frac{3}{4}$ replaces $d$. The farmer sold $\frac{3}{4}$ gal. of honey for each gallon he had. So we say that he sold $\frac{3}{4}$ of his supply.

When you solved Problem A, you compared the gallons that were sold with all the gallons. You compared part of a group with the whole group.

B Mr. Henry planted 34 acres in corn. Mr. Jackson planted 51 acres in corn. The amount of land Mr. Henry planted in corn was what fraction of the amount of land Mr. Jackson planted?

Problem B really asks you to find what fraction of an acre Mr. Henry planted for each acre that Mr. Jackson planted.

Again make an equation of ratios. Each ratio will compare acres planted by Mr. Henry with acres planted by Mr. Jackson.

$$\frac{34}{51} = \frac{a}{1}$$

Ratio expressing the comparison for ■ acres

Ratio expressing the comparison for ■ acre

How is the equation below found?

$51a = 34$

$34 \div 51 = \frac{2}{3}$

$\frac{2}{3}$ replaces $a$. For each acre of land that Mr. Jackson planted, Mr. Henry planted ■ acre. Mr. Henry planted ■ as much land as Mr. Jackson.

When you solved Problem B, you compared Mr. Henry's 34 acres with Mr. Jackson's 51 acres. You compared two groups.

C One morning a milkman started out with 25 qt. of orange juice. By noon





# 210

1 Have Problem A read. Then discuss the statement that explains the comparison of the part with the whole in terms of 1 gallon.

2 Explain that the second ratio in the equation can be made because the relationship of the fraction of a gallon to 1 gallon will be the same as the relationship of the amount sold to the total supply.

3 The pupils should see that this equation results from the use of the ratio test.

4 Make certain that the pupils understand each step of the computation.

5 Have someone read these statements. Be sure the pupils are able to interpret the answer correctly.

6 Follow the same procedure for Problem B as for Problem A. Be sure the pupils understand that the comparison in Problem B is of two groups.

he had sold $\frac{3}{5}$ of the 25 qt. How many quarts had he sold by noon?

Problem C tells you what fraction of a quart the milkman sold for each 1 qt. he had to start with. You are to find how many quarts of his total supply he had sold by noon.

Look at the equation below. Does each ratio show the comparison of quarts the milkman sold with quarts he had?

$$\frac{t}{25} = \frac{3}{5} \quad \text{(1)}$$

Ratio expressing the comparison for 1 qt.

Ratio expressing the comparison for 25 qt. (2)

How do you get the equation below?

$t = \frac{3}{5} \times 25$

$\frac{3}{5} \times 25 = 15$ (3)

15 replaces $t$. The milkman had sold ■ qt. of orange juice by noon.

In Problem C, was a part of a group compared with the whole group, or were two groups compared?

D Mrs. Price bought 18 yd. of cloth. Mrs. Cook said she bought $\frac{2}{3}$ as much cloth as Mrs. Price. How many yards of cloth did Mrs. Cook buy?

Does Problem D tell you what fraction of a yard Mrs. Cook bought for each yard that Mrs. Price bought? What are you to find?

What does each ratio in the equation below show?

$$\frac{\frac{2}{3}}{1} = \frac{w}{18}$$

Ratio expressing the comparison for ■ yd.

Ratio expressing the comparison for ■ yd.

Is the equation below correct?

$w = \frac{2}{3} \times 18$

$\frac{2}{3} \times 18 = 12$

What numeral replaces $w$? Mrs. Cook bought ■ yd. of cloth.

In Problem D, was a part of a group compared with the whole group, or were two groups compared?

E On a trip, Mr. Henry used $\frac{5}{8}$ of a full tank of gasoline. He used $12\frac{1}{2}$ gal. How many gallons does the gasoline tank hold? (7)

You know what fraction of a gallon Mr. Henry used for each 1 gal. he had. You know how many gallons he used. So you can make an equation of ratios that compare the gallons Mr. Henry used with the gallons he had.

$$\frac{\frac{5}{6}}{1} = \frac{12\frac{1}{2}}{n}$$

Ratio expressing the comparison for 1 gal.

Ratio expressing the comparison for the total number of gallons

Now turn the page.



Is the equation below correct?

$\frac{5}{6}n = 12\frac{1}{2}$

$12\frac{1}{2} \div \frac{5}{6} = 15$ (1)

What numeral replaces $n$? Mr. Henry's gasoline tank holds ■ gal.

In Problem E, was a part of a group compared with the whole group, or were two groups compared?

F Don has a collection of 200 pennies. He says that he has $\frac{2}{3}$ as many pennies as Andy. How many pennies does Andy have?

Do the ratios in the equation below express the comparison of Don's pennies with Andy's pennies?

$$\frac{\frac{2}{3}}{1} = \frac{200}{k}$$

Ratio expressing the comparison for ■ penny

Ratio expressing the comparison for the total number of Andy's pennies

Is the equation below correct?

$\frac{2}{3}k = 200$

$200 \div \frac{2}{3} = 300$

What numeral replaces $k$? Andy has ■ pennies.

In Problem F, was a part of a group compared with the whole group, or were two groups compared?

G Mrs. Price bought 15 yd. of cloth. She used $3\frac{3}{4}$ yd. of this cloth for a dress. The cloth she used was what fraction of all the cloth she bought? (4)

THINK One ratio you use to solve Problem G is $3\frac{3}{4}$ over 15. What is the other ratio?

H Pat built 20 model ships. Jim built $\frac{1}{4}$ as many ships as Pat. How many model ships had Jim built?

THINK One ratio you use to solve Problem H is $\frac{1}{4}$ over 1. What is the other ratio?

I John Price spent $8 for baseball equipment. This amount was $\frac{2}{5}$ of his savings. How much had he saved before he bought the baseball equipment?

THINK To solve Problem I, use $\frac{2}{5}$ over 1 as one ratio. Why can you use either 8 over $r$ or 800 over $r$ as the other ratio?

J Jim Cook said that in three months he had saved $7. He also said that this amount was $\frac{1}{8}$ of the amount he had earned during the three months. How much had he earned during the three months?

K Mrs. Stone baked 36 cookies one Saturday. She said, "This is only $\frac{3}{4}$ as many cookies as I baked on Tuesday." How many cookies had she baked on Tuesday?



# 211

1 Have Problem C and the first explanatory statement read. Be sure the pupils understand that the relationship of the amount sold to the total amount is the same as the relation of the fraction of a quart to 1 quart.

2 Let someone read the text and answer the question concerning the equation.

3 The pupils should know that the second equation resulted from using the ratio test.

4 Let pupils discuss the computation.

5 Help the pupils see that in Problem C a part of the group was compared with the whole group.

6 Follow the same procedure for Problem D as for Problem C, but point out that in Problem D two groups are compared.

7 In Problem E, help the pupils see that the first term of each ratio represents the number of gallons used and that the second term represents the number of gallons in all.

# 212

1 Let pupils explain the equation that resulted from using the ratio test.

2 Discuss the computation.

3 Pupils should understand that Problem F is the same type as Problem E, except that two groups are compared.

4 Help pupils analyze Problems G to K. Have them write an equation of ratios on the chalkboard for each problem, then apply the ratio test and solve the problems. They should be able to explain, for each problem, whether two groups are compared or whether a part of the group is compared with the whole group.

L On Monday, Mrs. Stone baked 60 cookies. She baked what fraction as many cookies on Saturday as she did on Monday?

M Mr. Parker had a pasture with an area of $6\frac{1}{4}$ acres. He plowed $\frac{3}{5}$ of the pasture to make a garden. How many acres did he plow for the garden?

N Mr. Parker's garden is $\frac{1}{3}$ as large as another field that is planted in potatoes. How many acres are in the field of potatoes?

O One winter Rock Falls had 21 in. of snow. Garden City, 20 mi. from Rock Falls, had only 14 in. of snow that winter. What fraction can be used to express the comparison of the snowfall in Garden City that winter with the snowfall in Rock Falls?

P A baker made 120 small cakes. He put chocolate frosting on $\frac{5}{8}$ of them. He put chocolate frosting on how many of the small cakes?

Q A road builder had $20\frac{3}{5}$ mi. of road to build. At the end of 8 months, he had finished $\frac{1}{2}$ of the road. How many miles of road had he built then?

R Nancy said her new dress cost $\frac{1}{2}$ as much as her new coat. Her coat cost $34. How much did her dress cost?

S Nancy spent 75¢ of her allowance for magazines. Her allowance is $2.50 per week. She spent what fraction of her allowance for magazines?

Now you should be able to solve comparison problems in which you compute with fractions.

## Keeping skillful

Find the area of each parallelogram whose dimensions are given below.

A base, 2 ft.; altitude, 18 in.
B base, $1\frac{3}{5}$ mi.; altitude, $1\frac{1}{8}$ mi.
C base, 15 yd.; altitude, 20 yd.
D base, $3\frac{1}{2}$ yd.; altitude, $1\frac{1}{4}$ yd.
E base, 35 in.; altitude, 2 ft.
F base, $2\frac{3}{10}$ mi.; altitude, $1\frac{7}{10}$ mi.
G base, $4\frac{7}{12}$ in.; altitude, $2\frac{3}{4}$ in.
H base, 50 yd.; altitude, 27 ft.
I 40-ft. square
J base, 4 mi.; altitude, $3\frac{1}{2}$ mi.
K base, 60 in.; altitude, 2 yd.
L base, $1\frac{1}{3}$ in.; altitude, $1\frac{7}{8}$ in.
M $5\frac{1}{4}$-ft. square
N base, 3 mi.; altitude, $7\frac{1}{3}$ mi.
O base, 4 in.; altitude, $4\frac{1}{2}$ ft.
P $17\frac{1}{8}$-ft. square

213

# 213

1 Assign Problems L to S as independent written work. Suggest to the pupils that they use an explanation such as was used for Problems A to F to help them in making the correct equations.

2 Verify each pupil's work so that corrections may be made and reteaching done, if necessary.

3 Assign the "Keeping skillful" exercises as a separate lesson. For each exercise have pupils make an equation of ratios. Provide answers so pupils may verify their work.

# 214-215 Exploring problems

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to make equations for sentences that involve the use of fractions to compare one number with another.

**Vocabulary**

New words page 214 iceberg, ocean*, estimated, floating*, scientist; page 215 drifted*, southward*

**Comments**

The use of ratio equations in the *Seeing Through Arithmetic* program makes it unnecessary to teach the traditional distinctions among the "three cases" of fractions. However, in standardized tests and abstract practice, the pupils will encounter sentences and questions written in such language as: 270 is what fraction of 2430? $\frac{9}{10}$ of 2430 is what? 420 is $\frac{1}{8}$ of what?

This lesson is planned to help pupils interpret such sentences. First they solve problems of the type they studied in the preceding lesson. Abstract sentences are written for these problems. Then they are given problems and asked to decide which of two sentences correctly describes the situation. Finally, the pupils are given abstract sentences for which they are to write and solve ratio equations.

**Answers**

Exploring problems:

(block 1)

A $270/2430 = [\frac{1}{9}]/1$
B $\frac{9}{10}/1 = [2187]/2430$
C $\frac{1}{8}/1 = 420/[3360]$
D $\frac{1}{9}/1 = 200/[1800]$; Sentence 1
E $20{,}000/1{,}500{,}000 = [\frac{1}{75}]/1$; Sentence 2
F $\frac{3}{5}/1 = [108]/180$; Sentence 1

(block 2)

A $\frac{7}{10}/1 = [420]/600$
B $165/660 = [\frac{1}{4}]/1$
C $\frac{4}{5}/1 = 12\frac{1}{2}/[15\frac{5}{8}]$

D $\frac{3}{4}/1 = [39\frac{3}{4}]/53$
E $\frac{3}{4}/1 = 48/[64]$
F $8\frac{3}{4}/17\frac{1}{2} = [\frac{1}{2}]/1$
G $225/675 = [\frac{1}{3}]/1$
H $\frac{7}{10}/1 = [10\frac{1}{2}]/15$
I $\frac{7}{8}/1 = 63/[72]$
J $114/138 = [\frac{19}{23}]/1$
K $\frac{7}{8}/1 = [1100\frac{3}{4}]/1258$
L $\frac{9}{10}/1 = 73/[81\frac{1}{9}]$
M $\frac{1}{4}/1 = 82/[328]$
N $115/575 = [\frac{1}{5}]/1$

**Keeping skillful:**

(block 1)

A 6184
B 122,010
C 10,061
D 5512
E 39,485
F 10,001
G 22,566
H 43,044
I 49,937
J 1886
K 8024
L 101,701
M 12,002
N 7289
O 96,661

(block 2)

A 45.526
B 14.61
C 712.8
D 21.69
E 1000.992
P 88,000
Q 5792
R 9185
F 22.510
G 8308.4
H 2087.4
I 705.714

## Exploring problems

A The height of an iceberg above the ocean surface was about 270 ft. Its total height was estimated as 2430 ft. The height above the ocean was what fraction of the total height?

THINK Problem A asks you to find what fraction 270 is of 2430. You can write this equation for the problem.

$$\frac{270}{2430} = \frac{d}{1}$$ ← Fraction of a foot above the water for each foot of total height

You can divide 270 by 2430 to get the answer.
270 is what fraction of 2430?

B If the iceberg described in Problem A had been floating in fresh water, about $\frac{9}{10}$ of it would have been below water. About how much of the estimated total height would have been below water?

THINK Problem B asks you to find the number that is $\frac{9}{10}$ of 2430.

$$\frac{\frac{9}{10}}{1} = \frac{k}{2430}$$ ← Number of feet below water

You can multiply 2430 by $\frac{9}{10}$ to find the answer.
$\frac{9}{10}$ of 2430 is ■.

C The height above water of a large iceberg was about 420 ft. A scientist estimated that this was only $\frac{1}{8}$ of the total height. What was the approximate height of the iceberg? 4

THINK 420 is $\frac{1}{8}$ of what number?

$$\frac{\frac{1}{8}}{1} = \frac{420}{t}$$ ← Number of feet in total height

You can divide 420 by $\frac{1}{8}$ to find the answer.
420 is $\frac{1}{8}$ of ■.

D The height above water of another iceberg was estimated as 200 ft. If you estimate the height above the water as $\frac{1}{3}$ of the total height, what was the total height of this iceberg? 5

THINK What equation can you make to find the answer for Problem D?

Which of these two sentences is correct for Problem D?
1 200 is $\frac{1}{3}$ of ■.
2 $\frac{1}{3}$ of 200 is ■.

E The weight of one large iceberg was estimated as 1,500,000 tons. At one time an estimated 20,000 tons of ice fell from this iceberg. The weight of the ice that fell was what fraction of the total weight of the iceberg?



# 214

1 Point out to the pupils that in the preceding lesson they learned to make an equation for this type of problem. Let someone use the ratio test and solve the resulting equation. Point out that the answer is the same as that obtained by the short cut of dividing 270 by 2430.
2 Be sure the pupils see that the answer for the problem also answers this question, since the question interprets the problem.
3 Adapt the procedure in Notes 1 and 2 for Problem B. Again, be sure the pupils understand how the sentence is related to the problem and the equation.
4 Proceed in much the same way with Problem C. Relate the sentence to the problem and the equation.
5 Use these problems for class discussion. Have the pupils read the problem, make and solve the equation, and decide which sentence correctly interprets the problem.

THINK What equation can you make to find what fraction 20,000 is of 1,500,000?

Which of these two sentences is correct for Problem E?

1 1,500,000 is what fraction of 20,000?

2 20,000 is what fraction of 1,500,000?

F The iceberg described in Problem E drifted southward. Its length one Monday was estimated as 180 ft. On Wednesday, its length was estimated as $\frac{3}{5}$ of its length on Monday. What was its estimated length on Wednesday?

THINK What equation can you make to find $\frac{3}{5}$ of 180?

Which of these two sentences is correct?

1 $\frac{3}{5}$ of 180 is ■.

2 180 is $\frac{3}{5}$ of ■.

For each exercise below, first make an equation of ratios. Then find the answer.

A $\frac{7}{10}$ of 600 is ■.

B 165 is what fraction of 660?

C $12\frac{1}{2}$ is $\frac{5}{8}$ of ■.

D $\frac{3}{4}$ of 53 is ■.

E 48 is $\frac{3}{4}$ of ■.

F $8\frac{3}{4}$ is what fraction of $17\frac{1}{2}$?

G 225 is what fraction of 675?

H $\frac{7}{10}$ of 15 is ■.

I 63 is $\frac{7}{8}$ of ■.

J 114 is what fraction of 138?

K $\frac{7}{8}$ of 1258 is ■.

L 73 is $\frac{9}{10}$ of ■.

M 82 is $\frac{1}{4}$ of ■.

N 115 is what fraction of 575?

Now you know how to work with problems and sentences that use fractions to compare one number with another number.

## Keeping skillful

A v + 14862 = 21046
B 89166 + 32844 = s
C a − 9215 = 846
D 7000 − t = 1488
E 9236 + k = 48721
F 11000 − 999 = m
G r + 27535 = 50101
H 45147 − b = 2103
I 4890 + 45047 = c
J 41731 − 39845 = d
K n − 6476 = 1548
L 96822 + 4879 = k
M 6340 + w = 18342
N 7403 − t = 114
O b − 95865 = 796
P 87001 + 999 = f
Q 70507 + y = 76299
R g − 8373 = 812
A z − 42.831 = 2.695
B r + 695.40 = 710.01
C 4611.5 − 3898.7 = t
D 178.36 + d = 200.05
E 986.005 + 14.987 = x
F f − 1.076 = 21.434
G 8492.3 − b = 183.9
H 6936.1 + q = 9023.5
I 735.729 − 30.015 = c

215

# 215

1 Use this problem for class discussion in the way you used Problems D and E.

2 Have the pupils make the equations and find the answers for Exercises A to N. Then discuss the exercises and have pupils replace the screens with the answers.

3 Assign the "Keeping skillful" exercises as a separate lesson.

# 216-217 Thinking straight

Expanded Notes for this lesson are on pages 253-255.

**Objectives**

The pupil uses decimal fraction numerals to express precision in measurement.

**Vocabulary**

New words page 216 magnified; page 217 indicating*, precise*, wire*

**Comments**

Although this lesson has a threefold purpose, it is primarily concerned with the idea of precision in measurement. Besides introducing the concept of precision, the lesson provides experiences with using decimal fraction numerals in measuring situations, and also provides opportunity to round off numbers.

It is important for the pupils to realize that the results of measurement are never exact. Exactness is possible only in a counting situation. If a person counts the apples on the table and says there are 6, he means *exactly* 6 apples, no more and no fewer. But when he says these 6 apples weigh 2 pounds 3 ounces, he does not mean this is *exact*. He is giving an approximation of the weight that here is correct to the nearer ounce.

Measurement provides only an approximation because it involves a comparison with a standard. Any measuring instrument is graduated to provide a certain precision, and some instruments provide more precise measures than others of the same type. A ruler graduated to sixteenths of an inch can be read more precisely than one graduated to fourths or eighths of an inch. In this lesson the pupils work with rulers that provide measures of different precision. However, the teacher should point out that the idea of precision applies to reading all measuring instruments—scales, clocks, odometers, etc. If some of these instruments are available, pupils may experiment with them in the classroom.

Decimal fraction numerals are convenient to use to express the precision of measures. The difference be-

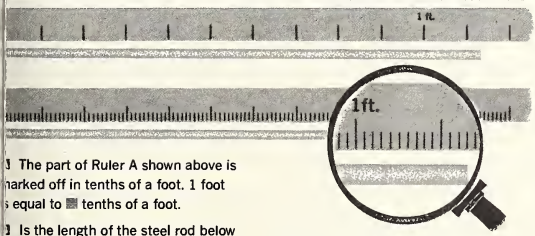
tween 1.0′ and 1.00′ is a difference in precision (1.0′ shows a measure precise to the nearer tenth of a foot; 1.00′ to the nearer hundredth.) Also, measures expressed with decimal fraction numerals are in a convenient form to round off to the nearer hundredth, tenth, etc.

The third purpose of this lesson is to give pupils practice in rounding off numbers. The pupils are shown that the principles of rounding off that apply to whole numbers also apply to numbers expressed by decimal fraction numerals. They see, for example, that .47 can be rounded up to .5 just as 47 can be rounded up to 50, or that 3.13 can be rounded down to 3.1 just as 313 can be rounded down to 310. The pupils also learn that .15 in., for example, is halfway between .1 in. and .2 in.; or that .365 in. is halfway between .36 in. and .37 in. From these experiences in measurement situations, the pupils ought to generalize that one half of .1 is .05, that one half of .01 is .005, and that one half of .001 is .0005.

**Answers**

A 10 tenths
B 1.1 ft.; yes
C 1 ft.
D 100 hundredths; 10 hundredths
E 1.13 ft.; yes
F Tenths; 1.1 ft.
G Yes
H 1.07 ft.; yes
I Yes; .06
J Nearer to 1.0 ft.
K Nearer tenth
L Yes; yes; yes
M In parts of watches, instruments in satellites, airplanes, etc.
N Yes; yes; yes; yes; yes
O Yes, yes; no; yes
P Tenths

## Thinking straight




A The part of Ruler A shown above is marked off in tenths of a foot. 1 foot is equal to ■ tenths of a foot.

B Is the length of the steel rod below Ruler A nearer to 1.1 ft. or to 1.2 ft.? Is the rod about 1.1 ft. long?

C When you say that the rod is 1.1 ft. long, you are giving the length to the nearer tenth of a foot. What is the length of the rod to the nearer foot?

D Look at Ruler B. The part shown is marked off in hundredths of a foot. 1 foot is equal to ■ hundredths of a foot. 1 tenth of a foot is equal to ■ hundredths of a foot.

E Now look at the magnified part of Ruler B and the rod below it. Is the length of the rod nearer to 1.12 or to 1.13 ft.? Is the rod 1.13 ft. long to the nearer hundredth of a foot?

F The part of Ruler C shown on page 217 is marked off in ~~~ of a foot. What is the length of the rod below Ruler C to the nearer tenth of a foot?

G Ruler D is marked off in hundredths of a foot. Is the magnified part of this ruler also marked off in hundredths of a foot?

H Now look at the rod below the magnified part of the ruler. Is the rod nearer to 1.08 or to 1.07 ft. in length? Is it about 1.07 ft. long?

I When you describe the rod on this page and the rod on page 217 as about 1.1 ft. long, you cannot tell how much longer the first rod is than the second. When you measure the rods in hundredths of a foot, can you tell approximately how much longer the first rod is? $1.13 - 1.07 = ■$

5

Decimal fractions: precision in measurement

# 216

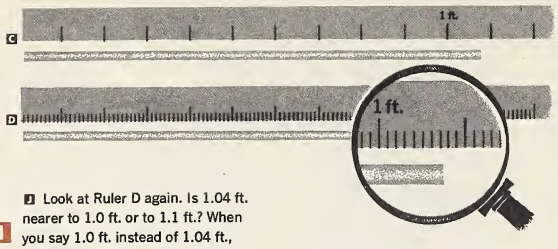
1 Have the pupils relate Exercises A and B to Picture A.

2 In connection with Exercise C, you may wish to have the pupils use rulers or yardsticks and measure the length and width of objects (desks, books, tables) to the nearer foot, the nearer inch, the nearer $\frac{1}{2}$ inch, the nearer $\frac{1}{2}$ foot, etc.

3 Have pupils note that each unit on Ruler B is only $\frac{1}{10}$ as long as each unit on Ruler A. Ask what kind of measuring unit would result if each hundredth on Ruler B could be marked off into 10 equal parts.

4 Be sure the pupils understand that the magnified part of the ruler is also marked in hundredths of an inch.

5 Be sure the pupils understand that the rulers marked in hundredths can be read more precisely than those marked in tenths.




J Look at Ruler D again. Is 1.04 ft. nearer to 1.0 ft. or to 1.1 ft.? When you say 1.0 ft. instead of 1.04 ft., you round 1.04 ft. down to 1.0 ft. When you express a measurement as 1.0 ft., you are indicating that the measurement is made to the nearer tenth of a foot. The measurement is precise to tenths of a foot. 1

K Use Ruler D to decide whether .68 ft. is nearer to .6 ft. or to .7 ft. When you say .7 ft. instead of .68 ft., you round .68 ft. up to .7 ft. Is .7 ft. precise to the nearer tenth or to the nearer hundredth of a foot?

L Is .45 ft. halfway between .40 ft. and .50 ft.? Can you say that .45 ft. is about .4 ft.? Can you also say that .45 ft. is about .5 ft.? 2

M Some measuring instruments can be used to measure thousandths of a foot or thousandths of an inch. What cases can you think of where such precise measurements are needed? 3

N Suppose you are told that the diameter of a piece of wire has been measured as .02 in. to the nearer hundredth of an inch. Could the diameter of the wire be .019 in.? Could it be .024 in.? Could it be .016 in.? Could it be .020 in.? Could the diameter of the wire be anywhere between .015 and .025 in.? 4

O The thickness of a strip of metal is measured as .065 in. to the nearer thousandth of an inch. Could the thickness of the metal be .0654 in.? Could the thickness be .0647 in.? Could it be .0658 in.? Could the thickness of the metal be anywhere between .0645 in. and .0655 in.?

P With which can you measure more precisely, a ruler marked off in tenths of an inch or a ruler marked off in fourths of an inch? 5

217

# 217

1 Have the pupils observe that they had to round *down* to find the nearer tenth in Exercise J, and they had to round *up* to find the nearer tenth in Exercise K. Pupils should also observe that to find a measure *precise* to the nearer tenth of a foot, a ruler marked in hundredths was used. Ask them how they could find a measure precise to the nearer hundredth of a foot.

2 Point out that a measure halfway between two units may be rounded either up or down.

3 As answers to Exercise M, pupils may suggest parts of watches, ball bearings, etc.

4 Pupils should observe that when a measure is given to a nearer unit, it may be any measure from $\frac{1}{2}$ unit below to $\frac{1}{2}$ unit above the given measure.

5 Exercise P points out that the finer the graduation of a measuring instrument, the more precise will be the measure it gives.

# 218-221 Learning how

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to multiply numbers expressed by decimal fraction numerals.

**Vocabulary**

New words page 218 ignore*; page 221 ordinary*

**Comments**

The only new skill needed when pupils learn to multiply numbers expressed by decimal fraction numerals is that of determining the position of the decimal point in the product. Traditionally, pupils have been told to count the total number of places in the factors and to mark off this number of places in the product. There is a sound mathematical basis for this short cut; but, like many short cuts, it is meaningless to the pupils until they understand why it works.

To help the pupils understand how the position of the decimal point in the product is determined, this lesson first shows them how to find the product when the fractions are expressed by common fraction numerals. For example, when tenths are multiplied by tenths, it is easy for the pupils to see that the product of the numbers in the denominators must be expressed in hundredths.

Once the pupils have seen how the position of the decimal point in the product can be determined by the use of common fraction numerals, they are shown that they can temporarily ignore the decimal point and find the product of the whole numbers indicated by the numerators of the common fraction numerals. The position of the point in the product can be determined by finding the product of the numbers indicated by the denominators of the equivalent common fraction numerals.

Most pupils will eventually see that the decimal point can be placed in the product as many places from the right as the total of the places from the right in the factors. There is no objection to the pupils' using this computational short cut once they have developed understanding, and skill in computing is the goal.

**Answers**

Pages 218-219:

Only the equations, which the pupils are asked to supply, are given for Problems A to C.

A $7.6/1=n/4.25$ or $7.6\times4.25=n$

B $2.14/1=n/4$ or $2.14\times4=n$

C $15/1=n/.9$ or $15\times.9=n$

Page 221:

(block 1)

| | | |
|---|---|---|
| A 2853.5 | I 12.1950 | Q .006 |
| B 20.75 | J .335 | R .3024 |
| C 16.51 | K 94.32 | S 5.265 |
| D 2.45 | L 5.994 | T 11.68 |
| E 19.9958 | M .72 | U .5538 |
| F 14.940 | N 2.10 | V 4.4 |
| G .0008 | O .644 | W 43.7500 |
| H 207.5 | P 9.765 | X .588 |

(block 2)

| | | | |
|---|---|---|---|
| A 1.6192 | D 24.0 | G 190.0 | J 14.2844 |
| B 24.39 | E .06 | H 18.00 | K 225.455 |
| C .420 | F .0056 | I 1.08 | |

Keeping skillful:

(block 1)

| | | | |
|---|---|---|---|
| A $18\frac{23}{24}$ | C $14\frac{2}{3}$ | E $16\frac{19}{24}$ | G 17 |
| B $25\frac{1}{6}$ | D $19\frac{7}{8}$ | F $21\frac{11}{16}$ | H $24\frac{5}{24}$ |

(block 2)

| | | | |
|---|---|---|---|
| A 3 | D $\frac{5}{18}$ | G 7 | |
| B $17\frac{1}{2}$ | E $28\frac{11}{16}$ | H $\frac{2}{25}$ | |
| C $14\frac{1}{4}$ | F $46\frac{7}{8}$ | I $198\frac{2}{5}$ | |

(block 3)

| | | | |
|---|---|---|---|
| A 2 | D $\frac{1}{8}$ | G $\frac{93}{100}$ | |
| B $2\frac{4}{7}$ | E $6\frac{2}{15}$ | H 60 | |
| C 3 | F $6\frac{3}{4}$ | I $14\frac{1}{4}$ | |

(block 4)

| | | | |
|---|---|---|---|
| A 9.843 | E 6.5 | I 18.43 | |
| B 11.5 | F 6.45 | J 10.381 | |
| C 4.72 | G 3.983 | | |
| D 3.1695 | H 12.0 | | |

## Learning how

In this lesson you will learn how to find the product of numbers expressed with decimal fraction numerals.

**see**

A The Wests' hallway is 7.6 ft. long and 4.25 ft. wide. What is the area of the hallway in square feet?

What equation can you make?

To solve your equation, you need to multiply 4.25 by 7.6. You can find the product by using common fraction numerals.

$\frac{76}{10}\times\frac{425}{100}=\frac{32300}{1000}$

$\frac{32300}{1000}=32\frac{300}{1000}$ or 32.300

4.25
7.6

Instead of using common fraction numerals, you can write the numerals as shown at the left.

Now think of 4.25 as 425, and think of 7.6 as 76.

3

425
76
2550
29750
32300

You are multiplying the same numbers that you did when you found the product of the numerators of the fractions.

4

4.25
7.6
2550
29750
32.300

The product of 4.25 and 7.6 must be $\frac{32300}{1000}$ because the product of $\frac{425}{100}$ and $\frac{76}{10}$ is $\frac{32300}{1000}$. So you must use a point to show the thousandths.

The area of the hallway is 32.3 sq. ft.

B One quart of milk weighs about 2.14 lb. About how much does one gallon of milk weigh?

5

What equation can you make?

To solve your equation, you need to multiply 2.14 by 4. Write 2.14 and 4 as common fraction numerals and multiply.

$\frac{4}{1}\times\frac{214}{100}=\frac{856}{100}$ or 8.56

You can find the product of 2.14 and 4 by writing the numerals as shown below.

6

2.14
4
856

When you compute, ignore the point in 2.14. You know that 856 is too large for the product.



# 218

1 Have a pupil write an equation on the chalkboard for Problem A.

2 Be sure the pupils understand that the common fraction numerals and the decimal fraction numerals are just different names for the same numbers. Discuss the computation and the fact that the product can be given in either of two forms.

3 Help the pupil see that the numbers multiplied are whole numbers.

4 The pupils should see that exactly the same computation is done when the numerals are written with the decimal points included, and the position of the decimal point in the answer is determined by the product of the tenths, hundredths, etc. that the denominators indicate.

5 Proceed with Problem B in the way suggested for Problem A.

6 Explain that 856 must be too large, because 2.14 has been multiplied as though it were 214.

**1** 2.14 × 4 = 8.56

The product of 2.14 and 4 must be $\frac{856}{100}$ because the product of $\frac{214}{100}$ and $\frac{4}{1}$ is $\frac{856}{100}$. So you must use a point to show the hundredths.

One gallon of milk weighs about 8.56 lb.

**c** The Millers' sidewalk is 15 yd. long and .9 yd. wide. What is the area of the sidewalk in square yards?

**2** What equation can you make?

To solve your equation, you multiply 15 by .9. You can write 15 and .9 as common fraction numerals and then multiply.

$\frac{9}{10} \times \frac{15}{1} = \frac{135}{10}$ or 13.5

You can find the product of 15 and .9 by writing the numerals as shown below.

**3**
```
 15
 .9
---
135
```
When you compute, ignore the point in .9. You know that 135 is too large for the product.

```
 15
 .9
---
13.5
```
The product of 15 and .9 must be $\frac{135}{10}$ because the product of $\frac{15}{1}$ and $\frac{9}{10}$ is $\frac{135}{10}$. So you must use a point to show the tenths.

The area of the sidewalk is 13.5 sq. yd.

**think**

**A** Find the product of .38 and .17.

**4** You multiply ■ by ■. Are .38 and .17 correctly written below as common fraction numerals?

$\frac{38}{100} \times \frac{17}{100}$

What is the denominator of the product? The product is $\frac{\blacksquare}{10000}$.

You can find the product of .38 and .17 by computing as shown below.

```
 .38
 .17
----
 266
 380
----
.0646
```
**5** The product of 38 and 17 is 646. How do you know that 646 is too large for the product? Why must you show ten-thousandths in your answer?

You must put a zero to the left of 646 before you can put in the point to show ten-thousandths.

**B** Find the product of .306 and 18.

```
 .306
   18
-----
 2448
 3060
-----
5.508
```
When you multiply 306 by 18, you get 5508. How do you know that the product of .306 and 18 is $\frac{5508}{1000}$? Why do you put a point in 5508 between 5 and 508?



**C** Find the product of .75 and 1.5.

**1**
```
 .75
 1.5
-----
 375
 750
-----
1.125
```
Is 1125 the product of 75 and 15? Why must you show thousandths in the product?

**D** Find the product of 2.7 and 4.3.

```
 2.7
 4.3
-----
  81
1080
-----
11.61
```
When you multiply 27 by 43, you get . Why must you show hundredths in the product?

**try**

**2**

**A** .6 × 7 = v

```
  7
 .6
----
4.2
```
How do you know that the product is 4.2 and not 42?

**B** 8.2 × 1.7 = g

```
  8.2
  1.7
------
  574
  820
------
13.94
```
How do you know that the product must be expressed in hundredths?

**C** 1.3 × 12.32 = s

```
 12.32
   1.3
-------
  3696
 12320
-------
16.016
```
The product of a number expressed in hundredths and a number expressed in tenths must be expressed in ~~~.

**D** 1.35 × .15 = p

```
 1.35
  .15
------
  675
 1350
------
.2025
```
Is the point in the product in the correct position? How do you know?

**E** .106 × .3 = b

```
 .106
   .3
------
.0318
```
How do you know that the product is not 318? Why must you put a zero to the left of 318 before you can put in the point?

**F** 6 × .092 = r

```
 .092
    6
------
 .552
```
The product of a whole number and a number expressed in thousandths must be expressed in ~~~.





# 219

1. Be sure the pupils see that the numbers multiplied are whole numbers, but that the point should be put in the answer because when the product is found by using common fraction numerals, the denominator expresses hundredths. Thus the answer must be expressed as hundredths.
2. Have a pupil write the equation for finding the area of the sidewalk.
3. Help pupils see why 135 is too large [because .9 is less than 1, and consequently the answer must be less than 15]. Help them understand that the answer must be expressed in tenths because the whole number is multiplied by a number expressing tenths.
4. Use the examples in the THINK step for class discussion. Pupils may take turns answering the questions.
5. Be sure the pupils understand why the zero is needed in the product.

# 220

1. Let pupils take turns answering the questions for Examples C and D. Make sure they understand why each point is placed where it is.
2. Assign Examples A to F in the TRY step as written work. Suggest to the pupils that they cover the computation for each example until they have worked the example themselves. Tell the pupils to write the numerals in correct computational form and find the answer. Then have them compare their work with that in the book. When the class has finished, let pupils take turns giving their answers and explaining any examples that caused difficulty.

**do**

A Bill read that the average load per freight car in the United States is 43.9 tons. At this rate, how many tons of freight will a train of 65 freight cars carry?

$\frac{43.9}{1}=\frac{t}{65}$

B An ordinary household pail holds about 2.5 gallons of water. Water weighs about 8.3 lb. per gallon. If you fill a pail with water, about how much does the water in it weigh?

$\frac{8.3}{1}=\frac{n}{2.5}$

**2**

C $12.7\times1.3=v$
D $.35\times7=n$
E $3.278\times6.1=w$
F $4.5\times3.32=a$
G $.02\times.04=k$
H $25\times8.3=y$
I $16.26\times.75=m$
J $.67\times.5=e$
K $13.1\times7.2=s$
L $3.33\times1.8=n$
M $.8\times.9=b$
N $6\times.35=p$
O $.7\times.92=z$
P $1.55\times6.3=c$
Q $.01\times.6=h$
R $.84\times.36=q$
S $27\times.195=f$
T $1.6\times7.3=r$
U $.39\times1.42=d$
V $.4\times11=n$
W $100\times.4375=t$
X $.6\times.98=g$
A $5.06\times.32=b$
B $27.1\times.9=z$
C $.35\times1.2=m$
D $48\times.5=h$
E $.6\times.1=n$
F $.08\times.07=a$
G $50\times3.8=k$
H $72\times.25=v$
I $.6\times1.8=e$
J $21.32\times.67=y$
K $33.5\times6.73=w$

Now you should be able to multiply numbers expressed with decimal fraction numerals.

## Keeping skillful **3**

Find the sum.

A $6\frac{1}{8}, 4\frac{1}{2}, 8\frac{1}{3}$
B $7\frac{2}{3}, 9\frac{1}{2}, 8\frac{1}{4}$
C $1\frac{1}{5}, 6\frac{1}{8}, 7\frac{3}{10}$
D $5\frac{3}{4}, 7\frac{5}{8}, 6\frac{1}{2}$
E $3\frac{2}{3}, 8\frac{3}{8}, 4\frac{3}{4}$
F $9\frac{3}{8}, 7\frac{7}{8}, 4\frac{7}{16}$
G $5\frac{5}{6}, 7\frac{2}{3}, 3\frac{1}{2}$
H $9\frac{1}{2}, 4\frac{1}{3}, 7\frac{1}{8}, 3\frac{1}{3}$

A $4\frac{1}{2}\times\frac{2}{3}=m$
B $3\frac{1}{8}\times5\frac{3}{5}=y$
C $19\times\frac{3}{4}=d$
D $\frac{5}{6}\times\frac{1}{3}=r$
E $2\frac{7}{10}\times10\frac{5}{8}=z$
F $6\frac{1}{4}\times7\frac{1}{2}=b$
G $42\times\frac{1}{6}=s$
H $\frac{4}{5}\times\frac{1}{10}=g$
I $20\frac{2}{3}\times9\frac{3}{5}=k$

A $1\frac{1}{2}\div\frac{3}{4}=p$
B $6\div2\frac{1}{3}=k$
C $\frac{3}{5}\div\frac{1}{5}=v$
D $\frac{5}{8}\div5=n$
E $7\frac{3}{5}\div1\frac{1}{4}=r$
F $13\frac{1}{2}\div2=q$
G $7\frac{3}{4}\div8\frac{1}{3}=b$
H $25\div\frac{5}{12}=x$
I $19\div1\frac{1}{3}=a$

A $6.754+3.089=x$
B $12.3-.8=q$
C $17.35-a=12.63$
D $.3219+2.8476=n$
E $t+.6=7.1$
F $z-3.48=2.97$
G $14.238-10.255=b$
H $181.5+e=193.5$
I $7.39+11.04=n$
J $w+47.792=58.173$

221

# 221

1 Assign Problems A and B and Exercises C to X as written work. Instruct pupils to write the numerals in each equation in correct computational form and multiply to find the answer. Verify the pupils' work and have them make any needed corrections.

2 Use the second block of exercises (A to K) for pupils who need further practice. Provide reteaching if necessary.

3 Use the "Keeping skillful" exercises as separate lessons, adjusting assignments to the needs and abilities of individual pupils. Provide answers so that they may verify their work and make needed corrections.

# 222-223 Using arithmetic

Exponded Notes are not considered necessary for this lesson.

**Objectives**

The pupil solves problems that require the multiplication of numbers expressed by decimal fraction numerals. He takes tests in multiplying numbers expressed by such numerals.

**Vocabulary**

New words page 222 swam*, backstroke*; page 223 meters*, English*, Channel, Michigan, Chicago*, stop*, course*, woman*

**Comments**

In this lesson the pupils apply their knowledge of multiplying numbers expressed by decimal fraction numerals to problem-solving equations. Be sure the pupils can write a ratio equation for each problem. However, if a pupil is capable of writing the equation he would get by using the ratio test without making a ratio equation first, and he understands what he is doing, permit him to write the short-cut form immediately. Also make sure the pupils can explain how the position of the decimal point in each product is determined.

The three "Checking up" tests can be used to test the pupils' understanding of and skill in multiplication of numbers expressed by decimal fraction numerals.

Assign as many exercises of each test as you see fit. After the pupils have done the work on one test, provide answers and discuss any exercises that have caused difficulty before assigning the next test. If any reteaching is needed, refer to pages 218-221.

**Answers**

A $60/1=[840]/14$ or $14\times60=[840]$; 840 ft.
B $14.8/1=[59.2]/4$ or $4\times14.8=[59.2]$; 59.2 sec.
C $3/1=[80.4]/26.8$ or $3\times26.8=[80.4]$; 80.4 sec.
D $.74/1=[23.976]/32.4$ or $.74\times32.4=[23.976]$; 23.976 yd.; about 24 yd.
E $.975/1=[.78]/.8$ or $.975\times.8=[.78]$; .78 yd.
F $.38/1=[14.06]/37$ or $37\times.38=[14.06]$; 14.06 yd.
G $7.6/1=[98.8]/13$ or $13\times7.6=[98.8]$; 98.8 ft.

H 12,148.9/1=[72,893.4]/6 or 12,148.9×6=[72,893.4]; 72,893.4 gal.

I 1.094/1=[109.4]/100 or 1.094×100=[109.4]; 109.4 yd.

J .655/1=[576.4]/880 or .655×880=[576.4]; 576.4 sec.

K .7/1=[14]/20 or .7×20=[14]; 14 hr.

L 1.3/1=[36.4]/28 or 1.3×28=[36.4]; 36.4 mi.

M 2880/1=[82,080]/28.5 or 28.5×2880=[82,080]; 82,080 strokes

Checking up:

Test 1

| | | | |
|---|---|---|---|
| A 81.00 or 81 | E 216.92 | I 20.0044 | M 2234.4 |
| B 15.2 | F 277.8 | J 243.25 | N 55.800 or 55.8 |
| C 4034.88 | G 482.848 | K 607.68 | O 19.182 |
| D .660 or .66 | H 69.7 | L 152.097 | |

Test 2

| | | | |
|---|---|---|---|
| A .5625 | E 19.4832 | I 1.8395 | M .9632 |
| B 48.493 | F 30.52 | J 13.3651 | N 20.01 |
| C 7.3676 | G 1.5525 | K 23.90 or 23.9 | O .02448 |
| D 64.7153 | H 16.089 | L 7.008 | |

Test 3

| | | | |
|---|---|---|---|
| A .062 | F .0729 | K .0903 | |
| B .096 | G .0690 or .069 | L .0090 or .009 | |
| C .0294 | H .076 | M .0361 | |
| D .0783 | I .0896 | N .0006 | |
| E .0988 | J .0686 | O .0030 or .003 | |

Problems incorrect.

L 32 miles

M 22.1

3600

1326

663

79,200 strokes

22.1 × 2 = 44.2

− 32.0

12.2 mi

## Using arithmetic

**1**

A The Stone School swimming pool is 60 ft. long. Jim swam 14 lengths of the pool. How many feet did he swim?

You can make an equation of ratios for Problem A.

$$\frac{60}{1}=\frac{t}{14}$$

How do you find the numeral that replaces $t$?

B It takes Jim 14.8 seconds to swim 1 length of the pool. At this rate, how many seconds does it take him to swim 4 lengths?

Can you make the equation below for Problem B?

$$\frac{14.8}{1}=\frac{k}{4}$$

What numeral replaces $k$?

C In a swimming test, Bob stayed under water 3 times as long as John. John stayed under water 26.8 seconds. How long did Bob stay under water?

You know that Bob stayed under water 3 seconds for every second that John stayed under water. Does the equation below show how to solve Problem C?

$$\frac{3}{1}=\frac{r}{26.8}$$

What numeral replaces $r$?

**2**

D The coach clocked Bob's swimming speed and found that it was .74 yard per second. At this rate, how many yards can Bob expect to swim in 32.4 seconds?

First write an equation of ratios for Problem D. Use $m$ to hold a place for the numeral you are to find. How do you find the numeral that replaces $m$?

When you know the numerals in both terms of one ratio, and one of these numerals is 1, do you need to write an equation of ratios?

E Pete swims faster than Bob. His average rate is .975 yards per second. How far can Pete swim in .8 second?

F The coach found that when Bob used the backstroke, he swam at a speed of .38 yd. per second. At that rate, how many yards can Bob swim in 37 sec.?

G When Bob used the side stroke, he swam 7.6 feet per stroke. How far can Bob swim in 13 strokes?

H The water in the pool is replaced at the rate of 12,148.9 gallons per hour. The coach said that the water is replaced completely every 6 hours. How many gallons of water does the pool contain?





# 222

**1** You may wish to review the writing of ratio equations and the use of the ratio test when discussing Problems A, B, and C. Have the pupils explain what each term represents.

**2** Assign Problems D to M as written work (I to M are on page 223). Instruct the pupils to write a ratio equation for each problem and solve it by using the ratio test. If any of the pupils understand the problem situation well enough so that they can write the equation resulting from the ratio test without writing the ratio equation, permit them to do this.

I In some swimming meets, distances are measured in meters. A meter is 1.094 yards. How many yards does a swimmer go in a 100-meter race?

J A world record for fast swimming was set by a man who swam 880 yards at an average rate of .655 seconds per yard. How many seconds did it take him to swim the 880 yards?

K A record for swimming the English Channel was set by a woman who swam the distance of 20 mi. at an average rate of .7 hr. per mile. How long did it take her to swim the Channel?

L A man who tried to swim across Lake Michigan from Chicago to Michigan City, first timed his rate for such a swim. His average speed was 1.3 mi. per hour. He hoped to reach Michigan City in 28 hours. How many miles did he expect to swim?

M He thought that in long-distance swimming he could average 2880 strokes per hour. He swam for 28.5 hr. before he discovered that he was off course and then had to stop. About how many strokes had he taken during the time he was swimming? 2

## Checking up 3

| Test 1 | Test 2 | Test 3 |
|---|---|---|
| A $12 \times 6.75 = p$ | A $.75 \times .75 = v$ | A $.31 \times .2 = q$ |
| B $38 \times .4 = n$ | B $6.83 \times 7.1 = h$ | B $1.6 \times .06 = y$ |
| C $42.03 \times 96 = w$ | C $11.3 \times .652 = t$ | C $.21 \times .14 = n$ |
| D $132 \times .005 = a$ | D $8.71 \times 7.43 = k$ | D $.3 \times .261 = h$ |
| E $.29 \times 748 = k$ | E $5.412 \times 3.6 = b$ | E $.76 \times .13 = w$ |
| F $.6 \times 463 = s$ | F $10.9 \times 2.8 = n$ | F $.27 \times .27 = r$ |
| G $79 \times 6.112 = y$ | G $6.21 \times .25 = d$ | G $3.45 \times .02 = c$ |
| H $4.1 \times 17 = e$ | H $9.3 \times 1.73 = s$ | H $.19 \times .4 = z$ |
| I $52 \times .3847 = m$ | I $.065 \times 28.3 = w$ | I $.32 \times .28 = n$ |
| J $35 \times 6.95 = r$ | J $4.27 \times 3.13 = p$ | J $.098 \times .7 = s$ |
| K $12.66 \times 48 = z$ | K $119.5 \times .2 = a$ | K $.21 \times .43 = a$ |
| L $419 \times .363 = n$ | L $7.3 \times .96 = m$ | L $.06 \times .15 = p$ |
| M $76 \times 29.4 = c$ | M $.688 \times 1.4 = z$ | M $.19 \times .19 = k$ |
| N $.186 \times 300 = q$ | N $2.3 \times 8.7 = e$ | N $.03 \times .02 = t$ |
| O $.278 \times 69 = k$ | O $.048 \times .51 = h$ | O $.06 \times .05 = a$ |

End-of-block tests on multiplication of decimal fractions 223

# 223

1 Have pupils continue solving the problems.
2 When the pupils have completed their work, provide equations and solutions for verification. Discuss any problems that caused difficulty. Be sure the pupils can explain how the position of the decimal point in their answers is determined.
3 Assign these tests as time permits. Discuss any exercises that the pupils find difficult. Do any reteaching that is necessary.

# 224 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil extends his knowledge of multiplying numbers expressed by powers of ten.

**Vocabulary**

There are no new words.

**Comments**

Pupils studied multiplication of whole numbers expressed by powers of ten on page 136. In this lesson multiplication of numbers expressed by powers of ten is extended to numbers expressed by decimal fraction numerals.

The pupils know that the effect of multiplying a whole number by 10 is shown by annexing zero to the numeral, for example, $43 \times 10 = 430$. This example could also be written as $43.0 \times 10 = 430.0$ or 430. Although decimal points are not used when writing numerals that express whole numbers, the point in the product of a whole number and 10 can be thought of as being one place to the right of the point in the original number. This generalization holds for multiplication involving numbers expressed by decimal fraction numerals.

This lesson shows that decimal fraction numerals are an extension of what the pupil has learned about the number system with base ten. The pupil will use the concepts and skills learned in this lesson when he divides numbers expressed with decimal fraction numerals.

**Answers**

A 1 (Example A); (Example F); 3 (Example K)
B Yes (Examples B and P)
C Yes (Examples H and Q)
D Yes (Examples N and R)  E Yes (Example S)
F Tenths (Example C)  G Hundredths (Example D)
H Tenths (Example I)

(block 2)

| | | | |
|---|---|---|---|
| A 635 | D 443.25 | G 100.58 | J 469 |
| B 279.8 | E 76 | H 345 | K 1 |
| C 3 | F 710 | I 1390 | L 3227 |

## 1 Thinking straight

2

A $43 \times 10 = 430$
B $4.3 \times 10 = 43.0$ or 43
C $.43 \times 10 = 4.30$ or 4.3
D $.043 \times 10 = .430$ or .43
E $.0043 \times 10 = .0430$ or .043
F $43 \times 100 = 4300$
G $4.3 \times 100 = 430.0$ or 430
H $.43 \times 100 = 43.00$ or 43
I $.043 \times 100 = 4.300$ or 4.3
J $.0043 \times 100 = .4300$ or .43
K $43 \times 1000 = 43000$
L $4.3 \times 1000 = 4300.0$ or 4300
M $.43 \times 1000 = 430.00$ or 430
N $.043 \times 1000 = 43.000$ or 43
O $.0043 \times 1000 = 4.3000$ or 4.3
P $14.3 \times 10 = 143.0$ or 143
Q $1.43 \times 100 = 143.00$ or 143
R $.143 \times 1000 = 143.000$ or 143
S $.0143 \times 10000 = 143.0000$ or 143

Use the examples above to answer the following questions.

3

A How many zeros do you put at the right of a whole number when you multiply the whole number by 10? By 100? By 1000?

B When a decimal fraction expressed in tenths is multiplied by 10, is the product a whole number?

C When a decimal fraction expressed in hundredths is multiplied by 100, is the product a whole number?

D When a decimal fraction expressed in thousandths is multiplied by 1000, is the product a whole number?

E When a decimal fraction expressed in ten-thousandths is multiplied by 10,000, is the product a whole number?

F When a decimal fraction expressed in hundredths is multiplied by 10, what decimal fraction is in the product?

G When a decimal fraction expressed in thousandths is multiplied by 10, what decimal fraction is in the product?

H When a decimal fraction expressed in thousandths is multiplied by 100, what decimal fraction is in the product?

4

Multiply each number below by 10.
A 63.5 B 27.98 C .3 D 44.325

Multiply each number below by 100.
E .76 F 7.1 G 1.0058 H 3.45

Multiply each number below by 1000.
I 1.39 J .469 K .001 L 3.227

224 Multiplication of decimal fractions by powers of 10

# 224

1 Review the short cuts on page 136, which show multiplication involving multiples of ten.

2 Have the pupils verify the examples shown at the top of the page, in which multiplication involves numbers expressed in decimal fraction notation.

3 Have the pupils answer the questions in Exercises A to H and discuss with them the groups of examples at the top of the page. Develop the generalizations and short cuts they can make.

4 Assign Exercises A to L as written work. Be sure the pupils can explain how the position of the decimal point in each product numeral is determined.

# 225 Thinking straight

Expanded Notes for this lesson are on pages 357-358.

**Objectives**

This lesson extends the idea that both dividend and divisor can be divided by the same number without affecting the quotient. The pupil studies its application to numbers expressed by decimal fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

To understand division involving numbers expressed by decimal fraction numerals, the pupil must realize that the same quotient may result from many different divisions, and that certain ways of expressing the division make it easier to locate the point in the quotient. In selecting an appropriate divisor and dividend, he must recognize that multiplying or dividing the numbers represented by the dividend and divisor by the same number does not affect the quotient. For example, $168 \div 12$, $336 \div 24$, $84 \div 6$, $1.68 \div .12$, $16800 \div 1200$, etc., all express the same division and, hence, the same quotient.

To develop this concept, the lesson provides a series of experiences in which the pupil first recognizes that he can multiply and divide both dividend and divisor by the same number without changing the quotient. Then he applies this concept to dividends and divisors expressed by decimal fraction numerals. Note that the pupils should *not* be required to compute in this lesson.

**Answers**

(block 1)

A By dividing both 168 and 12 by 12 B 2; yes
C Yes D 100; yes E .168 and .012; 14
F Both 168 and 12 have been multiplied by .0001, and this multiplication does not change the quotient.
G Yes; 12

(block 2)

A [I and P] B [E, J, Q, and R]
C [G, L, M, and T] D [H, K, and S]

## Thinking straight

A $168 \div 12 = n$ **1**

B $\frac{168}{12} = \frac{14}{1} = 14$

C $\frac{168 \times 2}{12 \times 2} = \frac{336}{24} = \frac{14}{1} = 14$ **2**

D $1.68 \div .12 = n$ **3**

E $\frac{1.68 \times 100}{.12 \times 100} = \frac{168}{12} = \frac{14}{1} = 14$

A The first part of Example B shows another way to write the division in Example A. How is 14 over 1 found?

B In Example C, both the 12 and the 168 are multiplied by ■. Do $336 \div 24$ and $168 \div 12$ have the same quotient?

C If you multiply 168 and 12 by .01, do you get the numbers in Example D?

D Look at Example E. 1.68 and .12 have been multiplied by ■. Is the quotient of $1.68 \div .12$ the same as the quotient of $168 \div 12$?

E If you multiply 168 and 12 by .001, what numbers do you get? What is the quotient of .168 and .012?

F Why should $.0168 \div .0012$ equal 14?

G The quotient of $1728 \div 144$ is 12. If you multiply both 1728 and 144 by .01 and then divide, will the quotient be 12? If you multiply both 1728 and 144 by .001 and then divide, what will the quotient be?

Look at Exercises A to D below.

| A | B | C | D |
|---|---|---|---|
| $\frac{64}{5}$ | $\frac{1.45}{1.9}$ | $\frac{6 \times 7}{14}$ | $\frac{3.27}{.054}$ |

For each exercise above, find others below that will have the same quotient. You should not do any computing.

E $\frac{1.45 \times 4}{1.9 \times 4}$ **5**

F $\frac{64 + 10}{5 + 10}$

G $\frac{6 \times 7 \times 10}{14 \times 10}$

H $\frac{3.27 \times 1000}{.054 \times 1000}$

I $\frac{64 \div 2}{5 \div 2}$

J $\frac{1.45 \div 3}{1.9 \div 3}$

K $\frac{3.27 \times 6}{.054 \times 6}$

L $\frac{6 \times 7 \times .01}{14 \times .01}$

M $\frac{6 \times 7 \div 2}{14 \div 2}$

N $\frac{64 \times .1}{5 \times .01}$

O $\frac{32.7}{54}$

P $\frac{6.4}{.5}$

Q $\frac{145}{190}$

R $\frac{14.5}{19}$

S $\frac{327}{5.4}$

T $\frac{.6 \times 7}{1.4}$

Extension of multiplying and dividing dividend and divisor by the same number; preparation for division of decimal fractions 225

## 225

**1** Call attention to Examples A and B. Get the pupils to explain that both the 12 and the 168 were divided by 12 to get 14/1.

**2** Let pupils verify the quotient in Example C. Ask them what would happen if the 168 and the 12 were *divided* by 2.

**3** Be sure the pupils see how Example D is related to Example A. Point out that, although they do not know how to divide numbers expressed as decimal fraction numerals, they should *expect* the quotient to be the same as the quotient for Example A, since both 168 and 12 have been multiplied by .01.

**4** Similarly, for Example E, pupils should see that the quotient for $1.68 \div .12$ is the same as the quotient for $168 \div 12$, since both dividend and divisor were multiplied by 100.

**5** Have the pupils find the exercises that have the same quotient. Be sure that in the discussion they explain how they know that the quotient will be the same.

# 226-229 Learning how

Expanded Notes for this lesson are on pages 358-359.

**Objectives**

The pupil learns how to divide numbers that are expressed by decimal fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

When dividing numbers expressed by decimal fraction numerals, the pupils apply the same procedure used for dividing whole numbers. Two additional steps are required: (1) changing the divisor to a whole number and also changing the dividend accordingly; (2) determining the position of the decimal point in the quotient.

To solve $24.48 \div 1.7 = z$ (the first example), both 24.48 and 1.7 are multiplied by 10, which results in $244.8 \div 17 = z$. Since the quotient for $24.48 \div 1.7$ is the same as the quotient for $244.8 \div 17$, the numbers in the second example are used for computing.

The computation for each example in the SEE step is shown both with common fraction numerals and decimal fraction numerals. Computing is always done with whole numbers, and the quotient is expressed by using a decimal point instead of writing 10 or a multiple of 10 in the denominator. When the divisor is a whole number, the position of the decimal point in the quotient is related to the position of the decimal point in the numeral that expresses the dividend. After studying several examples, the pupils reach the generalization that the numerals in the quotient and in the dividend have the same number of decimal places.

The traditional method teaches the placement of the decimal point by a rule. But this lesson, which first shows the computation with numbers expressed by common fraction numerals, gives the pupil reason for the position of the decimal point in the quotient.

**Answers**

Pages 227-228:

C Tenths; hundredths

A Yes; Multiply each number by 100. The number divided is a whole number.

B 100; yes; It was divided by 348. It was divided by 348. Hundredths; hundredths

C Yes; hundredths; hundredths

A The dividend and the divisor were multiplied by 100, which resulted in whole numbers.

B Yes

Page 229:

(block 1)

| | | | |
|---|---|---|---|
| A 75 | I .832 | | |
| B .06 | J 900 | | |
| C 25.4 | K .04 | | |
| D 6.12 | L .12 | | |
| E 1.32 | M 47.6 | | |
| F 7.5 | N 849 | | |
| G 17 | O .83 | | |
| H 720 | P 785 | | |

(block 2)

| | | | |
|---|---|---|---|
| A 17.2 | C 775 | E 3.6 | G 15 |
| B 3.18 | D 59.2 | F 3.4 | H 46.1 |

Keeping skillful:

(block 1)

| | | |
|---|---|---|
| A 19.506 | D 166.8 | G 2.268 |
| B 25.4329 | E .7337 | H 13.2 |
| C 20.36 | F 54.23 | I 40.12 |

(block 2)

| | | |
|---|---|---|
| A 6.61 | D .4950 | G 15.27 |
| B .438 | E 37.14 | H .845 |
| C 14.6 | F 3.9756 | I .5 |

(block 3)

| | | |
|---|---|---|
| A 17.51 | E 11.54 | I .6777 |
| B 13.7 | F .454 | J 1.332 |
| C 5.6489 | G 10.21 | |
| D 13.073 | H 88.97 | |

## Learning how

In this lesson you will learn how to divide numbers expressed by decimal fraction numerals.

### see

**A** The equation below was made for a problem.

$24.48 \div 1.7 = z$

Division will be easier if you change the divisor to a whole number. To do this, multiply 1.7 by 10. You must also multiply 24.48 by 10. $244.8 \div 17 = z$

You can divide by using common fraction numerals. $\frac{2448}{10} \div \frac{17}{1}$

$$\frac{\overset{144}{\cancel{2448}}}{10} \times \frac{1}{\cancel{17}_{1}} = \frac{144}{10} = 14.4$$

You can also divide by using decimal fraction numerals. You divide just as you do with whole numbers.

```
17)244.8 | 100
   1700
    748 |  40   You divided as though there were no tenths. The work with common fraction numerals shows that your answer should be expressed in tenths.
    680
     68 |   4
     68
      0 | 14.4 ← tenths.
```

$24.48 \div 1.7 = 14.4$

**B** The equation below was made for a problem.

$60.25 \div 5 = d$

The divisor is a whole number. You can divide by using common fraction numerals. $\frac{6025}{100} \div \frac{5}{1}$

$$\frac{\overset{1205}{\cancel{6025}}}{100} \times \frac{1}{\cancel{5}_{1}} = \frac{1205}{100} = 12.05$$

You can also divide by using decimal fraction numerals. Divide just as you do with whole numbers.

```
5)60.25 | 1000
   5000
   1025 |  200   You divided as though there were no hundredths. The work with common fraction numerals shows that your answer should be expressed in hundredths.
   1000
     25 |    5
     25
      0 | 12.05 ← hundredths.
```

$60.25 \div 5 = 12.05$

In Example B, the number you divided was expressed in hundredths. So the quotient was expressed in hundredths.



# 226

1 Have the pupils recall that multiplying the divisor and the dividend by the same number does not change the quotient. Explain why 10 is used. Have the pupils observe that $24.48 \div 1.7$ gives the same quotient as $244.8 \div 17$.

2 Discuss the computation. Point out that 2448 and 17 are crossed out because both have been divided by 17.

3 Have the pupils follow the division process using decimal fraction numerals. Point out that the dividend is treated as a whole number.

4 Compare the position of the decimal point in the quotient to the position of the point in the dividend and to the quotient obtained by using common fraction numerals.

5 Adapt Notes 1-4. Ask why a second equation need not be written. Have the pupils observe that the same process is used here as for Example A.

c This equation was also made for a problem.
2÷80=p

To solve the equation, you divide 2 by 80. Before you can divide, you need another name for 2. Use 2.000 as another name for 2.

You can now change the numerals to common fraction numerals and divide. $\frac{2000}{1000} \div \frac{80}{1}$

$$\frac{\frac{25}{2000}}{1000} \times \frac{1}{\frac{80}{1}} = \frac{25}{1000} = .025$$

You can also divide just as you do with whole numbers.

```
80)2.000 | 20
   1600  |
    400  |  5
    400  |
         | .025
```

You divided as though there were no thousandths. The work with common fraction numerals shows that your answer should be expressed in thousandths. 3

2÷80=.025
Both the number you divided and the answer were expressed in thousandths.

In Example A on page 226, the number you divided was finally expressed in tenths. The answer was also expressed in ~~~. 4
In Example B, both the number you divided and the answer were expressed in ~~~.

## think

A 201.6÷2.88=n

Can the equation below be used in place of the equation above?
20160÷288=n

What do you do to get 20160 and 288?

```
288)20160 | 60
   17280  |
    2880  |  10
    2880  |
          |  70 ← Why is no decimal fraction expressed in the answer?
```

201.6÷2.88=■

B 23.4204÷3.48=w

Before you divide, multiply both 3.48 and 23.4204 by ■.
2342.04÷348=w

Are 2342.04 and 348 correctly written below as fraction numerals? $\frac{234204}{100} \div \frac{348}{1}$

$$\frac{673}{\frac{234204}{100}} \times \frac{1}{\frac{348}{1}} = \frac{673}{100} = 6.73$$

Why was 234204 replaced by 673? Why was 348 replaced by 1?

Now turn the page. 227

Now divide 2342.04 by 348 just as you do with whole numbers.

```
348)2342.04 | 6.00
   208800   |
    25404   |  70
    24360   |
     1044   |   3
     1044   |
            | 6.73
```

The number that is divided is expressed in ~~~. The answer is also expressed in ~~~.

23.4204÷3.48=■ 1

C .702÷35.1=s

Can the equation below be used in place of the equation above?
7.02÷351=s

Now divide just as you do with whole numbers.

```
351)7.02 |  2
   702   |
         | .02
```

The number that is divided is expressed in ~~~. So the answer is expressed in ~~~.

.702÷35.1=■ 2

## try

A 8021÷24.68=p

```
2468)802100 | 300
   740400   |
    61700   |  20
    49360   |
    12340   |   5
    12340   |
            | 325
```

Why is no decimal fraction expressed in the answer?

B 3.00÷12=z

Is 3.00 another name for 3?

```
12)3.00 | 20
   240  |
    60  |  5
    60  |
        | .25
```

C 3.588÷.13=n

```
13)358.8 | 200
   2600  |
    988  |  70
    910  |
     78  |   6
     78  |
         | 27.6
```



# 227

1 Point out that by using 2.000 as a name for 2, the equation can be written 2.000÷80=p.
2 Ask the pupils to explain the computation involving common fraction numerals.
3 Now have pupils observe the division example. Relate the position of the decimal point in the quotient to the number divided and to the quotient obtained when using common fraction numerals.
4 Discuss the generalization resulting from these statements.
5 As the pupils answer the questions, have them observe that both the divisor and the number divided are whole numbers in the second equation.
6 Have pupils discuss the second equation and the computation with common fraction numerals.

# 228

1 Have the pupils explain the position of the decimal point in the quotient.
2 Get the pupils to observe that a zero is placed to the left of 2 in the quotient to express it in hundredths. Let pupils work this example, using common fraction numerals, and compare the quotient with that in the text.
3 Let pupils work Examples A to F (D to F are on page 229). Have the pupils write the equation as it first appears in the book, then write a second equation with the divisor changed to a whole number and the dividend changed accordingly, so that the quotient will be the same for both equations. (In Examples B and D, the divisors are already whole numbers.) Suggest that they cover up the computation in the book as they do each example, then verify their work by comparing it with the work in the text.

D 2.660 ÷ 35 = b

$$35\overline{)2.660}\quad 70$$
$$2450$$
$$210\quad 6$$
$$210$$
$$.076$$

E 67.50 ÷ 1.8 = h

$$18\overline{)67.50}\quad 300$$
$$5400$$
$$1350\quad 70$$
$$1260$$
$$90\quad 5$$
$$90$$
$$37.5$$

F 25.76 ÷ .28 = y

$$28\overline{)2576}\quad 90$$
$$2520$$
$$56\quad 2$$
$$56$$
$$92$$

1

## do

| | | |
|---|---|---|
| A 13.5 ÷ .18 = z | I 3.6608 ÷ 4.4 = b | A 275.2 ÷ 16 = c |
| B 28.50 ÷ 475 = w | J 5652 ÷ 6.28 = q | B 55.5546 ÷ 17.47 = n |
| C 93.98 ÷ 3.7 = k | K 3.1356 ÷ 78.39 = n | C 105.4 ÷ .136 = y |
| D 44.676 ÷ 7.3 = a | L .0144 ÷ .12 = s | D 17.76 ÷ .3 = h |
| E 11.1936 ÷ 8.48 = n | M 18.564 ÷ .39 = d | E 39.6 ÷ 11 = r |
| F 75.0 ÷ 10 = e | N 254.7 ÷ .3 = z | F .9248 ÷ .272 = f |
| G 652.8 ÷ 38.4 = v | O .2407 ÷ .29 = t | G 22.5 ÷ 1.5 = w |
| H 92.88 ÷ .129 = r | P 8368.1 ÷ 10.66 = m | H 1396.83 ÷ 30.3 = a |

Now you should be able to divide numbers expressed by decimal fraction numerals.

## Keeping skillful 2

Find the sum.

A 3.682, 7.425, 8.399
B .0037, 25.4292
C 17.93, .02, .64, 1.77
D 37.1, 19.8, 2.6, 107.3
E .1894, .3625, .1818
F 37.56, 7.39, 9.28
G .745, .631, .892
H 7.2, .6, 4.5, .9
I 19.52, 17.48, 3.12

Find the difference.

A 7.39, .78
B .055, .493
C 21.3, 6.7
D 3.7748, 3.2798
E 51.16, 14.02
F 8.5363, 4.5607
G 3.49, 18.76
H 1.784, .939
I 124.8, 125.3

A 36.46 − z = 18.95
B 6.2 + 7.5 = n
C w + .3596 = 6.0085
D a − 3.345 = 9.728
E 6.38 + m = 17.92
F 1.349 − .895 = y
G b − 6.72 = 3.49
H 72.04 + 16.93 = r
I .6832 − n = .0055
J 3.227 − 1.895 = x

229

# 229

1 When the pupils have completed the TRY examples and have verified their work, assign the first block of exercises in the DO section as individual work. Have pupils write another equation for the exercises whose divisors are not whole numbers, as they did in the TRY step, before doing the computation. Verify each pupil's work so that corrections may be made and reteaching done, if necessary. Use the second block of exercises for those pupils who need more practice.

2 Use the "Keeping skillful" exercises as separate lessons. Make the assignments according to the pupils' individual needs and abilities.

# 230-231 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns that the number of places in the quotient can be established by annexing zeros to the dividend. He also learns what to do with remainders.

**Vocabulary**

New words page 230 Sam*, shall*

**Comments**

In the preceding lesson the pupils were introduced to division of numbers expressed by decimal fraction numerals. The division examples, however, were confined to those that did not involve a remainder. In this lesson, the pupils learn what to do with remainders.

With decimal fraction numerals, it is inconsistent to use a remainder to make a common fraction numeral in the quotient. The pupil must remember that when he uses decimal fraction numerals, he should express his answer in decimal fraction numerals. If he has a remainder that is equal to or greater than one half of the divisor, he should round up the answer. He can drop the remainder without altering the answer if the remainder is less than one half of the divisor.

The pupil should always decide how many places he wants in his answer *before* he begins his computation. Gradually he will develop discrimination in this matter. In Exercises A to F, at the bottom of page 231, he is given practice in dividing so that the quotients contain a specified number of decimal places.

In connection with division involving decimal fraction numerals, there is the problem of checking the computation. The authors' position concerning checking of computation is that the best check of work is a careful recheck of each step in computation as it is performed or a recheck of the computation after it is finished. There is a special danger in using multiplication to check division when the remainder is either dropped or used to round up the quotient. Unless the pupil goes back to the quotient he had before he rounded it up

and remembers to add the remainder after he has multiplied, the check will not work. The authors recognize that checking division by multiplication reinforces the pupils' understanding that the two processes are inverse, but this idea has already been adequately established in this program. For best results, the teacher should encourage step-by-step examination of the division to ensure that the work is correct as the pupil proceeds with his computation.

**Answers**

Page 231:

(block 1)

A 4.9; because there is a 5 in hundredths' place

B To make the divisor a whole number

C To make the divisor a whole number; yes; because the remainder is more than one half of the divisor; 1.9

D 8.0; 7.99; 7.987; 623.0; 623.00; 623.000

(block 2)

A .11

B 55.3

C 10.686

D .2

E 19.86

F 15.897

## Thinking straight

I know that there will be hundredths in the answer when there are hundredths in the number I am dividing. So I know the answer is 3.83 and not 383.
A
That means that if I want hundredths in the answer, I must have hundredths in the number I am dividing. But here I have tenths. How can I get hundredths in the answer?
2
B
Well, Sam, you know that 26.30 is another name for 126.3. So you can just put a zero at the right of the 126.3.
I see! Now when I divide I will have hundredths in my answer.
C
I do have hundredths in my answer. But I also have a remainder. What shall I do with it, Miss Allen?
4
D
You can use the remainder to round off the answer. If the remainder is equal to, or greater than, one half of the divisor, you round up the answer. If the remainder is less than one half of the divisor, you ignore it. What will you do in this example?
E
Well, the remainder, 20, is greater than one half of the divisor, 26. So I round up the answer to 4.86. But if the remainder had been less than 13, I would ignore it.
F

Division of decimals: determining position of the point in the quotient, division with remainders

# 230

1 Have a pupil read what is said about the division example in Picture A. Then let another pupil work through the computation to verify the answer.

2 Call on a third pupil to read the dialogue in Picture B. Some pupils will see that to show hundredths in the answer, they must have hundredths in the dividend.

3 After the dialogue for Picture C has been read, point out that a zero has been annexed to the dividend. Be sure the class understands that the number has not been changed. The number is now expressed in hundredths rather than in tenths.

4 Have the dialogue read and the example worked through orally.

5 Let pupils read the dialogue in Pictures E and F. Point out that since the remainder is more than one half of the divisor, it is used to round up the answer.

A Now divide 126.3 by 26 so that you will have thousandths in the answer. You will need thousandths in the number you are dividing. So use 126.300 as another name for 126.3.

1

```
26)126.300|4000
  104000
   22300   800
   20800
    1500     50
    1300
     200      7
     182
      ~~18~~  ~~857~~
          |4.858
```

Round the answer up to 4.858 because 18 is greater than one half of the divisor, 26. Would the answer to the nearer tenth be 4.9 or 4.8? Why?

B Divide 69.62 by 7.25 so that you will have tenths in the answer. First multiply both 7.25 and 69.62 by 100. Why? Since you will need tenths in the number you are dividing, use 6962.0 as another name for 6962.

2

```
725)6962.0|90
   65250
    4370    6
    4350
      ~~20~~  9.6
```

You can ignore the remainder because 20 is less than one half of 725, the divisor.

C Divide 37.204 by 19.8 so that you will have hundredths in the answer. Why do you first multiply 19.8 and 37.204 by 10? Now you must find the quotient of 372.04 ÷ 198. Is the number to be divided expressed in hundredths?

3

```
198)3720.4|100
   19800
   17404   80
   15840
    1564     7
    1386
     ~~178~~  ~~187~~
          |1.88
```

Why do you round up the answer to 1.88? What would the answer be to the nearer tenth?

D Divide 623 by 78. Find the answer first to tenths, then to hundredths, and then to thousandths.

4

THINK What three names do you use for 623?

Find the answer for each exercise below. The word in parentheses tells what decimal fraction you are to have in the answer.

5

A 8.6 ÷ 75 = b (hundredths)
B 492.6 ÷ 8.9 = z (tenths)
C 897.6 ÷ 84 = n (thousandths)
D 23 ÷ 120 = s (tenths)
E 35.756 ÷ 1.8 = g (hundredths)
F 95.38 ÷ 6 = y (thousandths)

231

# 231

1 In discussing Exercise A, be sure the pupils understand that using 126.300 as another name for 126.3 will not affect the quotient. Explain why the 8 in tenths' place can be rounded up to 9.

2 Discuss why both numbers must be multiplied by 100 [to make the divisor a whole number]. Show that if the number that is being divided is expressed as tenths, this provides for an answer in tenths. Have the class note that the remainder is dropped because it is less than one half of the divisor.

3 Discuss how provision for an answer in hundredths is made. Also discuss why the answer is 1.9 to the nearer tenth.

4 Have the pupils work this exercise on paper. The class should see that for each requirement they must rework the example.

5 Assign Exercises A to F as written work.

# 232-233 Using arithmetic

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil solves problems that require division of numbers expressed by decimal fraction numerals. He tests his ability to divide numbers expressed by decimal fraction numerals.

**Vocabulary**

New words page 232 aluminum*, walnut; page 233 uranium, cork, lithium, balsa, ebony, slate

**Comments**

The purpose of this lesson is to give the pupils an opportunity to solve problems in which the data are expressed by decimal fraction numerals. Each problem situation can be described by an equation of ratios and solved by applying the ratio test.

This set of problems should make the value of ratios in problem solving obvious. Students of high-school physics are frequently required to solve problems of this type. When the students know how to make equations of ratios, such problems are greatly simplified. Thus, the sixth-grade pupil who has learned how to make equations of ratios can solve these problems.

The number of exercises in the "Checking up" tests has been limited because of the time needed to work them. Each test contains examples that represent the various types of difficulty common in division of numbers expressed by decimal fraction numerals. In each test, that is, the pupils are required to divide numbers expressed as decimal fraction numerals and whole numbers in various ways. Since both tests cover the same material, Test 1 may be given first and the pupils' learning evaluated. Then any necessary reteaching should be done. Test 2 may or may not be given, depending upon the performance of the pupils on Test 1.

**Answers**

**Using arithmetic:**

In many of these problems the pupils may write ratios that express comparisons of weight to weight and vol-

ume to volume. These ratios are not incorrect, although ratio equations that express weight per volume seem more natural and are given in the answer key.

A 2/5=[.4]/1; .4 lb.
B .4/.04=[10]/1
C 8.34/(8)=[1.04]/1; 1.04 lb.
D 848.8/62.4=[13.6]/1
E 848.8/7.48=[113.48]/1; 113.48 lb.
F 168.5/62.4=[2.7]/1
G 40.25/62.4=[.65]/1
H 62.4/1=1167.4/[18.7]; 18.7 cu. ft.
I 62.4/15=[4.2]/1
J 62.4/1=57.2/[.92]; .92 cu. ft.
K 416.25/12.5=[33.30]/1; 33.30 lb.
L 7.3/1=100/[13.7]; 13.7 cu. ft.
M 539.5/6.5=[83]/1; 83 lb.
N 62.4/1=22/[.4]; .4 cu. ft.
O 162.8/1=100/[.61]; .61 cu. ft.

Checking up:

Test 1

| | | | |
|---|---|---|---|
| A .75 | B 4.293 | C 17.778 | D 2.3 |
| E .05 | F .333 | G 687.5 | H .0220 |

Test 2

| | | | |
|---|---|---|---|
| A 2.4 | B 11.11 | C 2.403 | D .9 |
| E 7.50 | F 3.84 | G .0747 | H 30.769 |

## Using arithmetic

For the problems below, express each answer with a decimal fraction numeral if necessary. Each problem tells you how to express the answer.

**A** John had 5 cu. in. of lead that he found weighed 2 lb. What is the weight of 1 cu. in. of lead? Find the answer to tenths.

You can write an equation of ratios for Problem A. What does each term in the equation below stand for?

$$\frac{2}{5}=\frac{n}{1}$$

What numeral replaces $n$?

**B** John read in a science book that 1 cu. in. of water weighs approximately .04 lb. One cu. in. of lead is about how many times as heavy as 1 cu. in. of water?

$$\frac{.4}{.04}=\frac{s}{1}$$

Without dividing, how do you know that your answer will be a whole number? What numeral replaces $s$?

**C** John also read that 1 gal. of water weighs 8.34 lb. How much does 1 pt. of water weigh? Find the answer to hundredths.

THINK There are how many pints in 1 gallon?

In some of the following problems you will need to understand what 1 cubic foot means. Think of 1 cubic foot as meaning a cube 1 ft. long, 1 ft. wide, and 1 ft. high.

**D** John's science book also stated 4 that 1 cubic foot of water weighs 62.4 lb. and that mercury, when it is a liquid, weighs 848.8 lb. per cubic foot. Mercury is how many times as heavy as water? Find your answer to tenths.

**E** John's teacher said that there are 7.48 gal. in 1 cubic foot. How much will 1 gal. of mercury weigh? Find the answer to hundredths.

THINK How do you know from Problem D that 7.48 gal. of mercury weigh 848.8 lb.?

**F** Aluminum weighs 168.5 lb. per cubic foot. This metal is how many times as heavy as water? Find the answer to tenths.

THINK Which problem tells you the weight of 1 cubic foot of water?

**G** 1 cubic foot of walnut wood weighs 40.25 lb. The weight of 1 cubic foot of walnut wood is what fraction of the weight of 1 cubic foot of water? Find the answer to hundredths.



# 232

1 Have Problem A read and discuss the equation of ratios. Be sure the pupils understand what each of the terms represents.

2 Have the pupils read Problem B and observe that the answer to Problem A is one of the terms in the first ratio for this problem. Help the pupils see why the answer will be a whole number.

3 For Problem C, remind the pupils that 8 pints equal 1 gallon. Have the pupils give the equation of ratios $8.34/(8)=n/1$ and solve it.

4 Assign Problems D to O (H to O are on page 233) as written work. Instruct the pupils to write an equation of ratios for each problem. Give individual help where it is needed.

1

H Uranium, which is a heavy metal, weighs 1167.4 lb. per cubic foot. How many cubic feet of water are equal in weight to 1 cubic foot of uranium? Find the answer to tenths.

I Cork is a very light wood. It weighs only 15 lb. per cubic foot. 1 cubic foot of water is how many times as heavy as 1 cubic foot of cork? Find the answer to tenths.

J Ice, which is water in a solid state, is lighter than water as a liquid. Ice weighs 57.2 lb. per cubic foot. What fraction of a cubic foot of water will make 1 cubic foot of ice? Find the answer to hundredths.

K Lithium is a very light metal. 12.5 cubic feet of this metal weigh 416.25 lb. What is the weight of 1 cubic foot of lithium? Find the answer to hundredths.

L Balsa, a very light wood, weighs only 7.3 lb. per cubic foot. How many cubic feet of balsa weigh 100 lb.? Find the answer to tenths.

M 6.5 cubic feet of ebony weigh 539.5 lb. How much does 1 cubic foot of ebony weigh?

N White pine weighs about 22 lb. per cubic foot. What fraction of a cubic foot of water is equal in weight to 1 cubic foot of white pine? Find the answer to tenths.

O Slate weighs about 162.8 lb. per cubic foot. What fraction of a cubic foot of slate will weigh 100 lb.? Find the answer to hundredths. 2

## Checking up

The word in parentheses after each exercise tells what decimal fraction you are to have in the quotient.

Test 1 3

A $3 \div 4 = m$ (hundredths)
B $38.64 \div 9 = w$ (thousandths)
C $16 \div .9 = e$ (thousandths)
D $8.43 \div 3.622 = n$ (tenths)
E $5.8 \div 123 = p$ (hundredths)
F $9 \div 27 = f$ (thousandths)
G $11 \div .016 = d$ (tenths)
H $.27 \div 12.3 = z$ (ten-thousandths)

Test 2 4

A $14.4 \div 6 = n$ (tenths)
B $3 \div .27 = y$ (hundredths)
C $14.9 \div 6.2 = t$ (thousandths)
D $21 \div 24 = a$ (tenths)
E $.3 \div .04 = s$ (hundredths)
F $172.6 \div 45 = q$ (hundredths)
G $28 \div 375 = b$ (ten-thousandths)
H $8 \div .26 = x$ (thousandths)

End-of-block tests on division of decimal fractions 233

# 233

1 Follow the procedure suggested in Note 4 for page 232.
2 When the pupils have completed the problems, supply answers so they may verify their work. Discuss each equation of ratios.
3 Assign Test 1 as written work. Give pupils sufficient time to complete the exercises. When they have finished, verify each pupil's work to find out if any reteaching is needed.
4 Assign Test 2 only to those pupils who had difficulty with Test 1 and who have had some reteaching.

# 234 Side trip; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil learns how to use the caret to show the new position of the decimal point.

**Vocabulary**

New words page 234 caret

**Comments**

When dividing fractions expressed by decimal fraction numerals, which have already been written in computational form, the caret may be used as a short cut to show where the point is placed after the numbers represented by the dividend and divisor have been multiplied by some power of 10. Use of the caret makes it unnecessary to rewrite the numerals when they are already written in computational form. Emphasize that the caret shows the new position of the point.

Use the "Keeping skillful" exercises as a separate lesson.

**Answers**

(block 1)

A 75.4 has to be multiplied by 100 because .83 has been multiplied by 100. $75.4 \times 100 = 7540$; no
B Two zeros have been annexed to the dividend. Since the number to be divided is expressed in hundredths, you should have two decimal places in the answer.

(block 2)

A $.38\wedge)\overline{7.68\wedge40} = 20.22$
B $2.9\wedge)\overline{1137.0\wedge0} = 392.1$
C $3.46\wedge)\overline{.59\wedge370} = .172$
D $.472\wedge)\overline{6.700\wedge0} = 14.2$
E $1.85\wedge)\overline{13.92\wedge00} = 7.52$
F $36.2\wedge)\overline{278.4\wedge40} = 7.69$
G $.02\wedge)\overline{47.00\wedge0} = 2350.0$

**Keeping skillful:**

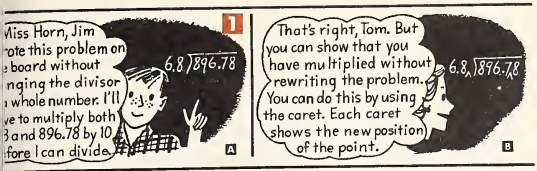
(block 1)

A $\frac{15}{32}$
B $5\frac{1}{3}$
C $10\frac{5}{16}$
D $\frac{18}{25}$
E $63\frac{1}{15}$
F $3\frac{5}{12}$
G $87\frac{1}{2}$
H $2\frac{5}{8}$
I $61\frac{19}{27}$
J $26\frac{1}{4}$

(block 2)

A $3\frac{7}{9}$
B 20
C $2\frac{7}{55}$
D $\frac{7}{36}$
E $1\frac{14}{25}$
F 3
G $2\frac{13}{16}$
H $10\frac{2}{3}$
I $2\frac{7}{11}$
J $4\frac{5}{18}$

## Side trip


Miss Horn, Jim wrote this problem on the board without changing the divisor a whole number. I'll have to multiply both 6.8 and 896.78 by 10 before I can divide.
6.8 ) 896.78
A
That's right, Tom. But you can show that you have multiplied without rewriting the problem. You can do this by using the caret. Each caret shows the new position of the point.
6.8 ) 896.78
B

ou can use the caret to show how you ould rewrite numerals for division.

1 In the example below, why do you rite a zero at the right of 75.4 before utting in the caret? Will there be any ecimal places in the answer?

8 3)7 5.4 .8 3^)7 5.4 0^

2 Now look at the example below. What has been done to the example hown in Exercise A? Where should you ut the point in your answer?

8 3^)7 5.4 0^0 0

For each exercise below, rewrite the numerals for division. Use the caret. Then find the answer. The word in parentheses tells how many decimal places you are to have in the answer. 3

A .3 8)7.6 8 4 (hundredths)
B 2.9)1 1 3 7 (tenths)
C 3.4 6)5 9 3 7 (thousandths)
D .4 7 2)6.7 (tenths)
E 1.8 5)1 3.9 2 (hundredths)
F 3 6.2)2 7 8.4 4 (hundredths)
G .0 2)4 7 (tenths)

## Keeping skillful 4

- $3\frac{3}{4} \times \frac{4}{9} = v$
- $2 \times 2\frac{3}{8} = m$
- $1\frac{1}{2} \times 6\frac{6}{8} = e$
- $\frac{8}{10} \times \frac{5}{8} = p$
- $8\frac{3}{8} \times 7\frac{1}{2} = s$
- $6\frac{5}{8} \times \frac{1}{2} = a$
- $14\frac{7}{12} \times 6 = z$
- $\frac{3}{8} \times 7 = q$
- $11\frac{1}{3} \times 5\frac{4}{5} = d$
- $3\frac{1}{2} \times 7\frac{1}{2} = n$
- $4\frac{1}{4} \div 1\frac{1}{8} = t$
- $18 \div \frac{9}{10} = c$
- $7\frac{5}{8} \div 3\frac{3}{8} = w$
- $\frac{7}{12} \div 3 = n$
- $6\frac{1}{2} \div 4\frac{1}{8} = z$
- $\frac{3}{5} \div \frac{1}{5} = b$
- $5\frac{5}{8} \div 2 = k$
- $8 \div \frac{3}{4} = r$
- $2\frac{9}{10} \div 1\frac{1}{10} = x$
- $17\frac{1}{3} \div 4 = f$

Division of decimals: use of the caret

# 234

1 Explain to the pupils that they will learn how they can avoid rewriting the decimal fraction numerals in a division exercise when someone has already written them without expressing the division as a whole number. Discuss the example in the pictures at the top of the page, and stress that the caret shows the new position of the decimal point.

2 Use the example in Exercise A to point out that sometimes zeros will have to be annexed before the caret, just as zeros had to be annexed before rewriting the numerals in the preceding lesson.

3 Discuss the rewriting of numerals for exercises similar to those in Exercises A to G. Then assign Exercises A to G as written work. When the pupils have finished, supply answers and discuss the position of the caret in each exercise.

4 Assign these exercises as a separate lesson or as time permits.

# 235 Side trip

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns how to divide numbers expressed as decimal fraction numerals in the traditional way.

**Vocabulary**

New words page 235 method

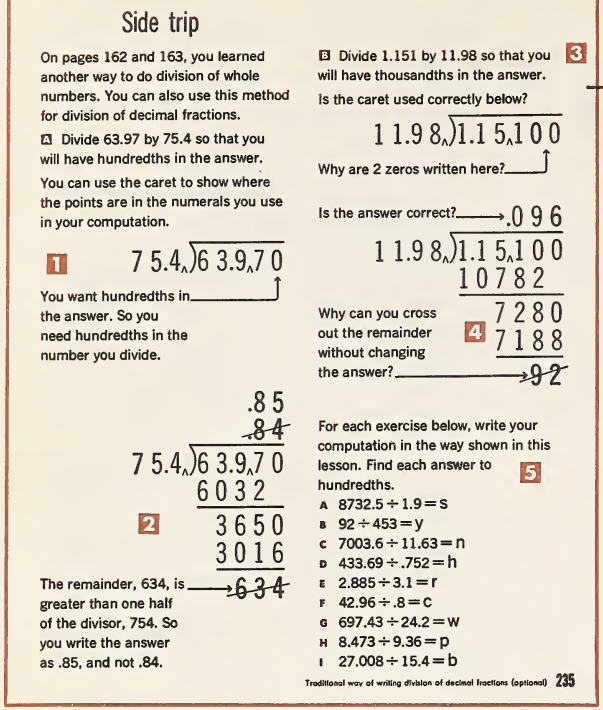
**Comments**

This lesson and the lesson on pages 162-164 are optional lessons in this program. The teacher may decide not to teach them. The pupils who mastered the traditional method of division taught on pages 162-164 will be interested in applying this method to division involving decimal fraction numerals. However, do not expect pupils who had difficulty with the earlier work or slow learners who omitted that lesson to profit from the work on page 235.

The work preliminary to computation is exactly the same as that with the method just taught (to make the divisor a whole number, it is multiplied by a multiple of 10, and the dividend is also multiplied by the same number). As the pupils have already learned, they decide how many decimal places they want in the answer *before* they divide. Accordingly, they can write the numeral in the dividend in the form that has the desired number of decimal places. Then they divide by the traditional method. Finally, the decimal point is placed in the answer in the predetermined place.

**Answers**

A 4596.05
B .20
C 602.20
D 576.72
E .93
F 53.70
G 28.82
H .91
I 1.75



Side trip

On pages 162 and 163, you learned another way to do division of whole numbers. You can also use this method for division of decimal fractions.

A Divide 63.97 by 75.4 so that you will have hundredths in the answer.

You can use the caret to show where the points are in the numerals you use in your computation.

1

$$75.4\overset{\wedge}{)}63.9\overset{\wedge}{,}70$$

You want hundredths in the answer. So you need hundredths in the number you divide.

$$\begin{array}{r} .85 \\ \cancel{.84} \\ 75.4\overset{\wedge}{)}63.9\overset{\wedge}{,}70 \\ 6032 \\ \hline 3650 \\ 3016 \\ \hline \cancel{634} \end{array}$$

2

The remainder, 634, is greater than one half of the divisor, 754. So you write the answer as .85, and not .84.

B Divide 1.151 by 11.98 so that you will have thousandths in the answer.

3

Is the caret used correctly below?

$$11.98\overset{\wedge}{)}1.15\overset{\wedge}{,}100$$

Why are 2 zeros written here?

Is the answer correct? .096

$$\begin{array}{r} .096 \\ 11.98\overset{\wedge}{)}1.15\overset{\wedge}{,}100 \\ 10782 \\ \hline 7280 \\ 7188 \\ \hline \cancel{92} \end{array}$$

Why can you cross out the remainder without changing the answer?

4

For each exercise below, write your computation in the way shown in this lesson. Find each answer to hundredths.

5

A $8732.5 \div 1.9 = s$
B $92 \div 453 = y$
C $7003.6 \div 11.63 = n$
D $433.69 \div .752 = h$
E $2.885 \div 3.1 = r$
F $42.96 \div .8 = c$
G $697.43 \div 24.2 = w$
H $8.473 \div 9.36 = p$
I $27.008 \div 15.4 = b$

Traditional way of writing division of decimal fractions (optional) 235

# 235

1 Discuss why a new place for the point in 75.4 is indicated by the caret. Then have a pupil explain the changes made in the numeral 63.97. Be sure the pupils understand that a zero is annexed to 63.97 to get hundredths in the answer.

2 Ask a pupil to explain the division. The pupils should understand that the computation is done as with whole numbers. Be sure they realize that they are not working with all of the digits of a numeral in each step. Be sure, also, that they understand why .84 is rounded to .85.

3 Adapt Note 1 in discussing the preliminary work before computation.

4 Let a pupil work through the division orally, and have him explain why the remainder is ignored.

5 Assign Exercises A to I as written work. Supply answers and discuss the exercises that cause the most difficulty.

# 236-238 Exploring problems

Expanded Notes for this lesson are on page 359.

**Objectives**

The pupil solves comparison problems by means of equations of ratios that involve the use of decimal fraction numerals.

**Vocabulary**

New words page 237 league, wins*; page 238 attendance

**Comments**

The pupils have already studied comparison problems on pages 122-124 and 210-213. In solving these problems they used equations of ratios with names of whole numbers or fractions as the terms. In this lesson the pupils learn how to solve comparison problems when the fraction is expressed by a decimal fraction numeral.

The comparison problem situations studied here are commonly described as the "three cases of decimal fractions." The need for these distinctions disappears when equations of ratios are used to express the problem situation. The pupil learns that in making an equation of ratios, one of the ratios will be based on a comparison with 1, and the other ratio must be equal to it. He can then solve the equation in the same way he has solved other ratio equations.

The "Keeping skillful" exercises will give the pupils more practice with inequalities, which they first studied on pages 104-106. Note that the pupils are asked to find only one solution that will make a true statement; they need not find the entire solution set.

**Answers**

D .762
E 120
F 16
G 40/120=[.33]/1
H 6/(9)=[.667]/1
I 4/(9)=[.444]/1
J .980/1=147/[150]
K .380/1=57/[150]

L .375/1=[15]/40
M 435/1562=[.278]/1
N .95/1=[76,000]/80,000

**Keeping skillful:**

The answers are limited to those that can be found by operations with positive numbers. Accept as correct any answer contained within each solution set as defined below.

A Any number smaller than $3\frac{1}{8}$
B Any number except $2\frac{1}{5}$
C 10
D Any number smaller than 35
E Any number except $1\frac{9}{11}$
F 1496.88
G Any number greater than $3\frac{39}{40}$
H 0
I Any number except $2\frac{8}{25}$
J Any number greater than 15.9
K Any number greater than $1\frac{9}{40}$
L $1\frac{1}{5}$
M Any number smaller than 17.59
N 1
O Any number smaller than $1\frac{19}{69}$
P 137.97
Q 0
R Any number greater than 210
S Any number smaller than $84\frac{9}{10}$
T Any number smaller than 1.5
U Any number except 7
V 113
W Any number except $2\frac{54}{91}$
X Any number greater than $5\frac{1}{6}$

## Exploring problems

A The Jackson baseball team has played 40 games so far this year. They have won .600 of these games. How many games has the team won?

When you say that the team has won .600 of its games, you may think of this as meaning that the team has won .600 of a game for each game played. You are to find how many of the 40 games the team has won.

First make an equation of ratios. Each ratio will express the comparison of games won with games played.

$$\frac{.600}{1} = \frac{t}{40}$$

Ratio expressing the comparison for 1 game

Ratio expressing the comparison for 40 games

$.600 \times 40 = 24.000$ or 24

24 replaces $t$. The Jackson baseball team has won 24 games.

B One player on the Jackson team has made 49 hits in 147 times at bat. This player's hits are what fraction of his number of times at bat? Find the answer to thousandths.

You are to find what fraction the player's hits are of the number of times at bat. You may think of this as finding what fraction of a hit he made for each time at bat.

Again make an equation of ratios. Each ratio will express the comparison of hits with times at bat.

$$\frac{49}{147} = \frac{z}{1}$$

Ratio expressing the comparison for ■ times at bat

Ratio expressing the comparison for ■ time at bat

3

$49 \div 147 = .333$

.333 replaces $z$. The player's hits are .333 of his number of times at bat.

C One player has made 16 single-base hits. These hits are .8 of all his hits. How many hits in all has this player made? 4

You can say that, for each hit, the player has made .8 of a single-base hit. You know he made 16 single-base hits. So you can make an equation of ratios that compare his single-base hits with all his hits.

$$\frac{.8}{1} = \frac{16}{k}$$

Ratio expressing the comparison for ■ hit

Ratio expressing the comparison for the total number of hits

6 Extending comparison problem situations to include fractions expressed with decimal fraction numerals

# 236

1 Have a pupil read Problem A. Discuss the ratio equation. Be sure the pupils understand that Problem A expresses a comparison situation and get them to explain each term in the ratio equation. Discuss how the ratio test can be used to find the numeral to replace $t$.
2 Apply Note 1 to Problem B.
3 Discuss how the .333 is found. Point out that .333/1 is a ratio with a decimal fraction numeral as its first term and that it is equal to the ratio 49/147.
4 Discuss Problem C and the ratio equation. Have pupils explain the meaning of each term.

$16 \div .8 = 20$ 1

In the equation for Problem C, what numeral replaces $k$? The player has made ■ hits.

D The team that is first in the league has won 32 of the 42 games it has played. This team has won what fraction of the games it has played? Find the answer to thousandths.

2

THINK You want to find what fraction of its games this team has won for each game it has played.

$$\frac{32}{42} = \frac{W}{1}$$

W: Ratio expressing the comparison for ■ games

1: Ratio expressing the comparison for ■ game

3 What numeral replaces $w$? This team has won what fraction of its games?

E Each year, the Jackson team plays .65 of its games at night. The team plays 78 games at night. How many games in all does the Jackson team play each year?

4

THINK You know what fraction of a game the Jackson team plays at night for each game that it plays. You also know how many games the team plays at night. In the equation below, what comparison does each ratio express?

$$\frac{.65}{1} = \frac{78}{b}$$

What numeral replaces $b$ in the equation for Problem E? The Jackson team plays ■ games each year.

F One player on the Jackson team has made 50 hits so far this year. Of the 50 hits, .32 have been extra-base hits. How many extra-base hits has this player made?

THINK You know that for each hit the player has made, he has made .32 of an extra base hit. You know how many hits he has made in all. In the equation below, what comparison does each ratio express?

$$\frac{.32}{1} = \frac{d}{50}$$

What numeral replaces $d$? The player has made ■ extra-base hits.

G So far, the Jackson team has played what fraction of its total number of games for the year? Find the answer to hundredths.

H One of the pitchers for the Jackson team has a record of 6 wins and 3 losses. This pitcher's wins are what fraction of his total number of wins and losses? Find the answer to thousandths.

THINK How can you find the total number of wins and losses? What equation of two ratios can you make for this problem?



I Another Jackson pitcher has a record of 4 wins and 5 losses. This pitcher's wins are what fraction of his total wins and losses? Find the answer to thousandths.

1

J The Jackson second baseman has fielded correctly .980 of the balls that have been hit to him. He has fielded 147 balls correctly. How many balls have been hit to him in all?

K The player who is first in the league in hitting has made a hit .380 of the times he has been at bat. He has made 57 hits. How many times has this player been at bat?

L The team that is last in the league has played 40 games so far. It has won .375 of these games. How many games has this team won?

M The Jackson team has made 435 hits in 1562 times at bat. The hits are what fraction of the number of times at bat? Find the answer to thousandths.

N Attendance at the Jackson games this year has been about .95 of what it was for the same number of games last year. By this time last year, attendance had been about 80,000. What has the approximate attendance been so far this year?

Now you should be able to solve comparison problems in which you use decimal fraction numerals to represent fractions. 2

## Keeping skillful

In each sentence below, replace $t$ with a numeral that will make a true statement.

3

| | | |
|---|---|---|
| A $24 \times t < 75$ | I $7\frac{1}{2} \div t \neq 3\frac{1}{8}$ | Q $29 \times t = 0$ |
| B $3\frac{5}{8} + t \neq 6$ | J $29.8 + t > 45.7$ | R $28 \times 7\frac{1}{2} < t$ |
| C $75 \div t = 7\frac{1}{2}$ | K $3\frac{5}{8} - 2\frac{3}{8} < t$ | S $t + 35 < 119\frac{9}{10}$ |
| D $48 - 13 > t$ | L $95 \times t = 114$ | T $10.80 \div 7.2 > t$ |
| E $t \times 5\frac{1}{2} \neq 10$ | M $37.23 - 19.64 > t$ | U $43.1 + t \neq 50.1$ |
| F $19.8 \times 75.6 = t$ | N $t \times 6 = 6$ | V $t - 113 = 0$ |
| G $13\frac{9}{10} + t > 17\frac{7}{8}$ | O $5\frac{3}{4} \times t < 7\frac{1}{3}$ | W $19\frac{2}{3} \div 7\frac{7}{12} \neq t$ |
| H $t + 2 = 2$ | P $t - 137.02 = .95$ | X $15\frac{1}{2} \div t < 3$ 4 |





# 237

1 Have a pupil explain how to solve the ratio equation on page 236 by using the ratio test. Have another pupil explain how this statement results from the ratio test.

2 Point out that when statistics of games are reported in newspapers, etc., the standings are traditionally expressed in thousandths.

3 After discussing the ratio equation, let the pupils explain the computation necessary to find the numeral to replace $w$. Again point out that .762/1 is a ratio with a decimal fraction numeral as its first term and that it equals the ratio 32/42.

4 Assign Problems E to N (I to N are on the following page) as individual work. Tell pupils to write a ratio equation for each problem. Remind them to pay special attention to the THINK questions. Give individual help as needed.

# 238

1 Use Note 4 on the preceding page for Problems I to N.

2 When the pupils have completed the problems, have individual pupils discuss their equations and solutions. Have all answers verified.

3 Review the interpretation of each of the signs of inequality before assigning these exercises as individual work.

4 When the pupils have completed the exercises, let them verify their work by discussing the solution set for each exercise.

# 239-240 Exploring problems

Expanded Notes for this lesson are on pages 360-361.

**Objectives**

The pupil learns to distinguish among and work with abstract statements for comparisons involving decimal fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

In the preceding lesson, pupils used equations of ratios to solve comparison problems involving fractions expressed by decimal fraction numerals. In this lesson the same types of problem situations are translated into expressions commonly used to compare two numbers. Abstract statements, such as "What number is .875 of 76?" will be met by pupils in standardized tests or in problem situations out of which the expression develops. Pupils should recognize such statements and should be able to solve them by using equations of ratios.

Problems A to C illustrate the three types of expressions that may result from comparison problems that involve decimal fraction numerals. These are the "three cases of decimal fractions." For Problems D to F the pupils are to make equations of ratios and then choose the sentence that correctly expresses the problem situation. In Exercises A to J on page 240, the pupils are given sentences for which they are to make equations of ratios and find the answers.

Often pupils are told to replace "is" by an equals sign and "of" by a multiplication sign to solve these abstract sentences. This contributes nothing to the pupils' understanding of the problem. Emphasize that in each situation, one number is compared with another. Therefore, it can be expressed by an equation of ratios, and the computation needed to solve the problem results from the ratio test.

**Answers**

Pages 239-240:

(block 1)

A .89 B 66.5 lb. C 60 in.

D 53.5/57=[.94]/1; sentence 2

E [55.1]/58=.95/1; 55.1 in.; sentence 1

F 66/[74.2]=.89/1; 74.2 lb.; sentence 2

(block 2)

A [34.56]/72=.48/1

B 95/127.3=[.75]/1

C 39/[312]=.125/1

D 12.8/49=[.3]/1

E .6/1=[76.32]/127.2

F .322/1=9/[27.95]

G [32.571]/98.7=.33/1

H 11.6/1160=[.01]/1

I .75/1=.892/[1.189]

J .24/1=[9.3096]/38.79

Keeping skillful:

(block 1)

A 21/75=[28]/100; 28%

B 45/100=[50.85]/113

C 128/100=16.4/[12.81]

D 67/100=[61.64]/92

E [130]/100=325/250; 130%

F 5/100=37/[740]

(block 2)

A 62.582

B .558

C 6.9216

D 156.4

E .0882

F .7125

G .0012

H .703

I .35862

(block 3)

A .04

B 21.56

C 17.58

D 85.81

E 29.59

F 170.40

G .14

## Exploring problems

A The sixth graders were weighed in health class. Jim weighs 76 lb. Carol weighs 68 lb. Carol's weight is what fraction of Jim's weight? Express the answer with a decimal fraction numeral in hundredths. 1

THINK To solve Problem A, you must find what fraction 68 is of 76. You can write the equation below for Problem A.

$$\frac{68}{76} = \frac{n}{1}$$ ← Fraction of a pound Carol weighs for each pound Jim weighs

To get the answer, you can divide 68 by 76. 2

68 is what fraction of 76? 3

B Jim's weight one year ago was .875 of his weight now. What was Jim's weight one year ago? 4

THINK In Problem B, you are to find the number that is .875 of 76. You can write the equation below.

$$\frac{.875}{1} = \frac{k}{76}$$ ← Number of pounds Jim weighed one year ago

To get the answer, you can multiply 76 by .875.

■ is .875 of 76.

C One year ago Cal was 57 in. tall. His height then was .95 of his height now. What is his height now? 4

THINK 57 is .95 of what number? You can write the equation below.

$$\frac{.95}{1} = \frac{57}{d}$$ ← Cal's height now

To get the answer, you can divide 57 by .95.

57 is .95 of ■.

D Ann is 53.5 in. tall. Sue is 57 in. tall. Ann's height is what fraction of Sue's height? Find the answer to hundredths. 5

THINK What equation can you make to find what fraction 53.5 is of 57?

Which of these sentences is correct for Problem D?

1 57 is what fraction of 53.5?
2 53.5 is what fraction of 57?

E The sixth-grade class found that the average height of the boys is 58 in. They also found that the average height of the girls is .95 of the average height of the boys. What is the average height of the girls? 6

Now turn the page.



THINK What equation can you make to find .95 of 58? 1

Which of these sentences is correct for Problem E?

1 .95 of 58 is ■.
2 58 is .95 of ■.

F Alice's weight is .89 of George's weight. Alice weighs 66 lb. What does George weigh? Find the answer to tenths. 1

THINK 66 is .89 of what number? What equation can you write for Problem F?

Which of these sentences is correct for Problem F?

1 .89 of 66 is ■.
2 66 is .89 of ■.

For each exercise below, first make an equation of ratios. Then find the answer. 2

A ■ is .48 of 72.
B 95 is what fraction of 127.3? Find the answer to hundredths.
C 39 is .125 of ■.
D 12.8 is what fraction of 49? Find the answer to tenths.
E .6 of 127.2 is ■.
F .322 of ■ is 9. Find the answer to hundredths.
G ■ is .33 of 98.7.
H 11.6 is what fraction of 1160? Find the answer to hundredths.
I .75 of ■ is .892. Find the answer to thousandths.
J ■ is .24 of 38.79.

Now you know how to work with problems and sentences that use decimal fraction numerals to compare one number with another number. 3

## Keeping skillful 4

If there is a decimal fraction numeral in the answer, express it in hundredths.

A 21 is ■% of 75.
B 45% of 113 is ■.
C 128% of ■ is 16.4.
D ■ is 67% of 92.
E ■% of 250 is 325.
F 37 is 5% of ■.

A $75.4 \times .83 = r$
B $.93 \times .6 = y$
C $10.3 \times .672 = n$
D $46 \times 3.4 = z$
E $.126 \times .7 = c$
F $.75 \times .95 = s$
G $.03 \times .04 = k$
H $1.9 \times .37 = w$
I $.086 \times 4.17 = b$

Find each answer to hundredths.

A $14 \div 386 = v$
B $77.2 \div 3.58 = q$
C $1.934 \div .11 = t$
D $532 \div 6.2 = b$
E $88.78 \div 3 = n$
F $42.6 \div .25 = y$
G $9 \div 63 = p$





# 239

1 Have Problem A read and discuss the equation of ratios. Point out that the ratio 68/76 compares Carol's weight to Jim's weight and that $n/1$ expresses a comparison of the fraction of a pound that Carol weighs to each pound that Jim weighs. Ask what kind of numeral replaces $n$ in the ratio $n/1$.
2 Have the pupils perform the computation that results from the ratio test. Have them replace $n$ in the equation with the numeral obtained from the computation. Point out that it is a decimal fraction numeral and that the ratio equal to 68/76 is .89/1.
3 Discuss the statement and relate it to the problem and to the equation of ratios.
4 Adapt Notes 1, 2, and 3 to Problems B and C.
5 Have the pupils read Problem D and supply the equation of ratios. Then have them find the answer and choose the correct sentence for the problem.
6 Have the pupils read Problem E.

# 240

1 Have the pupils supply equations of ratios for Problems E and F. Then have them find the answer and choose the correct sentence for each problem.
2 Assign Exercises A to J as individual written work. Instruct the pupils to write an equation of ratios for each exercise. Have them use the ratio test and compute to find the answer.
3 When pupils have completed the exercises, discuss their equations and supply answers so they may verify their work.
4 Assign the "Keeping skillful" exercises as a separate lesson or lessons, depending upon class needs. Note that although the number of exercises is limited, the computation required to complete them is time consuming.

# 241-242 Thinking straight

Expanded Notes for this lesson are on page 361.

**Objectives**

The pupil learns to express common fraction numerals as decimal fraction numerals, and decimal fraction numerals as common fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

Pupils will find that it is convenient to be able to express the same fraction with both a common fraction numeral and a decimal fraction numeral.

Expressing common fraction numerals as decimal fraction numerals is based on the principle that the numbers represented in the numerator and the denominator can be divided by the same number without changing the fraction. Thus $\frac{3}{8}$ can be expressed by a decimal fraction numeral by first dividing both 3 and 8 by 8. The new denominator is then the numeral 1, and the new numerator is found by the computation $3 \div 8$, which produces the desired decimal fraction numeral.

In these exercises, it is desirable to determine the denominator of the decimal fraction answer beforehand. If the decimal fraction numeral for $\frac{3}{8}$ is to be expressed in thousandths, the 3 is written as 3.000. If it is to be expressed in tenths, the 3 is written as 3.0.

On page 242, in Exercises H and K, pupils should be helped to recognize the repeating digits in the computation process, and that, for example, $\frac{1}{3} = .3333 \ldots$ to as many places as one wishes to carry it.

In Exercises A and B in the second column (white letters) on page 242, the pupils learn to replace decimal fraction numerals by common fraction numerals. It is desirable that the pupils memorize such equalities as $\frac{3}{4} = .75$, $\frac{1}{5} = .2$, etc., which are frequently used, so that they may simplify computation by using one in place of the other.

**Answers**

(block 1)

A *Both* the number represented in the numerator and in the denominator must be divided by the same number to get an equal fraction numeral. Yes

B Yes; The number divided shows that the quotient is to be expressed as thousandths.

C Yes

D To get 1 in the denominator

E 2; The quotient is expressed in thousandths because the number divided is expressed in thousandths.

F Yes; yes; yes; yes

G .4

H ——

I Yes; yes; yes

J In the computation of Exercise H, the last subtraction left a remainder of 1. But if there had been another zero in the number divided, the remainder would have been 10, and dividing 10 by 3 would result in another 3 in the quotient.

K Yes; yes

L When the numbers represented in the numerator and in the denominator of $\frac{7}{8}$ are both divided by 8, the denominator is 1 and the numerator is $7 \div 8$, which shows the necessary computation. .875

(block 2)

| | | |
|---|---|---|
| A .500 | E .833 | I .050 |
| B .250 | F .667 | J .429 |
| C .375 | G .900 | K .417 |
| D .111 | H .267 | L .091 |

(block 3)

A Yes

B $\frac{125}{1000}$; yes

(block 4)

| | |
|---|---|
| A $\frac{9}{25}$ | G .625 |
| B .563 | H $\frac{11}{25}$ |
| C $\frac{14}{25}$ | I .458 |
| D $\frac{17}{20}$ | J $\frac{27}{100}$ |
| E .583 | K .240 |
| F $\frac{4}{5}$ | L $\frac{19}{20}$ |

A $\frac{3\div8}{8\div8}=\frac{3\div8}{1}$

$$\begin{array}{r|l} 8\overline{)3.000} & 300 \\ 2400 & \\ \hline 600 & 70 \\ 560 & \\ \hline 40 & 5 \\ 40 & \\ \hline & .375 \end{array}$$

$\frac{3}{8}=\frac{.375}{1}$ or .375

B $\frac{2\div5}{5\div5}=\frac{2\div5}{1}$

$$\begin{array}{r|l} 5\overline{)2.000} & 400 \\ 2000 & \\ \hline & .400 \end{array}$$

$\frac{2}{5}=\frac{.400}{1}$ or .400 or .4

A Suppose you want to find a decimal fraction numeral to use instead of the common fraction numeral $\frac{3}{8}$. To do this, first divide the 8 in the denominator by 8 to get 1 as a new denominator. Why must you also divide the 3 in the numerator by 8? Look at the work labeled A. Does this show the result of dividing both the number in the denominator and in the numerator by 8?

B Now you must divide 3 by 8. You know that the answer can be expressed as a decimal fraction numeral. Usually, you will want to divide to thousandths. Is 3.000 another name for 3? Look at the work shown in red. Why is the answer expressed as .375 and not as 375?

C You can now write $\frac{.375}{1}$ in place of $\frac{3\div8}{1}$. Is .375 another name for $\frac{.375}{1}$? .375 is another name for $\frac{3}{8}$.

D The work labeled B shows how to find a decimal fraction numeral to use instead of the common fraction numeral $\frac{2}{5}$. Why are both the 5 in the denominator and the 2 in the numerator divided by 5?

E In the computation shown in red, 2.000 is used as another name for ■. How do you know that the quotient is .400, and not 400?

F Is $\frac{.400}{1}$ another name for $\frac{2}{5}$? Is .400 another name for $\frac{2}{5}$? Is .4 another name for .400? Is .4 another name for $\frac{2}{5}$?

G Suppose you had used 2.0 instead of 2 when you divided 2 by 5. What would your answer have been?



H The work below shows how to find a decimal fraction numeral to use in place of $\frac{1}{3}$.

$\frac{1\div3}{3\div3}=\frac{1\div3}{1}$

$$\begin{array}{r|l} 3\overline{)1.000} & 300 \\ 900 & \\ \hline 100 & 30 \\ 90 & \\ \hline 10 & 3 \\ 9 & \\ \hline 1 & .333 \end{array}$$

$\frac{1}{3}=\frac{.333}{1}$ or .333

I Can .333 be used instead of $\frac{1}{3}$? Can you use .33 instead of $\frac{1}{3}$? Can you also use .3?

J Suppose you want to find a decimal fraction numeral expressed in ten-thousandths to use instead of $\frac{1}{3}$. How do you know, without computing, that the decimal fraction numeral will be .3333?

K Use the way shown in this lesson to show that .1667 can be used for $\frac{1}{6}$. Can you use .167? Can you also use .17?

L When you find a decimal fraction numeral to use instead of $\frac{7}{8}$, why do you divide 7 by 8? $\frac{7}{8}$ is equal to what decimal fraction numeral expressed in thousandths?

For each common fraction numeral below, find a decimal fraction numeral expressed in thousandths that can be used in its place.

| | | | | | |
|---|---|---|---|---|---|
| A $\frac{1}{2}$ | D $\frac{1}{9}$ | G $\frac{9}{10}$ | J $\frac{3}{7}$ |
| B $\frac{1}{4}$ | E $\frac{5}{8}$ | H $\frac{4}{15}$ | K $\frac{5}{12}$ |
| C $\frac{3}{8}$ | F $\frac{2}{3}$ | I $\frac{1}{20}$ | L $\frac{1}{11}$ |

A Find the common fraction numeral that is equal to .75. Express the fraction numeral in lowest terms.

You know that 75 can be written as $\frac{75}{100}$.

$\frac{75}{100}=\frac{75\div25}{100\div25}=\frac{3}{4}$

Is $\frac{3}{4}$ another name for .75?

B Find the common fraction numeral that is equal to .125. Express the fraction numeral in lowest terms.

.125 can be written as $\frac{125}{■}$.

$\frac{125}{1000}=\frac{125\div125}{1000\div125}=\frac{1}{8}$

Is $\frac{1}{8}$ another name for .125?

For each common fraction numeral, find a decimal fraction numeral expressed in thousandths that can be used instead. For each decimal fraction numeral, find a common fraction numeral that is equal. Express it in lowest terms.

| | | | | | |
|---|---|---|---|---|---|
| A .36 | D .85 | G $\frac{5}{8}$ | J .27 |
| B $\frac{9}{16}$ | E $\frac{7}{12}$ | H .44 | K $\frac{6}{25}$ |
| C .56 | F .8 | I $\frac{11}{24}$ | L .95 |





# 241

1 The pupils should see that the numbers represented by both the numerator and the denominator are divided by 8 to get 1 in the denominator. To express the numerator with another name that is a decimal fraction numeral, the 3 is divided by 8.

2 In Exercise B, have the pupils observe that the decimal fraction numeral is to be expressed in thousandths. Therefore, 3 is written as 3.000. Point out that if in the computation the number to be divided is expressed in thousandths, the quotient will be expressed in thousandths.

3 Use Exercise C to explain why .375 is another name for $\frac{.375}{1}$.

4 Adapt Notes 1, 2, and 3 to Exercises D and E.

5 Point out that .4 is another name for .400 and that if 2.0 had been used instead of 2.000 when dividing, the quotient would have been .4.

# 242

1 Call attention to the computation in Exercise H. Point out the remainder 1 in the computation. Explain that the repeated occurrence of a numeral as a remainder indicates that the sequence of digits in the quotient will also repeat. Show this same process for $\frac{2}{3}$ and $\frac{4}{11}$ carried to ten-thousandths.

2 Get pupils to observe that .333 is less than $\frac{1}{3}$ because of rounding "down" and .167 is greater than $\frac{1}{6}$ because of rounding "up."

3 Assign Exercises A to L (at the top of the second column) as individual work. Discuss whether the quotients that are rounded are greater or less than the common fraction.

4 The pupils should see that Exercises A and B show the reverse process: replacing a decimal fraction numeral by a common fraction numeral. Point out that the common fraction numeral should be expressed in lowest terms.

5 Assign Exercises A to L as individual work.

# 243 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to replace ratios that express per cents by equal ratios that involve decimal fraction numerals or common fraction numerals.

**Vocabulary**

There are no new words.

**Comments**

When pupils were introduced to per cents they learned that the term "per cent" can be used in naming a ratio whose second term is 100. In this lesson they will learn to replace a ratio that expresses per cent, or whose second term is 100, by a ratio whose second term is 1. This is done by dividing both numbers by 100. The first term of the new ratio is then a decimal fraction numeral and the second term is 1. The first term can be renamed as a common fraction numeral.

Pupils are frequently taught that $25\% = \frac{1}{4}$, $30\% = \frac{3}{10}$, etc. It should be obvious, however, that a per cent, which is a ratio, cannot be equal to a fraction numeral. Rather, the per cent is a ratio that is equal to a ratio with either a common fraction numeral or a decimal fraction numeral in the first term and 1 in the second term. In computation, the 1 in the second term is ignored; and this is why it is frequently disregarded in teaching.

**Answers**

(block 1)

| | | | |
|---|---|---|---|
| C .42/1 | G 1.39/1 | K .06/1 | O .18/1 |
| D .63/1 | H 3.72/1 | L .85/1 | P .03/1 |
| E .01/1 | I 2.48/1 | M .50/1 | Q .55/1 |
| F .20/1 | J .19/1 | N .99/1 | R .70/1 |

(block 2)

| | |
|---|---|
| D .15/1, $\frac{3}{20}$/1 | J .76/1, $\frac{19}{25}$/1 |
| E .30/1, $\frac{3}{10}$/1 | K .35/1, $\frac{7}{20}$/1 |
| F .93/1, $\frac{93}{100}$/1 | L .02/1, $\frac{1}{50}$/1 |
| G 1.28/1, $\frac{32}{25}$/1 | M .45/1, $\frac{9}{20}$/1 |
| H 1.50/1, $\frac{3}{2}$/1 | N .08/1, $\frac{2}{25}$/1 |
| I 2.36/1, $\frac{59}{25}$/1 | O .90/1, $\frac{9}{10}$/1 |

## Thinking straight

$25\% = \frac{25}{100} = \frac{.25}{1}$

$80\% = \frac{80}{100} = \frac{.80}{1} = \frac{.8}{1}$

C $\frac{.25}{1} = \frac{\frac{1}{4}}{1}$

D $\frac{.8}{1} = \frac{\frac{8}{10}}{1} = \frac{\frac{4}{5}}{1}$

Look at Example A above. You know at 25% is a ratio that can be written as 25 over 100. Use the ratio test to show that the ratio .25 over 1 is equal to the ratio 25 over 100. Both 25 and 100 can be divided by ■ to get the ratio .25 over 1. Can 25% also be expressed by the ratio .25 over 1?

Look at Example B. Use the ratio test to show that the ratio .80 over 1 is equal to the ratio 80 over 100. Can .80 be written as .8? Can 80% also be expressed by the ratio .8 over 1?

Write the following per cents as ratios with a decimal fraction numeral in the first term and 1 in the second term.

| | | | |
|---|---|---|---|
| A 42% | C 139% | K 6% | O 18% |
| B 63% | H 372% | L 85% | P 3% |
| C 1% | I 248% | M 50% | Q 55% |
| D 20% | J 19% | N 99% | R 70% |

A Look at Example C. Is $\frac{1}{4}$ another name for .25? Can the ratio .25 over 1 be expressed as $\frac{1}{4}$ over 1? 4

B In Example A you saw that 25% can be expressed as .25 over 1. Can 25% also be expressed as $\frac{1}{4}$ over 1? 5

C Now look at Example D. Is $\frac{8}{10}$ another name for .8? Is $\frac{4}{5}$ another name for $\frac{8}{10}$? Can the ratio .8 over 1 be expressed as $\frac{4}{5}$ over 1? Can 80% be expressed as $\frac{4}{5}$ over 1? 6

For each exercise below, write a ratio with a decimal fraction numeral in the first term and 1 in the second term. Then write a ratio with a common fraction numeral in lowest terms in the first term and 1 in the second term. 7

| | | | |
|---|---|---|---|
| D 15% | G 128% | J 76% | M 45% |
| E 30% | H 150% | K 35% | N 8% |
| F 93% | I 236% | L 2% | O 90% |



## 243

1 Be sure the pupils understand that here 25/100 and .25/1 are to be thought of as *ratios*, not fraction numerals. Explain that 25/100 is read 25 to 100 and .25/1 is read .25 to 1.

2 Point out that .80, the first term of the ratio in Example B, can be expressed as .8.

3 Assign Exercises C to R as individual work and supply answers.

4 Discuss replacing .25 by $\frac{1}{4}$ as the first term of the ratio.

5 Point out that since 25% can be expressed as .25/1, it can also be expressed by the ratio $\frac{1}{4}$/1, and this ratio is read "one fourth to one."

6 Have the pupils observe that fraction numerals used as terms of a ratio should be expressed in lowest terms.

7 Assign these exercises as individual work. Supply answers so pupils may verify their work.

# 244-246 Exploring problems

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns to write equations of ratios that involve decimal fraction numerals or common fraction numerals for per cent problems.

**Vocabulary**

New words page 244 Miami, Florida, October*; page 245 Honolulu, Hawaii, December*; page 246 Britain, Texas*, Canada*, Brazil, television*

**Comments**

In this lesson the pupil recognizes that the conditions in a problem involving per cents can be expressed by an equation of ratios one of whose terms is a decimal fraction numeral or a common fraction numeral.

The pupil learns to write three sentences for each problem. For each of these sentences a corresponding equation of ratios is developed. In Problem A, for example, the equation for the sentence "65% of 48 is ▤" is 65/100=s/48. The pupil recognizes 65% as the ratio 65/100 and that he must find an equal ratio whose second term is 48. For the sentence ".65 of 48 is ▤," which expresses the same problem, the corresponding equation is .65/1=s/48. Here .65/1 should be recognized as another name for the ratio equal to 65%. It is important to call attention to the 1 in the second term. The traditional teaching of per cent, by failing to recognize the importance of this 1, short cuts understanding. The third sentence that expresses the same problem is "$\frac{13}{20}$ of 48 is ▤," and the corresponding equation is $\frac{13}{20}/1=s/48$. Here $\frac{13}{20}/1$ should also be recognized as another ratio equal to 65%. It is important to stress the 1 in the second term of this ratio also.

The pupil should be aware of the reason for the sentences and corresponding equations that can be made for a problem—namely, that for each of the three ratios that expresses the comparison originally expressed in terms of per cent, there is a corresponding sentence and equation of ratios. The pupils should also be aware that for each of these three equations there is a definite computational procedure, but that all of these procedures result in the same answer.

The three sentences and the related equations and computation are developed for all "cases." The pupil is then required to solve various per cent problems in these three ways. As he works with these problems, the pupil will come to see that, in any given problem, one of the three ways of computing is most convenient. This lesson should enable the pupil to choose the most convenient computational procedure when he solves per cent problems.

**Answers**

Pages 245-246:

D 75/100=[42]/56; .75/1=[42]/56; $\frac{3}{4}/1$=[42]/56

E 8/24=[33]/100, 33%; 8/24=[.33]/1; 8/24=[$\frac{1}{3}$]/1

F 10/100=56/[560]; .1/1=56/[560]; $\frac{1}{10}/1$=56/[560]

G 35% of 520 is ▤; 35/100=[182]/520; 182 acres

.35 of 520 is ▤; .35/1=[182]/520

$\frac{7}{20}$ of 520 is ▤; $\frac{7}{20}/1$=[182]/520

H 5 is what per cent of 20? 5/20=[25]/100; 25%

5 is what fraction of 20? 5/20=[.25]/1

5 is what fraction of 20? 5/20=[$\frac{1}{4}$]/1

I 525 is 15% of ▤. 15/100=525/[3500]

525 is .15 of ▤. .15/1=525/[3500]

525 is $\frac{3}{20}$ of ▤. $\frac{3}{20}/1$=525/[3500]

## Exploring problems

Miami, Florida, has an average yearly rainfall of about 48 in. About 65% of the rainfall comes during the months from June through October. About how much rainfall does Miami have during these 5 months?

THINK 65% can be expressed in the following ways.

$65\% = \frac{65}{100} = \frac{.65}{1} = \frac{13}{20}$

So you can write these three sentences for Problem A.

65% of 48 is ■.
.65 of 48 is ■.
$\frac{13}{20}$ of 48 is ■.

You can compute in the three ways shown below to solve Problem A.

$\frac{65}{100} = \frac{s}{48}$ $\quad 100s = 65 \times 48$

$\frac{65 \times 48}{100} = 31.20$ or 31.2

65% of 48 is 31.2.

$\frac{.65}{1} = \frac{s}{48}$ $\quad s = .65 \times 48$

$.65 \times 48 = 31.20$ or 31.2

.65 of 48 is 31.2.

$\frac{13}{20} = \frac{s}{48}$ $\quad s = \frac{13}{20} \times 48$

$\frac{13}{20} \times 48 = 31\frac{2}{10}$ or 31.2

$\frac{13}{20}$ of 48 is 31.2.

B Fifteen years ago, the average size of a farm in the United States was 175 acres. This is 70% of the average size of a farm in the United States now. What is the average size of a farm now? 4

THINK 70% can be expressed in the following ways.

$70\% = \frac{70}{100} = \frac{.70}{1} = \frac{.7}{1} = \frac{7}{10}$ 5

So you can write these three sentences for Problem B.

1 175 is 70% of ■.
2 175 is .7 of ■.
3 175 is $\frac{7}{10}$ of ■.

You can compute in the three ways shown below to solve Problem B.

1 $\frac{70}{100} = \frac{175}{n}$ $\quad 70n = 100 \times 175$

$\frac{100 \times 175}{70} = 250$

175 is 70% of 250.

2 $\frac{.7}{1} = \frac{175}{n}$ $\quad .7n = 175$

$\frac{175}{.7} = 250$

175 is .7 of 250.

3 $\frac{\frac{7}{10}}{1} = \frac{175}{n}$ $\quad \frac{7}{10}n = 175$

$\frac{175}{\frac{7}{10}} = 250$

175 is $\frac{7}{10}$ of 250.

C The Millers have a monthly income of $475. They spend an average of $133 per month for food. What part of their monthly income do the Millers spend for food?

THINK You are asked to find what part of their income the Millers spend for food. The answer to Problem C can be expressed as a per cent, as a decimal fraction numeral, and as a common fraction numeral.

So you can write the three sentences below for Problem C.

1 133 is ■% of 475.
2 133 is what fraction of 475? Express the answer as a decimal fraction numeral in hundredths.
3 133 is what fraction of 475? Express the answer as a common fraction numeral in lowest terms.

You can compute in the three ways shown below to solve Problem C.

1 $\frac{133}{475} = \frac{d}{100}$ $\quad 475d = 133 \times 100$

$\frac{133 \times 100}{475} = 28$

133 is 28% of 475.

2 $\frac{133}{475} = \frac{d}{1}$ $\quad 475d = 133$

$\frac{133}{475} = .28$

133 is .28 of 475.

3 $\frac{133}{475} = \frac{d}{1}$ $\quad 475d = 133$

$\frac{133}{475} = \frac{7}{25}$

133 is $\frac{7}{25}$ of 475.

D The population per square mile in the United States is now about 56 persons. The population per square mile 25 years ago was 75% of what it is now. What was the population per square mile 25 years ago? 4

THINK 75% can be expressed in the following ways.

$75\% = \frac{75}{100} = \frac{.75}{1} = \frac{\frac{3}{4}}{1}$

Are the three sentences below correct for Problem D?

1 75% of 56 is ■.
2 .75 of 56 is ■.
3 $\frac{3}{4}$ of 56 is ■.

Write an equation of ratios for each sentence above. Then compute to solve each equation.

E Honolulu, Hawaii, has an average yearly rainfall of about 24 in. An average of 8 in. of rainfall comes during the months of January and December. About what part of the average yearly rainfall in Honolulu comes during these two months?

Now turn the page.



Using per cents, common fraction numerals, and decimal fraction numerals in problem solving

# 244

1 Explain why 65% can be expressed in the three ways shown. Point out that 65/100 is a ratio with 100 as the second term, that .65/1 equals 65/100 [both numbers were divided by 100], and that $\frac{13}{20}$/1 is obtained by replacing the first term of .65/1 by $\frac{13}{20}$.
2 Relate the three sentences written in blue to the problem situation and to the three ratios that also express 65%.
3 Be sure the pupils see the relation between each of the three sentences and its corresponding equation of ratios. Discuss the computations. Explain that the statement written in blue after each computation is a way of expressing the answer, just as the corresponding sentence is a way of expressing the problem situation.
4 Have Problem B read. Ask the pupils whether the answer will be larger or smaller than 175.
5 Adapt Notes 1 to 3 to Problem B.

# 245

1 Have the pupils observe that Problem C asks them to make a comparison and that the ratio can be expressed in three ways—in terms of per cent, as a decimal fraction numeral, and as a common fraction numeral.
2 Discuss the three sentences for Problem C. Point out that, although Questions 2 and 3 are the same, the answers are required in different forms.
3 Have the pupils observe that an equation of ratios can be written for each sentence. Point out the equations that result from the ratio test and discuss the computation. Relate the statement after each computation to the three sentences written for Problem C.
4 Assign Problems D, E, and F as written work. Give individual help as it is needed. Note that the pupils are required to write an equation of ratios for each sentence.

THINK You are asked to find what part of the rainfall comes during January and December. You know that the answer can be expressed as a per cent, or as a decimal fraction numeral, or as a common fraction numeral.

Are the three sentences below correct for Problem E?

1 8 is ■% of 24. Find the answer to the nearer per cent.
2 8 is what fraction of 24? Express the answer as a decimal fraction numeral in hundredths.
3 8 is what fraction of 24? Express the answer as a common fraction numeral in lowest terms.

Write an equation of ratios for each sentence above. Then compute to solve each equation.

F The population of 56 persons per square mile in the United States is approximately 10% of the population per square mile in Great Britain. What is the approximate population per square mile in Great Britain?

THINK 10% can be expressed in the following ways.

$$10\% = \frac{10}{100} = \frac{.1}{1} = \frac{\frac{1}{10}}{1}$$

Are the following sentences correct for Problem F?

1 56 is 10% of ■.
2 56 is .1 of ■.
3 56 is $\frac{1}{10}$ of ■.

Write an equation of ratios for each sentence above. Then compute to solve each equation. 1

Make three sentences for each problem below. Make an equation for each sentence and solve each equation.

G The average size of an Illinois farm is about 35% of the average size of a Texas farm. The average size of a Texas farm is 520 acres. What is the average size of an Illinois farm? 2

H The population per square mile in Canada is 5 persons. The population per square mile in Brazil is 20 persons. What per cent, common fraction numeral, and decimal fraction numeral can you use to compare Canada's population per square mile with Brazil's?

I The number of television stations in the United States is 15% of the number of radio stations. There are 525 television stations. How many radio stations are there?

Now you should be able to write equations using per cents, decimal fraction numerals, and common fraction numerals when you solve per cent problems.

246

## 246

1 When the pupils have completed Problems D to F, supply answers so that they may verify their work. Discuss the equations used for each problem.
2 Assign Problems G to I as individual work. Point out that the pupils are to write three sentences and three equations of ratios for *each* problem.

## 247 Thinking straight

Expanded Notes for this lesson are on page 362.

**Objectives**

The pupil reviews the properties of the rectangular prism and learns to find the area of its faces.

**Vocabulary**

There are no new words.

**Comments**

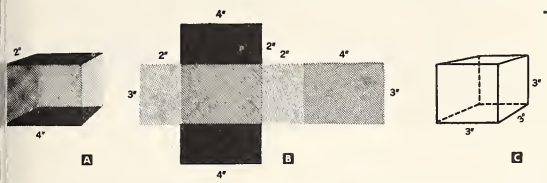
Pupils were introduced to rectangular prisms on pages 6-9. In this lesson, they review the properties of rectangular prisms before they learn in the next lesson to find the volume of such prisms. The notion of surface area is introduced to show pupils that the application of the concept of area is not confined to two-dimensional shapes and to increase their understanding of the properties of the rectangular prism. This should not be considered a formal treatment on finding surface area.

In presenting this lesson, use as a model a rectangular prism similar to that pictured in Diagram A. Construction of a model made of heavy paper that can be flattened out as shown in Diagram B would be helpful. The vocabulary and concepts developed during discussion of the model will supplement the exercises in the text.

**Answers**

A 2″; 3″
B 6; rectangle
C The other faces, or sides
D Yes
E 8 sq. in.; 16 sq. in.
F 3″; yes
G 36 sq. in.
H 9 sq. in.; All 6 faces have the same area.

## Thinking straight




A Picture A shows a rectangular prism. You can describe this prism as being 4 inches long. What is the width of Prism A? What is the height of Prism A?

B Prism A has how many faces? Two of these faces are usually called bases. Each face has the shape of a ——.

C Diagram B shows one way that Prism A would look if it were flattened out. The rectangles shown in solid blue represent the bases of Prism A. What do the four other rectangles represent?

D Can you use the equation below to find the area of one base of Prism A?

$$\frac{4}{1}=\frac{t}{2}$$

E You can also use the equation $4 \times 2 = t$ to find the area of one base. The area is ▩ sq. in. What is the total area of both bases?

F Now suppose you want to find the total area of the other faces. These faces form a rectangle 12″ × ▩″. Can you use the equation $12 \times 3 = n$ to find the area of this rectangle? 4

G You can find the total area of the faces of Prism A by adding 16 sq. in. and ▩ sq. in. The total surface area is 52 sq. in. 5

H Picture C shows a cube. What is the area of one face? Why is the total surface area of this cube 6 times the area of one face? 6



## 247

1 Show the class a model of a rectangular prism. Let the pupils see the model resting on different faces, so that they understand that any pair of parallel faces may be thought of as the bases. Review the terms *face*, *base*, and *side*.

2 If possible, show a model of Diagram B to help the pupils visualize the flattened-out prism.

3 Discuss in detail the work in Exercises D and E.

4 In discussing Exercise F, get the pupils to observe that four of the faces of the prism become one rectangle when flattened out as shown.

5 After the pupils have found the surface area of the prism in the diagram, have them find the surface area of a rectangular prism 3″ x 7″ x 10″ [242 sq. in.].

6 The pupils should recall that the faces of a cube have the same area and that, therefore, a short cut can be used in finding the surface area.

# 248-252 Exploring problems

Exponded Notes for this lesson are an poges 362-364.

**Objectives**

The pupil learns to compute the volume of a rectangular prism.

**Vocabulary**

New words page 252 Helen

**Comments**

Pupils are already familiar with the concept of volume from doing the work on pages 14-16. They know that the volume of a rectangular prism can be thought of as the number of one-inch cubes that fill it. They also understand what a cubic inch is. They are now ready to find the volume of a rectangular prism by computation.

The method for finding volume is similar to the method already used for finding area. Two ratios are involved. One ratio expresses the number of cubes for each layer, and the other expresses the number of cubes for the total number of layers. An equation of ratios is made, and the ratio test is applied to find the total number of cubes.

The traditional rule "volume is equal to the area of the base times the height or to length times width times height" is not presented to the pupils in this book. The pupils may discover the rule for themselves, since it follows directly from the ratio test. The rule, in itself, is merely a short cut; it does not aid in developing meaning. Therefore, pupils should learn to compute volume by writing an equation of ratios and applying the ratio test. Only then will any short cuts be meaningful to the pupils. Have the pupils observe that the second term of one ratio is the numeral 1 and they can use the short cuts learned on page 209.

**Answers**

When two answers are indicated, the answer in parentheses is the result of the computation. For page 252, the answer in brackets in the equation is the more acceptable one because of the issue of precision in measurement. See pages 216-217 of the pupils' book.

Page 250:

The volume of the piece of cheese is about 5 cu. in. ($4\frac{1}{2}$)

Page 251:

J 1000 cu. in.

K Yes (311.148)

L About 136 cu. in. ($135\frac{5}{8}$)

M About 98 cu. in.

Page 252:

(block 1)

A $3\times6.5/1=[318]/16.3$; (317.85)

B $15\times7/1=[735]/7$

C $12\times12/1=[1728]/12$

D $4\times8\frac{1}{2}/1=[340]/10$

E $11\times13\frac{1}{8}/1=[668]/4\frac{5}{8}$; ($667\frac{47}{64}$)

F $5.6\times7/1=[220]/5.6$; (219.52)

G $9\times9/1=[729]/9$

(block 2)

A $4\times4/1=[64]/4$

B $14\times8/1=[1008]/9$

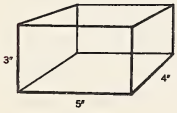
C $231/1=1008/[4]$; ($4\frac{4}{11}$)

D $20\times10/1=[2000]/10$; $231/1=(2000)/[9]$; ($8\frac{152}{231}$)

E Volume of first box: $7\frac{1}{2}\times15/1=[1238]/11$; ($1237\frac{1}{2}$)
Volume of second box: $8\times9\frac{1}{2}/1=[1026]/13\frac{1}{2}$
$1238-1026=[212]$
The volume of the first box is about 212 cu. in. greater.

## Exploring problems

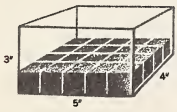
A A box is in the shape of a rectangular prism with a base 5″ x 4″ and a height of 3″. What is its volume in cubic inches? 1




Imagine that the box is being filled with 1-inch cubes. There are 20 cubes in the first layer.

Is the number of cubes in 1 layer the same as the number of square inches in the area of the base of the prism? Can you represent the number of cubes in 1 layer as $5\times4$? 2

B This is a rate of $(5\times4)$ cubic inches per 1 layer. Express this rate with a ratio.



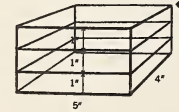

$$\frac{5\times4}{1}$$

$5\times4$ ← Number of cubic inches in each layer

1 ← Number of layers

3

C Now you must find how many layers of cubic inches there will be. The height of the prism is 3″. Each layer will use 1″ of the height. There will be 3 layers.




You do not know how many cubic inches will be used in all 3 layers, but you can express the rate as $w$ cubic inches per 3 layers. 4

$$\frac{W}{3}$$

$W$ ← Total number of cubic inches

3 ← Total number of layers





# 248

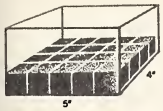
1 Have the problem read aloud. Ask the pupils what is meant by "the volume" of a prism. If possible, use a box and 1-inch cubes to demonstrate the solution of this problem.

2 Be sure the pupils understand that "20 cubes per 1 layer" is a *rate*. Have the pupils observe that the number of cubes per layer is the same as the number of square inches in the area of the base of the prism. Discuss the equation of ratios that tells that there are $(5\times4)$ square inches in the base.

3 Make certain the pupils understand that another way of writing the ratio $(5\times4)/1$ is $20/1$ and that this ratio tells the number of cubic inches in one layer.

4 Have the pupils observe that the height of the box is 3 in.; therefore, it will hold three layers of 1-inch cubes. Be sure the pupils can explain why the ratio $w/3$ can be written.

D




The rate at which cubic inches will be used will be the same for the 3 layers as for 1 layer. So you can make an equation of the ratios.

1

$$\frac{5 \times 4}{1} = \frac{w}{3}$$

2

Do these terms stand for number of cubic inches or number of layers? You must find the numeral that replaces $w$.

What do these terms stand for?

E

20 cubic inches are used for 1 layer. You can see that 60 cubic inches are used for all 3 layers. You can also compute to find how many cubic inches there are in the prism.

F

When you use the ratio test, you get this equation.

3

$W = (5 \times 4) \times 3$ or $W = 5 \times 4 \times 3$

60 replaces $w$.

The volume of the box is 60 cu. in.



# 249

1 Have the pupils recall that each layer has the same number of cubes. Therefore the rate expressed as cubic inches for one layer will be equal to the rate expressed as the total number of cubic inches for all the layers, and an equation of ratios can be written.

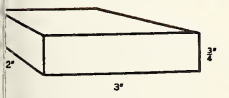
2 Get the pupils to explain that the first terms represent the number of cubic inches and the second terms represent the number of layers. Have them relate the ratios to the pictures. Point out that now there are three dimensions to consider instead of two as when computing areas.

3 Point out that the pupils can find the volume by thinking of three layers of 20 cubes each, or they may compute by applying the ratio test. Have them explain why w may be replaced by 60.

G

1

A piece of cheese is in the shape of a rectangular prism 3 in. long, 2 in. wide, and $\frac{3}{4}$ in. high. This piece contains how many cubic inches of cheese?




The blue dotted lines in Picture H show how the prism would look if it were 1 in. high. Imagine that you are cutting the prism into 1-inch cubes. The number of cubes in a 1-inch layer will be the same as the number of square inches in the area of the base of the prism. Can you represent the number of cubes as $3 \times 2$?

H

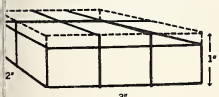
The rate is $(3 \times 2)$ cubic inches per 1-inch layer. Express this rate with a ratio.

2

$$\frac{3 \times 2}{1}$$

Number of cubes in 1 layer

Number of 1-inch layers



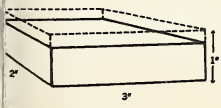

The prism is $\frac{3}{4}$ in. high. There will be $\frac{3}{4}$ cu. in. for every cubic inch in the 1-inch layer. Why can the rate also be expressed as $\frac{d}{3}$ cubic inches per $\frac{3}{4}$ layer?

I

$\frac{d}{3}$ What does this term stand for?

3

$\frac{3}{4}$ What does this term stand for?




Cubic inches will be used at the same rate for $\frac{3}{4}$ layer as for 1 layer.

How do you get the equation below?

4

$d = (3 \times 2) \times \frac{3}{4}$ or $d = 3 \times 2 \times \frac{3}{4}$

There are ■ cubic inches in the piece of cheese.

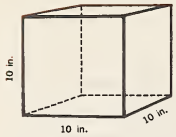
# 250

1 Discuss the problem and relate it to the picture. Have the pupils note especially the height of the piece of cheese.

2 Point out that if one layer of 1-inch cubes were placed on the base of a prism, there would be the same number of cubes as there are square inches in the area of the base. Be sure the pupils understand how to express the ratio for one layer of cubes.

3 Have the pupils explain that $d$ in the ratio stands for the total number of cubic inches. Point out that since the prism is only $\frac{3}{4}$ in. high, there is actually only $\frac{3}{4}$ cubic inch of volume for each square inch of the base. The number $\frac{3}{4}$ represents the number of layers.

4 Have the pupils tell how the two forms of the equation result from the ratio test, pointing out that the numerals in parentheses indicate the first term in the first ratio and that the equation can be written without parentheses.



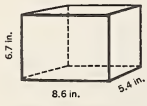

What are the dimensions of Cube J? What is the volume of this cube in cubic inches?

J

Why can you use the equation below to find the volume?

1 $n=(10\times10)\times10$

The volume of Cube J is ■ cu. in.



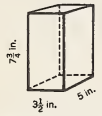

Prism K is 8.6″ long, 5.4″ wide, and 6.7″ high. Find the volume in cubic inches.

K

Why can you use the equation below to find the volume?

2 $m=(8.6\times5.4)\times6.7$

Is the volume of Prism K about 311 cu. in.?




Prism L has a base $3\frac{1}{2}$ in. by 5 in. It has a height of $7\frac{3}{4}$ in. What is the volume of this prism in cubic inches?

L

3 How do you get the equation shown below?

$h=(3\frac{1}{2}\times5)\times7\frac{3}{4}$

The volume of Prism L is ■ cu. in.

Prism M is 7 in. long, 4 in. wide, and $3\frac{1}{2}$ in. high. Find the volume in cubic inches.

M

3 How do you get the equation shown below?

$e=(7\times4)\times3\frac{1}{2}$

The volume of Prism M is ■ cu. in.

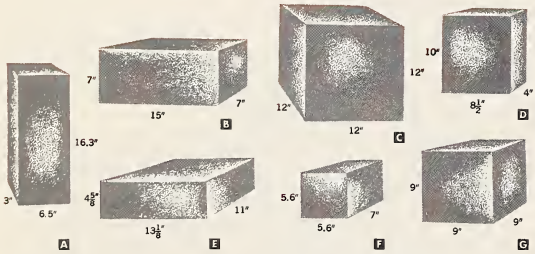


# 251

1 Have the pupils observe the dimensions of Prism J and explain why it is a cube. Discuss the equation of ratios $[10\times10/1=n/10]$ necessary to find the volume, and have the pupils explain the equation that results from the ratio test. Let a pupil solve the equation.

2 Apply Note 1 to Prism K.

3 Adapt Note 1 to Prisms L and M.




1 Each diagram above represents a rectangular prism. Find the volume of each prism in cubic inches.

In all the problems on this page, remember that you are working with rectangular prisms.

2 A A solid glass paperweight is a cube with an edge of 4 in. Its volume is how many cubic inches?

B Helen has a fish tank 14″ long, 8″ wide, and 10″ high on the inside. She keeps the tank filled with water to a depth of 9″. How many cubic inches of water are in the tank?

THINK The water is a prism ■″ high.

C There are 231 cubic inches in 1 gallon. About how many gallons of water does Helen keep in her fish tank? Give your answer to the nearer gallon.

$\frac{231}{1}=\frac{1008}{k}$

D Martha has a fish tank 20″ long, 10″ wide, and 12″ high. She keeps the fish tank filled to a depth of 10″. How many cubic inches of water are in this tank? To the nearer gallon, how many gallons of water are in the tank?

E One box measures $7\frac{1}{2}$″ x 15″ x 11″. Another box measures 8″ x $9\frac{1}{2}$″ x $13\frac{1}{2}$″. Which box has the greater volume? Its volume is how many cubic inches greater?

F What is the volume of a metal cube with an edge of 6.2 inches?

3 Now you should be able to find the volumes of rectangular prisms.





# 252

1 To find the volume of each prism, have the pupils write an equation of ratios and apply the ratio test to it.

2 Assign Problems A to F as individual work. Remind pupils that for some of the problems they may have to use information from previous problems. Point out that some problems require exact answers and others approximate answers.

3 When pupils have completed the problems, supply answers so that they may verify their work.

# 253-256 Moving forward

Expanded Notes for this lesson are on pages 364-365.

**Objectives**

The pupil extends his knowledge of volume to include the cubic foot and the cubic yard. He learns to reduce units of cubic measure.

**Vocabulary**

New words page 255 chest*, occupy

**Comments**

In the previous lesson on computing volume, only the cubic inch was used as the unit to express the volumes found. This procedure avoided certain complexities, and general principles related to finding volumes were established in the simplest way possible.

Now the computation of volume can be extended to include the cubic foot and the cubic yard. And since it is sometimes necessary to change from one unit of cubic measure to another, the reduction of cubic measures is included in this lesson. Since such reduction depends upon a knowledge of cubic measure equivalents, the cubic foot is introduced in terms of its volume in cubic inches. The cubic yard is introduced in terms of its volume in cubic feet.

Equations of ratios provide a meaningful way to convert cubic measures from one unit to another. The pupil does not have to rely on memorization. Instead, he has a method that he has used before in many different problem situations.

**Answers**

Pages 254-256:

(block 1)

G 3 yd. by 2 yd. by 2 yd.; Apply the ratio test to the equation $(3\times2)/1=n/2$ and multiply $3\times2\times2$ to find the numeral that replaces *n*.

H Yes; yes

I 2592 cu. in.

J 14′ x 12′ x 8′; by multiplying $14\times12\times8$

K ——

L $14\frac{2}{3}$ cu. ft.

M $24\times18/1=[10{,}368]/24$; 10,368 cu. in.

N $1728/1=10{,}368/[6]$ or $2\times(1\frac{1}{2})/1=[6]/2$; 6 cu. ft.

O Prism B: $(18)\times4/1=[288]/4$; 288 cu. ft.
Also, $6\times(1\frac{1}{3})/1=[10\frac{2}{3}]/(1\frac{1}{3})$; $10\frac{2}{3}$ cu. yd.
Prism C: $8\times(3)/1=[144]/(6)$; 144 cu. ft.
Also, $(2\frac{2}{3})\times1/1=[5\frac{1}{3}]/2$; $5\frac{1}{3}$ cu. yd.
Prism D: $2\times5/1=[20]/2$; 20 cu. ft.
Prism E: $(60)\times13/1=[15{,}600]/20$; 15,600 cu. in.
Also, $5\times(1\frac{1}{12})/1=[9\frac{1}{36}]/(1\frac{2}{3})$; $9\frac{1}{36}$ cu. ft.
Prism F: $39\times39/1=[59{,}319]/39$; 59,319 cu. in.

P $3\times2/1=[21]/3\frac{1}{2}$; 21 cu. ft.

Q $4\times2/1=[5\frac{1}{3}]/\frac{2}{3}$; about 5 cu. yd.

(block 2)

A $27/1=[567]/21$

B $1728/1=[5184]/3$

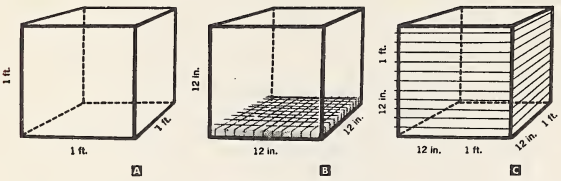
C $27/1=1180/[43\frac{19}{27}]$

D $1728/1=8640/[5]$

E $27/1=405/[15]$

## Moving forward

In this lesson you will learn more about the measurement of volume.




**1** Diagram A represents a box that is a 1-foot cube. The volume of the cube is 1 cubic foot. The work below shows how to find the volume in cubic inches.

**2** A Why can you think of each dimension of the box as 12 in.? Look at Diagram B. If you fill the box with 1-inch cubes, there will be (12 × 12) cubes in 1 layer. This is a rate of ■ cu. in. per 1 layer.

$\frac{144}{1}$ ← What does 144 represent?
← What does 1 represent?

**3** B Look at Diagram C. How do you know that there will be 12 layers of cubes? The rate can also be expressed as $z$ cubic inches per 12 layers.

$\frac{z}{12}$ ← What will the numeral that replaces $z$ represent?
← What does this 12 represent?

C Cubic inches will be used at the same rate for 12 layers as for 1 layer.

**4** $\frac{144}{1} = \frac{z}{12}$ ← How do you find the numeral that replaces $z$? When you find this numeral, you will know how many cubic inches there are in 12 layers.

Why can you also use the equation $z = 144 \times 12$? Can you use the equation $z = 12 \times 12 \times 12$?

The volume of the box is ■ cu. in.
1 cubic foot = 1728 cubic inches



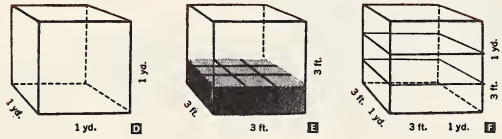



Diagram D represents another box in the shape of a cube. What are the dimensions of this box? The volume of the box is 1 cubic yard. The work below shows how to find the volume in cubic feet.

**1** D Why can you think of each dimension of the box as 3 ft.? Look at Diagram E. In one layer of 1-foot **2** cubes, there are (■ × ■) cubes. The rate is ■ cu. ft. per 1 layer.

E Look at Diagram F. How many layers of cubes will there be? The rate can now be expressed as $k$ cubic feet per ■ layers.

F Cubic feet will be used at the same rate for 3 layers as for 1 layer.

What does 9 represent?
How do you find the numeral that replaces $k$?

$\frac{9}{1} = \frac{k}{3}$ When you find this numeral, you will know how many cubic feet there are in 3 layers.

Why can you also use the equation $k = 9 \times 3$? Can you use $k = 3 \times 3 \times 3$?

The volume of the box is ■ cu. ft.
1 cubic yard = 27 cubic feet

**3** G Diagram G on page 255 represents a packing case that has the shape of a rectangular prism. What are its dimensions? You can express its volume as 12 cu. yd. How do you get the 12?

H You can also express the volume of the case in cubic feet. Find how many cubic feet there are in 12 cu. yd.

The rate of cubic feet per cubic yards is always the same. The rate of cubic feet for 1 cu. yd. is expressed as 27 per 1. Look at Diagram H. Can the rate of cubic feet for 12 cu. yd. be expressed as $e$ per 12?

**4**

You can use this equation to find the number of cubic feet in 12 cu. yd.

$\frac{27}{1} = \frac{e}{12}$

Can you multiply 12 by 27 to find the numeral that replaces $e$?

**5** 12 cu. yd. = 324 cu. ft.



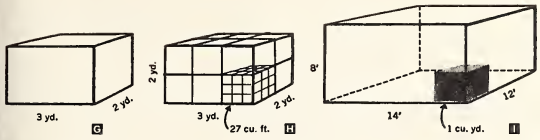


## 253

1. It will be helpful to have available a box in the shape of a one-foot cube and also approximately 150 one-inch cubes. Use these materials to demonstrate the action taking place in Exercises A to C.
2. Have someone explain why each edge of the cube can be thought of as 12 in. Get a pupil to explain why the first layer will contain 144 cu. in. Discuss the ratio that expresses the number of cubes per 1 layer.
3. Get the pupils to explain why there will be 12 layers. Have them explain the ratio that expresses the rate of all the cubes for all the layers.
4. Ask the pupils why they can write an equation of ratios. Point out the two equations ($z = 144 \times 12$ and $z = 12 \times 12 \times 12$) and let the pupils explain why either can be used. Point out that they now know the number of cubic inches equal to 1 cu. ft.

## 254

1. If possible, use cubes to demonstrate the ideas in Exercises D to F. Cubes of any size may be used if 1-ft. and 1-yd. cubes are not available. However, be sure the pupils understand that you are working with a scale model.
2. Adapt the Notes for page 253 to Exercises D to F. After the exercises have been discussed, point out that they now know the number of cubic feet in 1 cu. yd.
3. Have the pupils give the equation of ratios for this exercise and explain how to solve it.
4. Get the pupils to see that they can express the rate of cubic feet per one cubic yard as 27 per 1 and that the rate is the same for all the cubic yards. Explain Diagram H.
5. Have the pupils explain the equation of ratios and tell how to find the replacement for e. Let them verify the multiplication. See if they can suggest an alternative way of finding the volume in cubic feet.




I A piece of wood has a volume of 1.5 cu. ft. What is the volume of this piece of wood in cubic inches?

In 1 cu. ft. there are ■ cu. in.

$$\frac{1728}{1} = \frac{a}{1.5}$$

Can you find the number of cubic inches by finding the product of 1728 and 1.5?

J Diagram I represents a room. What are the dimensions of this room? You can say that the room contains 1344 cu. ft. of space. How do you get the 1344?

K You can also express the volume of the room in cubic yards. Find how many cubic yards there are in 1344 cubic feet.

Does 27 per 1 express the rate of cubic feet per cubic yards? There are 1344 cu. ft. in the room. Does 1344 per $b$ also express the rate of cubic feet per cubic yards?

You can use the equation below to find how many cubic yards there are in 1344 cu. ft. 3

$$\frac{27}{1} = \frac{1344}{b}$$

$27b = 1344$. To find the numeral that replaces $b$, divide ■ by ■.

1344 cu. ft. = 49⅓ cu. yd.

L A chest of drawers is 32″ wide, 18″ deep, and 44″ high. How many cubic feet of space does the chest of drawers occupy? 4

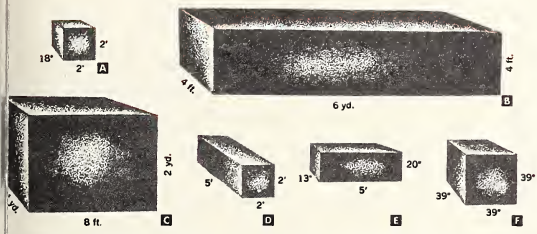
First find the volume of the chest in cubic inches. The volume is 25,344 cu. in. How do you get the 25,344?

Next express 25,344 cu. in. as cubic feet. ■ cu. in. will be used for each cubic foot. 5

$$\frac{1728}{1} = \frac{25344}{W}$$

$1728w = 25344$. To find the numeral that replaces $w$, divide ■ by ■.






M Each diagram above represents a rectangular prism. Look at Diagram A. Two of its dimensions are given in feet, but the third dimension is given in inches. To find the volume in cubic inches, you can use the dimensions 24″, 24″, and 18″. Why? What is the volume in cubic inches?

N You can also find the volume of Prism A in cubic feet. To do this, you can express 10,368 cu. in. as cubic feet. Or you can use the dimensions 2′, 2′, and 1½′. What is the volume in cubic feet?

O Find the volume of each of the other prisms above. You may have to change one dimension before you make your equation.

P A box containing a kitchen stove is 3′ wide, 2′ deep, and 42″ high. How many cubic feet of space does the box occupy?

THINK Why can you use 3½ feet as one dimension of the box?

Q Mr. Block has a rectangular pool about 4 yd. long, 2 yd. wide, and 2 ft. deep in his yard. He wants to fill it in to make a rock garden there. About how many cubic yards of fill will he need?

THINK Why can you use ⅔ yd. as one dimension of the pool?

A 21 cu. yd. = ■ cu. ft.
B 3 cu. ft. = ■ cu. in.
C 1180 cu. ft. = ■ cu. yd.
D 8640 cu. in. = ■ cu. ft.
E 405 cu. ft. = ■ cu. yd. 4

Now you should be able to find the volumes of rectangular prisms in cubic inches, cubic feet, and cubic yards.



# 255

1 Have Problem I read and let the pupils explain the equation of ratios. Have them compute to find the replacement for $a$.
2 Ask a pupil to give the dimensions of the room. Then discuss the equation of ratios needed to find the volume in cubic feet. Verify the answer.
3 Have the pupils explain the equation of ratios and how to find the replacement for $b$. Let them do the computation to verify the answer.
4 Get the pupils to observe that the dimensions are given in inches, and cubic feet are required in the answer. Let someone explain how to find the number of cubic inches in the chest of drawers, and have the class verify the answer.
5 Point out that the 25,344 cu. in. are to be expressed as cubic feet. Have the pupils explain the equation, tell how to find the replacement for $w$, and do the computation.

# 256

1 Have someone give the dimensions of Prism A. Then ask the pupils to give all the dimensions in inches. Have them find the volume in cubic inches by using an equation of ratios.
2 Have the pupils explain how $1\frac{1}{2}$ is found. Then have them find the volume in the two ways suggested, using equations of ratios.
3 Assign Exercises O to Q and A to E as individual written work. In Exercise O, where dimensions of the prisms are sometimes in different units, tell the pupils that they may use whichever unit seems simpler. Point out that fraction numerals may be used for dimensions.
4 When the pupils have finished the exercises, supply answers so they may verify their work.

# 257 Using arithmetic

Exponded Notes are not considered necessary for this lesson.

**Objectives**

The pupil solves problems involving square and cubic measures.

**Vocabulary**

New words page 257 paved, concrete*, dirt*, capacity

**Comments**

The problems in this lesson give the pupils practice in finding areas and volumes and in the reduction of measures. Problem B introduces the rod as a measure of length. Problems G to J involve capacity. Pupils should understand that the term *capacity* is often used when the volume of the inside of a container, such as a box or chest, is discussed.

**Answers**

A $9 \times 60/1 = [270]/\frac{1}{2}$; 270 cu. ft.

B $16\frac{1}{2}/1 = (528)/[32]$; 32 rods

C $20/1 = 160/[8]$; 8 rods

D $25/1 = [1712\frac{1}{2}]/68\frac{1}{2}$; $1712\frac{1}{2} - (1600) = [112\frac{1}{2}]$; more; $112\frac{1}{2}$ sq. rd.

E $\frac{3}{4} \times (96)/1 = [54]/\frac{3}{4}$; 54 cu. in.

F $50 \times 20/1 = [666\frac{2}{3}]/\frac{2}{3}$; about 667 cu. ft.

$27/1 = (667)/[24\frac{19}{27}]$; about 25 cu. yd.

G $36 \times 16\frac{1}{2}/1 = [9504]/16$; 9504 cu. in.

H $28 \times 19\frac{1}{2}/1 = [14,196]/26$; 14,196 cu. in.

I $231/1 = 14,196/[61\frac{5}{11}]$; about 61 gal.

J $4 \times 3.5/1 = [49]/3.5$; 49 cu. ft.

K $62.4/1 = [3057.6]/49$; 3057.6 lb.

## Using arithmetic 1

A A straight driveway is 9 ft. wide and 60 ft. long. It is to be paved with concrete 6 in. thick. How many cubic feet of concrete will there be in the driveway?

THINK Why can you think of the depth of the concrete as $\frac{1}{2}$ ft.?

B Sometimes length is measured in rods. A rod is equal to $16\frac{1}{2}$ ft. How many rods of fencing will be needed for a square garden, each side of which is 132 ft.?

C Mr. Parker has a rectangular truck garden with an area of 1 acre. Its area can also be given as 160 square rods. The garden is 20 rods long. How wide is it?

D Another rectangular truck garden is 25 rods wide and $68\frac{1}{2}$ rods long. Is the area of this garden more than or less than 10 acres? The area is how much more than or less than 10 acres?

E A steel bar is in the shape of a rectangular prism. It is $\frac{3}{4}$″ wide, $\frac{3}{4}$″ high, and 8′ long. How many cubic inches of steel are there in the bar?

F A garden 50 ft. long and 20 ft. wide is to be covered with black dirt to a depth of 8 in. About how many cubic feet of dirt are needed? About how many cubic yards of dirt are needed?

THINK When you find the number of cubic feet in Problem F, why can you use $\frac{2}{3}$ ft. as one dimension?

G Bob made a chest for his little sister's toys. He wanted to know the capacity of the chest, or how much it would hold. So he found the inside dimensions of the chest. These dimensions were 36″, $16\frac{1}{2}$″, and 16″. What was the capacity of the chest in cubic inches?

H A water tank in the shape of a rectangular prism is 28″ long, $19\frac{1}{2}$″ wide, and 26″ high. What is the capacity of this tank in cubic inches?

I 231 cu. in. of water are equal to 1 gal. When the tank described in Problem H is filled, about how many gallons of water will it hold?

J Another water tank in the shape of a rectangular prism is 4 ft. long, 3.5 ft. wide, and 3.5 ft. high. When this tank is filled with water, how many cubic feet of water does it hold?

K A cubic foot of water weighs 62.4 lb. What is the weight of the water in the tank described in Problem J? 2





## 257

1 Assign these problems as individual written work. Remind the pupils that before computing, they should express all dimensions in the same unit of measure. Also point out that Problems F and I ask for approximate answers.

2 Give the pupils ample time to work the problems. When they have completed the work, supply answers and discuss the equation for each problem.

# 258 Checking up

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil tests his skill in finding the volume of the rectangular prism and in reducing cubic measures.

**Vocabulary**

There are no new words.

**Comments**

Use these "Checking up" tests to discover how well the pupils understand the terminology used in expressing dimensions and how well they are able to compute volumes and reduce cubic measures. If Tests 1 and 2 reveal weaknesses, review the lessons on pages 248-252 and 253-256. If Test 3 causes trouble, review the lesson on pages 253-256.

The "Keeping skillful" exercises in division, addition, and subtraction of numbers expressed by decimal fraction numerals may be assigned according to individual needs.

**Answers**

Checking up:

Test 1

A $9 \times 9/1 = [729]9$; 729 cu. yd.

B $3\frac{1}{2} \times 10/1 = [122\frac{1}{2}]/3\frac{1}{2}$; $122\frac{1}{2}$ cu. in.

C $8 \times 4/1 = [160]/5$; 160 cu. ft.

D $(6) \times 11/1 = [330]/5$; 330 cu. ft. Also,
$2 \times (\frac{11}{3})/1 = [12\frac{2}{9}]/(\frac{5}{3})$; $12\frac{2}{9}$ cu. yd.

Test 2

A $4 \times 9/1 = [504]/14$; 504 cu. ft.

B $1\frac{3}{4} \times 2\frac{1}{2}/1 = [26\frac{1}{4}]/6$; $26\frac{1}{4}$ cu. in.

C $1.2 \times 3.5/1 = [21]/5$; 21 cu. yd.

D $42 \times 42/1 = [137,592]/(78)$; 137,592 cu. in. Also,
$(3\frac{1}{2}) \times (3\frac{1}{2})/1 = [79\frac{5}{8}]/6\frac{1}{2}$; $79\frac{5}{8}$ cu. ft.

E $9\frac{1}{4} \times 4/1 = [333]/(9)$; 333 cu. ft. Also,
$(3\frac{1}{12}) \times (1\frac{1}{3})/1 = [12\frac{1}{3}]/3$; $12\frac{1}{3}$ cu. yd.

F $25 \times 25/1 = [18,750]/(30)$; 18,750 cu. ft. Also,
$(8\frac{1}{3}) \times (8\frac{1}{3})/1 = [694\frac{4}{9}]/10$; $694\frac{4}{9}$ cu. yd.

Test 3

A $1728/1 = [5184]/3$; 5184 cu. in.

B $27/1 = 45/[1\frac{2}{3}]$; $1\frac{2}{3}$ cu. yd.

C $1728/1 = 864/[\frac{1}{2}]$; $\frac{1}{2}$ cu. ft.

D $27/1 = [63]/2\frac{1}{3}$; 63 cu. ft.

E $27/1 = 78/[2\frac{8}{9}]$; $2\frac{8}{9}$ cu. yd.

F $1728/1 = 1512/[\frac{7}{8}]$; $\frac{7}{8}$ cu. ft. Also,
$1728/1 = 1512/[.875]$; .875 cu. ft.

G $27/1 = [2160]/80$; 2160 cu. ft.

H $1728/1 = [25,056]/14\frac{1}{2}$; 25,056 cu. in.

I $27/1 = 144/[5\frac{1}{3}]$; $5\frac{1}{3}$ cu. yd.

J $1728/1 = 756/[\frac{7}{16}]$; $\frac{7}{16}$ cu. ft. Also,
$1728/1 = 756/[.44]$; .44 cu. ft.

K $1728/1 = [129,600]/75$; 129,600 cu. in.

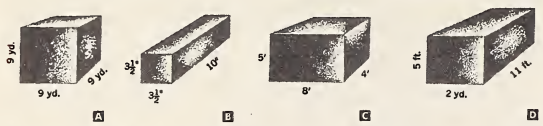
L $1728/1 = [26,352]/15\frac{1}{4}$; 26,352 cu. in.

M $27/1 = 774/[28\frac{2}{3}]$; $28\frac{2}{3}$ cu. yd. Also,
$27/1 = 774/[28.67]$; 28.67 cu. yd.

Keeping skillful:

| (block 1) | (block 2) | (block 3) |
|---|---|---|
| A 36.83 | A .774 | A 7.3894 |
| B 23.97 | B 1.6867 | B .6237 |
| C .15 | C 8.75 | C .240 |
| D .62 | D 182.48 | D .4632 |
| E 66.67 | E 10.63 | E 5.778 |
| F .06 | F 95.9000 | F .4836 |

## Checking up




1

Test 1

Find the volume of each rectangular prism shown above.

Test 2

Find the volume of each rectangular prism whose dimensions are given below.

| | Dimensions of base | Height |
|---|---|---|
| A | 4 ft. by 9 ft. | 14 ft. |
| B | 1⅜ in. by 2½ in | 6 in. |
| C | 1.2 yd. by 3.5 yd. | 5 yd. |
| D | 42 in. square | 6½ ft. |
| E | 9½ ft. by 4 ft. | 3 yd. |
| F | 25 ft. square | 10 yd. |

2

Test 3

A 3 cu. ft. = ■ cu. in.
B 45 cu. ft. = ■ cu. yd.
C 864 cu. in. = ■ cu. ft.
D 2⅓ cu. yd. = ■ cu. ft.
E 78 cu. ft. = ■ cu. yd.
F 1512 cu. in. = ■ cu. ft.
G 80 cu. yd. = ■ cu. ft.
H 14½ cu. ft. = ■ cu. in.
I 144 cu. ft. = ■ cu. yd.
J 756 cu. in. = ■ cu. ft.
K 75 cu. ft. = ■ cu. in.
L 15¼ cu. ft. = ■ cu. in.
M 774 cu. ft. = ■ cu. yd.

3

## Keeping skillful

4

Find each quotient to hundredths.

A 1565.25 ÷ 42.5 = r
B 3.5 ÷ .146 = d
C .624 ÷ k = 4.2
D 8.65 ÷ 14 = m
E 24 ÷ n = .36
F .522 ÷ 8.7 = t

Find the sum.

A .338, .160, .276
B .0438, 1.6429
C .29, .84, 7.62
D 84.66, 1.37, 96.45
E .19, 1.63, .55, 8.26
F 4.0064, 91.8936

Find the difference.

A 8.0491, .6597
B .7726, .1489
C 20.119, 19.879
D 1.3201, .8569
E 14.073, 8.295
F 16.3333, 15.8497

258 End-of-block tests on cubic measure

# 258

1 Have pupils work independently. Tell them to write an equation to find the volume of each rectangular prism. Remind them that when dimensions are given in different units, they should change one of the dimensions before computing and should label the answer to show the cubic units in which the volume was found.

2 Apply Note 1 to Test 2. Discuss the various ways in which volumes can be found and stated.

3 Have pupils write equations and compute to find the answers.

4 Assign these exercises as needed.

# 259-261 Exploring problems

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil learns how to solve problems comparing two groups of objects by using addition or subtraction.

**Vocabulary**

New words page 259 Canadian*; page 260 beetles*, canceled, Linda, Chuck*, autographs*; page 261 pearl*

**Comments**

The problems in this lesson can be classified as comparative-subtraction or comparative-addition situations. In one case, the pupils compare two groups to find out how many more or how many fewer objects there are in one group than in the other. Each object in one group is thought of as corresponding to one object in the other group. The situation is subtractive because the objects in the larger group that are corresponded to those in the smaller group are thought of as being removed, or subtracted, from the larger group. The number of objects remaining in the larger group tells how many more objects are in the larger group than in the smaller group. This number is the *difference* represented by the letter *t* in the equation for Problem A ($246 - 131 = t$).

If the number of objects in the larger group and the difference between the two groups are known, the problem leads to the same subtractive equation, as shown in Problem B by the equation $246 - n = 115$. The process used to compute in this case is, of course, subtraction.

When the pupil knows the number in the smaller group and the difference between the two groups, he again obtains the subtractive equation, but in this case the computational process is addition. The equation for Problem C, $k - 131 = 115$, illustrates this situation.

These two types of comparison problems complete the list of problem types that the pupils study in Grades 3 to 6. (A complete list of the problem types is found on pages 408-409 of this *Teaching Guide*.)

**Answers**

A Yes; 115

B The size of the larger group; yes; the size of the smaller group; 131

C The size of the smaller group; yes; the size of the larger group; 246

D The size of the larger group; the size of the larger group; 387

E The difference; the difference; 17

F The size of the smaller group; the size of the smaller group; 67

G The difference; 212−86=[126]

H The size of the larger group; [81]−63=18

I The size of the smaller group; 49−[31]=18

J The size of the smaller group; 119−[86]=33

K The size of the larger group; [60]−44=16

L The size of the smaller group; 128−[92]=36

M The size of the larger group; [105]−77=28

N The difference; 77−38=[39]

O The size of the larger group; [246]−208=38

P The size of the smaller group; 95−[68]=27

**Keeping skillful:**

(block 1)

The answers for Exercises C, H, J, and N are given to thousandths.

A 75/100=[36]/48

B 305.28/318=[96]/100; 96%

C 9.7/[60.625]=16/100

D 2.16/72=[3]/100; 3%

E 147/100=[216.09]/147

F 49/[98]=50/100

G 1.21/11=[11]/100; 11%

H 38/[13.818]=275/100

I [3]/300=1/100

J 32.3/[538.333]=6/100

K 138.32/76=[182]/100; 182%

L [17.25]/69=25/100

M [34.16]/244=14/100

N 75/[42.857]=175/100

O 8.41/29=[29]/100; 29%

P 88/[100]=88/100

Q 7.2/16=[45]/100; 45%

R [26.46]/42=63/100

S 77/[481.25]=16/100

(block 2)

A 48 pt.

B 35 bu.

C 32,000 lb.

D 32 tbs.

E 480 min.

F 540 in.

G 5 mi.

H 2 bu.

I 15,840 ft.

J 16 yr.

K 1200 oz.

L 60 qt.

M 48 qt.

N 144 things

O 21 wk.

P 9 qt.

Q 14 c.

R 5400 ft.

S 26,400 yd.

(block 3)

A 2304 sq. in.

B 18 sq. yd.

C 15,552 cu. in.

D 11 sq. ft.

E 8 cu. yd.

F 3 cu. ft.

G 342 sq. ft.

H 6480 sq. in.

I 162 cu. ft.

J 4 cu. yd.

K 20 sq. ft.

L 100 cu. ft.

M 63 sq. ft.

N 1728 sq. in.

O 32,832 cu. in.

P 729 cu. ft.

Q 69 sq. yd.

R 108 sq. ft.

S 864 sq. in.

## Exploring problems

A Dick has a collection of 246 rocks. He also has 131 shells. He has how many more rocks than shells?

In Problem A, do you know the size of each group? You are to find the difference of the numbers 246 and 131.

1

Number in larger group

$246 - 131 = t$

Number in smaller group

Number that tells the difference

Dick has ▪ more rocks than shells.

B Dick has a collection of 246 rocks. He has 115 more rocks than shells. Dick has how many shells?

2

In Problem B, do you know the size of the larger group or of the smaller group? Do you know the difference in the size of the groups? You are to find the size of which group?

Number in larger group

$246 - n = 115$

Number in smaller group

Number that tells the difference

Dick has ▪ shells.

C Dick has a collection of rocks. He also has a collection of 131 shells. He has 115 fewer shells than rocks. Dick has how many rocks?

3

In Problem C, do you know the size of the larger group or of the smaller group? Do you know the difference in the size of the groups? You are to find the size of which group?

Number in larger group

$k - 131 = 115$

Number in smaller group

Number that tells the difference

Dick has ▪ rocks.

D May collects Canadian and United States stamps. She has 260 Canadian stamps. She has 127 more United States stamps than Canadian stamps. She has how many United States stamps?

4

THINK Which do you have to find for Problem D, the difference, the size of the larger group, or the size of the smaller group?

$r - 260 = 127$

In the equation above, what does $r$ represent? What numeral replaces $r$?



1

E Tony has a collection of United States and Canadian pennies. He has 156 Canadian pennies and 139 United States pennies. He has how many fewer United States pennies than Canadian pennies?

THINK For Problem E, which do you have to find, the size of the smaller group, the size of the larger group, or the difference?

$156 - 139 = d$

What does $d$ represent in the equation above? What numeral replaces $d$?

F Bob has a collection of beetles and butterflies. He has 82 beetles. He has 15 fewer butterflies than beetles. Bob has how many butterflies?

THINK Which do you have to find for Problem F, the size of the larger group, the size of the smaller group, or the difference?

$82 - w = 15$

In the equation above, what does $w$ represent? What numeral replaces $w$?

For each problem that follows, first decide which you are to find, the size of the larger group, the difference, or the size of the smaller group. Then make an equation and find the answer for the problem.

2

G Paul collects stamped and canceled envelopes called covers. He has 212 United States covers and 86 covers from other places. He has how many more United States covers than covers from other places?

H Linda has many pictures of large cities. She has 63 pictures of New York. She has 18 more pictures of Chicago than she has of New York. Linda has how many pictures of Chicago?

I Chuck collects autographs of baseball and football players. He has 49 football autographs. He has 18 fewer baseball autographs than football autographs. He has how many baseball autographs?

J Chuck also collects baseball and football cards. He has 119 football cards. He has 33 more football cards than baseball cards. Chuck has how many baseball cards?

K Ann has a scrapbook with pictures of flowers and leaves in it. She has 44 flower pictures. She has 16 more leaf pictures than flower pictures. Ann has how many leaf pictures in her scrapbook?

L Jack and Don collect pictures of ships. Jack has 128 ship pictures. He has 36 more ship pictures than Don. Don has how many ship pictures?





# 259

1 Have Problem A read. Point out that Dick can find how many more rocks than shells he has if he makes each shell correspond to one of the rocks and then removes this group of corresponding rocks. Discuss how the equation describes this situation. Have the pupils observe that the equation is solved by subtraction.

2 Compare Problem B with Problem A. Point out that now the difference is known, but that the size of the smaller group is not. The equation is still of the form $a - b = c$, in which $a$ represents the size of the larger group, $b$ represents the size of the smaller group, and $c$ represents the difference. This equation is also solved by subtraction.

3 Study Problem C. Point out that the form of the equation is still the same, but that addition is used to find the size of the larger group.

4 Have the pupils tell how they will find the size of the larger group.

# 260

1 Discuss the equations for Problems E and F. Each problem involves the comparison of two groups of different size. The basic equation for each is the same as that for the problems on the preceding page. Get the pupils to see that subtraction will be used to solve both equations.

2 Assign Problems G to P (M to P are on the next page) as written work. Instruct the pupils first to write what they are to find, then to write an equation and find the answer.

N Chuck has 77 pictures of old autos. He has 28 fewer pictures of old autos than Paul has. Paul has how many pictures of old autos?

O Chuck also has 38 pictures of old sailing ships. He has how many more pictures of old autos than of old sailing ships?

O Sam has 208 match-book covers. He has 38 fewer match-book covers than Bill has. Bill has how many match-book covers?

P Carol has 95 glass buttons. She has 27 more glass buttons than pearl buttons. Carol has how many pearl buttons?

1

Now you should know how to solve problems in which two groups are compared by using the difference.

## Keeping skillful 2

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 75% of 48 is ■. | 6 gal. = ■ pt. | | | | | | | | | | | | | | | | | |
| B | 305.28 is ■% of 318. | 140 pk. = ■ bu. | | | | | | | | | | | | | | | | | |
| C | 9.7 is 16% of ■. | 16 T. = ■ lb. | | | | | | | | | | | | | | | | | |
| D | 2.16 is ■% of 72. | 16 fl. oz. = ■ tbs. | | | | | | | | | | | | | | | | | |
| E | 147% of 147 is ■. | 8 hr. = ■ min. | | | | | | | | | | | | | | | | | |
| F | 49 is 50% of ■. | 15 yd. = ■ in. | | | | | | | | | | | | | | | | | |
| G | 1.21 is ■% of 11. | 8800 yd. = ■ mi. | | | | | | | | | | | | | | | | | |
| H | 38 is 275% of ■. | 64 qt. = ■ bu. | | | | | | | | | | | | | | | | | |
| I | 1% of 300 is ■. | 3 mi. = ■ ft. | | | | | | | | | | | | | | | | | |
| J | 32.3 is 6% of ■. | 832 wk. = ■ yr. | | | | | | | | | | | | | | | | | |
| K | 138.32 is ■% of 76. | 75 lb. = ■ oz. | | | | | | | | | | | | | | | | | |
| L | 25% of 69 is ■. | 120 pt. = ■ qt. | | | | | | | | | | | | | | | | | |
| M | 14% of 244 is ■. | 6 pk. = ■ qt. | | | | | | | | | | | | | | | | | |
| N | 75 is 175% of ■. | 12 doz. = ■ things | | | | | | | | | | | | | | | | | |
| O | 8.41 is ■% of 29. | 147 da. = ■ wk. | | | | | | | | | | | | | | | | | |
| P | 88 is 88% of ■. | 36 c. = ■ qt. | | | | | | | | | | | | | | | | | |
| Q | 7.2 is ■% of 16. | 7 pt. = ■ c. | | | | | | | | | | | | | | | | | |
| R | 63% of 42 is ■. | 1800 yd. = ■ ft. | | | | | | | | | | | | | | | | | |
| S | 77 is 16% of ■. | 15 mi. = ■ yd. | | | | | | | | | | | | | | | | | |

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 16 sq. ft. = ■ sq. in. | | | | | | | | | | | | | | | | | | |
| B | 162 sq. ft. = ■ sq. yd. | | | | | | | | | | | | | | | | | | |
| C | 9 cu. ft. = ■ cu. in. | | | | | | | | | | | | | | | | | | |
| D | 1584 sq. in. = ■ sq. ft. | | | | | | | | | | | | | | | | | | |
| E | 216 cu. ft. = ■ cu. yd. | | | | | | | | | | | | | | | | | | |
| F | 5184 cu. in. = ■ cu. ft. | | | | | | | | | | | | | | | | | | |
| G | 38 sq. yd. = ■ sq. ft. | | | | | | | | | | | | | | | | | | |
| H | 45 sq. ft. = ■ sq. in. | | | | | | | | | | | | | | | | | | |
| I | 6 cu. yd. = ■ cu. ft. | | | | | | | | | | | | | | | | | | |
| J | 108 cu. ft. = ■ cu. yd. | | | | | | | | | | | | | | | | | | |
| K | 2880 sq. in. = ■ sq. ft. | | | | | | | | | | | | | | | | | | |
| L | 172,800 cu. in. = ■ cu. ft. | | | | | | | | | | | | | | | | | | |
| M | 7 sq. yd. = ■ sq. ft. | | | | | | | | | | | | | | | | | | |
| N | 12 sq. ft. = ■ sq. in. | | | | | | | | | | | | | | | | | | |
| O | 19 cu. ft. = ■ cu. in. | | | | | | | | | | | | | | | | | | |
| P | 27 cu. yd. = ■ cu. ft. | | | | | | | | | | | | | | | | | | |
| Q | 621 sq. ft. = ■ sq. yd. | | | | | | | | | | | | | | | | | | |
| R | 12 sq. yd. = ■ sq. ft. | | | | | | | | | | | | | | | | | | |
| S | 6 sq. ft. = ■ sq. in. | | | | | | | | | | | | | | | | | | |

261

# 261

1 When the pupils have finished the problems, supply answers so that they may verify their work.

2 The "Keeping skillful" exercises should be assigned as separate lessons. Block 1 consists of exercises in per cent. Have the pupils write an equation of ratios for each of these exercises. Block 2 covers reduction of various types of measures, and Block 3 requires reduction of square and cubic measures.

# 262-264 Side trip; Keeping skillful

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil learns how to compute with mixed measures.

**Vocabulary**

New words page 262 agree*; page 264 laps*

**Comments**

The teaching of computation with mixed measures (often referred to as "compound denominate numbers") is of only minor importance in the arithmetic program, but it is included because pupils will be required to perform these computations in standardized tests. It is possible to develop all four operations in three pages because the pupil has developed an understanding of the concept of regrouping and skill in the fundamental operations.

Three ways of writing the work are shown for addition and subtraction. It is important that the pupil observe that in all mixed measures the smaller or smallest unit plays the same rôle as the denominator of a fraction numeral. For example, in 3 ft. 7 in., the "in." plays the rôle of the 12 in $3\frac{7}{12}$ ft.

In each of the situations, the pupil should be aware of the regrouping involved. For example, in addition, 15 ft. 17 in. is regrouped and expressed as 16 ft. 5 in. Regrouping in subtraction is indicated by crossing out 5 ft. 1 in. and expressing the result as 4 ft. 13 in. The *base*, or "conversion factor," for the regrouping, of course, varies with the units involved. If weeks are regrouped to days, for instance, the base is 7; if yards are regrouped to feet, the base is 3.

The same fundamental concept of regrouping is stressed in the operations of multiplication and division. In multiplication, the pupil can determine the partial product for each unit and then regroup to get the answer in final form (see Example H, page 263). Similarly in division, the pupil uses the familiar pattern for division, but he must regroup as he performs the division (see Example I, page 263).

The pupil should understand all the patterns for processing as shown in the text. In standardized tests, however, the examples are usually given as shown on page 264; so the pupil should learn to compute within this framework.

**Answers**

Side trip:

(block 1)

A By adding the lengths of the sides

B $4\frac{5}{12}$, $2\frac{7}{12}$, $5\frac{1}{12}$

C For Example A, because $\frac{12}{12}$ equal 1 whole and $\frac{5}{12}$ remain; for Examples B and C, because 12 inches equal 1 foot, and 5 inches remain; the 15 is replaced by 16 because 1 more whole, or 1 more foot, must be added to the 15 ft.

D Yes

E 5 ft. 1 in.; 2 ft. 7 in.; by subtraction

F Yes; yes

G 5 ft. 1 in. has been renamed as 4 ft. 13 in.

H Yes; yes

I By multiplying the length of one side by 4

J Yes; yes

K Yes; yes; $22\frac{2}{3}$ yd.

L Yes; 8 feet equal 2 more yards, with a remainder of 2 ft.

M By dividing the perimeter by 5

N By multiplying 2 ft. by 5; by subtracting 10 ft. from 11 ft. 3 in.; so that it can be divided by 5; because 15 inches have been divided into 5 equal parts

O 4 hr. 50 min.; no; yes; yes; 4 hr. 50 min.

P 1 pt. 1 c./1=[1 qt. 1 pt.]/2 or
2×1 pt. 1 c.=[2 pt. 2 c.] or [1 qt. 1 pt.]

Q 11 lb. 12 oz./4=[2 lb. 15 oz.]/1 or
11 lb. 12 oz.÷[2 lb. 15 oz.]=4

R 5 ft. 4 in.−[1 ft. 10 in.]=3 ft. 6 in.

S 6 min. 30 sec./5=[1 min. 18 sec.]/1 or
6 min. 30 sec. ÷[1 min. 18 sec.]=5

(block 2)

A 7 wk.

B 4 T. 1200 lb.

C 6 min. 9 sec.

D 1 yd. 7 in.

E 1 T. 880 lb.

F 5 ft. $10\frac{1}{4}$ in.

G 12 qt. 1 pt.

H 35 yr. 10 mo. or $35\frac{5}{6}$ yr.

I 40 gal. 1 qt.

J 3 yd. 1 ft.

K 1 yr. 9 mo.

L 3 hr. 5 min.

Keeping skillful:

(block 1)

A 1

B 50

C 1388.9

D 4.6

E 12.48

F 9.1

G 160

H 10

I 177.87

J 3.65

(block 2)

A 13.710

B 35.8

C 11.250

D 15.28

E 1.688

F .125

G 9.4

H 7.740

I 17.3

J .502

K 14.03

L .438

M 2.825

N 6.360

O 2.79

P 3.5

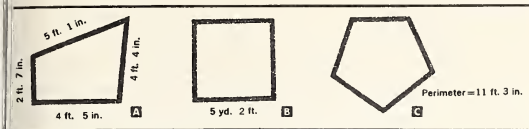
Q .871

R 2.271

S 170

T 25.85

# Side trip




A How do you find the perimeter of Polygon A?

B You can write the length of one of the sides as $4\frac{4}{12}$ ft. Write the other three dimensions in this way. 2

C Examples A, B, and C below show three ways to do the work to find the perimeter. When you find the sum in each example, why is the 17 replaced by 5? Why is the 15 replaced by 16?

D Do the answers agree in all three ways of doing the work?

| A 3 | B | C | |
|---|---|---|---|
| | | ft. | in. |
| $4\frac{5}{12}$ ft. | 4 ft. 5 in. | 4 | 5 |
| $4\frac{4}{12}$ ft. | 4 ft. 4 in. | 4 | 4 |
| $5\frac{1}{12}$ ft. | 5 ft. 1 in. | 5 | 1 |
| $2\frac{7}{12}$ ft. | 2 ft. 7 in. | 2 | 7 |
| $15\frac{17}{12}$ ft. | 15 ft. 17 in. | 15 | 17 |
| $16\frac{5}{12}$ ft. | 16 ft. 5 in. | 16 | 5 |

2 Using compound denominate numbers

E What is the length of the longest side in Polygon A? What is the length of the shortest side? How can you find the difference in the lengths?

F Examples D, E, and F below show three ways to do the work to find the difference. In Example D, is $4\frac{13}{12}$ another name for $5\frac{1}{12}$? Study the subtraction. Is it correct?

G In Example E, why are 4 and 13 written above 5 ft. 1 in.?

H Look at Example F. Is 4 ft. 13 in. another name for 5 ft. 1 in.? Does the answer in this example agree with the answers for Examples D and E?

| D 4 | E 5 | F | |
|---|---|---|---|
| | | ft. | in. |
| $4\frac{13}{12}$ | 4 13 | 4 | 13 |
| ~~$5\frac{1}{12}$~~ ft. | ~~5~~ ft. ~~1~~ in. | ~~5~~ | ~~1~~ |
| $2\frac{7}{12}$ ft. | 2 ft. 7 in. | 2 | 7 |
| $2\frac{6}{12}$ ft. | 2 ft. 6 in. | 2 | 6 |

| G | H 2 |
|---|---|
| $5\frac{2}{3}$ yd. | 5 yd. 2 ft. |
| 4 | 4 |
| $20\frac{8}{3}$ yd. | 20 yd. 8 ft. |
| $22\frac{2}{3}$ yd. | 22 yd. 2 ft. |

I 3

5)11 ft. 3 in. | 2 ft.
10 ft.
~~1 ft.~~ ~~3 in.~~
15 in. | 3 in.
15 in.
2 ft. 3 in.

I Polygon B on page 262 is a square. How can you use multiplication to find the perimeter of a square?

J Can 2 ft. be written as $\frac{2}{3}$ yd.? Can 5 yd. 2 ft. be written as $5\frac{2}{3}$ yd.?

K In Example G above, $5\frac{2}{3}$ has been multiplied by 4. Is the product $20\frac{8}{3}$ correct? Is $22\frac{2}{3}$ another name for $20\frac{8}{3}$? What is the perimeter of Polygon B?

L Look at Example H. Is 20 yd. 8 ft. the product of 5 yd. 2 ft. and 4? Why is it correct to replace 20 yd. 8 ft. by 22 yd. 2 ft.?

M Polygon C on page 262 has 5 equal sides. Its perimeter is 11 ft. 3 in. How can you find the length of a side?

N Look at Example I above. How is the 10 ft. found? How is 1 ft. 3 in. found? Why is 1 ft. 3 in. replaced by 15 in.? Why is 3 in. then written in the quotient? Each of the equal sides in Polygon C is 2 ft. 3 in. long.

O Mr. Page is planning a trip. If he goes by air, it will take 2 hr. 25 min. If he goes by train, it will take him 7 hr. 15 min. How much longer will the trip take if he goes by train? 4

THINK What numbers can you subtract to find this difference?

Can you subtract 25 min. from 15 min.? Is 6 hr. 75 min. another name for 7 hr. 15 min.? Can you subtract now? The trip by train will take ■ hr. ■ min. longer than the trip by air.

P Mrs. Cook has a recipe for a fruit drink that calls for 1 pt. 1 c. of grape juice. This recipe makes 1 gal. of fruit drink. How many pints of grape juice will Mrs. Cook need to make 2 gal. of the fruit drink? 5

Q Mrs. Cook bought 4 chickens. They weighed 2 lb. 12 oz., 3 lb. 7 oz., 2 lb. 9 oz., and 3 lb. What was the average weight of the chickens? 5



# 262

1 Discuss how to find the perimeter of a polygon.

2 Ask the pupils to write the lengths of the other three sides of Polygon A, as directed.

3 Point out that all the inches in Example A have been expressed as fractional parts of a foot. All the fraction numerals have a common denominator, and the fractions can be added. For Example B, point out that the 17 inches in the total are more than 1 foot and that changing 17 inches to feet gives 1 foot and 5 inches. For Example C, point out that the work is arranged in table form with headings. The work is done in the same way as for Example B.

4 Example D shows how to find the difference in lengths by subtraction. Let a pupil explain the regrouping.

5 Let a pupil explain how the 13 in. replaces the 1 in. in Example E. Then call attention to the table form of Example F and have the pupils note that the work is done in the same way.

# 263

1 Point out that multiplication can sometimes be used to find a perimeter. Ask a pupil to tell when this is possible. Then bring out why 2 ft. can be written as $\frac{2}{3}$ yd. Next ask a pupil to explain the multiplication in Example G and the reduction of $20\frac{8}{3}$ to $22\frac{2}{3}$.

2 Have the pupils notice in Example H that the number of feet is first multiplied and then the number of yards. Let someone explain how 8 feet are changed to 2 yards, 2 feet.

3 Discuss the type of division involved in Example I, and point out that the number of feet is divided first. Let a pupil explain why 1 ft. 3 in. is expressed as 15 in. Have the pupils note where the numeral expressing inches is placed in the quotient.

4 Have the pupils subtract to find the answer.

5 Some pupils may wish to solve this problem by using an equation of ratios. Allow them to do so.

R John has a board 5 ft. 4 in. long. How much should he cut from the board to get a board 3 ft. 6 in. long?

S A racing car was driven 5 laps around a track in 6 min. 30 sec. What was the average time per lap?

Find the sum.

A 3 wk. 2 da.
1 wk. 4 da.
2 wk. 1 da.

B 1 T. 1300 lb.
1400 lb.
2 T. 500 lb.

C 1 min. 28 sec.
2 min. 49 sec.
1 min. 52 sec.

Find the difference.

D 2 yd. 32 in.
1 yd. 25 in.

E 2 T. 380 lb.
1500 lb.

F $14'\ 7\frac{1}{2}''$
$8'\ 9\frac{1}{4}''$

Find the product.

G 2 qt. 1 pt. × 5

H 7 yr. 2 mo. × 5

I 5 gal. 3 qt. × 7

Find the quotient.

J $5\overline{)16}$ yd. 2 ft.

K $3\overline{)5}$ yr. 3 mo.

L $10\overline{)30}$ hr. 50 min.

## Keeping skillful 2

A $\frac{15}{45}=\frac{x}{3}$

B $\frac{b}{25}=\frac{100}{50}$

C $\frac{32.3}{1}=\frac{f}{43}$

D $\frac{2.3}{1.2}=\frac{v}{2.4}$

E $\frac{6.24}{z}=\frac{14}{28}$

F $\frac{14}{100}=\frac{a}{65}$

G $\frac{43.1}{16}=\frac{431}{n}$

H $\frac{14}{56}=\frac{c}{40}$

I $\frac{42.35}{1}=\frac{d}{4.2}$

J $\frac{100}{73}=\frac{5}{w}$

Find each quotient to thousandths.

A $6.2p=85$
B $42.3-r=6.5$
C $45\div k=4$
D $38.2\times .4=h$
E $5.4\div 3.2=t$
F $4z=.5$
G $8.2+c=17.6$
H $38.7\div w=5$
I $38.7+s=56.0$
J $3.26\div n=6.5$
K $t-.68=13.35$
L $m+.344=.782$
M $16\times p=45.2$
N $75a=477$
O $3.88-1.09=q$
P $.075+3.425=n$
Q $6.4\div 7.35=v$
R $17f=38.6$
S $25\times 6.8=y$
T $g-7.85=18.00$

264

# 264

1 Let the pupils write the equations for Problems R and S and solve them. Then have them work Exercises A to L. Provide answers so that pupils can verify their work. Discuss the exercises that caused difficulty.

2 Assign the exercises in "Keeping skillful" as separate lessons when time permits.

# 265 Thinking straight

Expanded Notes are not considered necessary for this lesson.

**Objectives**

The pupil extends his concept of number to include positive and negative numbers.

**Vocabulary**

New words page 265 negative, positive*, direction*

**Comments**

The purpose of this lesson is to develop further the pupils' concept of number by introducing negative and positive numbers on a number line. On pages 165-168 and 175-176 the pupil learned that there are many names for points on a number line and also that the line may be extended to show a point for any number. Now the number line is extended in the other direction from zero. Negative and positive numbers are associated with points on this line.

Positive and negative numbers ("signed numbers") are introduced at this time to broaden the pupil's concept of number and show that there are numbers other than the numbers he is familiar with. This lesson will also provide readiness for future work with "signed numbers."

It is important to point out that while it is conventional to associate positive numbers with the points to the right of zero and to associate negative numbers with the points to the left of zero, it is not necessary. Points on *either* side of zero could be considered "positive" and the points on the other side "negative." The pupil should be aware of the fact that zero is neither a positive nor a negative number. The teacher should put examples on the board showing number lines in all positions: horizontal, vertical, and oblique.

This lesson does not introduce operations with "signed numbers." The teacher should not attempt to teach addition, subtraction, multiplication, and division of such numbers.

The teacher should use examples like the following to show the pupils uses of "signed numbers." (1) The

scale on a thermometer ($-10°$ for 10 degrees below zero; $+72°$ for 72 degrees above zero). (2) The score on various games ($-10$ for 10 points in the hole, etc.). (3) Positive and negative numbers as used to represent directed distances. (A student lives 4 blocks east of school, and another student lives 5 blocks west of school. You can say one student lives $+4$ blocks from 0, the school, while the other student lives $-5$ blocks, or 5 blocks in the other direction.)

The pupils should be encouraged to give other examples of the use of positive and negative numbers from their own experience and to bring examples from other sources, such as newspapers and magazines.

**Answers**

(block 1)

A Negative 2, negative 3, etc.

B Negative 5

C Positive 2, positive 3, etc.

D Positive 1, positive 5

E Yes, you can extend the line to show a point for any positive or negative number.

F Negative and positive fraction numerals

G $-\frac{5}{2}$, $-\frac{4}{2}$, $+\frac{2}{2}$, $+\frac{5}{2}$

H Yes

(block 2)

A $-\frac{1}{2}$

B $\frac{0}{5}$

C Point c is $-\frac{3}{3}$; Point $d$ is $-\frac{5}{3}$.

## Thinking straight

Look at the number line above. At the right of zero this number line is like the others you have studied. At the left of zero this number line has been extended in the opposite direction. When the number line is extended in this way, other names are needed for the points on the line.

A The name for Point $p$ can be written as shown at the right. You can read it "negative 1." How do you read the numerals below?

$-1$

$-2$ $-3$ $-9$ $-4$ $-7$ $-5$

B What name can you use for Point $t$?

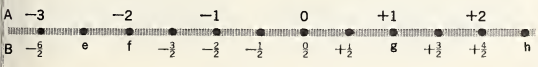
C When the number line is extended in the positive direction, the name for Point $r$ can be written as shown at the right. You can read it "positive 4." How do you read the numerals below?

$+4$

$+2$ $+3$ $+8$ $+6$ $+1$ $+7$

D What name can you use for Point $m$? For Point $v$?

E Could you extend the line to show points for $+10$, $-20$, $+500$, and $-1500$?

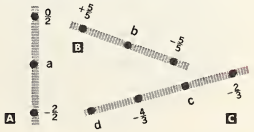



F More points are now shown on the number line. What kind of numerals have been put in the set in Row B to represent the points?

G What numerals can be used in Row B at Points $e$, $f$, $g$, and $h$?

H Could you put points on this line for $-\frac{1}{3}$, $-\frac{5}{6}$, $+\frac{13}{32}$, and $-\frac{13}{32}$?

Replace each letter with a pair of numerals that belongs in the set. 4




Extension of number line to include negative numbers 265

# 265

1 Review previous lessons on the number line (pages 165-168 and 175-176 of the pupils' book). Then ask the pupils to point out the differences between this number line and the ones previously studied.

2 The pupils should become aware of a need for new numbers that distinguish the points on either side of zero. Have them answer the questions in Exercises A to E. Then draw a number line on the chalkboard and ask the pupils to name other points.

3 Have the pupils answer the questions in Exercises F to H. Provide further experiences with positive and negative fractions by having pupils draw number lines and name points to the right of zero and to the left of zero.

4 When the pupils have replaced the letters with pairs of numerals, provide other examples like these. Point out that number lines do not have to be horizontal.

# 266-269 Moving forward

Expanded Notes for this lesson are on pages 366-368.

**Objectives**

The pupil learns to use number pairs, representing a rate, in a graph.

**Vocabulary**

New words page 268 prepared*, relation

**Comments**

The purpose of this lesson is to show how pairs of numbers can be expressed in a graph, and how a point in a two-dimensional graph can be associated with a pair of numerals.

The exercises on pages 267-269 describe two different uses of numbers in rates. The cost of balloons at 3 cents per 1 balloon is a rate that can be expressed by a series of unconnected points in the graph. Since the number of balloons and the number of cents do not involve fractional amounts, the points in the graph will have to be those that can be named by pairs of numerals representing whole numbers.

The rate of motion 5 miles per 10 minutes is expressed by a line in a graph. Since fractions of miles and fractions of minutes can be used, the pairs of numerals may be fraction numerals, and an infinite number of points are required to show this rate graphically. The line includes an infinite number of points.

Exercise G on page 267 shows the form used for writing pairs of numerals. This mathematical notation is one that the pupils will continue to use in later grades. Have the pupils observe that the first numeral of a pair corresponds to the numeral on the horizontal line directly below the point in the graph. The second numeral of the pair corresponds to the numeral on the vertical line directly to the left of the point in the graph.

In the next lesson, the pupils will draw graphs. The emphasis in this lesson should be on determining number pairs and locating them in a graph. Able pupils may wish to draw graphs for some of the rates in Exercise L, page 269.

**Answers**

Page 267:

A Yes

B 4, 12

C 5, 15; 6, 18; 7, 21; 8, 24; 10, 30; 13, 39

D Any two pairs in which the second number is 3 times the first, as (9, 27), (11, 33)

E Horizontal line; It is not possible to buy a fraction of a balloon.

F Vertical line; The cost will be a multiple of 3¢, since 1 balloon costs 3¢; or 1¢, 2¢, etc., would be the cost of a fraction of a balloon.

G The point located 4 units to the right and 12 units up; 12

H (1, 3); 1; 3

I (2, 6), (3, 9); any five pairs in which the second number is 3 times the first, as (6, 18); yes (although the scales may have to be extended)

J Extend the horizontal line to 10 and the vertical line to 30

K $2\frac{1}{2}\times 3 = 7\frac{1}{2}\times 1$; These numerals represent fractions of balloons and fractions of cents. The ratio 7/20 is not equal to 1/3.

Pages 268-269:

A 5 miles; (10, 5); (35, a)

B $10\times 10\frac{1}{2} = 5\times 21$

C $17\frac{1}{2}$; 45; 60; $8\frac{1}{2}$

D Any four points in which the first number is twice the second; no

E 5 would be halfway between zero and 10; 6, one interval beyond 5; 22, two intervals beyond 20; 38, eight intervals beyond 30; yes; yes; yes

F Yes; yes

G It is sensible to represent measures of time and distance by fraction numerals.

H Yes, the line includes points for all the pairs.

I No; yes, but since there is an infinite number of points, it would be impractical

J Yes, the line includes the infinite number of points.

K Any pairs in which the first number is twice the second

L (1) Any pairs in which the second number is 5 times the first, as (1, 5), (2, 10), etc.

(2) Any pairs in which the second number is $3\frac{1}{3}$ times the first, as (3, 10), (6, 20), etc.

(3) Any pairs in which the two numbers are the same, as (1, 1), $(\frac{1}{2}, \frac{1}{2})$, etc.

(4) Any pairs in which the second number is 4 times the first, as (1, 4), (2, 8), etc.

(5) Any pairs in which the second number is 50 times the first, as (1, 50), (2, 100), etc.

M Set 3 and Set 5 (In Set 5 only the numerals that represent pounds may be fraction numerals.)

Answers similar to the following may be given for Exercises N to Q:

N Candy bars at 6¢ each

O 3 articles for 25¢

P 2 people per table

Q A speed of 60 miles per hour

**Keeping skillful:**

(block 1)

A 150/75=[200]/100; 200%

B 200/100=122/[61]

C 62/100=[16.492]/26.6

D 16/1=[1600]/100; 1600%

E 10/100=2.3/[23]

F 2.75/6.25=[44]/100; 44%

G 80/100=100/[125]

H 20/100=[10.08]/50.4

(block 2)

A 1.575

B 255.0

C .0008

D 176.9599

E 25.2

F 63

G .78

H .2

(block 3)

A 2.66

B .71

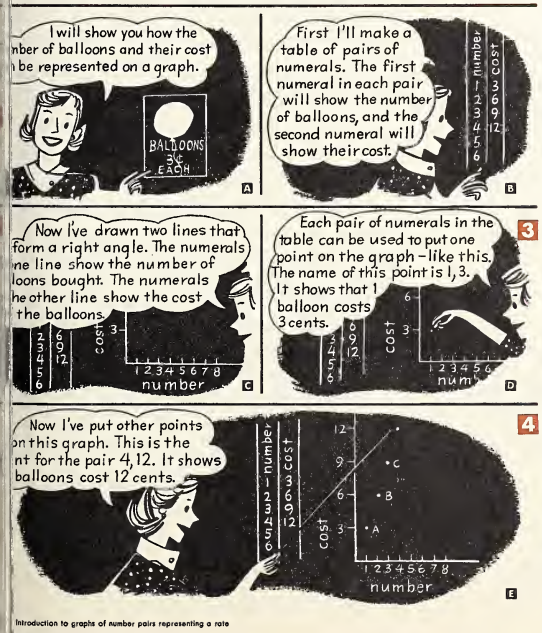
C 12.42

D 1.30

E 3.86

F 17.00

**Moving forward** Now you will learn how pairs of numbers can be expressed in a graph.


I will show you how the nber of balloons and their cost be represented on a graph.
BALLOONS 3¢ EACH
A
First I'll make a table of pairs of numerals. The first numeral in each pair will show the number of balloons, and the second numeral will show their cost.
number
cost
B
Now I've drawn two lines that form a right angle. The numerals ne line show the number of loons bought. The numerals he other line show the cost the balloons.
cost
number
C
Each pair of numerals in the table can be used to put one point on the graph – like this. The name of this point is 1, 3. It shows that 1 balloon costs 3 cents.
3
cost
num
D
Now I've put other points on this graph. This is the nt for the pair 4, 12. It shows balloons cost 12 cents.
4
number
cost
A
B
C
number
E

Introduction to graphs of number pairs representing a rate

# 266

1 Ask what information is given on the poster in Picture A. Then on the chalkboard make a table similar to the one in Picture B, letting the pupils supply the numerals for the number pairs.

2 Draw two lines at right angles on the chalkboard. On one line put numerals that represent the number of balloons bought, as shown in Picture C. Ask why only numerals for multiples of 3 are put on the line showing cost. Ask what numerals would be used if the balloons cost 5¢ each.

3 Put a point in the graph for the number pair representing the cost of 1 balloon. Name it (1, 3). Have the pupils put in points representing the cost of 2 balloons, 3 balloons, etc.

4 Ask how we could express the cost of balloons if they cost 5¢ apiece. Make a table for these pairs, draw two more lines at right angles, put in the scales, and have the pupils locate the points.

A Look at Picture A on page 266. The poster shows that 1 balloon costs 3¢. Does the equation of ratios below show how to find the cost of 4 balloons?

$\frac{1}{3}=\frac{4}{n}$ 1

B Now look at Picture B. Which pair of numerals in the table shows that 4 balloons cost 12 cents? 2

C For each expression below give the pair of numerals that belongs in the table.

5 balloons and their cost
6 balloons and their cost 2
7 balloons and their cost
8 balloons and their cost
10 balloons and their cost
13 balloons and their cost

D Find at least two more pairs of numerals that belong in the table. 2

E In Picture C, notice the two lines that form a right angle. On which line are the numerals that show number of balloons? Why are there no numerals on this line for $\frac{1}{2}$, $1\frac{1}{2}$, or $2\frac{1}{2}$ balloons? 3

F On which line are the numerals that show the cost of the balloons? Why are there no numerals on this line for 1 cent, 2 cents, or 4 cents?

G Look at Picture E. What point is the teacher talking about? You can write the name of this point as shown at the right. (4, 12) 4 The first numeral of the pair stands for the number of balloons. The second numeral stands for the number of cents these balloons cost. This pair shows that 4 balloons cost ■ cents.

H Find the point called 1, 3. How do you write its name? This point shows that ■ balloon costs ■ cents.

I Find Point B and Point C. How do you write the name of each of these points? These pairs of numerals can be thought of as part of a set of equal ratios. Give five other pairs that belong to this set of equal ratios. Could these five pairs of numerals be shown in this graph? 5

J Suppose you picked the pair (10, 30), which shows that 10 balloons cost 30¢. What would you have to do to the graph to show this point?

K There are many ratios equal to those in the table, but it would not be sensible to show some of these pairs of numerals in this graph. For example, the ratio $2\frac{1}{2}$ per $7\frac{1}{2}$ is equal to the ratio 1 per 3. Use the ratio test to show that these ratios are equal. Why would you not show Point ($2\frac{1}{2}$, $7\frac{1}{2}$) in this graph? Why would you not show Point (7, 20)? 6



Examine the table of number pairs below. In each pair, the first numeral stands for the number of minutes traveled. The second numeral stands for the number of miles traveled. The table shows a relation of time traveled to distance traveled. This relation is expressed by a set of equal ratios or pairs of numerals.

1

| MINUTES | 10 | 21 | 35 | b | c | 17 |
|---|---|---|---|---|---|---|
| MILES | 5 | $10\frac{1}{2}$ | a | $22\frac{1}{2}$ | 30 | d |

A What distance is traveled in 10 minutes? Write a pair of numerals to show that this distance is traveled in 10 minutes. What is used to show miles traveled in 35 minutes?

B Each pair of numerals in the table can be thought of as a ratio. Use the ratio test to show that the first two pairs are equal ratios.

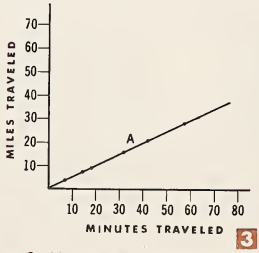
C If the pairs of numerals in the table above express equal ratios, what numeral replaces *a*? Replaces *b*? Replaces *c*? Replaces *d*? Be prepared to explain how you found these numerals.

D Find four other pairs of numerals that belong in the table. Is there any end to the pairs of numerals that could be put in this table?





E Look at the graph below. Notice the two lines that form a right angle. Find the line that shows time traveled. There are numerals on this line for 10 minutes, 20 minutes, 30 minutes, and so on. There could be numerals on the line for 5 minutes, 6 minutes, 22 minutes, and 38 minutes. Where would you put each of these numerals? Could you travel for $2\frac{1}{2}$ minutes? 3 For $5\frac{1}{4}$ minutes? Could the numerals $2\frac{1}{2}$ and $5\frac{1}{4}$ be put on this line? 2




3

F Could you travel 25 minutes and travel $12\frac{1}{2}$ miles? Could you put a point in this graph to represent this pair of numerals?

G It is not sensible to represent pairs like ($2\frac{1}{2}$, $7\frac{1}{2}$) in the graph on page 266. Why is it sensible to represent pairs like (25, $12\frac{1}{2}$) in this graph? 4

# 267

1 Use Exercise A to stress the fact that the pairs of numerals on the preceding page represented rates and that the numerals for each pair could have been found by using the ratio test.

2 Get the pupils to observe different ways of finding number pairs for Exercises B, C, and D (such as using the ratio test, noticing that the number represented by the second numeral is 3 times the first, or others).

3 Discuss the work in Exercises E, F, and G in connection with the pictures on page 266.

4 Call attention to the form used for writing pairs of numerals. Be sure the pupils know how to locate this pair in the graph.

5 Have the pupils locate points for some of their pairs of numerals in the graph on the chalkboard.

6 Emphasize again why only whole numbers can be used for number of balloons and number of cents.

# 268

1 Discuss the table and the text above. Then assign Exercises A to D as written work. When the pupils have finished, discuss each exercise. Be sure they understand why an unlimited number of pairs can be used.

2 Discuss Exercise E. Draw on the chalkboard a large graph similar to the one in the book, and have the pupils locate the points mentioned in the exercise. Then have them locate in the graph the points for the pairs of numerals in the table and some of the pairs they found for Exercise D.

3 After these points have been located, ask the pupils what the graph would look like if the unlimited number of pairs referred to in Exercise D were located in the graph [a line]. Draw the line as in the book, and locate the point A on it.

4 Discuss why it is sensible to use points for fractions in this graph when it was not for the graph on page 267.

J Look at the table again. Is each pair of numerals in the table represented on this graph?

K Is there any end to the number of points that can be shown in this graph at the left of Point A? Could the names of all these points be put in the table? Why?

L If it were possible to put all the points in the graph, would you have a line like the one shown?

M Give the names for five of these other points. Use the ratio test with any two of these pairs of numerals to show that they are equal ratios.

N At the top of the next column there are five rates. Write at least six pairs of numerals to express each one of these five rates. For example, you might write (2, 10) as one pair of numerals for Exercise 1.

1 Balloons at 5 cents each
2 Pencils at 3 for 10 cents
3 A speed of 5 miles in 5 minutes
4 Postage stamps at 4 cents each
5 Meat at 50 cents per pound

M In which of your sets of pairs of numerals would it be sensible to have fraction numerals?

Each of the four sets of pairs of numerals below expresses a rate. For each set, make a statement like the ones given in Exercise L.

| N | O | P | Q |
|---|---|---|---|
| (1, 6) | (3, 25) | (2, 1) | ($\frac{1}{2}$, 30) |
| (2, 12) | (6, 50) | (6, 3) | ($\frac{3}{4}$, 45) |
| (3, 18) | (12, 100) | (10, 5) | ($1\frac{1}{2}$, 90) |
| (4, 24) | (18, 150) | | (4, 240) |

Now you should see how pairs of numbers can be expressed in a table and in a graph.

## Keeping skillful

3

A 150 is ■% of 75.
B 122 is 200% of ■.
C 62% of 26.6 is ■.
D 16 is ■% of 1.
E 2.3 is 10% of ■.
F 2.75 is ■% of 6.25.
G 80% of ■ is 100.
H 20% of 50.4 is ■.

A .63 × 2.5 = n
B 42.5 × 6 = c
C .02 × .04 = t
D 62.53 × 2.83 = w
E 6 × 4.2 = y
F 100 × .63 = d
G .26 × 3 = b
H .002 × 100 = k

Find each answer to hundredths.

A 6.2 × m = 16.48
B .35 × e = .25
C y × .483 = 6
D 2 × z = 2.6
E 42.2 × h = 163
F a × 40.3 = 685.02

269

# 269

1 When the pupils discuss Exercises H to K, have them look at the graph in the book or the one on the chalkboard. In connection with Exercise I, explain that any point on the line could be put in the table, but it would be impractical to list all of them, since there is an unlimited number.
2 Assign Exercises L to Q as written work. Have the pupils explain how they determined each pair of numerals in Exercise L. Be sure they can explain why fraction numerals can be used in Exercises 3 and 5.
3 Assign the "Keeping skillful" exercises as a separate lesson. Encourage able pupils to solve some of the exercises in Blocks 1 and 2 without pencil and paper.

# 270-272 Moving forward

Expanded Notes for this lesson are on pages 368-369.

**Objectives**

The pupil learns how to make and use statistical tables and graphs.

**Vocabulary**

New words **page 270** scores*, frequency, Joe*, Dee, Clarence, Phyllis, Mike, Janice, Ross; **page 272** copy*, information*

**Comments**

Arithmetic test scores are used as statistical material in this lesson. A list of test results is shown first on page 270. The scores and the corresponding number of pupils attaining each score are arranged in a "frequency table." The column labeled "Total Points" shows the total number of points for each score.

Three graphs representing the data in the frequency table are shown on page 271. Each graph gives a picture of a set of number pairs. The pupils learn that a frequency *graph* helps in getting a picture of the trend or overall relationship. A frequency *table* gives the same information, but it is not so easy to recognize the trend. Both the bar graph and the broken-line graph do a better job of giving this picture than the graph showing only points.

The line in a broken-line graph is there to show relationships or changes, not to indicate an infinite number of points. The line merely connects the significant points that correspond to pairs of numerals in the frequency table. For a continuous-line graph, however, any point located on the line corresponds to a pair that satisfies a certain relationship (like the graph on page 268). When the pupils look at a line graph, they must always consider whether or not each point of the line actually represents a number pair.

The pupils' attention should be called to the fact that the set of numerals on the horizontal line in each graph does not start with 0 and that, since there are no test scores below 7, numbering from 0 is not necessary.

Notice that no grid, or lattice, is shown in these graphs. This permits the significant points to stand out. It is not the purpose of this lesson to teach the use of coordinate paper.

The pupils are also taught how to use the data in a frequency table to find an average. Finding averages by the frequency method will have a great deal of value in later statistical study.

On page 272 a different set of arithmetic scores is shown. The pupils should make rough graphs of these data on their own paper. It is important to stress the concept that the graph represents the pairs of numerals found in the table.

**Answers**

Moving forward:

A 15; 7; 8; 5

B 1; 1; 1; 1; 12

C Yes

D 15, 8, 7, and 13, 10; no score was lower than 7.

E 50; 96; 15

F 345; yes, 30; $11\frac{1}{2}$; no

G Point S shows that 5 pupils had a score of 10; (10, 5); Point T (12, 8)

H By a bar rising from 9 on the horizontal line to a height even with 2 on the vertical line; by a bar rising from 10 on the horizontal line to a height even with 5 on the vertical line; 1st, 2d, and 9th bars, and 4th and 7th bars

I Yes; no; no

J Graph B K No

L The lines give a better picture of the data.

M Total number of points is lower, there are no frequencies of 1, and only two scores have the same frequency.

N $9\frac{1}{30}$; 30 and 271 O ——

Keeping skillful:

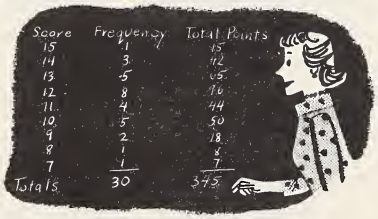
| | | | |
|---|---|---|---|
| A $22\frac{1}{2}$ | F $\frac{15}{16}$ | K 60 | P $\frac{1}{40}$ |
| B $42\frac{2}{3}$ | G $1\frac{5}{9}$ | L 24 | Q $1\frac{1}{8}$ or $\frac{9}{8}$ |
| C 64 | H 2 or $\frac{2}{1}$ | M $\frac{1}{3}$ | R $\frac{11}{42}$ |
| D $\frac{5}{12}$ | I $\frac{1}{10}$ | N $\frac{5}{16}$ | S $\frac{1}{7}$ |
| E 768 | J $13\frac{1}{2}$ | O 1 | T $\frac{1}{2}$ |

## Moving forward

Now you will learn more about tables and graphs.

Arithmetic Test Scores

| Section A | Monday |
|---|---|
| John | 12 |
| Mary | 14 |
| Jim | 11 |
| Joe | 14 |
| Ann | 11 |
| Dee | 12 |
| Nancy | 10 |
| Alice | 12 |
| Martha | 10 |
| Linda | 10 |
| David H. | 10 |
| George | 12 |
| Henry | 9 |
| Clarence | 15 |
| Bill K. | 10 |
| Jack | 13 |
| Jane | 13 |
| Phyllis | 13 |
| Kathy | 7 |
| Mike | 13 |
| Betty | 13 |
| Janice | 12 |
| Dick | 11 |
| Pete | 12 |
| Ross | 11 |
| Peggy | 14 |
| Paul | 8 |
| Don | 12 |
| Bill W. | 12 |
| David Y. | 9 |




1 A The test scores at the left were made by Miss Clark's arithmetic class. What was the highest score in the test? What was the lowest score? How many pupils had a score of 12? How many had a score of 13?

1 B How many pupils had a score of 8? Of 15? The number of pupils having a certain score is called the frequency for that score. What is the frequency for the score of 8? For the score of 15? Which score in the test had the highest frequency? 2

3 C The table above shows the frequency for each score in the test. Are the frequencies correct?

D Which scores have the same frequency? Why is no frequency shown for a score of 6 or lower? 4

E Look at the numerals in the column labeled "Total Points." The total points for the score of 11, for example, are 4 × 11, or 44, because 4 pupils had a score of 11 points. Find the total points for the score of 10. For the score of 12. For the score of 15. 5





# 270

1 Use this exercise to bring out that it is hard to interpret the scores in the typewritten list.

2 Discuss the meaning of *frequency*. Point out that it is hard to remember information about frequencies if only the list at the left is used.

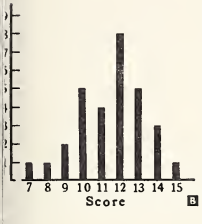
3 Point out that the table shown on the chalkboard gives the frequency for each score in the test. Be sure the pupils see that the scores are arranged from the highest to the lowest. Have the frequencies verified.

4 Emphasize the advantages in using this frequency table. The pupils should understand that if there had been scores lower than 7, such scores would have been listed.

5 Use the directions in Exercise E to discuss the column headed "Total Points."

F Look again at the table on page 270. The total number of points scored by all the pupils is shown in the table. What is this total? Can you find the total number of pupils from this table? Use these totals to find the class average. Did any pupil have this average score? 1

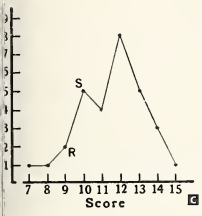
G Graph A shows the frequencies for the test scores on page 270. Point R on the graph shows that 2 pupils had a score of 9. We call this point (9, 2). What does Point S show? What is it called? What point represents the score with the highest frequency? 2




H Graph B is called a bar graph. It represents the same pairs of numbers as those shown on Graph A. How is Point R shown on this graph? How is Point S shown? Which bars represent scores with the same frequency? 3

I Could other bar graphs be made that would represent these same pairs of numbers? Would the bars have to be the same length as those in Graph B? Would they have to be the same width?

J Graphs are used to make comparisons. Which graph makes it easier to compare the frequencies of the scores, Graph A or Graph B?




K Graph C also represents the same pairs of numbers as those in Graphs A and B. If you put a point on the line between R and S, will it represent a frequency for a score in this test? 4

L Look at Graphs A and C again. Why do you think the points are joined by lines in Graph C?



# 271

1 In discussing the questions in Exercise F, have someone verify the total number of points by finding the sum of the scores in the typewritten list. Make clear why the total of the numbers in the frequency column gives the total number of pupils. Discuss how to find the average by using the scores in the typewritten list. Then have pupils find the average by using information in the frequency table. Point out the relative ease of finding the average in this way.

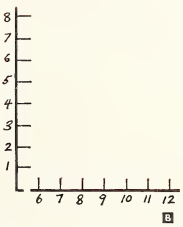
2 Discuss the pairs of numbers indicated by points in Graph A.

3 Use Exercises H to J to bring out that a bar graph pictures relationships.

4 Be sure pupils understand that other points in Graph C would be meaningless, and that the connecting line has been drawn only to give a picture for purposes of comparison.

| Score | Frequency | Total Points |
|---|---|---|
| 12 | | |
| 11 | | |
| 10 | | |
| 9 | | |
| 8 | | |
| 7 | | |
| 6 | | |
| Totals | | |

A




M The scores shown at the left were made by the arithmetic class in another test. Copy Table A above. Then complete the table, using the scores at the left. How is your completed table different from the table on page 270? 1

N Next find the class average for the test given on Friday. Which two numbers from your table do you use? 2

O Now make three copies of the number lines shown in Picture B. Use the information in Table A to make three graphs. Make a graph like Graph A on page 271. Make one like Graph B. Make one like Graph C. 3

Now you have learned how to show information in a table and in a graph.

## Keeping skillful 4

| | | | |
|---|---|---|---|
| $\frac{2}{3} \times n = 15$ | F $\frac{2}{3} \times m = \frac{5}{8}$ | K $\frac{8}{9} \times m = 72$ | P $\frac{3}{16} \times \frac{2}{15} = z$ |
| $y \times \frac{3}{4} = 32$ | G $p \times \frac{3}{5} = \frac{2}{5}$ | L $t \times \frac{7}{12} = 6$ | Q $n \times \frac{8}{9} = 1$ |
| $\frac{8}{9} \times w = 40$ | H $\frac{1}{2} \times b = 1$ | M $\frac{3}{7} \times \frac{7}{14} = s$ | R $\frac{11}{12} \times \frac{4}{7} = h$ |
| $\frac{3}{4} \times \frac{2}{5} = m$ | I $c \times 6 = \frac{3}{5}$ | N $\frac{5}{6} \times y = \frac{5}{8}$ | S $7 \times d = 1$ |
| $\frac{1}{16} \times p = 48$ | J $s \times \frac{2}{3} = 9$ | O $\frac{2}{11} \times a = \frac{2}{11}$ | T $\frac{10}{16} \times e = \frac{5}{16}$ |

# 272

1 Have the pupils study the arithmetic test scores listed at the left. They should note that the top score is 12. Have pupils supply the missing numerals in the table, and discuss this table with the class.

2 Have the average that was found by means of the frequency table verified by use of the test scores in the typewritten list.

3 Assign Exercise O as written work. Have pupils display their graphs and explain them.

4 Assign the "Keeping skillful" exercises as a separate lesson.

# 273 Side trip

Exponded Notes ore not considered necessary for this lesson.

**Objectives**

The pupil is introduced to the metric system and its relation to the English system of measurement.

**Vocabulary**

New words page 273 everybody*, metric, grams, kilograms, kilometers, centimeters, liters

**Comments**

This lesson introduces the metric system. The pupils apply the concept of rate to change measures in one system to equivalent measures in another system.

Ordinarily, pupils are given tables of equivalent measures that they are to memorize or refer to when needed. But pupils who use *Seeing Through Arithmetic* can apply their knowledge of rate and ratio and make conversions from one system to the other easily. The pupils should not be asked to memorize the equivalents. Able pupils may wish to learn more about the metric system and should be encouraged to do so.

**Answers**

In some of these exercises the pupils may make equations of ratios that express comparison rather than rate. For Exercise E, for example, the comparison equation would be 16/1=455/[28.4], or for Exercise K the comparison equation would be 3281/5280=[.62]/1. Such equations will, of course, produce the correct answer; and, so long as the pupils are able to justify their equations, they should be accepted as correct. Only rate equations are given below.

A 220 lb. B 45.45 kg. C .455 kg.
D 1000/2.2=[455]/1; 455 g.
E 455/16=[28.4]/1; 28.4 g.
F 28.4/1=1/[.035]; .035 oz.
G 36/1=39.37/[1.09]; 1.09 yd.
H 39.37/1=(71)/[1.8]; 1.8 m.
I 1/39.37=.01/[.4]; .4 in.
J 1/.4=[2.5]/1; 2.5 cm.
K 5280/1=3281/[.62]; .62 mi.
L 1/3281=[1.6]/5280; 1.6 km.
M 1/1.6=45/[72]; 72 km. N 1/1.06=[.94]/1; .94 liter

## Side trip

**1** A There are other systems of measures besides the one you use. Scientists in our country and everybody in some countries use the metric system of measures. In this system, weights are measured in grams and kilograms instead of in ounces and pounds. 1 kilogram is about 2.2 lb. You can use the equation below to find the number of pounds in 100 kilograms.

$$\frac{1}{2.2}=\frac{100}{t}$$

B Use the equation below to find the number of kilograms in 100 lb. Find the answer to hundredths.

$$\frac{1}{2.2}=\frac{n}{100}$$

C Use the equation below to find the number of kilograms in 1 pound. Find the answer to thousandths.

$$\frac{1}{2.2}=\frac{n}{1}$$

**2** D 1 kilogram is equal to 1000 grams. 1 lb. is about how many grams?

**3** E There are about how many grams in 1 oz.? Find the answer to tenths.

F 1 gram is about how many ounces? Find the answer to thousandths.

THINK You can use 28.4 over 1 for one ratio. What will the other ratio be?

**4** G Distances are measured in kilometers, meters, and centimeters. 1 meter is approximately 39.37 in. About how many yards are there in one meter? Find the answer to hundredths.

THINK You can use 36 over 1 for one ratio. What will the other ratio be?

H A man who is 5 ft. 11 in. tall is about how many meters tall? Find the answer to tenths.

**5** THINK 5 ft. 11 in. = ▩ in.

**6** I 1 centimeter is equal to .01 of a meter. 1 centimeter is about how many tenths of an inch?

J There are how many centimeters in 1 inch? Find the answer to tenths.

K 1 kilometer is equal to 1000 meters. This is about 3281 feet. A kilometer is about what fraction of a mile? Find the answer to hundredths.

L There are about how many kilometers in 1 mile? Find the answer to tenths.

M An automobile traveling at a speed of 45 miles per hour is traveling about how many kilometers per hour?

N Liquids are measured in liters. 1 liter is about 1.06 qt. Find the approximate number of liters in 1 qt. Find the answer to hundredths.





# 273

1. Discuss the equations for Exercises A, B, and C and get the pupils to point out that the ratios express "kilograms per pounds."
2. Get the pupils to write an equation of ratios expressing "grams per pounds." Have a pupil explain why the answer is 1000 times as large as that for Exercise C.
3. Point out that the ratios for Exercises E and F express "grams per ounces."
4. Have the pupils give an equation of ratios for Exercise G in which the second ratio will also express inches per yards.
5. Get the pupils to express the ratio for this exercise as "inches per meters."
6. Discuss the equations needed for Exercises I to N and have the pupils point out what the ratios in each equation will express. Have the pupils write the equations and solve them. Supply answers so they may verify their work.

# 274-278 Looking back

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil reviews the number system, area and volume, per cents, and other ratios, including some with fractions as terms. He also reviews measurement equivalents and some of the special vocabulary of arithmetic.

**Vocabulary**

There are no new words.

**Comments**

The purpose of these "Looking back" exercises is to stimulate thought rather than to test computational skills. (Tests on computation and problem solving are on pages 279-282.) The first group of exercises (pages 274-275) ties together the ideas of the number system by stressing the number line and place value and the concept of numerals as names for numbers. The meaning of area and of volume is emphasized in the exercises on page 276. The exercises on page 277 cover the use of ratios, with special emphasis on per cent. The final group of exercises (page 278) reviews the vocabulary relating to arithmetic and the reduction of measures.

Give instructions and have the pupils work these exercises individually. Do not assign more in one period than the class can be expected to finish. Supply answers and discuss each group of exercises *thoroughly*.

**Reteaching chart**

The pages listed in the chart below indicate the pages of the pupils' book that can be used for reteaching. All references are to Book 6 unless otherwise indicated.

**Pages 274-275:**

| Exercises | Pages | Exercises | Pages |
|---|---|---|---|
| A-H | 165-168, 175-176, 265 | R | 104-106, 165-168, 175-176 |
| I-J | Book 4: 11-15, 126-128 | S | 93-97, 104-106, 177-180, 186-191, 198-200, 218-221, 226-229 |
| K | 122-124, 198-200, 226-229 | T-X | 66-69, 165-168 |
| L-O | 169-172, 175-176, 241-242 | | |
| P-Q | 169-172, Book 4: 11-15, 126-128 | | |

**Page 276:**

| Exercises | Pages | Exercises | Pages |
|---|---|---|---|
| A | 7 | I | 8-9 |
| B-C | 148-149 | J-O | 248-252, 253-256 |
| D-H | 152-156, 157-160 | | |

**Page 277:**

| Exercises | Pages | Exercises | Pages |
|---|---|---|---|
| A-H | 134-135, 214-215, 239-240, 244-246 | K | 126-129 |
| | | L | 243 |
| | | M | 52-53, 126-129 |
| I | 52-53, 141-142 | N | 126-129, 143-145, 243 |
| J | 141-142 | | |

**Page 278:**

(block 1)

Reteaching references are not considered necessary for these exercises, since they are concerned with general vocabulary and terminology.

(block 2)

| Exercises | Pages | Exercises | Pages |
|---|---|---|---|
| A, B, D, E, F, G, H, I, K, M, N, O, Q | 283 | C, L | 253-256 |
| | | J, P | 157-160 |

**Answers**

**Pages 274-275:**

A [a] $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, etc.; [b] $\frac{3}{4}$, $\frac{6}{8}$, $\frac{12}{16}$, etc.; [c] $\frac{3}{2}$, $\frac{6}{4}$, $\frac{9}{6}$, etc.; [d] $\frac{2}{1}$, $\frac{4}{2}$, $\frac{6}{3}$, etc.; [e] $\frac{13}{6}$, $\frac{26}{12}$, $\frac{52}{24}$, etc.; [f] $\frac{7}{3}$, $\frac{14}{6}$, $\frac{21}{9}$, etc.; [g] $\frac{5}{2}$, $\frac{10}{4}$, $\frac{15}{6}$, etc.

B $\frac{30}{40}$, $\frac{18}{24}$, $\frac{3}{4}$, $\frac{45}{60}$, and $\frac{12}{16}$

C (1) $\frac{5}{8}$, $\frac{10}{16}$, etc. (2) 1, $\frac{2}{2}$, etc. (3) $\frac{7}{4}$ or $1\frac{3}{4}$ (4) $\frac{25}{12}$ or $2\frac{1}{12}$ (5) $\frac{11}{6}$ or $1\frac{5}{6}$ (6) $\frac{13}{8}$ or $1\frac{5}{8}$

D Any points with names

(1) greater than $\frac{1}{2}$ (.5) and less than $\frac{3}{4}$ (.75), such as $\frac{5}{8}$, $\frac{2}{3}$, .61

(2) greater than $\frac{3}{2}$ (1.5) and less than $\frac{2}{1}$ (2), such as $\frac{7}{4}$, $\frac{5}{3}$, 1.9

(3) greater than $\frac{7}{3}$ (2.34) and less than $\frac{5}{2}$ (2.5), such as $\frac{29}{12}$, $\frac{57}{24}$, 2.38

(4) greater than $\frac{1}{2}$ (.5) and less than $\frac{5}{2}$ (2.5), such as $\frac{3}{2}$, $\frac{5}{3}$, 1.7

(5) greater than $\frac{3}{4}$ (.75) and less than $\frac{3}{2}$ (1.5), such as 1, $\frac{4}{3}$, 1.23

(6) greater than $\frac{2}{1}$ (2) and less than $\frac{13}{6}$ (2.167), such as $\frac{25}{12}$, $\frac{51}{24}$, 2.13

E Greater than

F Any name for a negative number, such as $-1$, $-\frac{2}{3}$

G (Names for positive numbers are given, except for Part 7.)

(1) Any number less than 4, as 1, $\frac{1}{2}$, $\frac{7}{3}$

(2) Any number less than 15, as 10, 4.5, $\frac{8}{9}$

(3) Any number less than 6.1, as 1.3, 2.4, $6\frac{1}{20}$

(4) Any number less than .03, as .02, .011, $\frac{3}{1000}$

(5) Any number less than 1.2, as 1.1, .06, $\frac{7}{10}$

(6) Any number less than .001, as .0009, .0002, $\frac{1}{10,000}$

(7) Any negative number, as $-1$, $-\frac{3}{2}$, $-3.1$

(8) Any number less than .6, as .4, .001, $\frac{31}{100}$

(9) Any number less than .01, as .008, .0099, $\frac{5}{1000}$

H (Accept any correct name for a number. For example, accept $\frac{22}{4}$, $5\frac{3}{6}$, etc. for the number halfway between 5 and 6.)

(1) $5\frac{1}{2}$ or 5.5 (3) $\frac{9}{16}$ or .5625 (5) 5500

(2) $6\frac{1}{2}$ or 6.5 (4) .055 or $\frac{11}{200}$ (6) $2\frac{3}{4}$ or 2.75

I (1) 6 (3) 100 (5) 10 (7) 2 (9) 0

(2) 52 (4) 86 (6) 74 (8) 0

J (1) 50 (2) 1000 (3) 6 (4) 0

K (1) 10 (3) 2 (5) 3

(2) 100 (4) $6\frac{1}{2}$ (6) 2

L Yes; 2.33 M 2.17, 2.167, 2.1667

N No; because no matter how many places you have in the answer, there will always be a remainder of 4

O [a] .5, .50, .500; [b] .75, .750, .7500; [c] 1.5, 1.50, 1.500; [d] 2.0, 2.00, 2.000; [e] 2.17, 2.167, 2.1667; [f] 2.3, 2.33, 2.333; [g] 2.5, 2.50, 2.500

P (1) six thousand two hundred fifty

(2) four hundred two

(3) forty thousand six hundred fifty

(4) four and sixty-five hundredths

(5) three hundred sixty-five thousandths

(6) three hundred sixty-two ten-thousandths

(7) four hundred sixty-five and six hundredths

(8) four million (9) one and two ten-thousandths

Q (1) 6045 (5) .0300

(2) 400.6 (6) 6.006

(3) 32.085 (7) 9.0253

(4) 160,000,000 (8) 760.02

R (1) $>$ (5) $<$ (9) $>$

(2) $<$ (6) $=$ (10) $=$

(3) $<$ (7) $=$ (11) $=$

(4) $>$ (8) $>$ (12) $<$

S (Answers are limited to positive numbers.)

(1) Any number greater than 5

(2) Any number less than 9

(3) $1\frac{1}{2}$ (4) .97 (5) $\frac{17}{9}$

(6) 8 (7) .42

(8) Any number greater than 300

(9) 2.6 (10) Any number less than 4

T They are also equal.

U The denominator of the first is twice the denominator of the second.

V It is also zero.

W The other is also an improper fraction numeral or a mixed numeral.

X They are both equal to $\frac{1}{2}$.

**Page 276:**

A [A and B]; [A, B, C, and D]

B [A, B and D]; [A and B]; yes, B; yes, E

C [A] 42 ft.; [B] 32 ft.; [C] 42 in.; [D] 24 yd.; [E] 36 ft.

D [A] 90 sq. ft.; [B] 64 sq. ft.; [D] 21 sq. yd.

E 230 sq. ft.

F 1058 sq. in.

G 42 sq. ft. or $4\frac{2}{3}$ sq. yd.

H 120 sq. in. or $\frac{5}{6}$ sq. ft.

I All of them; Prism H

J 4; 2; 8

K 864; 108

L 270 cu. ft.

M 18 cu. ft. or $\frac{2}{3}$ cu. yd.

N 490 cu. in.

O 343 cu. in.

**Page 277:**

A $50/100=n/60$; $.5/1=n/60$

B $\frac{23}{100}/1=30/n$

C $.75/1=n/12$; $\frac{3}{4}/1=n/12$.

D $80/40=n/100$; $80/40=n/1$ $(2/1=200/100)$

E All three equations are correct.

F $60/100 = n/1$ G $.92/1 = n/65$

H $.5/1 = 60/n$; $60/n = \frac{1}{2}/1$

I $2/3\frac{1}{2}$; $20/35$; $1\frac{1}{3}/2\frac{1}{3}$; $.04/.07$

J (1) $\neq$ (2) $=$ (3) $=$ (4) $\neq$ (5) $\neq$ (6) $=$

K 100

L Seventy-five per cent, seventy-five to one hundred, three to four, three-fourths to one, seventy-five hundredths to one; other expressions that can be written in place of "to" are "out of," "for each," "in each," "over," and "of the."

M Many ratios can be written for each exercise.

(1) $40/100$; $4/10$; $2/5$; $\frac{2}{5}/1$; $.40/1$

(2) $20/100$; $2/10$; $1/5$; $\frac{1}{5}/1$; $.20/1$

(3) $50/100$; $5/10$; $1/2$; $\frac{1}{2}/1$; $.50/1$

(4) $65/100$; $6.5/10$; $13/20$; $\frac{13}{20}/1$; $.65/1$

(5) $120/100$; $12/10$; $6/5$; $\frac{6}{5}/1$; $1.20/1$

(6) $6/100$; $.6/10$; $3/50$; $\frac{3}{50}/1$; $.06/1$

(7) $14/100$; $1.4/10$; $7/50$; $\frac{7}{50}/1$; $.14/1$

(8) $200/100$; $2/1$; $20/10$; $\frac{2}{1}/1$; $2.00/1$

(9) $25/100$; $2.5/10$; $1/4$; $\frac{1}{4}/1$; $.25/1$

N (1) 30% (3) 60% (5) 72% (7) 80%
(2) 25% (4) 75% (6) 50% (8) 200%

Page 278:

(block 1)

A Product
B Difference or remainder
C Sum
D Numerator, denominator
E Ratio
F Equal
G Per cent
H Ratio test
I Reciprocal
J Smaller
K 2
L Solution
M Perimeter
N Equal
O Squares

(block 2)

A 128 pt.
B 420 sec.
C 3 cu. yd.
D $82\frac{1}{2}$ ft.
E 14,400 in.
F 150 min.
G 1500 lb.
H 26,400 ft.
I $3\frac{3}{8}$ lb.
J 27 sq. ft.
K $6\frac{1}{2}$ gal.
L 2 cu. ft.
M 312 wk.
N $9\frac{1}{3}$ min.
O $16\frac{2}{3}$ yd.
P 5 sq. ft.
Q 256 fl. oz.

## Looking back

**3** Look at the number line above. Give three fraction numerals that are names for each point that is indicated by a letter. **2**

**3** Which of the following fraction numerals are names for Point *b*?

$\frac{8}{8}$ $\frac{18}{24}$ $\frac{28}{56}$ $\frac{3}{4}$ $\frac{8}{12}$ $\frac{45}{60}$ $\frac{12}{16}$

**3** Give a name for a point halfway between each of the following points.

1 *a* and *b*
2 *a* and *c*
3 *c* and *d*
4 *d* and *e*
5 *c* and *e*
6 *b* and *g*

**3**

**D** Find the names for two other points that are between the following points on this line.

1 *a* and *b*
2 *c* and *d*
3 *f* and *g*
4 *a* and *g*
5 *b* and *c*
6 *d* and *e*

**E** If a point is to the right of Point *c* on the number line, does it represent a number that is less than or greater than the number for Point *c*?

**F** Give a name for a point to the left of zero on this line.

**G** Give the names for any three points that are to the left of each of the following points. **4**

1 4
2 15
3 6.1
4 .03
5 1.2
6 .001
7 0
8 .6
9 .01

**H** What number is halfway between the following numbers? **5**

1 5 and 6
2 3 and 10
3 $\frac{1}{2}$ and $\frac{5}{8}$
4 .05 and .06
5 1000 and 10,000
6 $2\frac{1}{2}$ and 3

**I** How many tens are there in each number below? **6**

1 60
2 520
3 1000
4 868
5 100
6 740
7 25
8 3
9 .1

**J** How many thousands are there in each number below?

1 50,000
2 1,000,000
3 6500
4 400

**K** The second number is how many times as large as the first number?

1 3.6 and 36
2 40 and 4000
3 $\frac{1}{4}$ and $\frac{1}{2}$
4 $\frac{2}{3}$ and $4\frac{1}{3}$
5 $\frac{5}{8}$ and $\frac{15}{8}$
6 $\frac{5}{8}$ and $\frac{5}{4}$



# 274

1 Give necessary instructions and assign these exercises (continued on page 275) as written work. Let the pupils have sufficient time to complete them. When they have finished the exercises, discuss each one *thoroughly*.

2 In discussing Exercises A and B, have the pupils explain *how* they found these names.

3 Get the pupils to observe that the mid-points can be found by dividing by 2 the sum of the numbers associated with the points. For example, in Part 1 of Exercise C, *a* is $\frac{1}{2}$ and *b* is $\frac{3}{4}$. To name the mid-point, divide the sum of $\frac{1}{2}$ and $\frac{3}{4}$ by 2.

4 Ask the pupils to give positive numbers wherever possible (for all except Part 7).

5 Adapt Note 3 to Exercise H.

6 Discuss how Exercises I, J, and K may be answered by using division.

L Look at the number line on page 274. Is $2\frac{1}{3}$ a name for Point *f*? Which of the following decimal fraction numerals is best to use as a name for Point *f*?
.33 2.34 .34 2.33

M Which of the following can be used as names for Point *e*?
2.16 2.17 2.167 2.1667

N Is it possible to write a decimal fraction numeral that is exactly equal to $\frac{1}{8}$? Why?

O For each point labeled with a letter on the number line on page 274, write three decimal fraction numerals to name the point.

P Write with words.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 6250 | 4 | 4.65 | 7 | 465.06 |
| 2 | 402 | 5 | .365 | 8 | 4,000,000 |
| 3 | 40,650 | 6 | .0362 | 9 | 1.0002 |

Q Write with numerals.
1 6 thousand 45
2 400 and 6 tenths
3 32 and 85 thousandths
4 160 million
5 300 ten-thousandths
6 6 and 6 thousandths
7 9 and 253 ten-thousandths
8 760 and 2 hundredths

R Make a true statement for each of the following exercises, using the sign <, or >, or =.

1 $28 \frown 27$
2 $.0625 \frown .625$
3 $\frac{1}{1000} \frown \frac{1}{100}$
4 $2.25 \frown 2.205$
5 $\frac{3}{8} \frown \frac{3}{4}$
6 $\frac{25}{8} \frown 2\frac{0}{8}$
7 $.2 \times 0 \frown 2 \times 0$
8 $.333 \frown .33$
9 $.02 \frown .002$
10 $\frac{1}{4} \frown .25$
11 $3 \times \frac{3}{2} \frown 3 + \frac{3}{2}$
12 $.333 \frown \frac{1}{3}$

S Find one solution for each sentence below.

1 $r + 5 > 10$
2 $12 > a + 3$
3 $\frac{1}{2} + d = 2$
4 $.03 + k = 1$
5 $\frac{9}{17} \times p = 1$
6 $\frac{8}{8} \times n = 5$
7 $.23 + r = .65$
8 $.02 \times m > 6$
9 $1.3 \div n = .5$
10 $\frac{1}{2} \times t + 3 < 5$

T If two equal fraction numerals have equal numerators, what must be true of their denominators?

U If two fraction numerals are equal, and the numerator of one is twice the numerator of the other, what must be true of their denominators?

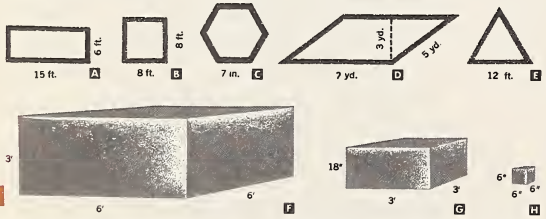
V If two fraction numerals are equal, and the numerator of one is zero, what must be true of the numerator of the other?

W If two fraction numerals are equal, and one is an improper fraction numeral, what is true of the other?

X What must be true of two equal fractions if their sum is equal to 1?



## Looking back




A Look at Polygons A, B, C, D, and E. Which polygons have right angles? Which polygons have parallel sides?

B Which polygons are parallelograms? Which are rectangles? Are there any squares? Any triangles?

C Polygon C has sides that are equal in length. Polygon E also has equal sides. Find the perimeter of Polygons A, B, C, D, and E.

D Find the area of Polygon A, of Polygon B, and of Polygon D.

Find the area of each rectangle whose dimensions are given below.

E 10′ x 23′
F 23″ x 46″
G 2 yd. by 7 ft.
H 2 ft. by 5 in.

I Pictures F, G, and H represent rectangular prisms. Which of these prisms have bases that are squares? Which prism is a cube?

J The area of the base of Prism F is how many times as large as the area of the base of Prism G? The height of Prism F is how many times as great as the height of Prism G? The volume of Prism F is how many times as large as the volume of Prism G?

K Prism F could contain how many cubes like Prism H? Prism G could contain how many cubes like Prism H?

Find the volume of each rectangular prism whose dimensions are given below.

L 6′ x 9′ x 5′
M 3 yd. by 2 ft. by 1 ft.
N 14 in. by 5 in. by 7 in.
O 7″ x 7″ x 7″




# 275

1 Have the pupils explain why 2.33 is a better name than 2.34 for Point *f* in Exercise L, and why 2.16 is not a good name for Point e in Exercise M.
2 Have the pupils observe that they write "and" only when there is a decimal point between digits.
3 In discussing Part 11, ask the pupils to find other pairs of numbers whose sums are equal to the products.
4 Restrict the answers to positive numbers for Exercise S.
5 Be sure the pupils observe that Exercises T to X involve *equal* fraction numerals.

# 276

1 Assign these exercises as individual work. When the pupils have completed them, discuss each exercise so that the pupils may verify their work.
2 Have the pupils explain how they know whether a polygon is a parallelogram, a rectangle, or a square.
3 Discuss how measures of area can be changed from one unit of square measure to another.
4 Have the pupils observe that Prism F could contain 8 prisms of the size of Prism G. (There would be 2 layers, with 4 prisms in each layer.)
5 Point out that the volume of a prism involves the rows and layers of cubes that it contains. Get the pupils to observe from this exercise that the volume of a 6-inch cube is only $\frac{1}{8}$ as much as that of a 12-inch cube.
6 Discuss how measures of volume may be changed from one unit of cubic measure to another.

## Looking back

Three equations are shown for each statement below. Copy each equation that fits the statement.

50% of 60 is ■.

$\frac{50}{100}=\frac{n}{60}$ $\frac{50}{100}=\frac{60}{n}$ $\frac{.5}{1}=\frac{n}{60}$

.23 of ■ is 30.

$\frac{23}{30}=\frac{n}{1}$ $\frac{\frac{23}{100}}{1}=\frac{30}{n}$ $\frac{23}{1}=\frac{n}{30}$

$\frac{3}{4}$ of 12 is ■.

$\frac{\blacksquare}{n}=\frac{12}{n}$ $\frac{.75}{1}=\frac{n}{12}$ $\frac{\frac{3}{4}}{1}=\frac{n}{12}$

80 is ■% of 40.

$\frac{\blacksquare}{100}=\frac{n}{100}$ $\frac{80}{40}=\frac{n}{100}$ $\frac{80}{40}=\frac{n}{1}$

25% of ■ is 352.

$\frac{\blacksquare}{n}=\frac{352}{n}$ $\frac{.25}{1}=\frac{352}{n}$ $\frac{352}{n}=\frac{25}{100}$

60 is what fraction of 100?

$\frac{60}{100}=\frac{n}{1}$ $\frac{100}{60}=\frac{n}{1}$ $\frac{n}{60}=\frac{100}{1}$

.92 of 65 is ■.

$\frac{92}{1}=\frac{65}{n}$ $\frac{.92}{100}=\frac{n}{65}$ $\frac{.92}{1}=\frac{n}{65}$

$\frac{1}{2}$ of ■ is 60.

$\frac{\blacksquare}{n}=\frac{n}{60}$ $\frac{.5}{1}=\frac{60}{n}$ $\frac{60}{n}=\frac{\frac{1}{2}}{1}$

I Which of the following ratios are equal to the ratio 4 over 7? **2**

$\frac{2}{3\frac{1}{2}}$ $\frac{20}{35}$ $\frac{45}{75}$ $\frac{1\frac{1}{2}}{2\frac{1}{3}}$ $\frac{.04}{.07}$ $\frac{.4}{.07}$

J Use the correct sign (=, ≠) for each of the following exercises. **2**

1 $\frac{.50}{1}$ ■ $\frac{400}{200}$

2 $\frac{.006}{.3}$ ■ $\frac{.02}{1}$

3 $\frac{2\frac{1}{3}}{9}$ ■ $\frac{7}{27}$

4 $\frac{\frac{2}{3}}{\frac{1}{2}}$ ■ $\frac{\frac{2}{5}}{\frac{1}{3}}$

5 $\frac{.42}{1}$ ■ $\frac{84}{100}$

6 $\frac{\frac{15}{17}}{15}$ ■ $\frac{\frac{1}{17}}{1}$

K Per cent is expressed by a ratio in which the second term is always ■.

L Write the words you would say in reading each of the following ratios. **3**

$75\%=\frac{75}{100}=\frac{3}{4}=\frac{\frac{3}{4}}{1}=\frac{.75}{1}$

M For each of these ratios, write five other ratios that are equal. **4**

1 40%
2 20%
3 50%
4 65%
5 120%
6 6%
7 14%
8 200%
9 25%

N Write each ratio below as a per cent.

1 $\frac{30}{100}$
2 $\frac{.25}{1}$
3 $\frac{3}{5}$
4 $\frac{3}{4}$
5 $\frac{72}{100}$
6 $\frac{.50}{1}$
7 $\frac{\frac{4}{5}}{1}$
8 $\frac{2}{1}$



## Looking back

When you multiply two numbers, e answer is called the ■■■.

When you subtract a number from other number, the answer is called e ■■■.

When you add two or more mbers, the answer is called the ■■■.

In the fraction numeral $\frac{2}{3}$, the 2 is lled the ■■■ and the 3 is called e ■■■.

The pair of numerals that expresses rate is called a ■■■.

Two fraction numerals that are mes for the same point are ■■■.

If a comparison is expressed by a tio that has 100 in its second term, e ratio may be called a ■■■.

Two ratios can be shown to be equal using the ■■■.

If the product of two fractions is ual to 1, one fraction is said to be e ■■■ of the other.

If two fraction numerals have the me numerator, the fraction numeral th the larger denominator expresses e ■■■ fraction.

When you divide 6 by 4, the mainder is ■■■.

L The set of numbers that makes true statements for the sentence $x+2>6$ is called the ■■■ set.

M The sum of the length of the sides of a rectangle is the ■■■ of the rectangle.

N A square is a rectangle whose sides are ■■■ in length.

O A cube is a rectangular prism whose faces are ■■■.

In some of the exercises below, you will have fraction numerals in your answers. **2**

A 64 qt. = ■ pt.
B 7 min. = ■ sec.
C 81 cu. ft. = ■ cu. yd.
D 5 rods = ■ ft.
E 400 yd. = ■ in.
F $2\frac{1}{2}$ hr. = ■ min.
G $\frac{3}{4}$ T. = ■ lb.
H 5 mi. = ■ ft.
I 54 oz. = ■ lb.
J 3 sq. yd. = ■ sq. ft.
K 26 qt. = ■ gal.
L 3456 cu. in. = ■ cu. ft.
M 6 yr. = ■ wk.
N 560 sec. = ■ min.
O 50 ft. = ■ yd.
P 720 sq. in. = ■ sq. ft.
Q 32 c. = ■ fl. oz.



# 277

1 Assign these exercises as written work. Tell the pupils that more than one answer may be correct for Exercises A to H. When the pupils have completed their work, discuss each exercise, and get the pupils to give a statement for each equation in each exercise to show why some of the equations do not fit the given statements. For example, the statement for the second equation in Exercise A is "50% of *n* is 60," not "50% of 60 is *n*."

2 Have the pupils use the ratio test to discover the equal ratios.

3 Be sure the pupils observe that these are ratios and that they are read "75 to 100," "3 to 4," etc.

4 Instruct the pupils to write ratios similar to those given in Exercise L.

# 278

1 Assign Exercises A to O as written work. Tell the pupils that a word or an expression is needed to make each sentence true. When the exercises have been completed, discuss them, and have the pupils give examples wherever possible.

2 The pupils should use these exercises to test their knowledge and skill in reducing measures. Assign these as individual work also.

# 279-282 Checking up

Expanded Notes are not considered necessary for these lessons.

**Objectives**

The pupil tests his ability to solve the types of problems presented in this book. He tests his ability to compute with whole numbers and with numbers expressed as common and decimal fraction numerals. He also tests his understanding of comparisons expressed by ratios with fraction numeral terms and by per cents.

**Vocabulary**

New words page 279 beef, aquarium*; page 280 mattress, hose*, regular*, Andrews, salad, oil*, Portland*, customers*, whom*, Route

**Comments**

These 19 tests can be used to determine diagnostically the pupils' ability to understand and solve problems, to compute with whole numbers and with numbers expressed by common fraction numerals and decimal fraction numerals (using the four processes), to interpret and solve equations, and to use per cent. The tests are not speed tests, and only as many of them as the class can reasonably be expected to complete should be used in one class period. It is best to give the tests one at a time and to follow each by any discussion and reteaching that may be necessary. The two tests on pages 279-280 cover essentially the same material. The first test should be given, and then, if necessary, any reteaching should be done before giving the second test.

**Reteaching chart**

References are to Book 6 unless otherwise indicated.

Test 1

| Problem | Pages | Problem | Pages |
|---|---|---|---|
| A | 112-116, 218-221, 222-223 | G | 26-30, 112-116, 259-261 |
| B | 130-133 | H | 205, 210-213 |
| C | 186-191, 192-193, 210-212 | I | 248-252 |
| D | 26-30, 80-85, 86, 198-200 | J | 226-229, 230-231, 232-233, 236-238 |
| E | 122-124, 186-191, 192-193 | K | 26-30, 112-116, 259-261 |
| F | 112-116, 143-145 | | |

Test 2

| Problem | Pages | Problem | Pages |
|---|---|---|---|
| A | 26-30, Book 4: 88-90 | H | 112-116, 186-191, 192-193 |
| B | 130-133 | I | 26-30, 80-85, 86, 112-116, 186-191, 192-193 |
| C | 112-116, 143-145 | J | 130-133 |
| D | 26-30, 112-116, 186-191 | K | 130-133 |
| E | 130-133 | L | 259-261 |
| F | 259-261 | M | 80-85, 86, 259-261 |
| G | 186-191, 192-193, 210-213 | | |

| Test | Pages | Test | Pages |
|---|---|---|---|
| 3 | Book 4: 24-27 | 5 | Book 4: 161-164, 167 |
| 4 | Book 4: 36-41, 45, 54 | 6 | 20-23 |

Test 7

| Exercises | Pages | Exercises | Pages |
|---|---|---|---|
| A, C, H, I | Book 4: 36-41, 45, 54 | D, F | Book 4: 24-27 |
| B, E, J | 20-23 | G | Book 4: 161-164, 167 |

| Test | Pages | Test | Pages |
|---|---|---|---|
| 8 | 74-79, 80-85 | 10 | 186-191 |
| 9 | 89-92, 93-97 | 11 | 198-200 |

Test 12

| Exercises | Pages | Exercises | Pages |
|---|---|---|---|
| A, C | 80-85 | D, E, F, H | 89-92, 93-97 |
| B, I | 198-200 | G | 186-191 |

| Test | Pages | Test | Pages |
|---|---|---|---|
| 13 | 177-180 | 15 | 218-221 |
| 14 | 177-180 | 16 | 226-229, 230-23 |

Test 17

| Exercises | Pages |
|---|---|
| A, C, D, F, G, H, I, J | 177-180 |
| B, E | 218-221 |
| Test 18 | 214-215 |
| Test 19 | 134-135 |

**Answers**

Test 1

A $14.5/1 = [229.10]/15.8$; 229.1 mi.

B $35/100 = [42]/120$ C $\frac{1}{3}/1 = [\frac{1}{2}]/1\frac{1}{2}$; $\frac{1}{2}$ yd.

D $(6\frac{1}{4}/7 = [\frac{25}{28}]/1$ or $(6\frac{1}{4}) \div [\frac{25}{28}] = 7$; $\frac{25}{28}$ hr.

E $3\frac{1}{2}/1=[42]/12$

F $45/2=215/[9\frac{5}{9}]$; $9\frac{5}{9}$ hr.

G (75¢)−(68¢)=[7¢] or (21)/3=[7]/1; 7¢

H $28/32=[\frac{7}{8}]/1$

I (200)/1=[2400]/12; 2400 cu. in.

J .3/1=9/[30]; $30

K ($8.25)−$3.15=[$5.10]

Test 2

A ($11.35)+[$3.60]=$14.95

B 6/20=[30]/100; 30%

C 630/50=[945]/75; $9.45

D 3/1=(52)/[$17\frac{1}{3}$]; $17\frac{1}{3}$ yd.

E 60/100=45/[75]; $75

F [80]−45=35; 80 min., or 1 hr. 20 min.

G $\frac{1}{2}/1=[\frac{3}{8}]/\frac{3}{4}$; $\frac{3}{8}$ c.

H 14/1=[$78\frac{3}{4}$]/$5\frac{5}{8}$; $78\frac{3}{4}$ mi.

I ($2.20)+($2.80)=[$5.00] or 60/1=[500]/$8\frac{1}{3}$; $5

J 15/(50)=[30]/100; 30%

K 5/100=9/[180]

L [$1.20]−$.85=$.35

M [$49\frac{1}{4}$]−$36\frac{1}{2}=12\frac{3}{4}$; $49\frac{1}{4}$ mi.

Test 3

| | | | |
|---|---|---|---|
| A 926 | C 77,948 | E 24,242 | G 173,263 |
| B 6286 | D 3649 | F 1338 | H 50,755 |

Test 4

| | | |
|---|---|---|
| A 57,498 | D 10,856 | G 9889 |
| B 78 | E 1001 | H 7682 |
| C 5718 | F 4125 | I 4058 |

Test 5

| | | |
|---|---|---|
| A 10,150 | D 619,742 | G 453,164 |
| B 62,006 | E 245,630 | H 46,416 |
| C 810,576 | F 394,580 | I 423,808 |

Test 6

| | | |
|---|---|---|
| A 846 | E 943 | I $384\frac{98}{187}$ |
| B $346\frac{53}{362}$ | F $406\frac{20}{29}$ | J $76\frac{38}{983}$ |
| C $74\frac{200}{683}$ | G 359 | |
| D $47\frac{63}{119}$ | H $509\frac{46}{83}$ | |

Test 7

| | | |
|---|---|---|
| A 6082 | E $926\frac{63}{89}$ | I 55,169 |
| B 84 | F 21,805 | J 848 |
| C 40,716 | G 764,307 | |
| D 44,845 | H 17,887 | |

Test 8

| | | |
|---|---|---|
| A $40\frac{11}{16}$ | F $24\frac{5}{9}$ | |
| B $1\frac{4}{5}$ | G $2\frac{23}{30}$ | |
| C $12\frac{7}{24}$ | H $74\frac{13}{16}$ | |
| D $6\frac{19}{30}$ | I $5\frac{11}{24}$ | |
| E 35 | | |

Test 9

| | | |
|---|---|---|
| A $\frac{11}{16}$ | F 9 | |
| B $2\frac{7}{12}$ | G $\frac{3}{4}$ | |
| C $7\frac{11}{20}$ | H $35\frac{5}{9}$ | |
| D $\frac{3}{4}$ | I $35\frac{3}{10}$ | |
| E $7\frac{1}{6}$ | | |

Test 10

| | | |
|---|---|---|
| A $\frac{13}{36}$ | F 27 | |
| B $1\frac{3}{10}$ | G 63 | |
| C $12\frac{1}{2}$ | H 34 | |
| D $8\frac{4}{5}$ | I 912 | |
| E 50 | | |

Test 11

| | | |
|---|---|---|
| A $\frac{11}{16}$ | F $1\frac{1}{2}$ | |
| B 4 | G $22\frac{10}{11}$ | |
| C $\frac{3}{20}$ | H $\frac{9}{10}$ | |
| D $6\frac{17}{18}$ | I $4\frac{4}{5}$ | |
| E $11\frac{43}{112}$ | | |

Test 12

| | | |
|---|---|---|
| A $13\frac{1}{8}$ | F $\frac{7}{10}$ | |
| B $3\frac{3}{26}$ | G 7 | |
| C $15\frac{15}{16}$ | H $9\frac{17}{30}$ | |
| D $\frac{3}{32}$ | I 36 | |
| E $3\frac{3}{4}$ | | |

Test 13

| | | |
|---|---|---|
| A 8.09 | F 142.6 | |
| B 37 | G 28.83 | |
| C 1.2979 | H 38 | |
| D 14.467 | I 5.8557 | |
| E 1.5023 | | |

Test 14

| | | |
|---|---|---|
| A .6868 | F 6.0086 | |
| B .47 | G 5581.8 | |
| C 185.08 | H .787 | |
| D .881 | I 19.6776 | |
| E .0816 | J .949 | |

Test 15

| | | |
|---|---|---|
| A .78 | F 93.968 | |
| B 11.1 | G 22.685 | |
| C .085 | H 11.077 | |
| D 40.071 | I .0288 | |
| E .3032 | J .000126 | |

Test 16

| | | |
|---|---|---|
| A 196.83 | F .07 | |
| B 4.26 | G 43.89 | |
| C .63 | H 4444.44 | |
| D .05 | | |
| E 6.39 | | |

Test 17

| | | |
|---|---|---|
| A 2.7318 | F 1.567 | |
| B .36 | G .8661 | |
| C .0007 | H 3.85 | |
| D 10.02 | I .9631 | |
| E 6 | J 75.9 | |

Test 18

A $\frac{2}{3}/1=3/[4\frac{1}{2}]$

B $\frac{4}{5}/1=48/[60]$

C $\frac{1}{3}/1=[41\frac{1}{3}]/124$

D $20\frac{1}{2}/82=[\frac{1}{4}]/1$

E $\frac{3}{8}/1=[51]/136$

F $1\frac{1}{3}/3=[\frac{4}{9}]/1$

G $\frac{4}{5}/1=[96]/120$

H $\frac{1}{3}/1=20\frac{2}{3}/[62]$

I $18\frac{2}{3}/21=[\frac{8}{9}]/1$

Test 19

A 45/100=[36]/80

B 14/100=35/[250]

C 164/82=[200]/100; 200%

D 325/312.5=[104]/100; 104%

E 80/100=[336]/420

F 25/100=180/[720]

G 45/300=[15]/100; 15%

H 123/100=24.6/[20]

I 130/100=[78]/60

J 14/280=[5]/100; 5%

## Checking up

Test 1 **1**

A Mr. Olds can drive an average of 14.5 miles on one gallon of gasoline. At this rate, how far can he drive on 15.8 gal. of gasoline?

B 35% of the people who work in Mr. Miller's office drive their cars to work. 120 persons work in Mr. Miller's office. How many of these people drive their cars to work?

C Mrs. Will made a skirt and a blouse for Peggy. She used $1\frac{1}{2}$ yd. of cloth for the skirt. She used $\frac{1}{2}$ as much cloth for the blouse as she used for the skirt. How much cloth did she use for the blouse?

D During one week Sue spent $1\frac{1}{2}$ hr. on her homework on Sunday, $1\frac{3}{4}$ hr. on Monday night, $\frac{3}{4}$ hr. on Tuesday night, and $2\frac{1}{4}$ hr. on Thursday night. She spent an average of how many hours per day on her homework during the 7-day week?

E Tom is 12 years old. His father is $3\frac{1}{2}$ times as old as Tom. How old is Tom's father?

F One evening Carol read 45 pages of a book in 2 hours. The book has 215 pages. At this rate, how long will it take her to read the whole book?

G Mrs. Jackson paid $2.25 for a 3-lb. roast of beef at the Beach Store. Mrs. Phillips paid $2.04 for a 3-lb. roast of beef at the Sanderson Store. The price of roast beef per pound was how much more at the Beach Store than at the Sanderson Store?

H 28 of the 32 sixth graders went on a trip to the zoo. What fraction of the sixth graders went to the zoo? Express the answer as a common fraction numeral in lowest terms.

I Jim's aquarium has the shape of a rectangular prism. The aquarium measures 20 in. x 10 in. x 12 in. What is the volume of the aquarium in cubic inches?

J Ted spent .3 of his savings for a new fishing rod. He spent $9 for the fishing rod. What was the amount of Ted's savings before he bought the fishing rod?

K Mr. Downs bought 3 rolls of black and white film at a total cost of $3.15. He also bought 3 rolls of color film at a cost of $2.75 per roll. He paid how much more for the 3 rolls of color film than he paid for the 3 rolls of black and white film? **2**



Test 2 **1**

A The air mattress that Cal wants to buy costs $14.95. His father has promised to pay $5 of the cost, and Cal has saved $6.35. How much more money does Cal need before he can buy the air mattress?

B In a basketball game, George made 6 baskets out of 20 tries. George's baskets were what per cent of his total tries?

C Mr. May bought 75 ft. of garden hose that was priced at $6.30 for 50 ft. At this rate, how much did he have to pay for the 75 ft. of hose?

D Mrs. Town needs eight $6\frac{1}{2}$-foot lengths of cloth for curtains. She needs to buy how many yards of cloth?

E Mrs. Anderson paid $45 for a radio that was on sale. The price she paid was 60% of the regular price. What was the regular price?

F Mr. Andrews can drive to work in about 35 min. less than the time it takes when he rides on the bus. He can drive to work in about 45 min. How long does it take Mr. Andrews to get to work when he rides on the bus?

G Mrs. Town made a salad dressing for which she used $\frac{1}{2}$ as much lemon juice as salad oil. She used $\frac{3}{4}$ cup of salad oil. How much lemon juice did she use in the dressing?

H On a map that Mr. Marks was using, 1 in. represented 14 mi. The distance on the map from New City to Portland was $5\frac{5}{8}$ in. What is the distance in miles from New City to Portland?

I Susan earns $.60 an hour for baby sitting. She worked for $3\frac{2}{3}$ hr. on Friday night and $4\frac{2}{3}$ hr. on Saturday night. How much did Susan earn on these two nights?

J George delivers newspapers to 35 customers each day. He also has 15 customers to whom he delivers only Sunday papers. His customers who take only Sunday papers are what per cent of his total number of customers?

K Alice missed 9 days of school during the entire school year. She missed 5% of the days in the school year. There are how many days in the school year?

L Ann's allowance is $.85 per week. Her allowance is $.35 less per week than her brother Bill's allowance. What is Bill's allowance per week?

M On Route 28, the distance from Grand City to Rock Falls is $12\frac{3}{4}$ mi. farther than it is on Route 11. The distance on Route 11 is $36\frac{1}{2}$ mi. What is the distance on Route 28? **2**





# 279

1 Have the pupils work the problems in Test 1. Direct them to write an equation for each problem using a letter to hold a place for the numeral they must find. Then have them do the computation indicated in the equation and rewrite the equation with the answer in place of the letter.

2 Provide answers for the pupils to use in verifying their work. Any discussion or reteaching that is necessary should be done before going on to Test 2.

# 280

1 Administer this test in the same way as Test 1.

2 Provide answers for the pupils to use in verifying their work. Do any necessary reteaching.

est 3
ind the sum.
44, 18, 206, 628, 30
1163, 4809, 215, 99
27638, 49544, 766
863, 2079, 633, 74
3675, 11483, 9084
89, 17, 206, 53, 973
94112, 78683, 468
316, 528, 49872, 39

Test 4
A 87046 − 29548 = t
B 10063 − 9985 = d
C 7582 − 1864 = a
D 49331 − 38475 = r
E 11000 − 9999 = w
F 53006 − 48881 = a
G 29438 − 19549 = h
H 11111 − 3429 = z
I 12006 − 7948 = b

Test 5
A 25 × 406 = n
B 721 × 86 = w
C 468 × 1732 = m
D 893 × 694 = d
E 29 × 8470 = y
F 905 × 436 = a
G 386 × 1174 = z
H 48 × 967 = b
I 896 × 473 = g

est 6 2
16074 ÷ a = 19
250610 ÷ 724 = g
50742 ÷ 683 = m
5656 ÷ w = 119
83927 ÷ 89 = b
294850 ÷ h = 725
17591 ÷ 49 = t
42293 ÷ d = 83
71906 ÷ 187 = f
74746 ÷ r = 983

Test 7 2
A n + 9382 = 15464
B 763 × k = 64092
C 58662 − y = 17946
D 36859 + 7986 = a
E 82477 ÷ m = 89
F w − 6385 = 15420
G 489 × 1563 = d
H 50461 − t = 32574
I 65003 − 9834 = a
J 56816 ÷ 67 = r

Test 8
A $40 + \frac{11}{16} = b$
B $\frac{19}{20} + \frac{3}{5} + \frac{1}{4} = r$
C $9\frac{5}{12} + 2\frac{2}{8} = k$
D $\frac{11}{15} + 5\frac{9}{10} = w$
E $21\frac{6}{6} + 13\frac{1}{3} = m$
F $18\frac{5}{8} + 6 = d$
G $\frac{14}{15} + \frac{2}{3} + 1\frac{1}{8} = n$
H $63\frac{3}{16} + 11\frac{2}{8} = a$
I $4\frac{1}{4} + \frac{7}{12} + \frac{5}{8} = x$

est 9
$1\frac{5}{16} - \frac{1}{4} = x$
$10\frac{7}{12} - 8 = r$
$8 - \frac{9}{20} = a$
$20\frac{1}{12} - 19\frac{1}{3} = w$
$15 - 7\frac{5}{6} = t$
$13\frac{2}{8} - 4\frac{7}{8} = m$
$\frac{13}{20} - \frac{1}{5} = d$
$47\frac{5}{8} - 11\frac{2}{3} = k$
$60 - 24\frac{7}{10} = v$

Test 10
A $\frac{5}{12} \times 1\frac{3}{15} = m$
B $\frac{1}{8} \times 10\frac{2}{5} = t$
C $4 \times 3\frac{1}{8} = r$
D $1\frac{5}{6} \times 4\frac{4}{5} = d$
E $16 \times 3\frac{1}{4} = n$
F $\frac{9}{20} \times 60 = w$
G $14\frac{2}{8} \times 4\frac{3}{8} = s$
H $12\frac{1}{2} \times 2\frac{18}{25} = a$
I $14\frac{1}{4} \times 64 = k$

Test 11
A $\frac{11}{12} \div 1\frac{1}{3} = b$
B $14 \div 3\frac{1}{2} = r$
C $\frac{9}{20} \div 3 = m$
D $8\frac{1}{3} \div 1\frac{1}{5} = w$
E $10\frac{5}{8} \div \frac{14}{15} = c$
F $\frac{9}{16} \div \frac{3}{8} = t$
G $21 \div \frac{1}{12} = a$
H $6\frac{3}{10} \div 7 = n$
I $34\frac{4}{5} \div 7\frac{1}{4} = d$

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Test 12
A $n - 1\frac{5}{8} = 11\frac{1}{2}$
$20\frac{1}{3} \div t = 6\frac{1}{2}$
$8\frac{11}{16} + 7\frac{1}{2} = d$
$\frac{31}{2} - r = \frac{7}{2}$
$w + 4\frac{1}{12} = 8\frac{1}{3}$
$30 - 29\frac{3}{10} = a$
$5\frac{3}{8} \times 1\frac{1}{4} = v$
$18\frac{1}{2} + m = 27\frac{9}{10}$
$60\frac{3}{4} \div 1\frac{11}{16} = f$

Test 13
Find the sum.
A .06, 6.87, .36, .80
B 7.3, 11.9, 13.2, 4.6
C .0463, .9215, .3301
D 9.124, .708, 4.635
E .8635, .0101, .6287
F 14.8, 34.6, .5, 92.7
G 8.44, 11.07, 9.32
H 21.6, 8.4, 7.1, .9
I .7063, .5230, 4.6264

Test 14
A 9.3343 − 8.6475 = d
B .6537 − .1837 = k
C 200.82 − 15.74 = x
D 4.3106 − 3.4296 = r
E .7412 − .6596 = t
F 8.3055 − 2.2969 = m
G 6000.4 − 418.6 = w
H 1.725 − .938 = a
I 24.0061 − 4.3285 = s
J 15.928 − 14.979 = g

Test 15
A .013 × 60 = w
B .6 × 18.5 = t
C 17 × .005 = r
D 42.18 × .95 = a
E .379 × .8 = d
F 56 × 1.678 = m
G 8.725 × 2.6 = k
H 1.007 × 11 = n
I .0009 × 32 = b
J .14 × .0009 = y

Test 16
Find each quotient below to hundredths.
A 1240 ÷ 6.3 = x
B .213 ÷ .05 = r
C 4.43 ÷ 7 = d
D 2 ÷ 40 = t
E 32.6 ÷ 5.1 = m
F .0986 ÷ 1.43 = a
G 9.13 ÷ .208 = k
H 4 ÷ .0009 = n

Test 17
A w + 4.3842 = 7.1160
B .075 × 4.8 = t
C .0086 − .0079 = n
D s − 1.95 = 8.07
E 8 × .75 = r
F 4.361 − 2.794 = d
G .9320 − x = .0659
H 11.06 + r = 14.91
I g + .0369 = 1.0000
J c − 24.6 = 51.3

Test 18
A 3 is $\frac{2}{5}$ of ■.
B 48 is $\frac{4}{5}$ of ■.
C $\frac{1}{3}$ of 124 is ■.
D $20\frac{1}{2}$ is what fraction of 82?
E $\frac{3}{8}$ of 136 is ■.
F $1\frac{1}{3}$ is what fraction of 3?
G $\frac{2}{5}$ of 120 is ■.
H $20\frac{2}{3}$ is $\frac{1}{3}$ of ■.
I $18\frac{2}{3}$ is what fraction of 21?

Test 19
A 45% of 80 is ■.
B 35 is 14% of ■.
C 164 is ■% of 82.
D 325 is ■% of 312.5.
E 80% of 420 is ■.
F 180 is 25% of ■.
G 45 is ■% of 300.
H 24.6 is 123% of ■.
I 130% of 60 is ■.
J 14 is ■% of 280.

2

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1 Instruct the pupils to write the numerals in computational form and find the answers.
2 Tell them to express the answers for the division exercises in Tests 6 and 7 as whole numbers with common fraction numeral remainders, not with decimal fraction numerals.
3 Evaluate the work on each test, and do any reteaching that is necessary.

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1 Assign these tests as written work. Have the pupils write equations of ratios for Tests 18 and 19.
2 Supply answers and do any reteaching that is necessary.

Your notes

This section of the *Teaching Guide* contains detailed suggestions for teaching the lessons. The Expanded Notes are really enriched lesson plans that go beyond the minimal essentials described in the Lesson Briefs (pages 21-269). Before using these Expanded Notes, read the Comments and examine the reproduced pages and keyed notes in the Lesson Briefs section. For your convenience, page references are made to the Lesson Briefs at the beginning of each set of notes in this section.

# Expanded notes

The Expanded Notes include:

**Overview**

A very brief survey of the lesson as it is developed from page to page in the pupils' book

**Teaching the whole class**

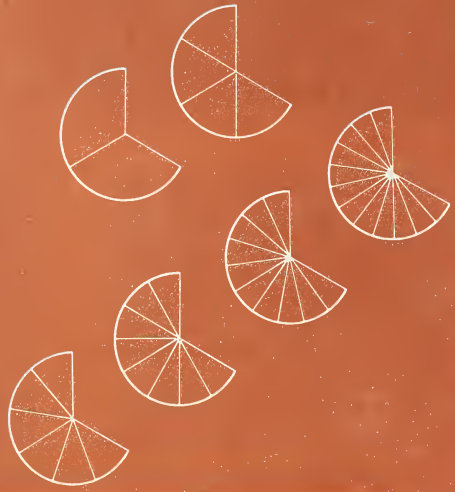
A plan of procedure to be used for the class as a whole before the children are separated into ability groups

**Providing for the able pupil**

Special suggestions for activities to extend the ideas of the lesson for abler pupils

**Helping the slow learner**

Suggestions to use in making clear the ideas of the lesson and for motivating interest therein



# Expanded notes

**A survey tour**

of theory and method
as applied to
the arithmetic content
of this course
is provided in the
Charting the Course sections
included in
the Expanded Notes.

CHARTING THE COURSE

## Geometric ideas—three-dimensional shapes and concept of volume

There are certain basic geometric ideas with which children can and should become familiar in their elementary-school years. A modern instructional program in mathematics must, therefore, provide learning activities to help the children acquire these geometric concepts. More than sixty years ago an influential committee (The Committee of Ten, 1893) recommended that informal geometry be taught beginning in Grade 5. The desirability of doing this has never been seriously questioned, but relatively little has been done to make the recommendation effective. Today it is much more urgent that an informal study of geometry begin in the elementary school.

The *Seeing Through Arithmetic* program introduces certain fundamental ideas of measurement and geometry in each grade. By the time pupils have reached the level of Grade 5, they have had experience that enables them to study a substantial body of geometric content. In Grade 5 the emphasis is upon two-dimensional or plane figures. The concepts of *perimeter* and *area* are introduced, and later in the year's work pupils are shown how to compute the perimeter and area of several simple polygons.

In *Seeing Through Arithmetic 6* the emphasis, as far as the geometric ideas are concerned, is upon three-dimensional shapes—for example, the cone and the pyramid—and the concept of volume. The pupils learn the names and elementary properties of the most commonly found "solid" shapes. Ideas of linear and area measurement are reviewed, and the concept of volume is developed.

The book begins with these ideas to take advantage of pupil interest and enthusiasm at the start of a new year. Review of computational arithmetic is postponed until after the class is off to a good start. Later in the year these geometric ideas are taken up again and the pupils learn how the volume of the rectangular prism may be computed.

## 3-5 Moving forward

Lesson Briefs for this lesson are on pages 22-25.

**Overview**

The work in this lesson has been planned to make the boys and girls aware of the variety of three-dimensional geometric shapes in the world around them. They learn to recognize differences among these shapes and begin to learn the names of the shapes as they hear you use them.

On pages 4 and 5, the pupils are introduced to some of the conventions used by artists in representing three-dimensional objects. They learn that it is possible to suggest distance in a picture by making objects in the foreground larger than objects in the background and by drawing lines that seem to intersect but, in reality, are parallel. They see that lines of various kinds (dotted, a different color, etc.) can be used to indicate how many surfaces a shape has, and that high lights can be used to suggest curved surfaces.

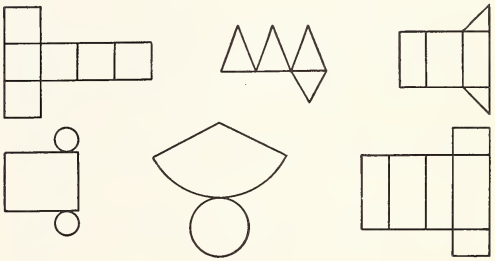
**Teaching the whole class**

As you know, some pupils need many concrete experiences to master arithmetical ideas. The abler pupils need, instead, more challenging—usually more abstract—activities to keep them interested. This section, "Teaching the whole class," contains detailed notes for working with the group as a whole. In "Providing for the able pupil" and "Helping the slow learner," activities based upon the ability of the pupils are suggested. (For this block of work, pages 3-5, you will find these special sections on page 275.) Be sure to look at these suggestions *before* you begin teaching the lesson.

To make reading easier, directions to the pupil have been kept at a minimum in *Seeing Through Arithmetic 6*. It is the purpose of this *Teaching Guide* to help the teacher give specific directions in connection with every page.

To help the pupils work with the pictures of three-dimensional shapes with which this lesson is concerned, certain conventions of drawing, such as perspective, are called to their attention. Such conventions are introduced and illustrated because the pupils may not be aware of them and may find themselves confused when encountering their use later in connection with geometric shapes.

Pupils will gain much by seeing and handling models of geometric shapes in the classroom. Models are available in wood and plastic.* The shapes can also be made of paper or cardboard. To make them, it is convenient to have a pattern of the geometric shapes as they look flattened out. The drawings below suggest how patterns look for a cube, triangular pyramid, triangular prism, cylinder, cone, and rectangular prism.



To make a rectangular pyramid, draw a pattern similar to the one for the triangular pyramid, but make the base a square instead of a triangle. For a hexagonal prism, the two bases must have six sides instead of four as shown for the rectangular prism.

**Page 3:** When the pupils have read the sentences at the top of the page, point out that all the things in the world, animate and inanimate, have shape and take up space. They have length, width, and either height or depth. They have three dimensions.

Next, encourage discussion of Picture A. When the pupils have spent a little time talking about what is taking place in the picture, ask them to describe the shapes they see. Guide them somewhat as follows:

* Commercial companies, such as W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago 10, Illinois, and Yoder Instruments, East Palestine, Ohio, will furnish descriptions and prices upon request.

*What objects resemble a ball? What objects resemble a box or a block? Would you say the bowl with the eggs in it has a flat or a curved surface? Name some other things in the picture that have curved surfaces.* [*Flour sifter, eggs, bowl of spoon, etc.*] *How would you describe the milk container? Do you see any other objects that have flat sides? How would you describe the clock? How would you describe the pile of flour that has been sifted? What shape do the apples have?*

Some of the students may know the names of several geometric shapes that are represented in the picture. They will be able to identify the sifted flour as being in the shape of a cone, the can at the right as being in the shape of a cylinder, etc. Accept these terms, but make no comment at the time.

Then, if possible, introduce models of geometric shapes represented in the picture. Show the pupils a rectangular prism, identify it, and set it in a central position where all can see it. Do the same for a sphere, cylinder, pyramid, hexagonal prism, and cone. Have a label ready to place beside each one. Then ask the pupils to look again at Picture A and choose one of the objects shown in the picture. When they are called upon, they can name and describe the object—then come to the table and show the class the geometric shape that the object most closely resembles. They should not be expected to name the geometric shape, but merely to select it. Be sure pupils see that the clock resembles a hexagonal prism.

Picture B may be treated in much the same way. Pupils may need help in seeing that the church steeple and the base of the windmill resemble pyramids. When you have finished with this picture, direct the pupils' attention to objects in the classroom. Let them find such objects as vases, chalk, erasers, crayon boxes, balls, aquarium, wastebaskets, etc., and describe their shapes. If geometric models have been provided, have them choose for each object the model it resembles.

By this time some of the boys and girls will be using the correct geometric names. Others will be describing the objects in their own words. Still others may point to the models the objects resemble.

You may want to conclude this part of the lesson by assigning some of the activities found under "Providing for the able pupil" and "Helping the slow learner" (page 275).

**Page 4:** Begin work on this page by asking the pupils to reread the last sentence after the heading "Moving forward" on page 3. Then have them look at Picture A on page 4. Let someone describe the scene. Ask the pupils if they can see any differences between the two stations. After they have noted that the station in the background of the picture is smaller than the one in the foreground, ask them which station appears to be closer. Then have them find other objects, some of which appear to be near and some of which appear to be farther away.

Direct attention next to the questions beneath the picture. Have the pupils read the first question in Exercise A. Then ask them to think of a railroad track they have seen. Get them to remember that the rails *seem* to meet in the distance. Now they should be able to answer the second question in Exercise A. Let the pupils measure the telegraph poles in the picture, as suggested in Exercise B, and then ask them to recall telegraph poles they have seen. Use Exercises C and D to help them see that on a flat surface objects cannot be represented as they actually are, but only as they appear to the eye or to the camera. Pictures cut from magazines and newspapers will further illustrate this idea.

Before considering Exercises E, F, and G, display a block or a cube with letters on it. Turn it about so the class can see that it has six sides and how it looks from a front view at eye level. Then, holding the block below eye level, have someone note the letter that is on the top side, also the letter on the side facing the class. Ask him to explain how the block looks to him. Bring out that the back part of the top side seems narrower. Then have the class look at Pictures B and C and decide, as Exercise E asks, which picture better suggests the actual shape of the block. Have the pupils discuss the use of perspective in Picture C. Be sure to develop carefully the meaning of the word *perspective*. The pupils should see that to represent the top side of the block, the lines must be drawn slanting together as if they might eventually meet. Have them note that the letter H also is narrower toward the back.

Using Exercises F and G, bring out that the top side of the block in Picture D is drawn so that the corners, which are actually square, look smaller than a right angle at the front and larger than a right angle at the back. Let the pupils compare Pictures C and D with the real block.

**Page 5:** Begin the work on this page by showing in turn all six sides of a cube. Then have the pupils look at the block in Picture A and answer the questions in Exercise A. They should realize that because the block is drawn in perspective they are aware that there are more sides, although the drawing shows only three sides.

In discussing Exercise B and the block in Picture B, the pupils should see that by drawing three additional lines, the artist can indicate the three sides that cannot be seen.

In considering Exercises C, D, and E, the pupils should note first that Picture C shows a rectangle. In Picture D, dotted lines have been inserted in the rectangle to suggest a third dimension. Picture D, then, is a drawing of a block having six sides. When they look at Picture E, they should see that now the dotted lines suggest a shape with five sides. Have the pupils find the five sides.

In considering Picture F, the boys and girls will probably think at first of something like the sector of a circle or a piece of pie. By the addition of a curved, dotted line, the drawing can indicate a cone resting on its circular base.

In discussing Exercise H and Picture H, the pupils should realize that the drawing shown can represent either a circle or a sphere. Bring out in connection with Picture I that to indicate roundness the artist must make use of highlighting or shading. A demonstration with a real ball and a lighted bulb or candle can illustrate vividly how shadows and lights vary on the surface of a rounded shape as the position of the light source is changed.

The class should see that in drawing Pictures J and K the artist used a dark shading to indicate roundness. To finish this lesson, have the concluding statements read aloud. Call on pupils to tell why pictures of objects are drawn in perspective and how parts that they do not see are suggested in the drawings.

**Providing for the able pupil**

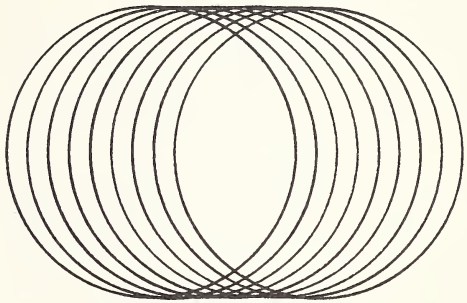
If you used cardboard or paper models of geometric shapes in this lesson, you might ask a committee of able pupils to make a more permanent collection of these models from plaster of Paris. These models can then be used in connection with work in the next lesson. See Activity 1, page 372, for complete details.

After the pupils have studied perspective and how it shows relative position and distance, they might be asked to bring in drawings, pictures from magazines and newspapers, and photographs that illustrate the use of perspective. These materials can be displayed on the bulletin board.

Some of the able students might like to do some extra reading on perspective in encyclopedias or art books and report to the class on what they find.

These students might also do some reading on optical illusions, which are closely akin to perspective. They might show the class examples of what they find or make drawings of optical illusions for display on the bulletin board. Some book and magazine references are given below.*

The following illustration is an example of what the pupils will find. Notice how the circles appear to form a cylinder. You might make a copy, or copies, of this drawing and ask the pupils if they can determine which end of the cylinder they are looking into.



**Helping the slow learner**

Ask the slow learners to bring from home one example each of an object with a curved surface (oatmeal box, soup can, ping-pong ball, top, spool) and with flat surfaces only (rectangular boxes, blocks, checker board). These may be displayed in a central location.

If cardboard or paper models of geometric shapes have been used, let the pupils fashion similar models of clay. Have each pupil make one shape, and when they have finished, display the models in the classroom. For each model have the pupils make a list of five objects that have similar shapes.

## 6-9 Moving forward

Lesson Briefs for this lesson ore on poges 26-28.

**Overview**

The "Comments" on page 26 of this *Teaching Guide* provide a good summary of this lesson.

**Teaching the whole class**

**Page 6:** The purpose of this page is to acquaint the pupils with some of the characteristics of the geometric shapes that are formally introduced on page 8. The word *shape* has been purposely used throughout

* Louis Grant Brandes, "Optical Illusions," *Math. Can Be Fun* (J. Weston Walch, Publisher, Box 1075, Portland, Maine), VI, pp. 114-132.
"Optical Tricks Train Yale Artists" (*Life* 40 [March 26, 1956]), pp. 71-72.
"See It Isn't So" (*Colliers* 131 [March 28, 1953]), pp. 66-67.

instead of the word *figure* because of the many meanings commonly attached to *figure*.

To start the lesson, ask someone to read the introductory statement. Then have the pupils discuss Exercises A and B in connection with Pictures A and B. Get the pupils to observe how the blue lines in Picture A and the dotted lines in Picture B make it possible to tell how many faces each shape has. Have someone explain what a rectangle is. Discuss why some of the angles in Pictures A and B do not appear to be right angles, although all of the faces of the rectangular prism are rectangles. If you have a classroom model of a rectangular prism (a block or a box), show it to the class and give its name at this time. Let the pupils examine it.

Exercises C and D concern prisms some of whose faces are not rectangles. Use the same type of questions for these prisms. Point out that the face on which each of these prisms is resting has the shape of a polygon that is not a rectangle. Then show models of triangular and hexagonal prisms to the class.

In considering Picture E, ask first how many flat sides, or faces, the cylinder has. Point out that the surface between them is curved. Then have the pupils answer the questions in Exercise E. Show a cylinder if you have one.

In connection with Exercise F, get the pupils to observe that the cone in Picture F has one flat side and that this side has the shape of a circle. Point out, too, that the curved surface comes to a point. Then display a cone.

Help the pupils find the four triangular faces of the pyramid in Picture G. Show a model of a pyramid if you have one.

Pupils should identify Shape H as a ball or sphere and observe that it has no flat faces.

Use Exercises I and J for class discussion or for written work. For each object, have the pupils point out the distinguishing characteristics.

**Page 7:** Pictures A to F at the top of the page illustrate some of the properties of geometric shapes. Discuss the pictures and the terms beneath them with the class, and ask pupils to find examples of parallel lines, curved lines, right angles, etc., in the pictures on page 6. When the pupils do Exercises A to F at the bottom of the page, they can refer to Pictures A to F if they are in doubt about any answers.

Point out to the class that Picture A represents a rectangle and that the opposite sides of a rectangle are always parallel.

In discussing Picture C, ask the pupils which angle is the right angle, and which lines are perpendicular.

In discussing Picture D, be sure pupils understand that a plane surface is flat, and that planes, like lines, can be parallel, that is, the same distance apart at all points. Ask the pupils to find planes that appear to be parallel (ceiling and floor, opposite walls, covers of a book, etc.).

Go on to Picture E and ask the pupils to point out right angles. Tell them that just as there are perpendicular lines that form right angles, so there are perpendicular planes. Have them look at the perpendicular planes indicated in blue.

In Picture F, be sure the pupils can pick out the curved surface.

Exercises A to C, which deal with Pictures A to O, can be treated as either oral or written work. If used as written work, ask the pupils to write on a sheet of paper the headings "Parallel lines," "Perpendicular lines," and "Curved lines." Under each heading tell them to write the labels of the pictures that fit the headings.

Exercises D to F, which are concerned with Pictures P to W, may be treated in the same way. Have the pupils use the headings "Parallel planes," "Perpendicular planes," and "Curved surfaces." Discuss the work afterward.

**Page 8:** On this page the pupils are expected to learn the names of geometric shapes that have so far been used only in discussion. Use Pictures A to C with Exercises A to E to help pupils see that a prism always has two bases that are the same shape and size and are parallel. Also tell them that the bases can be shaped like any polygon, but the other faces are usually rectangular. (The prisms shown in this book are right prisms, all of whose faces are rectangular.) Ask the pupils to note that the bases of the prisms in Pictures A to C are polygons, and let them suggest other polygonal shapes for bases of prisms (square, octagon, different types of triangles). If you have models of prisms, let the class examine them. If the pupils have any models of their own that they have made, ask them to show examples of prisms.

In considering Picture D in connection with Exercises F and G, help the pupils see that a cylinder also has two bases. Point out that the bases are usually circular in shape. Draw a circle on the board, and show that the diameter of a circle is measured on a straight line drawn from a point on the circle through the center to another point on the circle. Help the pupils see that the circular bases of Cylinder D have equal diameters and are also parallel. This is easier to demonstrate with models of cylinders.

Tell the pupils that a sphere has a curved surface only and that every point on the surface is the same distance from a point within called the center. We can informally compare the geometric shape called a sphere with the surface of a ball, an orange, etc.

In discussing Picture F, be sure the pupils understand that it shows half of a sphere. If you have two models of half spheres the same size, fit them together to show how they form one sphere.

In looking at Picture G the pupils should understand that the curved surface of the cone comes to a point.

Point out in Pictures H and I that the sides (lateral faces) of the pyramids meet at a point above the base, and that these faces are always triangles. Get the class to see that the bases of Pyramids H and I are two different kinds of polygons and that the number of sides the base has determines the number of triangular faces that meet at a point. Show models of different kinds of pyramids.

**Page 9:** Exercise A may be discussed in class or assigned as written work. Ask the pupils to write the letters A to N on their papers and after each letter write the correct name for the shape.

If you wish to continue practice in identification, ask the pupils to close their books so that they cannot refer to the printed names of the shapes. Then have them identify models that you display.

Before taking up Exercise C, be sure the pupils know what a polygon is. [In the pupils' language, a polygon is a *closed* geometric shape with any number of straight sides.] Then have the pupils answer the question. Permit them to name the polygons if they can. Review the meaning of *plane surface* before the pupils answer Exercises E and F. You may supplement this work by having the pupils answer similar questions about models that you display before them.

Finish the lesson by having a pupil read the concluding statement at the bottom of the page. If you think the pupils are still in doubt about the names and characteristics of some of the shapes, let them continue to identify and describe the models and pictures.

**Providing for the able pupil**

If you did not have the able pupils prepare a permanent set of geometric shapes for class use, as suggested in connection with the preceding lesson, you may wish to do so now. See Activity 1, page 372, for instructions.

After studying the characteristics of prisms and pyramids as developed on page 8, able pupils might like to draw (or make of cardboard) prisms and pyramids that have more sides than those shown in Pictures A, B, C, H, and I. They might also try making models of shapes or drawings of shapes all of whose edges are the same length—shapes made up of equilateral triangles and squares.

Able pupils will be interested in drawings of flattened-out three-dimensional geometric shapes. Draw patterns for different kinds of geometric shapes, one to a card, on 5″ x 7″ cards. (See pattern suggestions in connection with the preceding lesson, page 273.) Letter each card for identification. Then let the pupils write the identifying letters on a piece of paper and after each letter write the name of the shape that the flattened-out pattern makes when assembled.

**Helping the slow learner**

Ask each pupil to bring in an object to illustrate one of the geometric shapes studied. Give him an opportunity to show the rest of the class what he has found. As he displays the object, he should name the geometric shape it resembles.

If these pupils have not had opportunities to mold shapes of clay, as suggested in the preceding lesson, let them try this activity now. Ask them to make a label for each shape.

Another means of providing practice in making two- and three-dimensional shapes is described in Activity 2, page 373. This activity will be of especial value to the slow learners.

## 10-12 Moving forward

Lesson Briefs for this lesson are on pages 29-31.

**Overview**

The purpose of this lesson is to develop the pupils' intuitive knowledge of the important dimensions of the geometric shapes they have been studying (rectangle, triangle, circle, prism, cone, cylinder, pyramid, sphere). The dimensions mentioned in the lesson are, in general, the ones that will be used later in computing areas and volumes. Pupils should learn to distinguish such dimensions as the diameter of a cone, the altitude of a triangle or parallelogram, etc., by applying the principles of measurement demonstrated in this lesson. They should realize, for example, that the altitude of a parallelogram must be measured on a line perpendicular to the base, and that the height of a circular cylinder or cone is measured at right angles to the base.

In *Seeing Through Arithmetic 6*, the term *altitude* is reserved for polygons, and the term *height* for cylinders, prisms, and other three-dimensional shapes.

Do not expect the pupils to make this distinction correctly at all times.

**Teaching the whole class**

Rulers, compasses, and models of various geometric shapes can be used to good advantage to supplement the work in the book.

**Page 10:** Let the pupils look carefully at Pictures A and B to decide which demonstrates the correct way to measure a diameter. Help them recall that a diameter is a straight line that passes through the center of a circle. They should see that the edge of the ruler in Picture A appears to pass through the center of the circle.

Show the class how to use compasses to draw circles. Have them mark the center in some way, then draw several diameters in their circles, and measure them. Get them to observe that any circle has an unlimited number of diameters, and that these diameters are equal in length.

Next direct attention to Pictures C and D and Exercise B. The pupils should see that when the base is measured, the ruler must be held parallel with it. Point out that it is more convenient to begin measuring at the zero point on the scale. Emphasize also that when the altitude is measured, the ruler must be held at right angles, or perpendicular, to the base. Mention that any of the sides of a triangle can be used as the base, but that usually the side chosen is the one upon which the triangle appears to rest. Point out that since the altitude is being measured from Line $y$ in Picture D, Line $y$ is being used as the base of this triangle.

After the pupils have answered Exercise D, ask them to consider one of the other sides as the base and to describe how they would measure the altitude. If possible, provide a large cardboard triangle similar to the one in Pictures C and D, and have three pupils draw and measure three altitudes, each using a different base.

Next have the pupils look at Pictures E and F. Then discuss Exercise E. Even if the pupils have never measured the height of a cylinder, they should be able to see that Picture F shows the correct way, since the ruler is being held in such a way that only the height from the lowest point to the highest point can be measured. By observing Picture F the pupils should see that the ruler and the circular base are supposed to be perpendicular to one another.

Let the pupils take turns in measuring the height of actual cylinders and noticing the difference in results when the ruler is not held perpendicular to the base.

In discussing Exercise G, tell the pupils that the bases of this cylinder are circular. Then ask them how they are supposed to measure the diameter of a circle. Again, let the pupils use actual cylinders. Have them measure diameters of both circular bases so they can see that the results are approximately the same.

**Page 11:** On this page the pupils learn about important dimensions of other three-dimensional shapes.

Before discussing the rectangular prism shown in Picture G, you may want to talk about how to measure to describe the size of something. You might begin by holding up before the class a piece of cardboard in the shape of a rectangle and asking the pupils how they would describe its size. Some of them may remember that in the fifth grade they learned that the size of a rectangle is determined by two dimensions, the base and the altitude. Perhaps someone will suggest that the base and the altitude be measured to find the length of each in inches. Let one of the pupils make these measurements.

Now ask the pupils if they can reproduce the cardboard rectangle on the chalkboard by using the two lengths found, and let several pupils do so. Be sure they understand that in a rectangle the opposite sides are parallel and equal.

Next, have the pupils turn to page 11 and discuss Exercises H and I in connection with Picture G. Review the characteristics of a rectangular prism by asking such questions as:

*What shape are the faces of this prism? How many faces can we see? How many faces are there in all? How many of these faces can also be called bases at any one time?* [*Two*] *Do you think the faces that are opposite each other are alike in size and shape?* [*Most pupils will know this intuitively, but some will have to examine a block or box to satisfy themselves that the opposite faces are the same size and shape.*]

Continue the discussion somewhat as follows:

*Now give the letters that name the edges of the base we can see. Since this base is a rectangle, we know that to describe its size, we need to know just two dimensions. What are they?* [*Base and altitude*] *Does the prism have another dimension? Give a letter that names the third dimension of Prism G. How many dimensions do we need to know if we want to make a rectangular prism?*

If time permits, have the class measure model rectangular prisms and describe their sizes in terms of their dimensions. You may also want to let the pupils try making rectangular prisms, using cardboard and masking tape. Give different pupils different sets of

dimensions to use. Then let them show and discuss the prisms they have made.

In discussing Exercises J to L, remind the class of what they have learned about prisms. They should remember that the two bases of a prism can have the shape of any polygon—rectangle, triangle, pentagon, hexagon, etc.—but that the other faces are usually rectangles. Tell them that a prism is named according to the shape of its base. Therefore, the shape in Picture G is called a rectangular prism, and the shape in Picture H can be called a triangular prism. Help the pupils see that Prism H has two triangular faces (the bases) and three rectangular faces. Let the class examine a model triangular prism to verify this. If any pupils have trouble answering Exercise K, refer them to Picture D on page 10.

Next take up Exercise M and Pictures I and J. Tell the pupils that the black lines represent book ends in the two pictures. Have the pupils note where the ruler is placed in each picture. Help them decide that in Picture I the ruler does not measure the same distance that it would if it were placed so it could be used to measure the distance through the center of the sphere. Picture J shows the correct way to measure the diameter, since the book ends determine that the distance through the center of the sphere is being measured. Point out that if the ruler were placed higher, it would still give the correct measurement, because the two book ends are placed so that the distance between them is the same at all points. Let pupils place a ball between book ends and measure its diameter. Have them experiment by turning the book ends slightly and then measuring.

If the pupils understand that the base of the cone in Picture K is circular in shape, they should have no difficulty in knowing that the diameter is being measured.

Picture L shows a correct way to measure the height of a cone. To emphasize that the ruler is placed in the proper position, have the pupils compare this picture with Picture F on page 10. Such a comparison will point out that, while the height of a cylinder can easily be measured, there is a special problem in measuring the perpendicular distance from the base to the point (vertex) of a cone. It should be obvious that measuring from the base to the top of the cone along the curved side of the cone would not give the perpendicular height (not to be confused with the *slant height,* a term that has purposely been omitted).

Make clear to the pupils that holding a piece of cardboard across the point of the cone in a position that is parallel to the base, and then measuring from the base to the cardboard will give the actual height of the cone. Let the pupils measure the height of a cone in this way. Have them measure at different points on the cardboard, and let them find out what happens when the ruler is not held at right angles to the base. Also let them find out what happens when the cardboard is not parallel to the base.

For Exercise P and Picture M, the pupils should understand that it is necessary to measure only one side of the base because it is supposed to be a square. The length of the base and the width of the base are the same.

The pupils should see in Picture N that the height of the pyramid is being measured in exactly the same way as the height of the cone pictured above it. Again, let the pupils work with models in measuring the base and height of a "square" pyramid. Write the three dimensions on the chalkboard to emphasize that a pyramid has three dimensions.

**Page 12:** The work on this page will test the pupils' knowledge of the dimensions he has been studying. The exercises can be used as either oral or written work, but each should be thoroughly discussed. Besides using Exercise A as a test of whether or not the pupils can recognize a diameter, use it to emphasize what kinds of shapes have diameters. Ask the pupils to name one other shape that has a diameter [cone]. In discussing the sphere in Picture D, explain that Line *b* is a line that passes completely around the sphere on the surface and that Line *a* passes through the center. Note that the part of Line *b* that is shown broken is the part of the line that is on the opposite part of the sphere.

Exercise B is a test of the pupils' knowledge of altitudes of parallelograms and triangles. In connection with Picture E, bring out that one of the sides of a rectangle can be used for the measurement of the altitude because it is perpendicular to the base. Consider either Line *m* or Line *o* as correct for the altitude.

Exercises C to F are a test of the pupils' understanding of various dimensions of solid shapes. For Picture J, be sure the pupils do not confuse the altitude of the triangular base with the height of the prism. Point out that for a prism or other three-dimensional shape, we use the term *height,* not altitude. Tell the pupils to use the word *altitude* for plane shapes only.

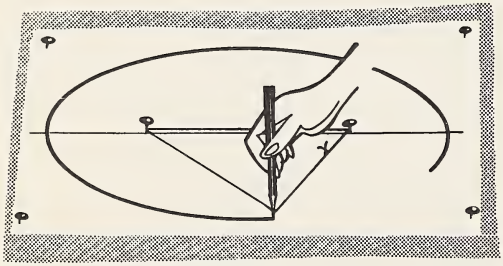
**Providing for the able pupil**

After the pupils have done some thinking about the dimensions that can be used to construct a rectangle

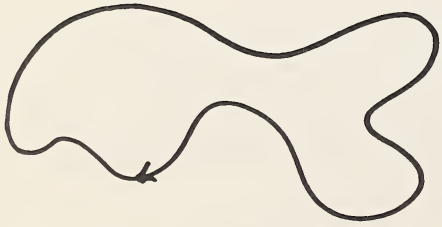
and a rectangular prism (as suggested in connection with the work on page 11), they might do some experimenting with polygons. Activity 3, described on page 373, will help them see that angles as well as number and length of sides determine the shapes of polygons.

Let the pupils (including the slow learners) practice drawing circles with compasses. If you wish, you can tell the pupils that a line from the center to any point on a circle is called a *radius* of the circle. They may observe that the diameter consists of two radii in a straight line.

Next show the pupils how to draw an ellipse. To do this, fasten a piece of heavy paper to a board. Put two thumbtacks a few inches apart. Tie the ends of a piece of string together, making a loop, and lay the loop around the thumbtacks. Place a pencil point inside the loop and pull the string taut. Keeping the string taut at all times, move the pencil inside the string and around the tacks to draw a curved line. Call the pupils' attention to the fact that the size of the ellipse is governed by the distance between the tacks and the length of the loop of string. (See drawing below.) The pupils will want to try making ellipses of their own.



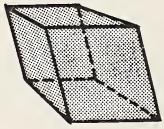
Now draw on the chalkboard a free form shape similar to the one illustrated below.



Ask the pupils how they would describe such a shape to a friend so that he could draw it. They should see that such a shape would be very difficult to describe. Then let the pupils take turns dropping a loop of string so that it falls in a closed curve. Let them try to describe the curve. From such experiments they should decide that the circle and the ellipse are closed curved lines that can be easily described, but that other curved lines are difficult to describe.

Have the pupils make a one-sided surface, or Moebius strip, in the following way. First cut a strip of paper about ½ inch wide. Then fasten the two ends of the strip together with tape; but, before doing so, turn one end over. Next, draw a line on the strip by holding the pencil still and pulling the strip. Without lifting the pencil from the paper, a line that meets the starting point can be drawn. Tell the pupils that, in a sense, the piece of paper has only one surface, and get them to discuss it.

The able pupils might also make a six-sided prism whose faces are parallelograms, but not rectangles. They could try to draw one or make one of paper.



**Helping the slow learner**

To give slow learners experiences in measuring and to help them get the correct terms into their vocabulary, try this activity. Collect objects such as large blocks of wood of assorted shapes, a large rubber ball or a basketball, a toy drum, an oatmeal box, and a cone made of construction paper. Then ask questions like the following:

*Which of these objects is in the shape of a cylinder? Show how you would measure its height and*

*its diameter. Which is in the shape of a sphere? Does a sphere have a diameter?*

As the pupils follow these directions, encourage them to tell what they do and what they find. For example: "The cylinder is 8 inches in height. I hold the ruler perpendicular to the base to find the height. I put the ruler across the center of the base of the cylinder to find its diameter."

For the pupils who have difficulty in knowing where to measure certain dimensions, try the following activity. Use polygons and three-dimensional shapes with lines and edges labeled with letters. Some of the letters should represent the dimensions the class has been studying; some should not. Ask the pupils to use the letters to name diameters, altitudes, heights, bases, etc., and then to measure them.

## 13-16 Moving forward

Lesson Briefs for this lesson are on pages 31-34.

**Overview**

In this lesson a brief review of linear and square measure precedes an introduction to the concept of volume and cubic measure. By relating this new type —cubic measure—to linear and square measure, the pupils should get a good understanding of volume as a measure of capacity. The boys and girls already know that linear measure may involve the repeated use of a linear unit to determine length, and that square measure may involve the repeated use of a square unit to find area. In this lesson the pupils learn that cubic measure may involve the repeated use of a cubic unit (cubic inch) to determine volume.

**Teaching the whole class**

It is important for the pupils to understand that anything that can be thought of as a container—no matter what its shape or size—also has a volume that can be determined. At the conclusion of this lesson, explain that it is often difficult to determine the volume of containers with irregular shapes, but this does not mean that such containers do not have a volume.

**Page 13:** After someone has read the statement that tells what the pupils are going to study in this lesson, explain that on this page they will review the measurement of length and area before learning about volume. There are two important concepts the pupils must recall or get from this page. First, in measuring length and area, it is necessary to have units of measure. And, second, the pupils should see that in determining length and area, the unit of length and the unit of area are used repeatedly. Thus, in Picture A, enough strips are needed to make a row of strips as long as the stick; and in the other pictures, enough small squares are needed to cover all of the rectangle, triangle, and circle. These concepts are necessary to an understanding of volume as developed on the following pages.

Use Exercises A and B and Picture A to help the pupils review the meaning of linear measure. The 9 one-inch strips that correspond to the length of the stick are shown simply to help the pupils see that the measurement of length involves the repeated use of a standard unit of measure. The ruler below the one-inch strips represents a refinement of the more primitive method of measuring the stick by counting one-inch strips.

Similarly, with Exercises C to G and Pictures B, C, and D, the pupils should see that square measure also involves the repeated "laying down" of a standard unit—in this case the square inch—and then counting the number of times this unit is used to find the total area. In comparing the area of the triangle and the circle, the pupils can see that they must count parts of square inches and estimate area. They should also see that the interior of the rectangle, triangle, and circle must be entirely covered by squares or parts of squares in order to find the total area.

**Page 14:** Begin the work with this page by telling the boys and girls that they are now going to learn about another kind of measure—that is, how to measure volume. Help them see that just as the one-inch square may be used in finding area, the one-inch cube—the three-dimensional shape related to the one-inch square—may be used in finding volume.

Then proceed with the exercises and pictures on this page in much the same way you did with the work on page 13. Be sure the pupils accept the boxes in these pictures as rectangular prisms. The pupils should see that when a box is filled with cubes and the cubes counted, the number of cubes is the volume. They should see that measurement of volume involves the repeated use of the cubic unit just as measurement of area involves the use of the square unit and measurement of length, the linear unit. They should begin to understand that shapes with different dimensions may nevertheless have the same volume.

**Page 15:** Tell the pupils that on this page they are going to try to find the volume of the cylinder shown in Picture I. They know that to find the volume, they must find how many one-inch cubes the cylinder will hold. They should see that, because of the nature of the shapes involved, it is impossible to fit whole one-inch cubes into a cylinder and find the exact volume.

In examining Pictures I and J they should become aware of the fact that the height of the cylinder and the height of the pile of cubes is the same and the diameter of the cylinder and the width of the pile of cubes is the same, but that the volume of the cylinder is less than the volume of the pile of cubes. Picture K shows that a cylinder with the same dimensions as Cylinder I has been cut out of the pile of cubes. Picture K should make clear that approximately one half of each cube has been cut away. The pupils can now count to find about how many cubes the cylinder contains.

Discuss Pictures L, M, and N and Exercises U and V with the pupils, helping them see how the standard units of measure used in linear, square, and cubic measurement are related.

For Exercise W the pupils should see that the dimensions of the prism can be found by counting the number of cubes in a row (length), the number of rows of cubes (width), and the number of layers of cubes (height).

**Page 16:** In finding the volume of the prisms, have the pupils count the cubes each prism contains. Tell them that in a rectangular prism there will always be the same number of cubes in each layer. The pupils will then begin to realize that a short cut to counting *all* the cubes is simply to count the number of cubes in one layer and multiply this number by the number of layers. Thus they are given some insight into how volume can be found by computation, but no computing should be done at this time.

Do not expect the pupils to count the cubes and parts of cubes in the cylinder and cone in Pictures E and F. They do not need to know the volumes of these shapes in order to compare them with the cube in Picture D.

**Providing for the able pupil**

Able pupils will be interested to learn that the volume of a cone is ⅓ the volume of a cylinder having the same diameter and height as the cone. If it is possible to find (or make) containers in the shape of cylinders and cones with the proper dimensions, the pupils can use sand to demonstrate that it takes three cones of sand to fill the cylinder.

The able pupils can assist you in providing rectangular prisms for use in helping the slow learners understand volume. Collect or make boxes that can be filled with one-inch cubes. The able pupils can also help in making one-inch cubes.

**Helping the slow learner**

In learning to understand the concept of volume, the slow learners will profit most from filling containers of various sizes and shapes with water or sand. They can compare the sizes of these containers by finding out which of two containers will hold more.

To help them find volume more precisely, it is best to use only rectangular prisms that will hold an exact number of one-inch cubes. The slow learners can then count to find that a prism holds 16 cu. in., 20 cu. in., 32 cu. in., etc.

## 17 Thinking straight

Lesson Briefs for this lesson are on pages 35-36.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

## Computation with whole numbers—inventory

The ability to compute accurately and with reasonable speed is universally accepted as a fundamental objective of the arithmetic program. In the past, this objective has so monopolized the time and attention of teachers and pupils that other important objectives have been neglected. This does not mean, however, that achievement of computational skill should now be ignored. The fact is that other objectives, such as an understanding of the number system, are also emphasized nowadays in part because they contribute to growth in computational ability and to its retention.

It is customary to take inventory of pupils' computational abilities early in the school year. *Seeing Through Arithmetic 6* provides materials for this purpose on pages 18-25. Thus page 18 provides short tests cov-

ering the processes of addition, subtraction, multiplication, and division with whole numbers. Also included is a simple set of addition and subtraction exercises presented in equation form, and a test of knowledge of common measures. In planning the lessons, most teachers will arrange to spread the work on these eight tests over several different days. In the text they are gathered on one page for the convenience of pupils and teachers in finding them.

Remedial instruction in the abilities required by the tests on page 18 is not given in *Seeing Through Arithmetic 6*. The authors believe that any pupils who need intensive remedial instruction should get it through restudy of the complete lessons in earlier books of this program. These pupils can rarely learn what they need to know by studying the typical condensed "review" materials if they failed to learn it at the time they were supposed to. Such condensed review materials when included tend to dilute the work of the grade level for which the book is intended without accomplishing their purpose. They are not included in this program. Procedures for carrying on remedial instruction are suggested in the Expanded Notes, which follow.

The *Seeing Through Arithmetic* program does provide *complete* materials for reteaching of fundamental work first introduced in the preceding grade. This plan is illustrated by pages 19-24, where the process of "long division" is retaught. The same procedures as were used in *Seeing Through Arithmetic 5* are here followed in full detail. The basis for the process as taught in these books is given in the "Comments" on pages 74-75 of the *Teaching Guide* for *Seeing Through Arithmetic 5*. These should be read carefully by the teacher. The way of writing the work that has become traditional in this country is shown on pages 162-163 of *Seeing Through Arithmetic 6*. A study of those pages will make clear why long division by the traditional method is unquestionably a difficult process to understand and learn.

## 18 Checking up

Lesson Briefs for this lesson ore on poges 36-38.

**Overview**

In *Seeing Through Arithmetic 6*, there are 89 tests that may be used to inventory the abilities of the pupils or to measure achievement. These tests will be found under the heading "Checking up." The first set is on page 18 of the pupils' book and consists of tests on some of the work done during the previous year.

Of these sets of tests, 40 are End-of-block tests that measure the pupils' knowledge of topics just studied or abilities just acquired. It is thus possible for the pupils and the teacher to use the tests *diagnostically*. If a pupil cannot respond, or if he makes errors that show lack of understanding, he needs more learning experiences before going ahead with new work. These tests make it possible to know on which areas to focus further teaching.

It is true that pupils have difficulty in reading or studying typical arithmetic texts, with the result that both teacher and pupils come to regard the textbook chiefly as a source of practice exercises. In preparing the *Seeing Through Arithmetic* program, much attention was given to making books that pupils could read and study and understand. The pictures are designed to relate the teaching to concrete situations. The explanations are written in a carefully controlled vocabulary. The best material for any needed reteaching, therefore, is to be found on the pages where the explanation was originally given. Pupils should be encouraged to reëxamine materials they have studied earlier.

Since the tests on page 18 are based on material taught in *Seeing Through Arithmetic 4* and *5*, these two books will provide the best sources of material if any reteaching is necessary. (A Reteaching Chart, with page references, will be found under "Comments" in the Lesson Briefs for this lesson.)

**Providing for the able pupil**

Able pupils might find it interesting to try adding numbers in a different way. For example, they might try adding the ones, tens, hundreds, thousands, etc., separately, and then adding these partial sums for the final answer. In using this method the pupils should see that they can add in any order. They can start by adding the ones, as shown in the first example, or by adding the hundreds first, as shown in the second example.

| | |
|---|---|
| 373 | |
| 628 | 741 |
| 470 | 895 |
| 11 (sum of the ones) | 1500 (sum of the hundreds) |
| 160 (sum of the tens) | 130 (sum of the tens) |
| 1300 (sum of the hundreds) | 6 (sum of the ones) |
| 1471 (final sum) | 1636 (final sum) |

The pupils can make up their own examples for this activity, or they can use the exercises in Test 1

on page 18. After they have worked a few exercises in this way, they should be able to see that the way in which they ordinarily add is simply a short cut for this method.

The pupils might also be interested in the way of multiplying shown at the right. They will realize that the way they usually multiply is a short cut for this method. Exercise B in Test 5 on page 18 is used for this example.

| | |
|---|---|
| 481 | |
| 336 | |
| 6 | 6×1 |
| 480 | 6×80 |
| 2400 | 6×400 |
| 30 | 30×1 |
| 2400 | 30×80 |
| 12000 | 30×400 |
| 300 | 300×1 |
| 24000 | 300×80 |
| 120000 | 300×400 |
| 161616 | |

In connection with the review and reteaching of the four processes, the able pupils should enjoy working with number sequences in which it is necessary to discover the relationship between the numbers in an arithmetical progression. In Activity 4, page 374, the pupils are to find how each number in a given sequence is arrived at and then find several numbers in the sequence beyond those already given. They may even make up some number sequences of their own for their classmates to work with.

Activity 4 may be adapted for each "Checking up" section that reviews the four processes. If the test contains a review of subtraction of fractions, for example, the numbers in the series may be determined by subtracting fractions. If you make up a number sequence using fractions, use simple numbers such as halves, fourths, tenths, etc., so that the pattern can be discovered without much computation.

Able pupils might enjoy experimenting with primitive forms of multiplication that have been used in other parts of the world. Several examples of early methods are described in Activity 5 on page 374. Exercises from page 18 may be used in connection with this activity also, if you wish.

**Helping the slow learner**

You will be able to judge weaknesses in the pupil's preparation by the types of errors he makes. For those who are still uncertain about some of the basic facts, the following activities are suggested: Activity 6 (Basic fact lists) on page 376, which can be adapted to include basic facts in addition and subtraction; Activity 7 on page 376, (Target toss); Activity 8 (Multiplication and division chart) on page 377; Activity 9, (Trouble spots) on page 377.

Should weaknesses appear in the pupil's method of computation, the Reteaching Chart provided under "Comments" in the Lesson Briefs on page 37 will be helpful.

## 19 Thinking straight

Lesson Briefs for this lesson are on pages 38-39.

Expanded Notes are not considered necessary for this lesson, except for a suggestion for able pupils.

**Providing for the able pupil**

Let pupils use almanacs and other books of facts, financial sections of newspapers, geography texts, etc., to find large numbers to round off in the three ways shown on page 19. Have them write the figures on the board or on paper. They should check one another's figures.

## 20-23 Learning how

Lesson Briefs for this lesson are on pages 40-44.

**Overview**

If you are not already familiar with the division method taught in this book, be sure to read "Charting the Course" on pages 282-283 and "Comments" in the Lesson Briefs, pages 40-41. Then examine pages 20-23 of the pupils' book. You will see that the first partial quotient, written at the right of the dividend in the first example on page 20, represents the number of groups of 78 that have been subtracted from the dividend. Note that this method is less likely to lead to frustration through failure to estimate the correct quotient figure. For example, if a pupil begins his work as shown at the right, he must erase and try a larger figure. With the procedure taught in the *Seeing Through Arithmetic* program, no work needs to be discarded. He can continue the process and obtain the correct answer. Also, at each step, *all* of the partial quotient is written (50 and not just 5 as in the example above). Thus the question of where to place the "5" in the answer does not arise.

```
   5
78)6472
  390
  257
```

The arrangement of the computation (partial quotients at the right of the dividend, separated by a vertical line from the rest of the work) makes it easy for the pupils to add the partial quotients to find the complete quotient. The sum of the partial quotients is written beside the remainder, if there is one. The complete answer is then in one place. Writing the partial quotients in this way enables the pupil to add

in a column and to place his sum underneath. The computation is written in a way that contributes to understanding, since the meaning of every figure is clear.

Much of the difficulty in the traditional way of writing division is in estimating the correct quotient figures. Even the most skilled and mature computers are not always able to select the correct partial quotient on the first trial. With the method used in the *Seeing Through Arithmetic* program, a low estimate can always be used, and the computation can be adjusted as one goes along. In order to avoid an estimate that is too high, special procedures in rounding off numbers are suggested. It should be obvious that by rounding the divisor *up* and the dividend *down*, the pupil can always avoid selecting too large a trial quotient.

**Teaching the whole class**

**Page 20:** Give the pupils time to study Picture A and read the adjoining text. You might then say something like this:

*What is Henry trying to find out?* [*How many groups of 78 there are in 6472*] *Did he have any special reason for choosing 50 groups of 78 as a start? He made a guess, didn't he? Is this a better guess than 100 groups of 78?* [*Yes*] *Why?* [*100 groups of 78 would be 7800, which is too large.*] *When Henry had subtracted 50 groups of 78, or 3900, how much was left?* [*2572*] *How many groups did he subtract the next time?* [*30*] *Why did he write 2340?* [$30 \times 78 = 2340$] *When 2340 was subtracted from 2572, what was left?* [*232*] *How many more groups of 78 could be subtracted?* [*2*] *What is the remainder?* [*76*] *Now all the groups of 78 that Henry subtracted were added to find how many in all. How many groups were there?* [*82*] *What is the answer?* [*82 and 76 remainder*] *Could Henry have found the right answer if he had subtracted 10 groups of 78 at a time?* [*Yes*] *How many times would he have had to subtract groups of 10?* [*8*]

Then direct attention to Picture B. Let the pupils relate the rounding off illustrated here to the previous lesson on page 19. Stress the fact that Bill is rounding the divisor up and the dividend down, and then explain the method he is using. Point out that this method makes it impossible to choose a trial quotient that will be too large. Discuss Bill's computation and bring out the fact that his method is more efficient than Henry's.

**Page 21:** Let pupils verify the computation in Ruth's example to be sure that it is correct. Ask them if they think Ruth used good judgment in choosing her first partial quotient. Then discuss Sally's method of estimating. Encourage the pupils to discuss and compare Ruth's work with Sally's work.

**Page 22:** Have the pupils read the four lines of text at the top of the page. Then work through the two methods of computing that have been selected for illustration, and have the pupils answer the questions in the text. Perhaps the able pupils can do some of the work mentally, but allow the pupils to use pencil and paper if they wish to do so.

The TRY step at the bottom of the page gives the pupils a chance to make their own estimates of partial quotients. They should cover the examples shown and look at them only after they have worked the example. Make it clear that it does not matter if their work differs step by step from that in the book so long as they get the correct answer. Explain that the example printed in blue at the right is the shortest computation and that it is achieved by using the largest possible partial quotient.

**Page 23:** Able pupils should be permitted to proceed immediately to the work in the DO step. Have the other pupils work Examples B and C and compare their work with the solutions in the book. When they are sure they understand the method of estimating and computing, they should be permitted to go on to the DO step, where they will work completely on their own. Provide answers for Exercises A to U so that pupils may verify their work, and then discuss the solutions.

**Providing for the able pupil**

Able pupils who feel confident about the division process will be interested in tests of divisibility. Many of the pupils will remember, or know intuitively, how to tell whether or not a number is divisible by 2, by 5, or by 10. Give them a list of numbers—for example, 38, 145, 200, 75, 41, 56, 80, 2476, 1430, etc., and let them find, by experimenting, which of these numbers are divisible by 2, by 5, or by 10.

A number is divisible by 4 if the number formed by the last two figures is divisible by 4. For example, the number 1836 is divisible by 4 because 36 is divisible by 4. (Numbers whose numerals end in two zeros are also divisible by 4.)

A number is divisible by 8 if the number formed by the last three figures is divisible by 8. The number 709416 is divisible by 8 because 416 is divisible by 8.

Give the pupils a list of rather large numbers, such as 128, 2724, 5000, 6340, 3872, 2596, 5640, 10272, 7579, and have them use the tests for divisibility by 4 and by 8. Some pupils may observe that every num-

ber that is divisible by 8 is also divisible by 4, but that the reverse is not necessarily true.

To test the divisibility of a number by 3, first divide the number by 9. If there is a remainder that is divisible by 3, the original number is also divisible by 3. To test 51, for example, divide it by 9. $51 \div 9 = 5$ and 6 remainder. 6 is divisible by 3. Hence 51 is divisible by 3.

Ask the pupils, by using what they already know, to make up tests of divisibility by 6 and by 12. Some of them will see that a number is divisible by 6 if it is divisible by both 2 and 3, and that a number is divisible by 12 if it is divisible by both 3 and 4.

**Helping the slow learner**

The slower pupils may have several kinds of difficulties, even in the sixth grade. One is in understanding the reasons for each step of the division process. Another is in doing the necessary multiplication. If lack of ability to multiply is causing the trouble, first determine whether the pupils know the multiplication basic facts. If not, have them review these. They may also need to review multiplication with 10, 100, 1000, etc., as the multiplier.

A good technique for helping the slower pupils understand the division process is to let them work out problems with objects and write the computation for each step. It may be necessary to let the very slow learners take out one group at a time until they understand exactly what they are doing.

## 24 Checking up; Keeping skillful

Lesson Briefs for these lessons ore on poges 44-45.

Expanded Notes are not considered necessary for these lessons.

## 25 Checking up

Lesson Briefs for this lesson ore on poges 46-47.

Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupils and the slow learners.

**Providing for the able pupil**

Problems of the types the pupils have studied may be cut from old textbooks and pasted on 3″ x 5″ cards. From a file of such cards the pupils may select problems to solve. Be sure the problems used do not exceed the range of the work done up to this time. See Activity 10, page 378, for complete details.

A group of able boys and girls may play a game entitled "Make up a problem." A pupil must make up a problem for an equation written on the board by another pupil. See details in Activity 11, page 378.

**Helping the slow learner**

For the pupils with the most serious deficiencies in problem solving, you may for a time need to provide work at the fourth-grade level. Find out which types of problem situations give these pupils the most trouble. Sometimes it is helpful to ask the slow learner questions concerning the method to be used in solving a problem. Do not be too concerned with the computational aspects, but rather with the kind of equation that can be made for the problem. Then, to help these pupils, refer them to the appropriate lessons in *Seeing Through Arithmetic 4.**

Activity 10 may be adapted for these pupils by using only problems that require simple computation. If the numbers in the problems are small, the pupils will benefit from using objects to make the groups indicated in the problems. Have them demonstrate the action (real or imagined) to find the answer.

An activity called "Solve it" (see page 378, Activity 12) can give the pupils practice in recognizing the proper equations for problem situations.

CHARTING THE COURSE

## Problem solving—the equation

The most important objective of instruction in arithmetic is the development of ability to solve problems. The *Seeing Through Arithmetic* program has been designed

**Seeing Through Arithmetic 4* by Maurice L. Hartung, Henry Van Engen, and Lois Knowles. Fourth-grade book in The Basic Mathematics Program. Scott, Foresman and Company.

with this objective always in view, and distinctive methods have been introduced to help pupils achieve the objective. Prominent among these methods is the use made of equations. To recall some of the reasons that justify this emphasis, the teacher should reread now the section on "Methods of Analysis and Communication" under "Guiding Principles" on pages 11-14 of this *Teaching Guide*.

Pupils who have studied books of the *Seeing Through Arithmetic* program in preceding grades have learned to analyze various types of problem situations (see the chart on pages 408-409 of this *Teaching Guide*). Pupils have learned how to record their analysis by writing an equation—at first using a "screen" (▤) for the idea "what number." Starting in Grade 5, the use of the letter *n* to hold a place for an unknown numeral was begun. In that grade, also, the solution of multiple-step problems was taught by using screens to indicate that certain numbers need to be found by preliminary computations before the main "step" or process of the problem can be carried out. Pages 26-30 of *Seeing Through Arithmetic 6* provide materials for reteaching these procedures. As was pointed out in the section on *Guiding Principles*, the aim is to help the pupil learn to use one of the basic mathematical tools for analysis and communication, instead of starting to compute without really seeing clearly what steps are required and why these steps are appropriate.

It is convenient to call attention now to the extension or generalization of the use of symbols in problem solving that is taught later on pages 49-51 of the pupils' book. On these pages the pupil learns that he can use any letter of the alphabet to hold a place for a numeral in a mathematical sentence such as an equation. With this idea in mind, the use of the screen as a place holder in equations can be abandoned, since hereafter a letter may be used. In multiple-step problems several letters are needed to write the equation. The teacher should not regard such equations as "difficult" or "too advanced." The idea of using a letter as a temporary place holder for a numeral is quite simple. The analysis of most problem situations is made easier, not harder, by the use of such devices. It is only through the use of such devices that one can learn to solve any but the simplest of mathematical problems. The failure to teach such devices is one of the reasons that problem solving has always been so difficult in the past.

## 26-30 Exploring problems

Lesson Briefs for this lesson are on pages 47-50.

**Overview**

The "Comments" on pages 47-48 of this *Teaching Guide* contain a useful summary of this lesson.

**Teaching the whole class**

**Page 26:** Have someone read the problem opposite Picture A aloud. Then ask what we must think about doing with the two groups of jars in order to show how to get the answer. Ask the pupils why we cannot do the addition immediately and why we cannot add 26 and 19.

Then have the pupils proceed to Picture B and the text opposite it. Explain what the dotted background indicates [that the two groups of jars are to be thought of as combined]. Ask someone to explain how the equation fits the problem situation. Be sure the pupils see that even before they know how many quarts there are in 26 pints, they can use a screen to stand for this number that must be added to 19. They can make an equation that symbolizes the whole problem situation before they do any computation. Be sure they see that *n* stands for the final answer. After someone has read the statement at the bottom of the page, ask the pupils how they would find the number of quarts in 26 pints.

**Page 27:** In Picture C the quart jars are dimmed off because the emphasis is on finding how many quarts can be made from 26 pints (the first step in the computation). The rings in the picture show that each group of 2 pints is equal to 1 quart. Have someone count the groups of two to verify the computation. Ask where 13 belongs in the equation.

Call attention to the equation opposite Picture D. Ask someone to find the numeral that will replace *n*. Then turn back to the original problem so that the class can see whether or not the question asked in the problem has been answered.

**Page 28:** After someone has read the problem opposite Picture A, find out what the pupils remember about the meaning of average. If they have forgotten what an average is, explain that finding the average of several numbers involves "evening up" the situation; it means finding a number that would describe the situation if Peggy and her mother had made the same amount of jelly each day.

Proceed to Picture B and the text opposite it. You might continue in this way:

*The picture shows that we are going to think of all the glasses of jelly as being in one large group. We do this because we must know how many*

*glasses there are in all before we can divide them into 3 equal groups. Now look at the equation. What does the screen stand for? We have to find this total before we can find the average. What does the* n *stand for? The number that* n *stands for also represents the average we are trying to find.*

When you discuss the text opposite Picture C, ask someone to tell why the sum, 42, replaces the screen, and not the *n*.

**Page 29:** The equation opposite Picture D has now become a partitive division equation, which the pupils have seen before.

In case some pupils still have difficulty in understanding the meaning of average, you might use objects to illustrate the problem, following the steps shown in the pictures. For Picture D, you would make the 3 groups of 14 by putting the objects into the 3 groups one at a time.

You might also show the pupils another way of finding the average for this problem. Instead of putting all 42 objects into one group, keep the objects in their original groups and then begin equalizing the groups by taking from the larger group and adding to the smaller groups until all the groups become equal. When the groups have all been equalized, the number in each group will represent the average. Explain to the pupils that this is what finding an average really means; the computational process we use is a short cut for the above way of equalizing the groups.

Have someone read Problem A. Then discuss the equation in blue—why it fits the problem situation, and what each symbol in the equation stands for. Then ask why $(2 \times \$2.75)$ can be used in place of the screen. Tell the pupils that the work in parentheses must always be done first.

In discussing Problem B, point out that the 17 and 9 within parentheses must be added before the final answer can be found. Have someone divide $3.45 by 26 at the board. Help the pupils see that the remainder is small enough to be disregarded, and that averages are frequently approximations.

**Page 30:** Have the pupils work independently on these problems. After they have verified their answers, discuss any problems that caused difficulty.

**Providing for the able pupil**

A game called "Problem relay," which will give the pupils experience in solving multiple-step problems, is described in Activity 13, page 379. This game can be used for both the able pupils and the slow learners.

Many of the activities and interests of pupils of this age can be used to provide practice in computing averages. For example, the pupils might be asked to find the following averages:

(1) The average height of the pupils in the class. Have them find the height of each pupil to the nearer inch, then compute the average. (Pupils should disregard remainders if they occur in finding the average.)

(2) The average weight of the pupils in the class. Have them find the weight of each pupil to the nearer pound.

(3) The average daily attendance of the class, based on the record for a month.

(4) The average time members of the class spent in watching TV during a week.

(5) The average number of blocks the pupils have to come to school.

(6) The average number of hours spent on hobbies by the members of the class during a month.

For further practice in computing averages, see "Call the average," Activity 14, page 380.

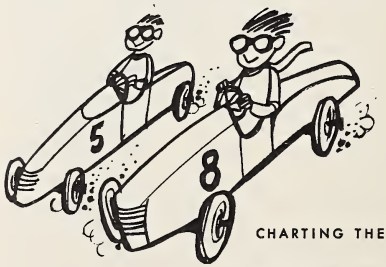
**Helping the slow learner**

All the exercises and activities described for the able pupil can be used for the slow learners if properly supervised.

## 31 Thinking straight

Lesson Briefs for this lesson are on page 51.

Expanded Notes are not considered necessary for this lesson.




CHARTING THE COURSE

Number pairs—rates, comparisons, and fractions

The idea of rate and the idea of comparison are both of great importance in the applications of arithmetic. Problems that involve one or the other of these ideas

using proportionality relations, occur very frequently in everyday living. Traditionally there has been little or no definite teaching of the general idea of rate in the middle grades, even though use has been made of the idea in problems. Similarly, the kind of comparison that uses a proportionality relation has not been adequately presented. If, however, understanding is to be achieved, it is essential that these ideas be carefully developed. *Seeing Through Arithmetic 5,* therefore, provided activities for this purpose, and these ideas are retaught on pages 32-46 of *Seeing Through Arithmetic 6.*

The idea of rate is involved whenever one measure is paired with another. Suppose, for example, that 4 batteries were sold for 56 cents. (See page 32 of the pupils' book.) This statement expresses a rate. To write this rate, two numerals are required. The numeral 4 shows the number of batteries that were sold. The numeral 56 shows the number of cents needed to buy them. There is a pairing, or correspondence, of 4 batteries to 56 cents. This same rate can be stated in other ways. For example, the rate can be expressed as 2 batteries for 28 cents, or 8 batteries for 112 cents.

In stating the rate as 4 batteries for 56 cents, the numerals 4 and 56 are used to express an abstract numerical relationship. A pair of numerals used in this way is called a *ratio.* In this example the ratio is 4 per 56. The same relationship is also expressed by the ratio 2 per 28 and by the ratio 8 per 112. The words *batteries* and *cents* are used to express concrete aspects of the problem situation from which this ratio came. A relationship of this type is called a *rate.*

In general, every rate also involves a ratio. As another example, suppose a train travels 120 miles in 2 hours. A rate of 120 miles in 2 hours is the same as 60 miles in 1 hour, or 60 miles per hour. The ratios, in this example, are 120 per 2 and 60 per 1.

Ratios are often written with one numeral above the other, as in the example shown at the right. In this case, the symbol does not represent a fraction. In the example above, the batteries and the cents are two entirely different kinds of objects. One cannot say that "the batteries are a fraction of the money."

$$\frac{4}{56}$$

In general, every rate can be expressed by using any one of many different ratios—that is, pairs of numerals. For example, in *Seeing Through Arithmetic 6,* pupils learn that 1/14, 2/28, 3/42, and other ratios all express the same rate in the battery-buying situation discussed above. Consequently, it is correct to write $1/14=2/28=3/42=4/56$. This statement is obtained by developing an understanding of rate through using various ways of grouping sets of batteries and cents. Although computational methods exist for showing that two different expressions, such as $2/28=3/42$, actually represent the same rate, they are not introduced at this point. The focus of attention is upon the concepts, rather than upon computational details.

Comparison situations involve two groups of objects, and two numerals are required to express the comparison. For example, suppose Mary has 9 spoons and Joan has 12 spoons. (See page 38 of the pupils' book.) If Mary says, "Joan, I have 9 spoons to your 12," she states a comparison. The ratio 9 to 12 expresses the abstract numerical relationship in this situation. The same relation is also expressed by other ratios, such as 3 to 4 or 6 to 8. The same situation also gives rise to a different ratio, namely 12 to 9, if Mary's spoons, rather than Joan's spoons, are made the basis of the comparison. The comparison idea grows out of *group* to *group* relationships.

The *relationship* remains the same no matter which of the many possible ratios are used to express it. Consequently, it is correct to write $3/4=6/8=9/12=12/16=15/20$. A relationship of this sort is appropriately called a *proportionality relationship.* These ideas (but not the term *proportionality relationship*) are developed in *Seeing Through Arithmetic 6* on pages 38-42.

Fractions play a very important rôle in arithmetic. Pupils were introduced to fractions in earlier grades, and in Grade 5 the study of fractions became much more intensive. A discussion of some of the central features of the way fractions are taught in *Seeing Through Arithmetic* may be found in the section "Getting acquainted with this book" on pages 8-9. Teachers are urged to read those paragraphs now before continuing with the comments that follow.

The fraction idea is used in situations where only a part of something is being considered. A fraction numeral tells *how much* of it is being considered if the whole thing is represented by 1. For example, suppose something is divided into three equal parts and then two of these parts are taken. The child learns to symbolize the portion of the original whole by using a new kind of numeral, namely, $\frac{2}{3}$. In this numeral, the 3 shows the number of equal parts into which the whole was divided. The 2 shows the number of these parts

that are being considered. The entire numeral tells how much of the original object is being considered.

These ideas are the basic meanings for situations that involve fractions. Here, as elsewhere, the child can best acquire these meanings by carrying out the actions with objects, and then using the number symbols to show what he has done. Pictures not only can help him to acquire these ideas, but also can guide him to the actions that are basic to the meaning. The pictures and text on pages 43-45 of *Seeing Through Arithmetic 6* provide materials for a careful review and extension of the fraction idea and the way fraction numerals are used to express it.

There are three types of problem situations that are symbolized by using number pairs. One of these types involves rates, another involves comparisons, and the third involves fractions. It is not possible to tell from the symbol alone which of these situations the symbol is intended to represent. The meanings are based on the concrete situations and are not inherent in the symbols. If pupils are to deal successfully with each of these types of problem situations, they must learn to distinguish among them regardless of how they are symbolized by numerals. Page 46 is of special significance because it helps the pupils to do this.

## 32-37 Looking back

Lesson Briefs for this lesson are on pages 52-55.

**Overview**

An understanding of the use of ratios is essential to the understanding of the many kinds of problems that can be most quickly and efficiently solved by means of a proportion, or equation of ratios. Before ratios can be used in problem solving, it is necessary for the pupils to learn how to recognize ratios and the concepts they can be used to express. In this lesson the pupils are taught to use ratios to express the familiar number relationship often referred to as *rate*, e.g., "3 pairs for $5," "78 revolutions per minute," "3 eggs for 2 cups of milk," etc.

There is no problem solving as such in this lesson, but the pupils see that many different ratios can be used to express the same rate, and that these different ratios are equal when they express the same rate. For example, if 3 pairs of gloves can be bought for $5, and larger quantities can also be bought at the same rate, the ratios 3/5, 6/10, 9/15, and 12/20 are all equal and express the same rate.

**Teaching the whole class**

You might introduce this lesson by helping the pupils discover how frequently rates enter into their daily experiences. For example, in the mixing of frozen fruit juice, 3 cans of water per 1 can of juice is a rate that can be represented by the ratio 3/1 (or 1/3, depending on whether one thinks first of the juice or of the water). Cookbooks will provide many examples of rates. The direction "Cook 20 minutes per pound" for timing a roast can be expressed by various ratios such as 20/1, 40/2, 60/3, etc. Miles per hour also express a rate, as do such common phrases as "3 cans for 29¢." Ask the pupils to suggest other examples of the use of rate.

**Page 32:** Let two boys act out Movie A and have a third pupil read the adjoining text aloud. Let someone else give the two numbers that express the rate at which Tom bought the batteries.

Now have a pupil read the first four lines of text opposite Picture B and ask him to count the batteries and the money in the picture. Next have the whole class read the rest of the text to themselves. By asking them questions, get them to observe that the picture, the phrases "4 for 56 cents" and "4 per 56," and the pair of blue numerals (the ratio) all say the same thing, namely, the rate at which the batteries were sold. Emphasize that the word *ratio* is the mathematical name for a pair of numerals used to express a rate. If you use the word *ratio* often when you discuss how rates are expressed, the pupils will soon accept it as the mathematical symbol.

Before going on to page 33, have the pupils look at Picture B again. Ask them if they can see any way to find out how many cents they would need if they wanted to buy only 1 battery. Try to get some suggestions from the pupils. Then go on to page 33.

**Page 33:** The work on pages 33-37 is intended to illustrate the principle that the numbers expressed in both terms of a ratio may be divided or multiplied by the same number without changing the relationship of the terms. Hence many different ratios can be used to express the same rate. Reducing ratios by multiplication or division is taught on pages 52-53 of the pupils' book, by which time the pupils have had careful preparation for understanding the computation. Many pupils, especially the able ones, will remember the computational procedure from *Seeing Through Arithmetic 5*, but do not stress computation at this time.

Tell the pupils to study Picture C. Have them count the batteries and the money, so they will see that there are still 4 batteries and 56 cents. Ask someone

to explain how this picture differs from Picture B on page 32. Ask someone to explain how Picture C could be obtained from Picture B. Then call on a pupil to read the text for Picture C. Let someone else read the first two lines of text for Picture D. Have him explain why some groups are dimmed off in the picture. You might then continue somewhat in this manner:

*If you can buy 1 battery for 14 cents, what two numerals can you use to express the rate? Look at the ratio shown in blue. What does each numeral stand for? How do you read the ratio? [1 per 14] Does "1 per 14" express the rate illustrated in Picture D? Does this ratio also express the rate shown in Picture B?*

Now ask the pupils how much 2 batteries would cost. Let them find out by examining Picture E. Then discuss the text in connection with Picture E.

Before going on to page 34, you can suggest other situations by asking questions similar to these:

*Now imagine that you want to buy 6 batteries. How much would you pay for them? Or perhaps you want to buy 5 batteries. How much would they cost?*

**Page 34:** By this time several of the pupils may see that the rate at which the batteries are sold can be expressed as 1 per 14, no matter how many are sold. Let these pupils help explain Picture F to the class.

Tell the pupils to count all the batteries in Picture F and then all the money. When they have agreed that there are 6 batteries and 84 cents, have someone read the text and explain the ratio given in blue.

Ask a pupil to read Exercise A and use Picture F to find the answers. Stress again that each group of batteries and coins in Picture F contains 1 battery and 14 cents, and that the amount of money paid for the batteries depends upon how many groups are sold. Therefore, a different ratio can be written to express each way of saying the rate, and all the ratios listed in Exercise B are illustrated in Picture F. Let several pupils supply the numerals to replace the screens in the ratios in Exercise B.

Then tell the class that another set of ratios is illustrated in Picture F. First point out that the ratios discussed so far were written to express the number of batteries first and then the amount of money paid for them. This was shown by placing the numerals representing the batteries over the numerals representing the money. Explain that if they now think of the money first, they should place the numerals that represent the money over the numerals that represent the batteries. Have someone read the paragraph starting at the bottom of the first column. Then have someone else explain what each numeral in the ratio 84/6 stands for.

Then have someone read Exercise C and, with the help of Picture F, supply the numerals that belong where the screens are as well as explain the rate 14 per 1 and the ratio 14/1. Draw attention to the fact that in this lesson the word *per* has been used when a ratio has been read.

Let another pupil read Exercise D and give the answer.

When you discuss Exercise E, again emphasize that all the ratios express the same rate. Use the groups in Picture F to help clarify this for the pupils.

**Page 35:** A new situation is introduced on this page, but the same steps are followed as for the situation just studied. Therefore, you may adapt the procedures for page 32 and for Picture C on page 33 to this page.

**Page 36:** Procedures suggested for Pictures D and E on page 33 and for page 34 may be adapted to this page. When you discuss the last three lines of text on page 36, make sure that the pupils see that 2 stands for the number of persons and 5 for the number of cups. Ask the pupils if the ratios 4/10 or 6/15 could also be used, and why.

**Page 37:** Give the pupils a little time to study the pictures and identify the objects in them. Be sure everyone understands the directions. Then discuss Exercises A to T, or have the pupils write each ratio, and beside it, the letter of the picture that shows the rate expressed by the particular ratio. When the pupils have finished these exercises, discuss each rate with them.

Assign the second block of exercises as written work. Be sure the pupils understand the directions. When they have finished, discuss their answers with them. Ask them to tell what each numeral in a ratio stands for.

#### Providing for the able pupil

The able pupils might enjoy playing "Ratio." See Activity 15, page 380. The game is played by groups of four, and picture cards are provided to aid the pupil in seeing the formation of rates and ratios. You may wish to arrange the pupils so that one or two who understand rates and ratios are grouped with a couple of pupils who are having difficulty in understanding them. In this way, the abler pupils will help verify the work done by the others.

#### Helping the slow learner

The pupil who does not understand this work very readily will be helped if he is given extra practice in

using materials to illustrate rates and ratios. Objects and coins may be used to illustrate the rates expressed in the pictures on page 37.

You might write on a sheet of paper a group of rates expressed in the form 3 per 2, 4 per 2, 5 per 7, etc., then have the pupils make circles, X's, etc., to illustrate them and express the rates by writing the ratios in the form 3/2, etc. (Be sure the pupils use a horizontal line, not a diagonal line, when they write ratios.) In this case, also, the pupils may understand the meaning more readily if they use objects.

The game "Ratio" (Activity 15, page 380) may be played by all the pupils. Since it is played in small groups, the slow learner may be put with pupils who understand rates and ratios better than he does and who, therefore, can help him. This game will give the slow learner good practice in matching rates illustrated by pictures with ratios that express the rates.

## 38-42 Looking back

Lesson Briefs for this lesson are on pages 56-59.

**Overview**

The "Comments" on page 56 of this *Teaching Guide* provide a summary of this lesson.

**Teaching the whole class**

**Page 38:** Before you review the use of ratios to express comparisons, you will want to be sure that the pupils know what a comparison is. For such a discussion, use situations and experiences familiar to the pupils. (For example, you might compare the number of cousins one pupil has with the number another pupil has.) Let the pupils give as many other comparisons as they can. Be sure the words *compare* and *comparison* are used.

Then proceed to the review, beginning on page 38. When discussing the dialogue in Picture A, emphasize the fact stated in the text that *two* numbers are used to make a comparison—in this case, 9 and 12.

Tell the pupils to study Picture B while reading the accompanying text. Discuss the meaning of each numeral in the ratio. Emphasize the use of the word *to* when reading a ratio that expresses comparison.

Have the pupils look next at Picture C and get them to note the total number of Joan's spoons and the total number of Mary's spoons. Then ask someone to explain what has happened. [The spoons have been put into equal groups, in each of which there are 3 of Mary's spoons and 4 of Joan's.] Have someone read the text.

The way in which the equal groups are formed from the total group may be clearer to the pupils if you demonstrate with objects. You can show that 9 objects of one kind and 12 objects of another kind can be separated into 3 equal groups with 3 of one kind and 4 of the other kind in each group. You also might let each pupil use 12 small pieces of construction paper of one color and 9 pieces of another color to represent the spoons in the picture. Emphasize that, starting with groups of 9 and 12, subgroups consisting of 3 of Mary's spoons and 4 of Joan's are the smallest subgroups that can be made.

**Page 39:** Tell the pupils that Picture D shows how other numbers besides 9 and 12 can be used to compare Mary's and Joan's spoons. Direct attention to the group in strong color and have someone read the statements beside the arrow. Let another pupil read the text that explains the numerals in the ratio 3/4. Be sure the pupils understand that this comparison is based on *one* group of 3 black spoons and 4 blue spoons, and that the proper way to read the ratio that expresses the comparison is "3 *to* 4."

For Picture E, get the pupils to notice that now they are thinking about two of the groups. Have them read the text and see what ratio can now be made to compare Mary's and Joan's spoons. Point out that Picture E, which shows the comparison of 6 to 8, indicates the same relationship that was developed in Picture D and its text.

**Page 40:** Tell the class that Picture F shows what happens when Joan has collected 8 more spoons and Mary has collected 6 more. If the work is being illustrated with pieces of construction paper, have the pupils use 15 pieces for Mary's spoons and 20 pieces for Joan's. Then let them group the pieces so that each group will represent 4 of Joan's spoons and 3 of Mary's. This will help them in answering the questions in Exercises A and B.

Continue somewhat as follows:

*So far in this lesson you have thought of Mary's spoons first when making the comparison. But you also may compare the girls' spoons by thinking of Joan's spoons first. Then the ratio you make must show the comparison of Joan's spoons with Mary's. In this ratio, 20 will be written above the line and 15 below the line. In other words, you think of Joan's 20 spoons first and then Mary's 15. You would say, "Joan has 20 spoons to Mary's 15," and you would read the ratio as "20 to 15."*

*When we look at only* one *group of spoons and think of Joan's spoons first, we write 4 over 3 as a ratio and read it "4 to 3."*

Have the pupils read the related text. Then discuss Exercises C and D.

**Page 41:** Ask the class to study the pictures and the related text. For each picture, have the pupils write all the ratios they can to show the comparison. Note that while Exercises A to E all apply to Picture A, they also outline steps that can be adapted to a discussion of Pictures B and C.

You will probably want the pupils to do Exercises H and I independently. They should study Picture C carefully and then select the ratios that apply to the picture. The use of pieces of construction paper to represent the objects in Picture C will help the slow learners in selecting the correct ratios.

**Page 42:** Give the pupils time to study the six pictures at the top of the page. Then take one picture at a time, and discuss with the pupils the different ratios that can be written for it. For example, for Picture A the ratios 5/4, 10/8, 15/12, and 20/16 are among the ratios that express the comparison.

When the pupils are familiar with the pictures, read the text for Block 1 (Exercises A to P) and make sure that all the pupils understand the example that is given. Then have them write each ratio, accompanied by the letter of the picture that shows the relationship expressed by the ratio. Verify the answers and discuss any exercises that seem difficult.

If the slow learners have trouble with Block 2, allow them to make circles, X's, etc., to illustrate each statement and then write one ratio that expresses the comparison. The able pupils will be able to write two ratios for each comparison—one by thinking of the objects mentioned first as being compared to the objects mentioned second, the other by taking the objects in the reverse order. Some of these pupils will begin to realize that an infinite number of ratios can be written to express the relationship indicated in each statement.

**Providing for the able pupil**

Activity 15, entitled "Ratio" and described on page 380, can be adapted to this lesson. Have the pupils make new pictures on cards that show comparisons between groups of objects. The groups may be distinguished by color, or they may differ in kind.

The able pupils also can make up statements expressing comparisons similar to the statements in Block 2 on page 42. The pupils should write their statements on 3″ x 5″ cards, five statements on a card. These cards can be used by the slow learners for extra practice in writing ratios. Number or letter the cards for easy identification.

**Helping the slow learner**

Duplicate a work sheet containing several pictures of objects similar to the pictures on page 42. Under each picture indicate the number of ratios the pupils are to write for the picture (2 ratios, 3 ratios, etc.). Label each picture with a letter. Have the pupils write the letter for each picture and then opposite the letter write the ratios.

For further practice these pupils may use the 3″ x 5″ cards containing statements of comparison made by the abler pupils. Tell the pupils to draw a picture to illustrate each statement and then write a ratio for each statement.

Use the adaptation (see notes for the able pupil above) of Activity 15, page 380, with these pupils. You may wish to pair a couple of slow learners with a couple of pupils who are not having difficulty in writing ratios for comparisons.

## 43-45 Looking back

Lesson Briefs for this lesson are on pages 59-61.

**Overview**

This lesson reviews the meaning of proper fractions, improper fractions, and mixed numbers. It also reviews the reading and writing of numerals to symbolize fractions and mixed numbers. The only material on these pages that is new at the sixth-grade level is the introduction of the word *terms* to represent the numerator and the denominator of a fraction numeral.

On page 43 such ideas as these are reviewed: a fraction represents one or more equal parts of a unit; different fraction numerals may be used to symbolize a fraction; a fraction numeral in which the number represented in the numerator is larger than the number represented in the denominator stands for more than one whole thing.

On page 44 the pupils review the idea that an improper fraction can be expressed as a mixed number or, in some cases, as a whole number.

On page 45 the pupils are asked to distinguish among numerals that represent proper fractions, improper fractions, and mixed numbers and also to write numerals that represent these types of numbers.

**Teaching the whole class**

A flannel board can be used very effectively in this lesson to give an understanding of the meaning of proper fractions, improper fractions, and mixed numbers. It can also provide opportunities for the pupils to demonstrate what they *know* about fractions and mixed numbers. Activity 16, page 381, tells how to make a flannel board.

If you want all the pupils (or a group, such as the slow learners) to work at the same time at their desks, provide disks of construction paper (all of the same

size) that have been cut into fractional pieces. The pupils can make their own sets of these fraction disks, as suggested in the instructions in Activity 17, page 381. Envelopes in which the pupils can keep these disks should be provided. The pupils may manipulate these disks as you direct, or they may make their own discoveries by using the disks.

**Page 43:** To start this lesson, remind the pupils that they have used pairs of numerals to make ratios that express rate and that express comparison. Now they are going to use pairs of numerals as fraction numerals to express fractions.

To discover what the class knows about proper fractions, call attention to the disk in Picture A. The pupils will note that the disk is partitioned into thirds and that two of these thirds are blue. Have someone read the text at the right. Discuss the interpretation of the fraction numeral $\frac{2}{3}$. Point out that the fraction numeral compares the 2 parts to the 3 parts, or whole. It is read *two thirds*. Also call attention to the way $\frac{2}{3}$ is written in words.

After the three lines immediately below have been read, discuss the new word *terms*. Help the pupils recall that they have used *term* in connection with ratios. Remind them that they called the upper numeral "the first term" and the lower numeral "the second term." Point out that in fraction numerals, the upper numeral is called the numerator, and the lower numeral is called the denominator. Then have the pupils refer to Picture A to explain how they know that $\frac{2}{3}$ is less than 1. Point out that in this fraction the number expressed by the numerator is less than the number expressed by the denominator.

When the pupils study Picture B, establish that this disk is the same size as that in Picture A. Explain that since the disk has been marked off into a different number of equal parts, a different fraction numeral must be used to tell how much is blue. The class should note that the denominator expresses the total number of equal parts, and the numerator shows the number of these parts that are blue. Since the same amount of each disk is blue, and since both $\frac{2}{3}$ and $\frac{4}{6}$ tell how much is blue, the two fraction numerals are different ways of expressing the same thing. Thus, since $\frac{2}{3}$ is less than 1, $\frac{4}{6}$ must also be less than 1.

If you have a flannel board (see Activity 16, page 381), you might put on it a disk composed of thirds and beside it another disk of the same size composed of sixths. Remove one of the thirds and two of the sixths, so that the class can see that the amounts left are equal. Or, if the pupils have fraction kits (see Activity 17, page 381), ask them to assemble a disk of thirds and a disk of sixths. Then tell them to take from the disk of sixths enough of the sixths to cover one of the thirds. Then ask them to repeat the procedure so that two thirds are covered. In this way they can see for themselves that two thirds are equal to four sixths.

The class should realize that while $\frac{2}{3}$ and $\frac{4}{6}$ can represent the same amount, $\frac{2}{3}$ of one thing is not necessarily the same amount as $\frac{2}{3}$ of something else. For an activity to clarify this idea, have the pupils fold a piece of paper to show thirds. Ask them to shade the lower two thirds of the paper. Have them refold the paper into thirds, and then fold the thirds in half (when unfolded, the paper will show sixths). The shaded amount that was $\frac{2}{3}$ of the paper is now also $\frac{4}{6}$ of the paper. The pupils can see, too, that the $\frac{2}{3}$ of the paper is not the same size or shape as the $\frac{2}{3}$ of the disk that was on the flannel board or on their desks.

Before going on to Exercise C, you might ask the pupils to give other examples of fractions in which the number expressed by the numerator is less than the number expressed by the denominator. Let them use fractional parts of disks to represent these proper fractions.

The first five lines of Exercise C should be read with reference to Picture C. Note that the parts of the disks are slightly separated. When considering the improper fraction $\frac{5}{3}$, be sure the pupils understand that the denominator, 3, shows that each disk is cut into thirds, and that the numerator, 5, shows how many of the thirds they are considering. Have them note how $\frac{5}{3}$ is written in words.

After the pupils understand why $\frac{5}{3}$ is more than 1, have them give examples of a few other improper fractions. Again, they may use disks to illustrate their examples. As a variation, you may assemble fractional pieces on the flannel board and ask the class to name the improper fractions represented. Occasionally, put up fractional parts that represent proper fractions.

**Page 44:** To begin this page, ask the pupils to compare the disks in Picture D with those in Picture C (on page 43). The class should observe how the disks are alike (they are the same size; both pictures show thirds) and how they differ (in Picture D the thirds are not separated from one another).

This might be a good time to point out that a fractional part of anything—for example, $\frac{2}{3}$ of a disk—does not have to be any particular portion. To demonstrate this fact, put on the flannel board a disk composed of fourths. Have two of the fourths in one

color and the other two in another color. Ask a pupil to point out the two fourths that are colored red, for example. Then raise the question, "What if we switch the parts about? Can we still say that the amount of the disk colored red is ¾?" Rearrange the parts so that the fourths that are red are separated by the other two fourths. Point out that the statement "The amount of this disk colored red is ¾" is still correct.

Now tell the pupils that they are going to learn another way to think about the amount in Picture D that is blue. Have them read the first three lines of text and answer the questions about each of the numerals. Let them find in the picture the quantity represented by each numeral. Be sure they realize that the entire numeral represents the total amount shown in blue. Also have them note how $1\frac{2}{3}$ is written in words.

To demonstrate once more that $\frac{5}{3} = 1\frac{2}{3}$, put one whole disk on the flannel board and beside it place two thirds of another disk of the same size. Then, below the $1\frac{2}{3}$ disks, place five thirds of the same size in a row. Help the pupils interpret the top disks as $1\frac{2}{3}$ disks, and the bottom disks as $\frac{5}{3}$ disks. Write the numerals on the board. Help the class see that $1\frac{2}{3}$ and $\frac{5}{3}$ are equal because the five separate thirds can be placed over the original whole disk and two thirds; so $1\frac{2}{3}$ and $\frac{5}{3}$ stand for the same quantity.

Next direct attention to Picture E. Let the class discover that the two disks are divided into sixths and that ten sixths in all are blue. Ask them to imagine that the separated parts have been put together. Then have them answer the question in Exercise E as to whether or not $1\frac{4}{6}$ of the disks are blue. The pupils should see that $1\frac{4}{6} = 1\frac{4}{6}$.

Let the class refer to Pictures C, D, and E when they consider whether or not, in Exercise F, $1\frac{4}{6} = \frac{5}{3}$ and whether $1\frac{4}{6} = 1\frac{2}{3}$.

Give the pupils, especially the slower ones, many opportunities to work at the flannel board to show other mixed numbers. They will need whole disks and fractional pieces of disks of the same size. Let the able pupils show equalities among such mixed numbers as $1\frac{4}{6}$ and $1\frac{1}{2}$.

In discussing Exercises G, H, and I, have the pupils refer to Picture F. Get the pupils to see that the original whole piece of blue paper has 12 equal parts. Help them decide that 12 twelfths are blue. If necessary, allow them to count the squares to find that $1\frac{12}{12} = 1$.

In working with Exercise J, help the pupils see how the papers in Picture G can be thought of as divided into ninths (if the small squares are considered) and into thirds (if rows of 3 squares are considered). Then, as the pupils select in Exercise J the mixed numbers and improper fractions, justify each choice in terms of the picture.

**Page 45:** Discuss the first five lines of text. Be sure the class understands that in a proper fraction the number expressed by the numerator is smaller than the number expressed by the denominator. Write a few examples on the chalkboard. Then call on a pupil to read the fraction numerals in Exercises A to H that represent proper fractions.

Exercises I to P and the sentences above that pertain to them can be treated in the same way. Stress that in improper fractions, the number expressed by the numerator is larger than, or equal to, the number expressed by the denominator. Write some examples on the board, and let a pupil choose the improper fractions in Exercises I to P. To extend this exercise, let someone select the improper fractions in Exercises A to H and the proper fractions in Exercises I to P.

Exercises Q to X may be discussed and treated in the same way. When the pupils have identified the mixed numbers, let them tell what the other numerals represent.

For Exercises A to C in the next block of work, allow the pupils ample time to write the numerals called for. Some of the pupils may write their examples on the board, and the rest of the class can verify the work.

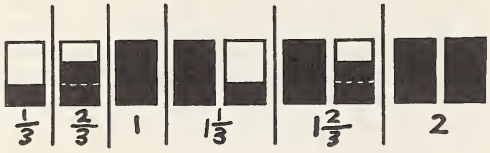
The pupils may also write their responses to the activity concerned with Pictures A to F. When they have finished, discuss what they have written. Allow for variations in interpretation. For example, Picture B can be interpreted as $1\frac{3}{6}$, $\frac{9}{6}$, $1\frac{1}{2}$, and $\frac{3}{2}$. Help the pupils see that Pictures C and F illustrate proper fractions. Spend a little time discussing the mixed number illustrated in Picture D. Point out that while the two wholes are separated from each other by a fractional part of a disk, this does not matter.

**Providing for the able pupil**

Ask the able pupils to find numerals that represent fractions and mixed numbers in newspapers and magazines. Tell them to copy or cut out and paste on 4″ x 6″ cards any statements that include such numerals. The cards can then be arranged as a bulletin-board display.

Able pupils can also make a wall panel of drawings that represent fractions, whole numbers, and mixed numbers in graduated order, increasing or decreasing in size. Several committees might each prepare a series of drawings for a specific set of numbers (½, 1,

1½, 2; ⅓, ⅔, 1, 1⅓, 1⅔, 2; ¼, ¾, ¾, 1; etc.). A series should end with a whole number, and each drawing should be labeled with the numeral for the number it represents. An example of a series based on thirds is shown below.




**Helping the slow learner**

Slow learners will benefit most from the use of visual aids. If a flannel board and cutouts of fractional parts are available, let these pupils take turns illustrating the numerals in Exercises A to X on page 45. Mix up the fractional parts so that if a pupil cannot recognize a fractional part—⅙, for example—he must first build a complete disk of 5 equal parts. Then he can either remove fractional parts from the disk, or use other fractional parts to make more whole disks or parts of disks. Able pupils can supervise the work of the slow learners.

Another activity makes it possible for the pupils to make some discoveries about equivalent fractional parts without computing. Have the pupils assemble fractional parts to illustrate the fractions in Exercises B, G, L, O, and S (page 45) and then find equivalent fractional parts for each, as follows: twelfths for B, tenths for G, eighths for L, thirds for O, and fourths for S. They can do this by putting the smaller fractional parts over the larger fractional parts. As they find the equivalent fractional parts for each, have the pupils use numerals to write the exercise on paper in this manner: B $\frac{5}{6}=\frac{10}{12}$.

As another activity for the slow learners, have the able pupils make a set of cards containing numerals for proper fractions, improper fractions, and mixed numbers. Have the slow learners sort these cards into three piles—one pile for proper fractions, one for improper fractions, and one for mixed numbers.

As still another activity, divide the pupils who are to participate into two teams. An able pupil can name a proper fraction, a mixed number, or an improper fraction and ask a member of one team to illustrate it on the flannel board. If that pupil cannot do it, after an adequate time limit, he must sit down and give a member of the other team a chance to illustrate the fraction or mixed number. The winning team is the side with the most players left at the end of a designated time.

## 46 Thinking straight

Lesson Briefs for this lesson ore on poge 62.

Expanded Notes are not considered necessary for this lesson except for suggestions for a class activity and suggestions for the able pupil.

**Teaching the whole class**

To give practice in distinguishing among rates, comparisons, and fractions, Activity 18, described on page 382, may be used.

**Providing for the able pupil**

In order to prevent any memorization of problem situations for Activity 18 on page 382, and to keep the game interesting and challenging, able pupils could write their own rate, comparison, and fraction problems to be used when the game is played.

## 47-48 Using arithmetic

Lesson Briefs for this lesson ore an poges 63-65.

Expanded Notes are not considered necessary for this lesson.

## 49-51 Exploring problems

Lesson Briefs for this lesson are an pages 65-67.

**Overview**

The "Comments" on pages 65-66 provide an extensive overview of this lesson.

**Teaching the whole class**

**Page 49:** Direct the attention of the class to Picture A. Have the dialogue read and then the equation. If any question arises about referring to the screen, explain that "what number" can be used. Deal with Picture B in the same way. Ask if a screen could be substituted for the question mark in the equation. Then have someone write the equation for Picture A on the chalkboard, with a question mark in place of the screen.

Go on to Pictures C and D, treating them in the same way. Ask questions such as these, which are suggested for use with Picture C:

*Could we use a question mark in place of the line? Could we use a screen? Jane, go to the board and write the equation with a question mark and then with a screen. Bob, write the equation for Picture B with a line. Write the equation for Picture A with a line, Sue.*

Treat Pictures E and F in the same way. Point out that while $n$ is the only letter that has been used in equations up to this time, here $x$ and $a$ are used. Make clear to the pupils that *any* letter can be used to hold

a place for a numeral. Then ask the class to suggest other letters that could be used in place of those in Pictures E and F. You may wish to suggest that *o*, *x*, and *i* are not the best letters to use and then let the pupils discuss why. (*O* could be confused with zero, *x* with a times sign, and *i* with 1.) Have some of the pupils write the equations in Pictures A to C on the board, using a letter for the place holder.

Before going on to Problems A to F below the pictures, you may wish to try the following activity.

Using the equations below, which involve basic facts only, make up some simple verbal problems. Have a pupil write an equation on the board for each problem you give. Have the pupils use the letter *n* for the numeral they are to find. The idea here is to include problems of different types that have been reviewed. (See the problem chart on pages 408-409 for samples of problem situations to fit each equation.)

| | |
|---|---|
| $6+n=14$ | $12\div n=6$ |
| $12-n=9$ | $n-4=8$ |
| $16\div 4=n$ | $3+5=n$ |
| $n+6=14$ | $9-6=n$ |
| $3\times 7=n$ | |

After the equations have been written on the board and solved, ask the class to think of another way to represent in each case the numeral they need to find. Then erase the equations and repeat the problems in a different order. This time have the pupils use any letter except *n* in the equations and find the answers.

Direct attention next to Problem A in the text. After the problem has been read, discuss the equation provided. Then have someone compute at the board to solve the equation.

Discuss Problem B in the same way. Then assign Problems C to F as written work. Tell the pupils to make an equation for each problem and compute to find the answer. Then have the equation and solution for each put on the board and discussed.

**Page 50:** After Problem G has been read, discuss it. Help the pupils see that the situation is one of combining two amounts of money, one of which is unknown. Let someone then read the information given after the heading THINK, and discuss the first writing of the equation. Have the pupils note that two letters are used to hold places for two numerals —*a* for the cost of the paper and *d* for the total cost of the paper and the pencils.

Then have the pupils look below, where the equation has been repeated, but $(2\times\$.37)$ is shown in black ink above it. Explain that the work indicated in parentheses shows what must be done to find the amount that *a* stands for and that this numeral must be found before the final answer, *d*, can be found. Next have them look at the equation $(2\times\$.37)+\$.49=d$, where the preliminary work is incorporated in the equation. After the computation indicated within the parentheses has been done, the equation is rewritten with the numeral that replaces *a* inserted. All that remains to be done is to add to find the replacement for *d*. Let someone do the addition at the board and answer the question.

Tell the pupils that in working problems with more than one step (multiple-step problems), they should always first make an equation. They may use any letters to indicate the numerals they are to find. Next they should rewrite the equation and indicate the preliminary work by placing it within parentheses. The next rewriting, then, should show numerals instead of the work indicated in parentheses. Finally, the equation should be written with the answer.

Have the pupils read Problem H carefully and study the equation that is provided. Be sure they understand what *m* represents and that *t* represents the final answer. Get someone to explain why they should think of $(6\times 16)$ as a way to get the numeral that replaces *m*. Then have everyone write the equation (with the preliminary computation indicated in parentheses), compute, and rewrite the equation as shown. Have someone tell what numeral will replace *t*.

Problem I can be treated in much the same way. The pupils have already studied averages in this book, but if they have any trouble in making a basic equation, refer them to the lesson beginning on page 28.

**Page 51:** Discuss Problem J with the pupils. Call attention to the three letters in the equation. Help them see that the problem suggests the combining of two groups of money. Since they do not know at first what two amounts are to be added, they can use letters in the first equation.

Remind the class that the computation indicated in parentheses shows what must be done to find the numerals to replace *a* and *b*. Have them write the equation with the preliminary computation indicated in parentheses and do the computation. Call on a pupil to tell what numeral will replace *x*.

Problems K, L, and M may be handled in the same way. Discuss the fact that in making equations *any* letters they choose may be used, but that, first, what each letter is to stand for must be decided.

At the end of this lesson pupils should begin to see the generalizing value of this method of problem solving. Certain basic equations are used and applied in many similar problem situations. It will be helpful to put an equation such as $a+b=x$ on the board and

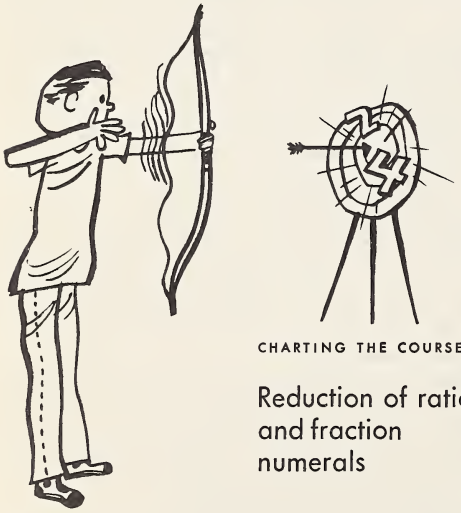
ask the pupils to suggest problems for which it is appropriate. They will soon begin to see similarities in problem situations and will begin to understand and appreciate how the problem-solving method can be generalized through equations.

**Providing for the able pupil**

"Problem tag" (Activity 19, page 383) is an activity that will give the pupils experience in symbolizing problem situations. In this activity a pupil reads a problem, puts on a flannel board an equation that describes the problem, and finds the answer.

**Helping the slow learner**

Simple one-step problems, written or pasted on 3″ x 5″ cards, can be used with Activity 19, page 383, by these pupils. Have a pupil explain what each symbol means as he forms an equation that describes a problem situation. The rest of the group can then study the equation to see if the problem has been symbolized correctly. If the equation is correct, the pupil may compute on the board to solve the problem. An able pupil may supervise the activity.



CHARTING THE COURSE

## Reduction of ratios and fraction numerals

Rates, comparisons, and fractions play a very fundamental rôle in arithmetic. Much of the computational work that arises when these concepts are used involves replacing a given ratio or fraction numeral by another that is equivalent to it.

Pupils were introduced on page 33 to the idea that different ratios can be used to express the same rate. By means of pictures showing the grouping of objects, they learned that the rate 4 per 56 can also be expressed as 1 per 14 and as 2 per 28. At that time no attempt was made to show that the ratio could be "reduced" by dividing or multiplying both members of the number pair by the same number. Similarly, pupils learned on later pages (see page 39) that different ratios can be used to express the same comparison. In none of these examples were formal computational methods used to replace one pair of numerals with another equivalent pair. In Grade 5 pupils were taught that the same fraction can be expressed by different fraction numerals.

Another way of stating the situation could be used. We could say that "$\frac{4}{5}$" and "$\frac{8}{10}$" are two different names for the same fraction. This is similar to saying that "4," "four," and "IV" are different names (or symbols) for the same number. This form of expression is mathematically sound and emphasizes the distinction between the concept and different symbols, or names, for it.

Computational methods of replacing ratios with equivalent ratios are taught on pages 52-53. Similarly, computational methods of replacing fraction numerals with equivalent fraction numerals are taught on pages 54-55. On these pages (52-55) the pupil is shown how he can find an equivalent number pair by dividing both terms of a given pair by the same number. For example, the pair 6/10 is "reduced" to 3/5 by dividing both 6 and 10 by 2. Also, the pair 6/10 can be replaced with the equivalent pair 12/20, which is found by multiplying both 6 and 10 by 2. This change is also usually referred to as a "reduction." Since equivalent numeral pairs are different symbols for the same relationship or fraction, they may be used to form equations (6/10=3/5).

Often a particular ratio is given, and at the same time one of the terms of an equivalent ratio is specified. The problem of finding the other term of the equivalent ratio then arises. The same situation occurs, of course, with fraction numerals. In *Seeing Through Arithmetic 5*, pupils were shown how to find the unknown term of a number pair in such situations. The importance of the ability to do this can hardly be overestimated, since it enters fundamentally into the solution of many types of problems. To cite but one example, it is used in replacing fraction numerals by others that have a common denominator before the fractions are added or subtracted. This basic skill is, therefore, carefully retaught on pages 56-58 of *Seeing Through Arithmetic 6*.

## 52-55 Looking back

Lesson Briefs for this lesson are on pages 68-71.

**Overview**

In this lesson the pupils review the computation involved when it is necessary to replace a ratio or a fraction numeral with an equal ratio or fraction numeral. Reduction of ratios is studied on pages 52 and 53; reduction of fraction numerals on pages 54 and 55. Page 55 presents reduction to lowest terms.

**Teaching the whole class**

In the lessons on pages 32-37 and 38-42, the pupils reviewed the meaning of rates and comparisons and learned how to write ratios to express these rates and comparisons. On pages 43-45 they reviewed fraction numerals. They learned that more than one pair of numerals can be written to represent a rate, the result of a comparison, or a fraction. Now the class reviews the computation by which equivalent pairs of numerals can be found.

As in the lessons on ratios and fractions, objects, the flannel board, and fraction disks can be used to help the pupils understand the computation. See Activity 16, page 381, for a description of the flannel board, and Activity 17, page 381 for a description of oaktag disks.

At all times you will want to keep the pupils aware of the fact that when a ratio or a fraction numeral is reduced, it still expresses the same rate or comparison or fraction. Stress, too, that when a ratio or a fraction numeral is reduced, the numbers represented by both terms must be multiplied or divided by the same number. Do not expect all the pupils to use the term *reduce*. Most of them will find it easier to talk about finding equal ratios or equal fraction numerals in lower or higher terms.

**Page 52:** Have the pupils read the statement beside the heading "Moving forward." Tell them that on pages 52 and 53 they are going to study ratios. Remind them that a rate or a comparison can be expressed in written form by many ratios and that they have seen this demonstrated by means of pictures and objects. Tell them that now they will learn how to compute to find ratios that are equal to other ratios.

Have the class read the text for Picture A and examine the picture. Then have them interpret the ratio in terms of the picture. Emphasize that 6/4 expresses the rate of eggs per cups of milk.

Now have the pupils read the text for Picture B and examine the picture. Ask them to think of one of the equal groups, as the text directs. Emphasize that the ratio 3/2 expresses the same rate of eggs per cups of milk as the ratio 6/4. Point out that in the ratio 3/2 the numeral standing for the number of eggs is the first term, just as it is in the ratio 6/4. Get the pupils to see that because the eggs and cups of milk have been divided into two equal groups, the corresponding computational procedure is to divide both 6 and 4 by 2. Then we have a new pair (3/2) that expresses the same rate as 6/4.

Going on to Picture C, the class should see that the original number of eggs and cups of milk has been doubled. Ask the pupils why the number of eggs must be doubled if the number of cups of milk is doubled. Have the pupils read the last two lines of text and supply the missing numerals. Tell them that on the next page they will see how to write a new ratio to express the rate.

**Page 53:** Have the pupils read the text at the top of this page, referring to Picture C as they do so. They should see why the rate at which Mrs. Stone used eggs and cups of milk can be expressed by a new ratio. Remind them again that the rate has not changed, but that different numerals are being used to express the rate. In the explanation of the computation, most pupils will understand why 2 is used as the multiplier; others will need to refer to Picture C.

Discuss Exercises A to D with the class. Keep emphasizing the fact that the new ratios are all equal to the first ratio, 16/20. You may need to help some of the pupils by using objects and setting up a rate or comparison situation to show that $16/20=4/5=8/10$, etc. By the time Exercise D has been completed, most of the pupils will begin to see that there is no limit to the equal ratios that can be found by multiplication, but that there is a limit to those found by division unless we permit the use of fraction numerals as terms for the ratio. Fraction numerals as ratio terms will be considered later in this book.

In discussing Exercises E to H, help the pupils see why dividing the numbers represented by both terms of a ratio by the same number gives an equal ratio in lower terms and why multiplying by the same number gives an equal ratio in higher terms. Assign Exercises A to P as written work. When the pupils have finished, discuss their answers.

**Page 54:** Remind the pupils that different fraction numerals can be used to express any fraction. Tell them that now they are going to compute to find fraction numerals that are equal to other fraction numerals.

Have the class look at the three paper disks shown on this page. Show them that the yellow portion is the same in each disk. Get them to observe that the

light, the heavy, and the broken lines mark off different fractional parts. Then focus attention on the disk in Picture A. Ask: "Into how many equal parts is this disk divided?" Let someone read the accompanying text, supply the missing numeral, and answer the question.

Next have the class inspect the disk in Picture B. Help the pupils see that the heavier lines mark off five equal parts in the disk. Ask how many fifteenths are contained in each fifth. Then have someone read the first five lines of text opposite the arrow and answer the questions. Discuss why it is still correct to say that $\frac{9}{15}$ of the disk is yellow. Have the pupils explain why the yellow part can also be expressed by the fraction numeral $\frac{3}{5}$. Write on the board what they have discovered—namely, $\frac{9}{15}=\frac{3}{5}$. Take some time here to discuss the terms of these fraction numerals. Have the pupils identify the numerators and the denominators.

Now discuss how to compute to find a fraction numeral that is equal to $\frac{9}{15}$. Point out that the disk was first divided into fifteenths, and then into larger equal parts. Since the division into fifths results in fewer equal parts, the pupils should see that, in computing, the process of division should be used. Since there are $\frac{3}{15}$ in each $\frac{1}{5}$, both 9 and 15 of $\frac{9}{15}$ are divided by 3.

Next have the class look at Picture C and proceed somewhat as follows:

*Is the same part colored yellow here? What lines divide the disk into fifths? What lines divide the disk into fifteenths? Notice the broken lines. They divide the disk into parts smaller than fifteenths. The broken lines divide each fifteenth into how many parts? Jean, read the first three lines of the text and answer the question. How many thirtieths are there in the part colored yellow? Bob, read and answer the next two questions. How does Picture C show that $\frac{9}{15}$ of the disk is yellow? Can we say that $\frac{9}{15}=\frac{18}{30}$? [Write $\frac{9}{15}=\frac{18}{30}$ on the board.] Let's see how we can change $\frac{9}{15}$ to $\frac{18}{30}$ by computation. Look at Picture C again. There are how many thirtieths in one fifteenth?* [2] *When we compute, what shall we do?* [*Multiply*] *Alice, read the next four lines. By what number should we multiply the 9 and 15 of $\frac{9}{15}$ to get $\frac{18}{30}$? Bill, read the last two lines. What fraction numerals are illustrated by the yellow part in each picture?*

The pupils can be helped to see the reasons for multiplying or dividing to find equal fraction numerals if equivalents are demonstrated on the flannel board or if they use cardboard fractional pieces at their desks. Since fractional pieces for fifteenths and thirtieths are rather small, it is suggested that other fraction equivalents be used.

In showing equivalents, follow the sequence used in the book. For example, if you wish to show $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$, start with $\frac{2}{4}$ of a disk. If the pupils are working at their desks, have them assemble two fourths and then find the number of halves equal to this part of a disk and superimpose them on the fourths. If you are using the flannel board, place the halves beside the fourths, so the equivalence can be easily seen. Have the class note that $\frac{2}{4}=\frac{1}{2}$, and that where there were two parts, there is now one part. Then discuss how to find this equality by computation. Write $\frac{2}{4}$ on the chalkboard and ask for suggestions. Many of the pupils will see that they should divide both 2 and 4 by 2 to change $\frac{2}{4}$ to $\frac{1}{2}$. Next have them find the number of eighths that are equal to $\frac{2}{4}$. Have them remove the half and superimpose eighths over the fourths to find that there are 4 eighths. Then write $\frac{2}{4}=\frac{4}{8}$ on the board. Help the pupils see that where there were fourths, there now are more parts that are smaller. For each fourth there are now two eighths. They should see that to get this result by computation, they must multiply both the 2 and 4 of $\frac{2}{4}$ by 2.

**Page 55:** To prepare the pupils for the work on this page, discuss the idea that a fraction numeral is equal to many other fraction numerals. Explain that when both terms are divided by the same number, a fraction numeral equal to the first one is produced. Thus, if both 12 and 24 of the fraction $\frac{12}{24}$ are divided by 4, a different name for the fraction will be produced. Show the class that many other equivalents may be found by multiplying the terms by any number from 2 up. [The 12 and 24 of $\frac{12}{24}$ may be multiplied by 2, 3, 4, 5, 6, etc.] Explain why we say that fraction numerals are in lower terms and why we say they are in higher terms.

Point out that in Exercises A to L at the bottom of Column 1 on page 55, division is to be used, and in Exercises M to X (at the top of Column 2) the fraction numerals are to be reduced by multiplication. Let the pupils work independently on these exercises, but go over their work with them and discuss it.

Use Exercises A and B that follow next for oral discussion to bring out the meaning of the expression *lowest terms.* Remind the pupils that this is another way of saying that fraction numerals are in simplest form, which they learned about last year. Then discuss Exercises C to N. Be sure the pupils understand how to determine whether or not a fraction numeral is in lowest terms.

The final exercises, A to T, should be assigned as written work. Discuss Exercise A with the class beforehand as follows, so that they can see the kind of thinking that is necessary.

*To find the fraction numeral equal to $\frac{12}{16}$ that is in lowest terms, you must divide both terms by the same number. Can 12 and 16 both be divided by 2? When this has been done, the new fraction numeral is $\frac{6}{8}$. Can $\frac{6}{8}$ be reduced still further? Try dividing by 2 again. The new fraction numeral is $\frac{3}{4}$. Is $\frac{3}{4}$ in lowest terms? How do you know? Since you divided by 2 twice, you could have done this by dividing by 4. Divide both terms of $\frac{12}{16}$ by 4. Do you get $\frac{3}{4}$ this way?*

Point out to the class that by dividing by the largest possible number, 4, they can get the smallest possible fraction numeral that is equal to $\frac{12}{16}$. Discuss the other fraction numerals in the exercises when the pupils have finished their work.

**Providing for the able pupil**

The following activity can give the able pupils experience in reducing fraction numerals to lowest terms and also in finding other fraction numerals that are equivalent. Prepare mimeographed sheets on which you have written the fraction numerals shown below. Omit the lines under the fraction numerals, but leave space so that the pupils can put in lines to show their answers.

$\frac{1}{2}$ $\frac{3}{5}$ $\frac{2}{4}$ $\frac{21}{35}$ $\frac{1}{3}$ $\frac{6}{18}$ $\frac{12}{20}$

$\frac{7}{14}$ $\frac{9}{15}$ $\frac{15}{25}$ $\frac{7}{21}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{4}{12}$

$\frac{2}{6}$ $\frac{18}{30}$ $\frac{3}{9}$ $\frac{6}{12}$ $\frac{3}{6}$ $\frac{5}{15}$ $\frac{4}{8}$

Give one sheet to each pupil. Tell the pupils to draw one line under all fraction numerals equal to ½ and two lines under all fraction numerals equal to ⅜. Then tell them that there is one other fraction numeral in lowest terms for which they are to find fraction numerals that are equal. Have them find this fraction numeral [⅛] and underline it three times; then find those that are equal to it.

Some of the better students may want to construct other sets of fraction numerals. Such sets can be mimeographed and then worked by others in the group.

For another activity, the game, "Spin-a-rate," Activity 20, on page 384, is devised to give pupils practice in recognizing among various ratios those which express a specific rate or comparison.

These pupils will also profit from playing "Fraction row," described in Activity 21, page 385. This game will help them recognize equivalent fraction numerals.

**Helping the slow learner**

When the slow learners begin to handle ratios more easily, let them join the able pupils in a game of "Spin-a-rate," Activity 20, page 384.

For another activity for the slow learners, sets of "fraction papers" may be made. See Activity 22, page 385.

## 56-58 Looking back

Lesson Briefs for this lesson are on pages 71-73.

Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil and the slow learner.

**Providing for the able pupil**

The game of "Ratio bingo," Activity 23, page 386, can be used to give the pupils practice in finding the missing term in a pair of equal ratios.

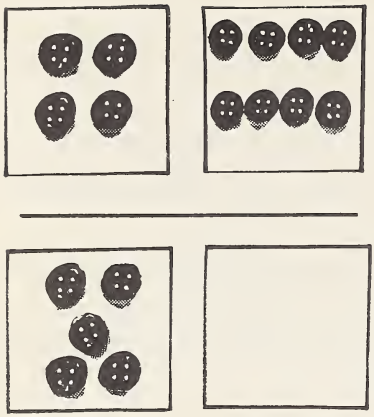
**Helping the slow learner**

Slow learners may need many opportunities to work with objects in ratio situations. The activity described below will give them practice in solving problems in *comparison* situations.

On 3″ x 5″ cards, write information like that shown in the illustration below.

| | |
|---|---|
| Pupil A | 12 |
| Pupil B | 4 |
| Pupil A | 6 |
| Pupil B | ? |

Provide a large group of objects (buttons will do). Place them on a desk or a table in a central location. Let the pupils pair off (as Pupils A and B) and allow one pupil (Pupil A) in each pair to draw a card. Explain that the set of numerals above the line shows that Pupil A has 12 objects and Pupil B has 4 objects. Tell Pupil B that he is to find how many objects he



will have if Pupil A has 6 and the relationship is kept the same.

The procedure may be as follows: Pupil A takes as many objects as the card indicates (12) and puts them in a row on the desk. Pupil B takes as many objects as the card indicates (4) and puts them in a row beneath the others. Pupil A compares the two groups and says "I have 12 objects to your 4." Pupil A then looks at what is written below the line and says "If I have 6 objects, how many will you have?" Then Pupil B must decide how many objects he should place under the 6 to show the same relationship. Allow Pupil B to use any means he wants to find how many objects he should place below the 6 of Pupil A. He might divide his partner's 12 objects so that there are 6 in a group. In this way, he finds that there are 2 groups of 6. Then he can divide his 4 objects into 2 groups to match the 2 groups of 6. He finds that there are 2 in each of his 2 groups and sees that his partner has 6 objects to 2 of his. Pupil B then states the new ratio as 6 to 2. Then suggest that he write the equation $[12/4=6/n]$ and show how to solve it.

Vary the information on the cards so that occasionally Pupil A will have to find the missing numeral.

The following comparison activity with groups of objects is somewhat similar. Four sheets of construction paper may be used to keep the groups separated. Place them on the arithmetic table or in some central spot where a group of pupils can work and watch. On each of two of the sheets, place a group of small objects. Note the illustration above.

Let three pupils work together. Have one of the pupils make a statement comparing the two groups displayed and write a ratio for them on the board. Then ask one of the other pupils to think of another ratio equal to this one. Have him illustrate one term of the new ratio by placing objects on one of the unfilled sheets of paper. Remind him that it will be wise to put the objects on the sheets in the same relative position to illustrate the ratio he has in mind. The sheets may be placed as shown in the picture.

The third pupil should then be instructed to place the correct number of objects on the fourth sheet to complete the illustration of the new ratio. Finally, the ratios should be written on the board as an equation.

Each time a new ratio equal to the first one is decided upon, be sure that the comparison does not require a fraction numeral in a term of the ratio. For instance, if, in the first comparison illustrated above, 10 buttons had been put opposite the 4 buttons (rather than 8 or 16 or 24), the third pupil would have had difficulty in completing the ratio. If this does occur, tell the pupils that they have not yet learned how to solve a situation of this type.

If the slow learners need more work in finding ratios, write several sequences of ratios on the board. Each ratio should be easy to derive from the ratio at the left of it. For example, you might write:

$$\frac{4}{6}=\frac{m}{3}=\frac{8}{a}=\frac{24}{c} \qquad \frac{2}{10}=\frac{6}{b}=\frac{24}{d}=\frac{k}{60}$$

Let the pupils explain what they must do to find the numeral to replace the letter in each case.

CHARTING THE COURSE

## The remainder in division

The remainder in a division example can always be used to form a fraction, but the number so obtained is

not always meaningful in the original situation. In *Seeing Through Arithmetic 6*, the interpretation of the remainder is carefully taught to help pupils avoid answers that are not sensible. Pages 59-62 show pupils how the remainder may, in appropriate situations, be used to form a fraction. Pages 63-65 are devoted to examples that help them to develop judgment as to when this is or is not a sensible thing to do.

The way a remainder is used depends solely upon the problem situation from which the need for a division comes. If the situation involves objects that can be partitioned into fractional parts, the remainder is often used to form a fraction. Two boys may share 11 candy bars equally if each takes $5\frac{1}{2}$ bars. However, if three girls have 16 balloons, it is not sensible to share them equally; to offer $5\frac{1}{3}$ balloons as an answer to such a problem reveals a lack of attention to and understanding of the situation. The common practice of instructing children, as soon as they have studied fractions, to use the remainder to form a fraction is not good teaching.

In exercises not arising from problem situations, the pupil may be required to use a remainder to make a fraction in the quotient. The pupil may or may not be required to express the fraction "in lowest terms." Sometimes, in problem situations, it is not sensible to reduce the fraction numeral.

Teachers should be aware of the fact that competent writers sometimes use remainders in ways that are not easily interpreted. Averages are an example, and one may see in print statements such as "The average number of persons was 91.3." A common-sense interpretation would demand that this answer be expressed as 91. Exceptions to "common-sense" expressions that arise in statistical and similar applications should be pointed out and explained to the children.

## 59-62 Looking back

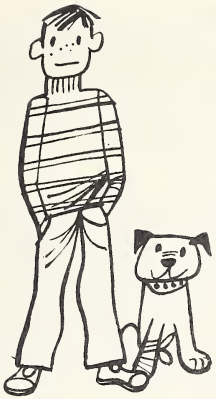
Lesson Briefs for this lesson ore on pages 74-76.

Expanded Notes are not considered necessary for this lesson.

## 63-65 Thinking straight; Keeping skillful

Lesson Briefs for these lessons ore on poges 77-79.

Expanded Notes are not considered necessary for these lessons.



CHARTING THE COURSE

## Common denominators

Fractions would not be useful as numbers if it were not possible to tell when one fraction is larger than another, or to add two or more fractions, or to subtract one fraction from another. One basic idea underlies the comparison, the addition, and the subtraction of fractions—namely, the use of a common denominator.

If one fraction is to be compared with another, both fractions are first expressed by fraction numerals that have the same denominator. Then the numerators may be compared, and the larger numerator belongs to the larger fraction. Similarly, if fractions are to be added, they are first expressed by fraction numerals with the same denominator. Then the numerators are added, and their sum is used as the numerator of a new fraction numeral that also has the common denominator. This new fraction numeral represents the sum of the fractions. Subtraction, of course, is also accomplished by using the common denominator idea.

Children must learn these principles of arithmetic. In the *Seeing Through Arithmetic* program, the common denominator idea is taught before the children are asked to add fractions, rather than as a step of the process itself. If children first gain a clear understanding of the common denominator idea, and some skill in replacing fraction numerals with others that have a common denominator, then learning the process of adding or subtracting fractions is greatly simplified.

In *Seeing Through Arithmetic 5*, a "number line" was used to show fraction numerals arranged in order. On a "number line," equal fraction numerals are associated with the same point on the line. For example, $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{4}{16}$ are each associated with a single point. In *Seeing Through Arithmetic 6* the "number line" is used again

(see pages 165-168 of the pupils' book) to develop the same idea in a more general case. In this book a table is used (see page 66 of the pupils' book) to reteach the common denominator idea and develop it further.

The table is a particularly useful device for this purpose because it enables the pupil to see a whole set of equivalent fraction numerals at once, and not just one or two at a time. By merely looking in the table, he can find the numerals for two (or more) different fractions that have the same, or a *common*, denominator. When the concept of common denominator has become well understood, the pupil is ready to learn how common denominators can be found by computational methods.

In the *Seeing Through Arithmetic* program the pupil is taught to decide first whether the "largest" denominator of two or more fraction numerals will serve as a common denominator. If so, he uses this and replaces other fraction numerals by equivalents that have that denominator. If the "largest" denominator will not serve as a common denominator, the pupil is taught to try a multiple of it until he finds a denominator that will serve. Although this method is not always the most direct, it is simple and will always produce a common denominator. The most extreme case arises when the denominators are "relatively prime" to each other, as in an example like $\frac{5}{13}$ and $\frac{8}{17}$. In such an example the first multiple of 17 that will serve is $13 \times 17$, which is, of course, the product of the two denominators.

The development of the common denominator idea as presented in this book should not be regarded as merely a set of lessons preliminary to the addition of fractions. It has important values in its own right in that it teaches pupils the fundamental procedure for comparing fractions and it introduces them to the idea of *sets of numbers* and the use of the word *set* in this way. This is one of the most fundamental ideas in the entire realm of mathematics. The pupil will have many opportunities to use it from this point onward.

## 66-69 Looking back

Lesson Briefs for this lesson are on pages 80-83.

**Overview**

In the chart on page 66 fraction numerals are arranged by rows into sets of equal fraction numerals. Each row, or set, begins with a fraction numeral in lowest terms. This chart can be used to compare two or more fractions by comparing equal numerals whose denominators are alike. In the exercises on page 67 the meaning of *lowest common denominator* is developed. On page 68 computational methods of finding common denominators are reviewed. On page 69 the pupils answer questions devised to produce important generalizations. They also work some exercises for practice.

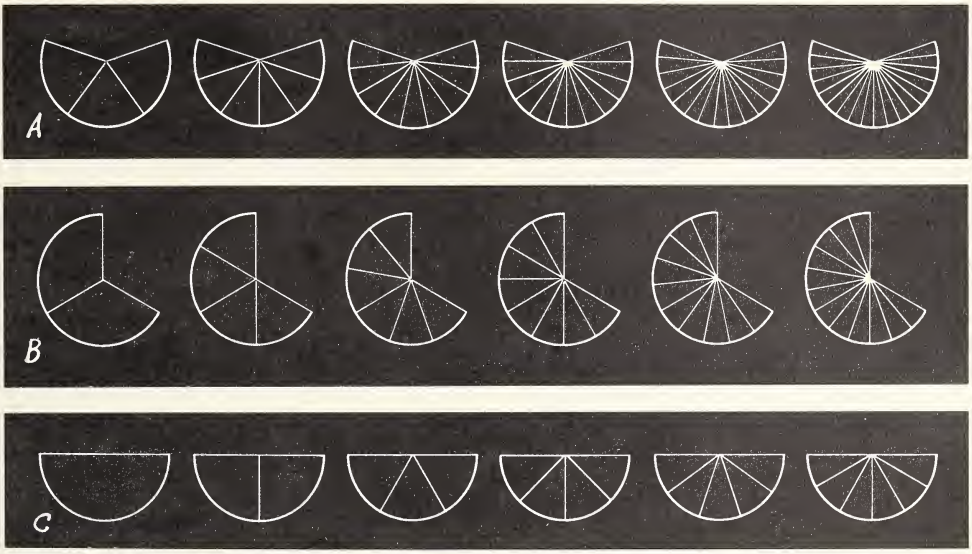
**Teaching the whole class**

**Page 66:** To assist boys and girls in understanding equal fraction numerals and common denominators, a portion of the chart on page 66 can be set up with fraction disks on a bulletin board, or drawn on a chalkboard, or set up on a flannel board. The first six equal fraction numerals of Sets A to C are used as a sample. (See chart on the next page.)

Ask the pupils if they see any relationship between the chart on page 66 and the demonstration display. Then direct their attention to Set A in the text and ask if they can discover why these fraction numerals have been placed together. Verify the pupils' answers with the visual materials. Discuss the equality of $\frac{3}{5}$ and $\frac{6}{10}$ to bring out the idea that both represent the same amount, but that $\frac{6}{10}$ has a different name because there are twice the number of parts and the parts are smaller. Discuss the equality of $\frac{3}{5}$ and $\frac{9}{15}$. Some pupil will discover that both represent the same amount, but that in $\frac{9}{15}$ there are three times as many parts and the parts are still smaller. Discuss, then, the principle that if both numbers involved in a fraction are multiplied by the same number, the fraction will be the same but it will have a different name; that is, the fraction numerals will be equal. Experiment with more fraction numerals in Set A and relate them to the fraction disks on the bulletin board. By this time some pupil will be ready to observe that each fraction numeral following $\frac{3}{5}$ can be found by multiplying both 3 and 5 by numbers that are consecutively higher—in this case, 2, 3, 4, . . . 10. Ask if Set A could be extended, and give pupils an opportunity to do so. Explain that since the fraction numerals in each row have a definite relationship to each other that shows they stand for the same thing, they are called a *set*, or family.

Examine each succeeding set and relate the numerals to the visual material as far as it applies, and then rely only upon the principle of multiplying both numbers involved in the terms by the same number to complete, and if desired, to extend each set.

**Page 67:** Have a pupil read Exercise A and answer the question. In connection with Exercises B to D, ask a pupil to compare $\frac{3}{5}$ and $\frac{2}{3}$ to learn which is




smaller. From the rows of fraction disk equivalents, he may be able to see that $\frac{3}{5}$ is smaller. Ask if anyone can tell, exactly, how much smaller it is. Through discussion some pupil will be able to suggest that it is difficult to tell, because $\frac{3}{5}$ and $\frac{2}{3}$ have different denominators. Then suggest to the pupils that they notice the $\frac{6}{10}$ in the $\frac{3}{5}$-set of the chart and look in the $\frac{2}{3}$-set to learn if it has a fraction numeral with 10 as its denominator. Relate this to the concrete representation with fraction disks. Try $\frac{9}{15}$ in the $\frac{3}{5}$-set and look to see if the $\frac{2}{3}$-set has a numeral whose denominator is 15. Relate this step to the fraction disks also so that the pupils can see that $\frac{3}{5}$ can be replaced by $\frac{9}{15}$, and that $\frac{2}{3}$ can be replaced by $\frac{10}{15}$. To show that $\frac{2}{3}$ is $\frac{1}{15}$ larger than $\frac{3}{5}$, a pupil can place the $\frac{9}{15}$ section over the $\frac{2}{3}$ and see that $\frac{1}{15}$ is still visible.

Use Exercises E to G to develop the idea that other common denominators can be found for fifths and thirds. Discussion will bring out the point that the chart shows 30 as a common denominator, and some pupil may see that the chart can be extended so that 45 could be a common denominator.

Use Exercise H to bring out that 15 is the *lowest common denominator* that can be used to compare $\frac{3}{5}$ and $\frac{2}{3}$, as well as any other fractions in the sets of fifths and thirds. Let the pupils answer the question and summarize the findings.

At this point the text can be supplemented by another example that will relate the chart and the fraction disks by comparing $\frac{2}{3}$ and $\frac{3}{4}$ to determine which is larger. The procedure used for Exercises B to H can be applied in this instance, also.

Use the same suggestions outlined above for Exercises I to K, in which the largest of three fractions is to be found, and for Exercises L to N in which the smallest of three fractions is to be found. Make use of visual comparisons whenever possible.

The chart on page 66 may be used to make other comparisons by finding common denominators. Let the pupils describe how they reach their conclusions and urge them to determine which denominator in a set is the *lowest common denominator.*

**Page 68:** Point out to the pupils that so far they have been finding common denominators by reading them on a chart or seeing them illustrated. Tell the pupils that Exercise A shows how to find a common denominator by computation. Have them note by referring to the chart that the common denominator produced in this way (80) is not the lowest common denominator for these fractions. Exercise B should show them that sometimes such multiplication does produce the lowest common denominator. Use Exercise C to demonstrate that in some cases the larger denominator is the lowest common denominator.

Exercises D to F show another way to find the lowest common denominator by computation. Tell the pupils first to think somewhat as follows to see if $\frac{5}{8}$ can be expressed as eighths:

*What do I do when I want to find an equivalent fraction numeral? I multiply or divide both 5 and 6 by the same number. Can I multiply 6 by any whole number to get 8? No. Then 8 is not a common denominator for ⅝ and ⅞.*

Tell the pupils that it is not possible to express a higher denominator in terms of a lower denominator; so eighths cannot be expressed as sixths. Demonstrate this fact with fraction disks. Show that an eighth cannot be expressed as sixths because a sixth is larger. The denominator is said to be higher because the unit, or whole, is thought of as divided into a greater number of equal parts. Next, they should try the first number after 8 that can be divided by 8, which is 16. When they find that ⅝ cannot be expressed as sixteenths, they should try 24, the next number after 16 that can be divided by 8. They find that 24 can be divided by 6. Therefore, 24 is the lowest common denominator for sixths and eighths.

For Exercises H to K, use the same procedure as outlined above for finding the lowest common denominators.

**Page 69:** In discussing Exercises L to O, be sure the pupils realize that the two fractions are the same ones worked with in Exercise K. Ann used the lowest common denominator (24) to find which fraction is larger. Joan used 48. Ask the pupils which number they think is easier to work with.

Exercises P and Q may be used as a review in finding common denominators by computation. Assign Exercises A to T as written work. Provide answers, and discuss the exercises.

The first set of exercises in "Keeping skillful" gives more practice in finding lowest common denominators. The second and third blocks provide practice in subtraction, addition, and division.

#### Providing for the able pupil

Able pupils might make a chart of sets of equivalent fraction numerals. Give them a list of fraction numerals not included in the chart on page 66 and let them find 10 equivalent fraction numerals for each numeral in the list.

Two versions of a game called "Fraction numeral set-up" (Activity 24, page 387) may be used by these pupils to get practice in assembling sets of equivalent fraction numerals.

#### Helping the slow learner

The slow learners may need help in understanding that, to make an exact comparison, it is necessary to express fractions with a common denominator. Concrete illustrations, provided by fraction disks placed on a flannel board, should be helpful.

You might begin by comparing one half of a disk and one third of a disk. Have someone put on the flannel board a disk composed of halves and a disk of the same size composed of thirds. Ask a pupil to point out one half of the first disk and one third of the other one. Discuss what has to be done to compare the two fractional parts. The pupils should decide that to make a comparison, the one half and the one third must be divided into equal parts. Explain that when they divide the fractional parts into equal parts to make a comparison, the parts to be compared must be of the same size. They are now using a common denominator. Have the pupils find the lowest common denominator for ½ and ⅓. When they decide that it is 6, put 2 disks composed of sixths on the flannel board. Let a pupil take enough sixths from one disk to equal one half and place them beside the one half disk, then take enough sixths from the other disk to equal one third and place them beside the one third disk. The pupils should observe that one half of a disk is larger than one third of a disk by one sixth. Have them compare the fractions expressed as fraction numerals with 6 as a denominator in the same way.

As another activity, the slow learners might make a fraction numeral chart, too. Give them a list of fraction numerals with 1 in the numerator and have them make a set of equivalent fraction numerals for each unit fraction in the list.

"Finding the common denominator," described in Activity 25, page 388, will give slow learners practice in finding common denominators by computation.

One of the versions of the game of "Fraction numeral set-up" will be helpful to the slow learners in getting practice in assembling sets of equivalent fractions. See page 387, Activity 24.

## 70-71 Looking back

Lesson Briefs for this lesson are on pages 83-85.

Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil and the slow learner.

#### Providing for the able pupil

The game of "Fraction match," described in Activity 26, page 389, will provide able pupils with practice in replacing improper fraction numerals with equal mixed numerals. For these pupils, the improper fraction numerals and the equal mixed numerals that are written on cards should express large numbers.

After a pupil has completed Exercises A to R at the top of page 71, he could verify a few of his answers

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| halves | $\frac{3}{2}$ $1\frac{1}{2}$ | $\frac{4}{2}$ 2 | $\frac{5}{2}$ $2\frac{1}{2}$ | $\frac{6}{2}$ 3 | $\frac{7}{2}$ $3\frac{1}{2}$ | $\frac{8}{2}$ 4 | $\frac{9}{2}$ $4\frac{1}{2}$ | $\frac{10}{2}$ 5 |
| thirds | $\frac{4}{3}$ $1\frac{1}{3}$ | $\frac{5}{3}$ $1\frac{2}{3}$ | $\frac{6}{3}$ 2 | $\frac{7}{3}$ $2\frac{1}{3}$ | $\frac{8}{3}$ $2\frac{2}{3}$ | $\frac{9}{3}$ 3 | $\frac{10}{3}$ $3\frac{1}{3}$ | |
| fourths | $\frac{5}{4}$ $1\frac{1}{4}$ | $\frac{6}{4}$ $1\frac{1}{2}$ | $\frac{7}{4}$ $1\frac{3}{4}$ | $\frac{8}{4}$ 2 | $\frac{9}{4}$ $2\frac{1}{4}$ | $\frac{10}{4}$ $2\frac{1}{2}$ | | |
| fifths | $\frac{6}{5}$ $1\frac{1}{5}$ | $\frac{7}{5}$ $1\frac{2}{5}$ | $\frac{8}{5}$ $1\frac{3}{5}$ | $\frac{9}{5}$ $1\frac{4}{5}$ | $\frac{10}{5}$ 2 | | | |
| sixths | $\frac{7}{6}$ $1\frac{1}{6}$ | $\frac{8}{6}$ $1\frac{1}{3}$ | $\frac{9}{6}$ $1\frac{1}{2}$ | $\frac{10}{6}$ $1\frac{2}{3}$ | | | | |
| sevenths | $\frac{8}{7}$ $1\frac{1}{7}$ | $\frac{9}{7}$ $1\frac{2}{7}$ | $\frac{10}{7}$ $1\frac{3}{7}$ | | | | | |
| eighths | $\frac{9}{8}$ $1\frac{1}{8}$ | $\frac{10}{8}$ $1\frac{1}{4}$ | | | | | | |
| ninths | $\frac{10}{9}$ $1\frac{1}{9}$ | | | | | | | |

by using the fraction kit mentioned in the activities for the slow learner that follow. For example, for the improper fraction numeral 1½, the pupil might count out 11 halves, place pairs of halves together, and see that he has five wholes with 1 half left over.

**Helping the slow learner**

Individual kits of disks cut into halves, thirds, fourths, and eighths are useful in helping the pupils experiment and learn how many fractional parts equal a whole. (See Activity 17, page 381.) When several pupils have demonstrated that a specified number of fractional parts are required to make a whole, let other volunteers demonstrate with a room kit, which contains a greater variety of fractional parts. With these visual aids the pupils should more readily understand that if the parts are halves, two of them make a whole; if the parts are eighths, eight make a whole; and so on.

Slow learners may also like to construct a chart showing equal mixed numerals for improper fraction numerals using denominators through ninths. Each pupil can take one or more series of improper fraction numerals with the same denominators and with numerators through 10 and compute to change them to the equal mixed numerals or whole numeral form. Then all the numerals can be arranged on a chart for display. Such a chart will give the pupils an idea of the order (and relative size from smallest to largest) of a few improper fractions. At the same time the pupils will begin to acquire the ability to recognize certain equivalents without computing. A completed chart is shown at the left.

If you decide to use the game "Fraction match" suggested below, you should transfer the numerals from the chart above to sets of cards used in playing the game. Some pupils can help you. If small numbers are used, the game of "Fraction match" will provide the slow learners with needed practice in replacing improper fraction numerals with equal mixed numerals by computation. See page 389, Activity 26.

The game of "Make 1" will give these pupils practice in making visual discoveries as to the number of various fractional parts it takes to make a whole. See Activity 27, page 389.

## 72 Checking up

Lesson Briefs for this lesson are on pages 85-86.

Expanded Notes are not considered necessary for this lesson.

## 73 Using arithmetic

Lesson Briefs for this lesson are on pages 86-87.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

# Addition and subtraction involving common fraction numerals

The processes of addition and subtraction of fractions have traditionally been regarded as very difficult to

learn and to remember. In part, this is because teaching procedures have traditionally relied chiefly upon stimulus-response learning theories. Here again the process was analyzed into an extensive set of "specific habits" that were then taught one by one through drill methods. The teacher is urged to read again the paragraphs on "Simplified organization" found in the section on "Getting acquainted with this book," pages 14-15.

In contrast to these traditional procedures, *Seeing Through Arithmetic* puts the emphasis on developing an understanding of the general principles of adding and subtracting fractions before the pupil is expected to develop skill through practice. Children are shown by visual, rather than computational, methods what adding or subtracting fractions means (pages 74-85 and 89-97). Addition is associated with the act of combining parts of concrete objects. Subtraction is associated with the act of taking part of an object away from a larger part or from the whole object. Since the idea of a common denominator has already been introduced, the presentation is not complicated by giving attention to the details of how fraction numerals that have the same denominators may be found computationally.

When the process of adding fractions is presented by the methods described above, pupils can learn the process itself without experiencing undue difficulty. In the past, some of the difficulty has usually been postponed by using at first only examples in which the fraction numerals have the same denominator. *Seeing Through Arithmetic 6* introduces the process with examples in which the fraction numerals do not have the same denominator. This is in conformity to the principle that, to promote the development of insight, processes should be introduced by examples that are as representative of the complete process as is possible.

Similar principles govern the teaching of the addition of numbers expressed as mixed numerals. In the first example the sum of the fractional parts of the two mixed numbers exceeds 1, so that a form of the process of "carrying" is required. Once this major feature of the process is understood, the teacher does not need to be careful to select only examples that do not require "carrying," and the pupil may proceed without encountering later a fundamental modification of the process.

The teaching of the process of subtraction of fractions is organized according to the plans outlined above for addition. Although pupils at this stage should not be expected to achieve immediately either perfect accuracy or a high rate of performance in adding and subtracting fractions, there is no reason why they should not be expected to have a good understanding of what they are doing. When this is reasonably well established, attention may be turned to the development of acceptable skill with the processes.

## 74-79 Learning how

Lesson Briefs for this lesson are on pages 87-91.

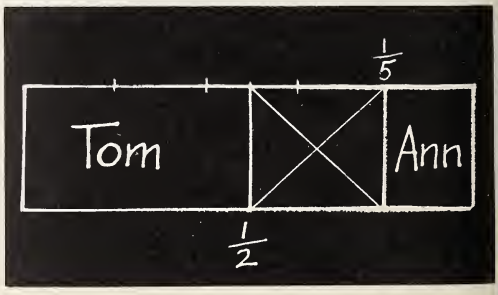
**Teaching the whole class**

The teacher may observe that grammatical difficulties often arise when fractions are being considered. For example, the *singular* form of the verb is used when a portion of a whole is the subject of a sentence as in "Seven tenths of the bar *was* given away." Here the name of the fraction is used to describe the *amount* that was given away. One would say "Part of the bar *was* given away," not "Part of the bar *were* given away." If one asks "What part?" the answer is "seven tenths," which seems to be plural, but may be considered singular. This should not be taken up with the pupils.

**Page 74:** To begin the lesson, have the text for Picture A read and relate the fractions to the two pieces of the candy bar. Discuss the meaning of the equation. Be sure the pupils understand why a new name is needed for the part of the candy bar given away. Emphasize that when the sum of the numbers is found, a name for the total has been found.

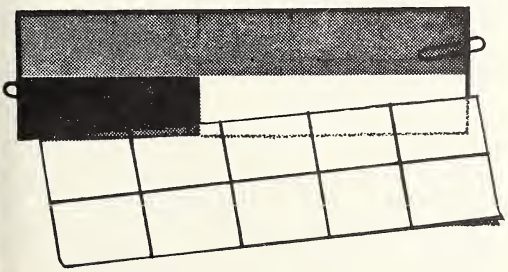
Then proceed with the text for Picture B. Discuss that, to find out how much of the bar Susan gave away, it is necessary to put the two pieces together.

Draw on the board (see illustration below) a rectangle to represent a candy bar. On the longer dimension of the bar mark off sections showing fifths on one side and halves on the other side. Draw a line across the bar at the ½ mark and another line across the bar at the last ⅕ mark, as shown in the illustration.



Indicate the parts that were given to Ann and Tom. Cross out the part of the bar that Susan kept. Someone in the class will soon observe that the ½ mark is midway between two of the ⅕ marks and will be able to reason that tenths may be used as a common name for both halves and fifths.

Pieces of colored paper may also be used. Over a white rectangle representing the whole candy bar, superimpose a piece of colored paper equal to one half and a piece of paper of another color equal to one fifth. These colored pieces may be fastened to the white rectangle with paper clips (see illustration below). Now you can illustrate to the pupils that before it is possible to know what fraction they can use to express the sum of the two fractional parts, something else must be done. Mark off into tenths a transparent tissue rectangle of the same size as the white rectangle. Put the tissue over the pieces already assembled, and let the pupils tell what they see. They should see that the tenths' marks divide the whole rectangle into equal parts—that the one-half and the one-fifth sections are also divided into equal parts. They can, therefore, say that fraction numerals with 10 as the denominator can be used to express the parts of the candy bar given away.



If necessary, review the procedure of multiplying the numbers represented in the terms of a fraction numeral by the same number to arrive at an equal fraction numeral. Relate the reduction process as used with comparison of fractions to the addition of fractions.

**Page 75:** Discuss Picture C and let pupils relate the text to the picture. The pupils know that $\frac{2}{10}$ is equal to ⅕, and that $\frac{5}{10}$ is equal to ½. Have them note that Picture C shows $\frac{2}{10}$ of the bar and $\frac{5}{10}$ of the bar put together. Now, since all the parts are the same size, it is possible, just by looking at the picture, to tell how much of the bar was given away.

Read the equation and emphasize that the denominator 10 in each fraction numeral indicates that the original candy bar was thought of as marked off into 10 equal parts. Let a pupil explain what the numerator of each fraction numeral indicates and why the numbers indicated by the numerators are added to find the sum of the two fractions. Then let another pupil read the original equation with $n$ replaced by $\frac{7}{10}$.

Point out that there are other fraction numerals besides $\frac{2}{10}$ and $\frac{5}{10}$ that are equal to ⅕ and ½ (for example, $\frac{1}{20}$ and $\frac{10}{20}$) and that these fraction numerals could have been used to find the answer. However, computation is usually simpler when the lowest common denominator is used. You may wish to illustrate how the fractional parts in Picture C can be divided into twentieths.

Note that the SEE step on page 75 shows the addition of *three* fractions. To illustrate this example, use 3 strips of paper, each a yard long. (Paper ribbon, adding machine paper, or strips of brown wrapping paper can be used. The width of the material used is unimportant, since this is not an important aspect of the problem situation.) Mark one strip in thirds, one in fourths, and one in halves. Then fold back each strip to the desired length, so that one strip shows ⅔ yd., one shows ¾ yd., and one shows ½ yd. Do not cut off the folded-back parts, since the whole strip will be needed later. Mark each folded-back strip close to the fold with its appropriate fraction numeral— ⅔, ¾, or ½.

Let pupils read the first paragraph of the text in the SEE step and the equation that follows. Then ask someone to arrange the folded-back strips of paper in a line and in the order shown by the numerals in the equation. Ask pupils to look at the equation again and tell in what way the fractions in the problem differ from one another. Pupils should see that since ⅔, ¾, and ½ have different denominators, these fractions cannot be added in their present form. Pupils should also see that there is no way of telling how many yards these combined strips represent just by looking at them. The pupils should decide that each fraction must be expressed in another way. Have the pupils read the rest of the text and study Picture A.

**Page 76:** Review with the pupils the idea that a common denominator represents a number that is divisible by all the numbers represented in the denominators. Help the pupils decide that they will first try 4, the largest of these numbers, and they will next try $2\times4$, or 8. Then they will try $3\times4$, or 12. In finding the common denominator some pupils will readily recognize the relationship between numbers and will immediately see that 12 is a common denominator for these three numbers.

Refer to what the class knows about linear measurement by working out with the pupils how many inches there would be in each part if a yard were divided into twelve equal parts. Then let volunteers unfold the three strips and mark them off into 12 three-inch sections. Let other volunteers refold the strips to equal $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ of their respective wholes, as originally marked. It is important that the pupils observe that a division line for one of the 12 equal parts of the whole coincides with the original fold of each strip.

Let someone again arrange each folded-back strip of paper to represent one of the pieces of cloth in Picture B. Have pupils examine Picture B and tell what it shows. (The separate lengths of cloth have been put together. The picture also shows how many twelfths each piece of cloth contains and how much of a yard each piece is.) Read and discuss all the text for Picture B, making certain that the class understands the computational method for finding the missing numerators for the three fraction numerals. Also emphasize that the new fraction numerals permit them to add the fractions.

Through the discussion of measurement in the preceding picture, various members of the class will be able to reason from Picture C that each piece of cloth, in itself, is *less* than a yard, but when the pieces are placed end to end, the total length is *more* than a yard. Others will reason further by noticing that the total length is 1 twelfth less than two yards. From the computational viewpoint, relate the fraction numerals in Picture C to those in A on page 75. Let pupils read and discuss how the sum 23 for the numerator of the new name is obtained. Because of the visual presentation of the lesson, most of the pupils will be ready to accept and understand that 12 twelfths equal 1 yard; therefore, 23 twelfths would be equal to 1 and 11 twelfths of a yard. Pupils will now be able to read aloud the equation at the bottom of the page and relate the steps needed to arrive at an understandable conclusion.

**Page 77:** Have pupils read the text opposite Picture A. Let a pupil relate the equation to the picture. Have the pupils explain why the last question cannot be answered at the moment. Go on to Picture B and relate the text to the illustration. Encourage pupils to furnish any needed information in supplying the missing sixths.

Look at Picture C and discuss its related material. Pupils will discover that the numbers represented by the numerators can be added because a common denominator has been found. Let a pupil explain how the answer $\frac{3}{6}$ was obtained. Another class member could show with fraction kit segments that this answer can be reduced to $\frac{1}{2}$.

**Page 78:** Discuss Example A in detail and direct the pupils' thinking as to why the fraction numerals need not be replaced, how the 8 in the numerator is obtained, and why and how the answer is reduced to lowest terms. Point out that the horizontal form of computation is easy to use when the denominators of the fraction numerals are the same. Note that the vertical arrangement is introduced as a preparation for the addition of numbers expressed with mixed numerals. Either arrangement is satisfactory for adding numbers expressed with proper fraction numerals.

In Example B, call attention to the number of fractions, how and why 24 was arrived at as the common denominator, the method and understanding employed in replacing the improper fraction numeral $\frac{57}{24}$ with the mixed numeral $2\frac{9}{24}$, and the reason why this mixed numeral is to be replaced by $2\frac{3}{8}$. Have the pupils explain how they know that $2\frac{3}{8}$ is in lowest terms.

Have pupils work Examples C and D independently on paper and then compare their answers and work with the text. Any pupils who have difficulty understanding the examples on this page should be encouraged to use fractional parts of disks, chalkboard illustrations, or scratch paper drawings to assist them in their thinking.

**Page 79:** When you are sure pupils understand the examples on page 78, let them work Exercises A to P in the DO step on page 79 independently. You will want to make sure that the pupils know how to use both the equation form and the vertical form in showing their work, and that they also know how to express their answers in lowest terms. When this work has been completed, have the answers put on the board. If any answer is incorrect, let the class discuss the solution to find out what is wrong. Pupils who make many errors may, after this discussion, try Exercises A to N. Those who still make errors will need the help described below in the section "Helping the slow learner."

The three blocks of work in the "Keeping skillful" exercises should be used as separate lessons. Supply answers, and discuss the equations and computation with the pupils, if you think it is necessary.

**Providing for the able pupil**

The game "Fraction race," described in Activity 28, page 390, will give able pupils practice in adding mixed numbers.

**Helping the slow learner**

To help the slow learner, you will have to observe his work closely to see what his specific difficulty is. If it is inability to find common denominators, review with him the lesson on common denominators, pages 66-69; if necessary, go back to page 54 and review replacing fraction numerals with equal ones. Also let him try the activities suggested for these lessons. If his difficulty is inability to replace an improper fraction numeral with a mixed numeral, have him review the work in the lesson beginning on page 70, and let him try again the activities suggested. For practice in expressing fraction numerals in lowest terms, have him work again the exercises in "Keeping skillful" on page 65. If he needs help in adding proper fractions, ask him to go back to page 74 and do the work with objects or paper cut-outs whenever possible.

## 80-85 Learning how

Lesson Briefs for this lesson ore on pages 92-95.

**Teaching the whole class**

Since the work in this lesson requires the replacing of improper fraction numerals, you may wish to review briefly the idea that 8 eighths are needed to make 1 whole, that 6 sixths are needed to make 1 whole, and so on. Some pupils will need demonstrations using the flannel board, oaktag fractional pieces, etc.

Be sure pupils understand that when a fraction numeral is replaced, the fraction itself is not being changed. That is, they must understand that a *name* of the fraction is being replaced by another name for the same fraction. The fraction is not changed. For example, two fourths, four eighths, eight sixteenths, three sixths, fifty hundredths, etc., are other names for one half.

Pupils are unlikely to have trouble later on if they are encouraged to see that the addition of fractions uses some of the same ideas that they used in the addition of whole numbers. A chart such as the one below might be put on the board. Point out that when

the sum is a fraction that equals a whole or two wholes or three wholes, these wholes are put with the numerals in the ones' column, just as tens in the sum of the ones' column are put with the numerals in the tens' column.

**Page 80:** The pupils should first identify the pints of ice cream in Picture A in terms of the data given in the problem. Be sure they understand that because the situation is additive, the fractional parts of the pints, as well as the whole pints, must be combined, and some way found to express the total. Discuss the equation in relation to the picture.

When Picture B is discussed, be sure the pupils understand why the groups of whole pints are dimmed (they are not being considered at the moment). Get the pupils to observe that the $\frac{2}{3}$ pint is replaced by $\frac{4}{6}$ pint, and the $\frac{1}{2}$ pint is replaced by $\frac{3}{6}$ pint. Have them explain why this replacement has been made. Then have them look at the way the computation is written. They should note that the mixed numerals are written first as they appear in the problem, then with the fraction numerals replaced with equal fraction numerals with a common denominator. Discuss why 6 was chosen as the common denominator. Get someone to explain how Picture B shows that the sum of the two fractions is $\frac{7}{6}$. Then tell the pupils to write the mixed numerals in computational form on their papers.

**Page 81:** Have the pupils read the text for Picture C and observe the computation. Be sure they observe where the $\frac{1}{6}$ is written. They may now add the fractions by using the numerals they have written on their papers. Ask someone to explain how he knows that $\frac{7}{6}$ is more than 1. Tell them to remember the 1 and explain that the amount left over ($\frac{1}{6}$) is all that should be written as a fraction numeral in the answer.

Call attention to Picture D. Discuss where the whole new pint came from, and why it is being put with the other whole pints. Remind the pupils that only the $\frac{1}{6}$ has been recorded so far in the sum. Have them note where the 1 (representing the whole new pint in the picture) is written in the ones' column. Point out that $\frac{6}{6}$ has been replaced with 1, and that this 1 has been put with the numerals in the ones' column. Call attention, too, to the way this computation parallels that used in the addition of whole numbers.

Then have the pupils look at Picture E to see that the groups of whole pints are being combined. Ask someone to tell how many whole pints there are in all. Have the pupils next add the whole numbers, including the carried 1, and note where the 6 is written.

Let them write the carried 1 above the ones' column on their papers, and then add. Call on someone to read the answer. Then have the final equation read.

**Page 82:** Proceed with the work on this page in much the same way as was outlined for the SEE step. Now, however, the pupils should answer the questions provided in the text as a guide to their thinking. The only new aspects here are that three numbers are being added and that one of the numbers is expressed with a proper fraction numeral. The procedures for adding are no different from those developed for the addition of numbers expressed with mixed numerals only. You may wish to discuss the use of 10 as the common denominator. Get the pupils to explain why it is better to use 10 than 50, for example. When the pupils come to the last line of the page, remind them that the $2\frac{2}{10}$ may be remembered or recorded on scratch paper.

**Page 83:** Continue by adapting the procedures suggested for page 81. The pupils are to answer the questions in the text. They should note that the numeral for 2 wholes is put with the numerals in the ones' column. Be sure they can explain, in connection with Picture D and the computation beside it, where this 2 came from.

**Page 84:** Have pupils answer all the questions opposite Pictures E and F. Let someone supply the numeral that replaces *n* in the equation (opposite Picture F). Finally, have pupils reread the problem on page 82 to see if the question has been answered. Pupils should use Problem A in the TRY step on this page and Exercises B to G on page 85 to see how well they understand the addition of numbers expressed with mixed numerals.

**Page 85:** Beginning with Problem A on page 84, let pupils work through Exercise G. Tell them that if they have any difficulty, they may look at the worked-out example and compare their own work with it. Let pupils use scratch paper to write any information they feel they may not remember and to do any computing they feel necessary.

As the pupils work, observe their progress. Help those who seem to be having trouble. Afterward, discuss the examples briefly. Ask questions that will enable the pupils to make the following observations: the sum of the fractions may be expressed by a proper fraction numeral that need not be changed to a mixed numeral (Examples B and C); the number to be carried does not always need to be written above the ones' column (Example D); proper fraction numerals sometimes should be expressed in lowest terms (Example E); sometimes the replacement of the improper fraction numeral is a numeral for a whole number instead of a mixed numeral (Example G); sometimes it is not necessary to find a common denominator (Example E).

Assign Problems A and B and Exercises C to W in the DO step as written work. Remind pupils to use the vertical form when computing. Also remind them that sometimes they will not need to find a common denominator. Afterward, have the work put on the board for any exercises that caused general difficulty. The pupils who made many errors may, after the class discussion, try Exercises A to G (the second block). Those who still make many errors will need the help suggested in the section "Helping the slow learner."

#### Providing for the able pupil

The game "Fraction race," described in Activity 28, page 390, will give able pupils practice in adding mixed numbers.

#### Helping the slow learner

To help the slow learner, you will have to observe his work very closely to see what his specific difficulty is. If it is inability to find common denominators, review with him the lesson on pages 66-69, and let him use "Finding the common denominator" (Activity 25, page 388). If it is necessary to review changing fraction numerals to equivalents, Activity 22, page 385, will be helpful. Other activities he might use here are "Fraction match" (Activity 26, page 389) and "Make 1" (Activity 27, page 389).

## 86 Using arithmetic

Lesson Briefs for this lesson are on page 96.

Expanded Notes are not considered necessary for this lesson.

## 87-88 Looking back

Lesson Briefs for this lesson are on pages 97-98.

#### Overview

Unless considerable time is devoted to developing an understanding of how mixed numbers are expressed by equivalent forms, many pupils will resort to rules without understanding what they are doing.

#### Teaching the whole class

Individual fraction kits (see Activity 17, page 381) can be used to help pupils grasp the meaning underlying this lesson. You might write a numeral such as $2\frac{5}{8}$ on the chalkboard. Each pupil could then arrange some of the paper sections found in his kit to show how this number may be expressed. First he might

show two disks, each composed of 8 one-eighth pieces, along with 5 one-eighth pieces. He could then rearrange all these parts to show an improper fraction (in this case, $2\frac{1}{8}$) or another mixed number with an improper fraction element ($1\frac{13}{8}$). The numerals you write on the board, of course, must conform to the limitations of the fraction kits, unless there is some provision for pupils to make additional paper parts as needed.

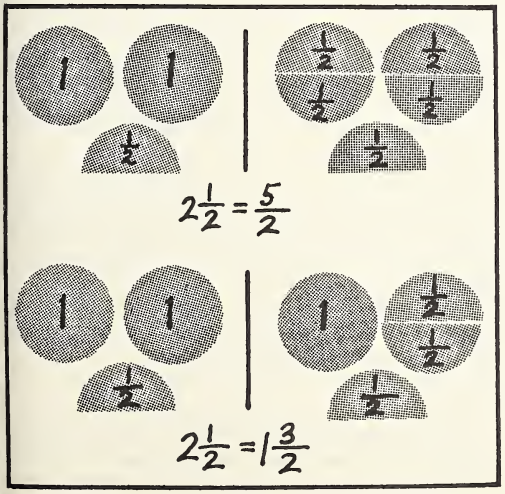
**Providing for the able pupil**

Draw rulers similar to the one on page 87 of the pupils' book, but with the inches marked off into tenths and sixteenths. (The scales $\frac{1}{8}$": $\frac{1}{16}$" and $\frac{1}{8}$": $\frac{1}{10}$" will be convenient to use.) Duplicate these drawings so that there are enough copies for all. These worksheets may be used in various ways. You might ask the pupils to choose a number of points on a ruler and show both the improper fraction numerals and the mixed numerals that name each of the points. Or you might ask questions such as: "Find $2\frac{3}{16}$ on your rulers. What improper fraction numeral also expresses this length?"

The game of "Fraction bingo," described on page 391 in Activity 29, may be used for these pupils to give them experience in replacing mixed numerals with improper fraction numerals and with other mixed numerals containing improper fraction numerals.

**Helping the slow learner**

Pupils who still cannot remember fractional equivalents for one whole should try the following activity.




Let them use fractional parts of paper disks to work out solutions to such examples as $1=?/2$, $1=?/3$, $1=?/4$, etc. (You could give them a list of equivalents through sixteenths, with numerators missing.)

Slow learners might also paste paper disks and fractional parts of paper disks on cardboard to show such equivalents as $2\frac{1}{2}=\frac{5}{2}$ or $2\frac{1}{2}=1\frac{3}{2}$.

The game of "Fraction bingo," described in Activity 29 on page 391, will provide slow learners with practice in replacing mixed numerals with equivalent improper fraction numerals or with mixed numerals like $2\frac{3}{2}$.

## 89-92 Learning how

Lesson Briefs for this lesson are on pages 99-101.

**Overview**

In this lesson the pupils learn how to subtract numbers expressed by proper fraction numerals. Pictures help pupils see that before they can tell exactly how much remains they must think of the original quantity and the quantity removed as divided into parts that are equal in size. They then see that the fractions must be expressed by numerals with a common denominator before one number can be subtracted from the other.

**Teaching the whole class**

**Page 89:** After you have started this lesson by having a pupil read the introductory statement aloud, review with the class what they already know about subtracting fractions. Remind pupils that in an earlier lesson they used sets of equivalent fraction numerals to compare fractions to see if one was larger or smaller than the other. Recall that they were able to tell how much larger or smaller one fraction was by replacing the fraction numerals with those having a common denominator. If necessary, pupils may refer to the lesson beginning on page 66.

When calling attention to the first problem on page 89 and its accompanying picture, be sure the pupils understand that $\frac{3}{4}$ of a yard of material is shown in the top scene. Two thirds of the original *whole* yard is to be used for place mats. The pupils can see the piece that will be left, but they have no way of knowing exactly what fraction of the whole yard it is. Then point out how the accompanying equation relates to the situation. Have someone explain each symbol in the equation.

Continue by asking for suggestions as to how to find what fraction of a whole yard will be left. By this time the pupils should realize that it is necessary to think of the original piece of material and the part to

be taken away as though both were marked off into the same sized pieces, which are fractions of one whole yard. Have pupils look at Picture B to see that a whole yard of material has been marked off into twelve equal parts, that $\frac{3}{4}$ yd. is equal to $\frac{9}{12}$ yd., and that $\frac{2}{3}$ yd. is equal to $\frac{8}{12}$ yd.

Now discuss what should be done with the numerals in the equation to make them correspond to the changes in the picture. In working with the fraction numerals, the pupils should see that they must find a common denominator. You might ask:

*What are the two fractions we want to express with a common denominator?* [*Write the numerals on the chalkboard.*] *We know that to replace a fraction numeral with an equivalent fraction numeral, we can multiply or divide both terms of the fraction numeral by the same number. Remember that we want a common denominator for both fraction numerals. Should we start working with the larger or the smaller denominator? Why?*

If necessary, refer to the work on pages 66-69 of the pupils' book for a review of the techniques of finding the common denominator.

Have the pupils look at the fraction numerals in the text to verify the replacements. Be sure they see that the equation can be rewritten with these new fraction numerals in place of $\frac{3}{4}$ and $\frac{2}{3}$.

**Page 90:** Next call attention to Picture C. The pupils should see that $\frac{8}{12}$, or $\frac{2}{3}$, of one yard of the material has been removed. They can see that $\frac{1}{12}$ of a whole yard is left. Have them read the text at the right carefully to find out that the same answer can be obtained by subtracting the numerators of the fraction numerals, now expressed as twelfths, and writing 12 as the denominator of the answer. Have them note how and where the answer is written.

You may want to make up a similar problem about a pie, and use disks on the flannel board. Let the pupils write the fraction numerals on their papers, make the necessary replacements, and do the computation.

When the problem in the THINK step has been read, have the pupils relate it to the equation and to what is happening in Picture A.

**Page 91:** Finish the problem by having the pupils answer the questions and supply the numerals that replace the screens. Be sure the pupils can explain why 8 is selected as the common denominator and how the fraction numerals are replaced with equivalent fraction numerals having the same denominator.

For the examples in the TRY step, discuss thoroughly the vertical computational arrangement presented in Example A. Then let the pupils write the example in the vertical form and replace the fraction numerals with others that have a common denominator, as shown in the text. Have them try Examples B to E (C to E are on the next page) and compare their work with that in the book after they have finished each example. In the discussion afterward, point out that in Example B they do not have to find a common denominator because the fraction numerals already have a common denominator.

**Page 92:** In discussing Examples D and E, point out that the fraction numerals in the answers should be reduced to lowest terms.

For the DO step, have the pupils write their work and answers for the exercises labeled A to T. Discuss the work afterward. Then, for those pupils who seem to be having difficulty, assign the second block of exercises, labeled A to L. Help these pupils if necessary.

**Providing for the able pupil**

All pupils will benefit by seeing concrete illustrations of subtraction of fractions. The able pupils can help out here especially by demonstrating on the flannel board. After discussion of the examples in the DO step on page 92, they can take turns illustrating those examples that seemed to cause trouble for some of the pupils. Such demonstrations and explanations by the abler pupils may help clear up difficulties for the slow pupils, and also will benefit those who explain and demonstrate.

For an example like $\frac{3}{4} - \frac{1}{3}$, an able pupil should proceed somewhat as follows: Put $\frac{3}{4}$ of a disk on the flannel board and then explain that he cannot yet remove $\frac{1}{3}$ of the disk. He should discuss finding a common denominator for $\frac{3}{4}$ and $\frac{1}{3}$ and then tell why he chooses 12. On the chalkboard he should show the class how he computes to replace $\frac{3}{4}$ and $\frac{1}{3}$ by fraction numerals with the same denominator. Then on the flannel board he should put $\frac{9}{12}$ of a disk to replace the $\frac{3}{4}$ of a disk. Next he should take away $\frac{4}{12}$, explaining that this is equivalent to taking away $\frac{1}{3}$. He can let a classmate who had trouble with the example count the remaining parts to find that there are $\frac{5}{12}$.

Able pupils will welcome a challenge to find a quick method for getting the answers to subtraction examples such as the following:

$\frac{2}{3} - \frac{1}{2} = [\frac{1}{6}]$ $\qquad$ $\frac{4}{5} - \frac{3}{4} = [\frac{1}{20}]$

$\frac{3}{4} - \frac{2}{3} = [\frac{1}{12}]$ $\qquad$ $\frac{5}{6} - \frac{4}{5} = [\frac{1}{30}]$

Put these and similar examples (without the answers) on the chalkboard. Ask the pupils to work them in the usual way and then try to discover, first,

similarities among the examples and, second, a quick method for getting the answers.

The pupils should be able to discover that the examples above are all alike in that the numerator of the fraction that is to be subtracted is one less than the numerator of the fraction from which it is to be subtracted; the denominator of the fraction to be subtracted is one less than the denominator of the fraction from which it is to be subtracted. Also in each case the numerator of the answer is 1. It should not take the pupils long to discover that they can subtract the numerators and multiply the denominators to get the answers. Since the denominators in each example are consecutive numbers, the lowest common denominator for each example will be the product of the denominators. (This is true because consecutive numbers have no factors in common except 1.) Let those who wish make up similar examples.

The game "Fraction draw" makes use of addition and subtraction of fractions. See Activity 30, page 392, for details.

**Helping the slow learner**

Activity 25, page 388, "Finding the common denominator," can be adapted to give the slow learners practice in subtracting fractions. The same pack of cards with two fraction numerals on each card may be used in this version. One pupil turns up a card, and all the pupils write a subtraction exercise in equation or vertical form, using the fraction numerals on the card. Whoever first succeeds in replacing these fraction numerals with others having a common denominator, subtracting, and finding the correct answer, wins that round. The pupils may keep track of the number of rounds they win. An able pupil might act as a checker.

## 93-97 Learning how

Lesson Briefs for this lesson are on pages 102-104.

**Teaching the whole class**

**Page 93:** After someone has read the problem aloud, discuss the equation. Then direct attention to Picture A, making sure the pupils see that it represents the original $9\frac{1}{2}$ lb. of fudge from which $4\frac{7}{8}$ lb. must be removed.

Now have the class read the first three lines opposite Picture B. Discuss with the class why the $\frac{1}{2}$ lb. of fudge (in strong color) is marked off into eighths of a pound and why it will be necessary to cut into the large piece of fudge to get more eighths. [A common denominator must be found. Also, it is necessary to "borrow" from the whole number.] Now have the class read the next four lines and study the computation. Ask someone to explain why 8 is selected as the common denominator.

**Page 94:** When the pupils have read the text opposite Picture C, have them study the picture and explain how it illustrates what has happened in the computation. [$9\frac{1}{8}$ has been replaced with $8\frac{1}{8}$.] The pupils should understand why it is now possible to take away seven eighths, both in the picture and in the computation.

Picture D illustrates what has happened in the fraction column in the computation. Have the pupils count the one eighths that are left after seven eighths are removed.

**Page 95:** Picture E shows what is happening in the last step of the computation. The pupils should be able to see that there were 8 whole pounds, from which 4 pounds were removed.

Ask a pupil to read aloud the problem in the THINK step. Then discuss the equation and the data shown in Picture A. The pupils should see immediately that the fraction numerals already have a common denominator, but that it is again necessary to "borrow" from the whole number, since $\frac{3}{4}$ cannot be subtracted from $\frac{1}{4}$.

Discuss Picture B and have the class answer the questions in the text opposite it. Ask someone to explain why the picture shows a whole stick being cut into fourths.

**Page 96:** For the work on this page, have the pupils use the pictures to help them answer the questions in the text. Be sure they see that the two fourths left can be thought of as one half.

**Page 97:** With the TRY step, tell the pupils to work each example without help from the book, then to verify their work by comparing it with that in the book. Discuss Example E, since this example contains a fraction in the minuend from which nothing is to be subtracted. Since nothing needs to be borrowed from the whole number, the numeral expressing it need not be changed.

When all pupils have finished the first block of work in the DO step, discuss any exercises that caused difficulty. Pupils who continue to have difficulty will need additional help, as suggested in the section "Helping the slow learner." After such work, you might have them do Exercises A to J in the second block.

**Providing for the able pupil**

If the playing board, fraction cards, and markers suggested for Activity 28, "Fraction race," have been made and have been used to practice the addi-

tion of numbers expressed by fraction numerals and mixed numerals, the same materials may be used and the game adapted to subtraction. (For a description of the materials, see page 390.)

In the subtraction version of the game, the pupils try to move their markers on the number line from 4 to 0 by subtracting the fraction on one card at a time. As usual, 7 cards are distributed to each of 2, 3, or 4 players. The first player draws a card from the top of the pile and tries to subtract the fraction shown on one of his cards from 4 so that the remainder equals one of the places marked on the line on the board. He should, of course, try to use a fraction card that results in his moving as far as possible toward the goal of zero. After moving his marker, he must put down the card bearing the fraction he subtracted, and explain his answer to the other players. Each player should have scratch paper and pencil available so that he can figure his own moves and verify those of his competitors. Anyone found to be in error loses his play and cannot move his marker that round. After moving his marker, the first player discards one card from his hand. If he cannot make a play, he simply discards.

Each player in turn does the same thing. On the second time around, the player must subtract one of his fractions from the number at the point where his marker is. For example, if he reached the $3\frac{5}{8}$ mark the first round, he must subtract from $3\frac{5}{8}$ on the second round, and so on. The round is over when a player's marker rests at 0. His final answer must be exactly 0. The round is also over when one player has put down all his cards. This will happen often. However, this does not mean that the first player to lay down all his cards wins. The winner of each round is the one who has reached the lowest number on the board. A game should consist of five rounds.

**Helping the slow learner**

Slow learners will have to be observed carefully to determine just where their individual weaknesses lie. If they are still having trouble in finding common denominators, refer them again to the lesson on pages 66-69. If necessary, refer them to pages 54-55 on replacing fraction numerals by equivalent fraction numerals. Also let them play the game "Finding the common denominator" (Activity 25, page 388).

If the difficulty lies in "borrowing" from the ones' column and replacing the mixed numeral by an equivalent form so that the fractions can be subtracted, refer the pupils to the lesson on pages 87 and 88. Let the pupils use objects at their desks or work with the flannel board wherever possible.





Lesson Briefs for this lesson are on page 105.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the able pupils and slow learners.

**Providing for the able pupil**

An activity that able pupils might enjoy involves adding and subtracting numbers expressed by mixed numerals to complete a magic square. See Activity 31, page 393, for complete details.

Problems involving the situations in which subtraction with fractions will be used may be cut from old textbooks and pasted on 3″ x 5″ cards, or the pupils may make up such problems. The cards may then be used as suggested in Activity 10, page 378.

**Helping the slow learner**

Slow learners will be helped if the teacher uses objects to demonstrate the situations with fractions on page 98.

Use whatever fractional parts you have—felt pieces on the flannel board or fraction pieces made of construction paper for the pupils' individual use. For Problem A, discuss what the equation should be. Then put down $\frac{3}{4}$ of a disk and $\frac{2}{3}$ of a disk and have the pupils pretend that these fractional amounts represent similar amounts of cups of sugar. Discuss the fact that this is a comparison situation, and that it is necessary to find out how much larger $\frac{3}{4}$ of a cup of sugar is than $\frac{2}{3}$ of a cup. Help the pupils see that both fractions must be changed to twelfths. Have fractional parts available so that this change can be made. Then, to facilitate comparison, suggest that the fractional parts that make up $\frac{9}{12}$ of a disk and the parts that make up $\frac{8}{12}$ of a disk be lined up in rows. (See the diagram below.)

Get the pupils to see that 8 of the 9 twelfths of a cup of sugar used for lemon drops match the 8 twelfths of a cup of sugar used for spice bars. These matched twelfths should be removed from the total

number of twelfths used for lemon drops. Then even the slow learners can see that $\frac{1}{12}$ of a cup *more* of sugar was used for lemon drops than for spice bars. Finish by having the pupils do the necessary computation.

### 99 Checking up; Keeping skillful

Lesson Briefs for these lessons are on pages 106-107.

Expanded Notes are not considered necessary for these lessons.

### 100-101 Using arithmetic

Lesson Briefs for this lesson ore on pages 107-108.

Expanded Notes are not considered necessary for this lesson.

### 102-103 Checking up

Lesson Briefs for this lesson ore on poges 109-111.

Expanded Notes are not considered necessary for this lesson.

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CHARTING THE COURSE

## Inequalities—solution sets

The relation of *equality* plays a dominant rôle in mathematics. At the same time, it must be recognized that the full significance of equality cannot be grasped unless relations of *inequality* are also recognized, understood, and appreciated. In the past, relations of inequality—for example, "3 is less than 5"—have been observed more or less incidentally in arithmetic programs. In a truly modern program such relations not only must be noticed, but also the symbolism and the language forms commonly used to denote them must be learned.

The teacher should know that an expression such as $6+w=10$ is called a sentence-*form*. In this sentence-form the letter $w$ is a placeholder for a numeral. When $w$ is replaced by a numeral, the result is a sentence. For example, if $w$ is replaced by the numeral 8, the sentence $6+8=10$ is obtained. This sentence states something that is false. If, however, $w$ is replaced by 4, the sentence $6+4=10$ states something that is true. Sentences that can be judged "true" or "false" are commonly called *statements*. A sentence-form such as $6+w=10$, which cannot be judged true or false until $w$ has been replaced by a numeral, is called a *sentence*, for short.

In a similar way, an expression such as $9 > k$, which is read "9 is greater than $k$," is called a sentence-form, or, for short, a sentence. When $k$ is replaced by the numeral 5, a true statement is made. When $k$ is replaced by 12, a false statement is made. These statements are examples of an "inequality," or "*not* equal," relation.

The set of numbers whose numerals, or names, make *true* statements when used as replacements in an equation or an inequality is called the *solution set*. It may consist of many numbers, of only a few, of only one, or of none. Up to this point the pupil has solved many equations. In each case, the solution set consisted of just one number. It is time that the pupil learned that the mathematical world is not so simple as this suggests. Many problems have solution sets that consist of more than one number. Pages 104-106 of the pupils' book introduce these ideas in an elementary way. In the later work of a modern mathematics program the pupil will encounter these ideas more and more often.

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### 104-106 Thinking straight

Lesson Briefs for this lesson are on poges 111-114.

**Teaching the whole class**

**Page 104:** As the pupils read the dialogue in Pictures A to D, have them notice especially the difference in the signs for *is greater than* and *is less than*. They

should see that the sign points to the smaller quantity. In reading from left to right, if the point is at the left, the sign is read "is less than." If the point is at the right, the sign is read "is greater than." These signs of inequality should be written on the board, along with their names, to help the pupils become familiar with them.

After the pupils have worked Examples B to H, have them read their answers aloud to the class. Point out that when the correct sign has been inserted, the statements are true even if they are read backward. For example, $13 + 10 > 9 + 12$ can be read 13 plus 10 is greater than 9 plus 12, or it can be read 12 plus 9 is less than 10 plus 13. Both of these statements are true if the signs are read correctly. Some of the pupils may be interested in making exercises like Exercises B to H.

**Page 105:** Beginning with Exercise J, the pupil is introduced to a sentence, a statement, and a solution set. The teacher should point out that a sentence like $x + 3 > 6$ is neither true nor false. There are many possible replacements for $x$, and, therefore, there are many possible statements. The teacher should list and discuss many of these statements and help the pupils find methods of deciding whether these statements are true or false. For example, from the sentence $k < 9$, you might form the following statements by replacing $k$ with numerals: $1 < 9$, $2 < 9$, $3 < 9$, $8 < 9$, $9 < 9$, $10 < 9$, $11 < 9$, $12 < 9$, etc. The teacher should point out that certain of these statements are true. The set of replacements for $k$ that make true statements determines the solution set. In future work in mathematics, the names of members of the solution set will be enclosed with braces. For example, the solution set of whole numbers for the sentence $x + 1 < 5$ will be written in this way: $\{0, 1, 2, 3\}$. For the sentence $x + 1 = 5$, the solution set will be written in this way: $\{4\}$. The braces show that the numerals enclosed are names of a set. If the teacher wishes, this method of indicating the solution set can be used at this time.

The concepts of sentence, statement, and solution set are important concepts in mathematics and are introduced at this time to provide readiness for future work.

Exercises J to P should be discussed in class. The teacher should emphasize the sentence, the statements formed by replacements, and the solution set. Point out that the only numbers considered in these exercises are whole numbers.

The exercises following P should be done individually by the pupils and discussed in class. The pupils should read their answers to help them become familiar with the language and symbols.

**Page 106:** On this page the pupil makes statements for sentences in which the replacements are names of fractions and mixed numbers. The pupils should see that for sentences like $5 > n + 4$ there is no end to the number of fractions in the solution set, because the solution set includes all of the proper fractions. A similar situation is true for the sentence $3 - d < 1$, where the solution set includes all mixed numbers between 2 and 3.

Exercises A to V provide further practice and should be worked individually by the pupils.

**Providing for the able pupil**

The able pupil could be shown that solution sets can be graphed on a number line. To graph a solution set, you draw a number line and show the part of this line that represents the numbers in the solution set. Consider the three examples shown below. Let the pupils observe that the three sentences are alike except for one symbol.

Example 1 Sentence: $x + 3 = 7$

Solution set: $\{4\}$

GRAPH:

Circle indicates that 4 is a member of solution set.

Example 2 Sentence: $x + 3 > 7$

Solution set: {all numbers greater than 4}

GRAPH:

( Shows that 4 is not in set

Wavy line indicates that all numbers greater than 4 are in solution set.

Example 3 Sentence: $x + 3 < 7$

Solution set: {all numbers less than 4}

GRAPH:

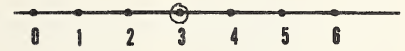
Wavy line indicates that all numbers less than 4 are in solution set.

) Shows that 4 is not in solution set

The able pupil should note that even though we cannot list all of the members of the solution set, we can show them on a graph. If we consider only the whole numbers, the graph would just have circled points and not wavy lines to show the numbers in the solution set.

It is of interest to note that if you put the above three solution sets on one number line, it would include all of the line. It should also be pointed out that there is no end to the numbers that can be put on this number line and included in the solution set for a sentence like $x + 3 > 7$.

The pupil's experience with graphing a solution set can lead him to discover another method of finding this solution set. For example, if the pupil needs to find the solution set for $x + 2 > 5$, he can first graph the solution set for $x + 2 = 5$, as shown below.




He should note that the solution set for $x + 2 > 5$ is either the part of the line to the right or the part to the left of 3, but not both. To determine which part of the line represents the solution set, he needs only to select a numeral, such as 5, and use it to replace $x$ to form a statement. If the statement is true, that part of the line including 5 represents the solution set. If the statement is false, the part of the line on the other side of 3 represents the solution set.

Introducing a graph of a solution set to the able pupil will help him understand the concept of a solution set and will provide background for further work with other graphs.

**Helping the slow learner**

The slow learner should be given many experiences in determining whether a statement is true or false. The teacher should write a series of sentences and ask the slow learner to make statements by replacing the letter with names of both whole numbers and fractions. The pupil should then determine which of the statements are true and which are false. This type of experience will give the pupil further work with the basic operations and use of the symbols in sentences and statements.

Examples of arithmetic statements with missing signs like those on pages 104-105 can be put on large pieces of cardboard, one to a sheet, and used for a group activity by the slow learners. Each pupil participating, except for the leader (the teacher or an able pupil), should provide himself with one 4″ x 6″ card or piece of paper. Instruct the pupils to draw a large "equals" sign on one side and the sign for "is less than" on the other side. Point out that by turning the card around, they will have the sign for "is greater than."

The leader holds up before the group a card with a statement on it. The group then vies to see who can first hold up his card to show the sign that should be inserted. (Warn the pupils to hold their cards at their sides, instead of in front, so that they can be sure which sign they are showing.) Whoever holds up the correct sign first must also be able to verify the statement.

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CHARTING THE COURSE

## The distributive law—mental computation

Some mathematical sentences have a solution set that consists of all the numbers used in arithmetic. For example, $5+n=n+5$ states something that is true, no matter what numeral replaces $n$. Some basic sentences are called "laws." Among the very important laws of mathematics is the one called "the distributive law." An example is: $13 \times (9 + 30) = (13 \times 9) + (13 \times 30)$. Briefly, this law asserts that the same number is named by the groups of symbols on both sides of the equals sign. Suppose two different numbers (for example, 9 and 30) are added, and the sum is multiplied by another number (for example, 13). The answer can also be found by multiplying the first number (9) by the last (13), then multiplying the second number (30) by the last (13), and finally adding the two products. There are variants of the law that replace addition by subtraction and multiplication by division. Examples of these are given on pages 108 and 109 of *Seeing Through Arithmetic 6.*

There are many uses of this fundamental law of mathematics. In elementary arithmetic, one of the main

uses is in mental computation. It is possible, for example, to find the product of 8 × 33 mentally. One way is to think of 33 as 30 + 3. Then both 30 and 3 are multiplied by 8. The results, 240 and 24, can easily be added "in your head," and the product is found to be 264.

In the *Seeing Through Arithmetic* program, attention has been given to mental computation in a number of places. Understandings of the principles underlying mental computation are important because they enable a person to "figure the answer out in his head" in situations where the use of paper and pencil is not necessary or is not convenient. In *Seeing Through Arithmetic 6,* the use of the distributive law as an aid in mental computation is suggested to the pupil on page 110.

One place where mental computation is useful is in the process commonly called "short division." In this process, which is used only with one-digit divisors, much of the work is done mentally and not recorded. In *Seeing Through Arithmetic 6,* this short cut is taught on page 111. One place where it is useful is in reducing the dividend and divisor in certain types of division examples, such as those considered later on page 137.

## 107-109 Thinking straight

Lesson Briefs for this lesson ore on pages 114-117.

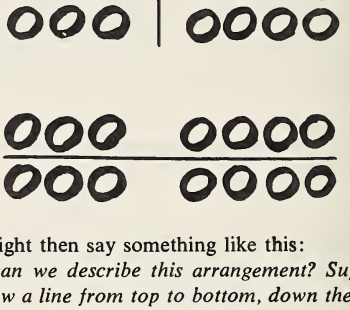
**Overview**

These pages present some aspects of the distributive law. One reason for presenting these ideas at this grade level is to provide a foundation for the study of mathematics. Since pupils will be unaware of this, one way of interesting them is to suggest that the ideas contained in this lesson will be useful later as short cuts in computation. The expression "distributive law" should not be used at this time.

**Teaching the whole class**

**Page 107:** Examples A to D and their related exercises A to G should be discussed individually. These exercises and examples have been written to show how symbols of grouping (parentheses) are used in mathematics. In Example A [13 × (9 + 30) = *d*], you may add the 9 and the 30 and then multiply to get 507. You may also multiply the 9 and the 30 separately by 13, and add the products to get 507. Bring out that multiplying the ones and tens of a two-figure numeral separately and then adding the products (as in Exercise D) is nothing new, but simply what everyone in the class has been doing in computational multiplication. Pupils should also understand that when using parentheses, it is not necessary to use the multiplication sign, since any numeral immediately to the left of the parentheses is understood to be a multiplier.

You may find it helpful to demonstrate some of these ideas, either with objects or a drawing on the chalkboard. It will be best to keep the quantities small, as in the arrangement pictured below.



You might then say something like this:

*How can we describe this arrangement? Suppose we draw a line from top to bottom, down the middle. On one side of the line we see two groups of 3, and on the other side, we see two groups of 4. We might say this picture shows two groups of 3 and two groups of 4. Can we describe it in another way? Suppose we erase this line and draw another one, but this time we will make it go from side to side. What is above the line?* [*A group of 3 and a group of 4*] *What is below the line? Can we say this picture shows two groups, each group being 3 plus 4? How much is 3 plus 4? How much is two groups of 3 plus 4? How much is 2 times 3 plus 2 times 4? If you multiply 2 by 3, do you get the same answer as when you multiply 3 by 2?*

When you feel pupils understand these ideas, assign Exercises H to O as written work.

**Page 108:** A discussion of the question in Exercise A, together with an examination of Example E, should bring out the idea that the equation requires both subtraction and multiplication for its solution. In Exercise B the pupils will discuss ways of rewriting this equation. They should observe that 11 is now written before the parentheses, which have replaced the multiplication sign. Bring out by discussion that the order of the numbers to be multiplied does not matter. Follow through on the suggestions made in Exercises C and D by having a pupil do the work at the chalkboard.

Assign Exercises F to M as individual written work.

Instruct pupils to read Exercise A of Block 2 silently. Ask for a volunteer to work the equation at the chalkboard and to answer the questions, with the assistance of other pupils if necessary. Discuss Exercise B and its related Example J (on page 109). Observe Example K, as instructed in Exercise C, and discuss how the 50 was obtained. Read Exercise D together, and guide discussion toward the observation that here 50 over 5 means 50 divided by 5. Stress that the answer in division is called the quotient.

**Page 109:** Suggest that it is possible to illustrate the steps developed in Examples I to K. Provide 50 sticks and ask for a volunteer to give the class a demonstration. 15 sticks (1 bundle of ten and 5 ones) can be combined with 35 sticks (3 bundles of ten and 5 ones) to make a total of 50 sticks (5 bundles of ten). 50 divided by 5 means, then, that the 50 sticks must be divided into groups of 5, to make 10 groups.

First read, then discuss Exercises E and F and their related Examples L and M to answer the questions in each. Here, again, make sure the pupils understand that both equations give the same information, but that they are written in different forms. Two pupils can illustrate this method of solving an equation. One pupil can divide 15 ones into groups of 5. The other pupil can divide 35 ones into groups of 5. The two pupils can then count the number of groups they have in all. Two pupils can also team up to work the equation at the chalkboard while the class watches and evaluates the process.

For Exercise G, have the pupils compare Example N with Example I to help them answer the question. Carry out the instruction of Exercise H by having a pupil write the equation in its second form on the chalkboard while the class watches and evaluates. To complete Exercise I, have the class compare Example O with Example K, and then discuss how the 32 in Example O was obtained. Let pupils take turns answering the questions in Exercises I to K. In discussing Exercise L and answering the question, try to determine if the ideas presented in this lesson have been clearly understood by the pupils. Use simple examples to clear up any misunderstandings that may still exist. Adjust the lesson to the needs and abilities of the class members.

Assign Exercises M to U as individual written work.

**Providing for the able pupil**

The able pupils should be given opportunities to work with equations (similar to those on page 107) that have three or more addends. For example, give them the equation $13 \times (9 + 7 + 30) = d$ and ask them to solve it in two ways. Provide them also with equations having three or more addends that are to be divided by the same number—for example, $(15 + 10 + 25) \div 5 = x$ — and ask them to show two ways of solving them.

Ask these pupils also to think of problem situations that could be solved by equations of the type studied in this lesson. Select the best ones and have them written or typed on 3″ x 5″ cards for use as suggested in Activity 10, page 378.

**Helping the slow learner**

If the slow learners have difficulty with the work on these pages, provide a series of equations with easy number facts in them. Begin with equations such as $4 \times (3 + 2) = n$, $(8 - 5) \times 6 = r$, $(6 + 8) \div 2 = c$, and $(12 - 6) \div 3 = p$. Have the pupils solve and illustrate these equations. Then discuss with them another way of writing and solving them.

Some pupils will also profit from seeing equations of the type described in this lesson related to problem situations. You might give them this problem: "Jack bought 15 apples for 12¢ each and then bought 14 more apples at this same price. How much did he pay in all?" Help them see that the equation could be $(15 \times \$.12) + (14 \times \$.12) = d$, but that another way to write it is $\$.12(15 + 14) = d$. Have them solve this equation in two ways. Next change the price of the apples, but keep the number of apples the same. This problem can be solved in the same two ways. Finally, present this problem: "Jack bought 15 apples at 10¢ and 14 apples at 8¢. How much did he pay in all?" Help the pupils see that there is only one way to solve this problem; that is, multiply \$.10 by 15 and \$.08 by 14 and add the results. Make up other similar problems and present them as suggested above.

## 110 Thinking straight

Lesson Briefs for this lesson are on pages 117-118.

**Teaching the whole class**

Activity 32 on page 394 provides practice in mental computation with multiplication. Also, use Activity 33 on page 394 for review of mental computation with addition.

No special materials need be supplied for mental calculating because each pupil should be encouraged to calculate without the use of a pencil whenever he can, as he does his day-by-day work in arithmetic.

**Providing for the able pupil**

Able pupils may like to try multiplying a two-figure number by a two-figure number without paper and pencil. As an example, ask them if they can think of a way to multiply 96 by 98 in their heads. If necessary, give them some clues. Tell them that 98 is almost 100 and that 100 × 96 equals 9600. Then, if you remind them that this is, of course, too much, some of the pupils may realize that the product, 9600, is two 96's too many, or 192 too much. Subtracting 192, they will get the correct answer, 9408.

Help the pupils find an easier way of figuring how much larger 9600 is than the correct answer. Point out that 96 is nearly 100. Instead of subtracting 192 from 9600, they can subtract 200 (two 100's). But then they have subtracted too much. 96 is 4 less than 100. There are two 96's; so 8 must be added to 9400. When the answer has been figured out by these two methods, pupils can write on the chalkboard the equations that show what has been done. For the first method, they will write $100(96) - 2(96) = 9408$; for the second method, $9600 - 2(100) + 2(4) = 9408$. Other two-figure numerals may be multiplied in this same way.

Able pupils can make themselves a slightly more difficult version of Activity 32 by using only two-figure numerals that end in 6, 7, 8, or 9, such as 37, 58, 69, 86, and so on. To make the activity more difficult still, the card-pack numerals from 5 to 9 only could be used.

**Helping the slow learner**

An easier version of Activity 32 can be used by the slow learner in which the two-figure numerals on the disk end in 1 to 5 and the numerals 1 to 5 only are used in the card pack.

## 111 Thinking straight

Lesson Briefs for this lesson are on poges 118-119.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the able pupils and the slow learners.

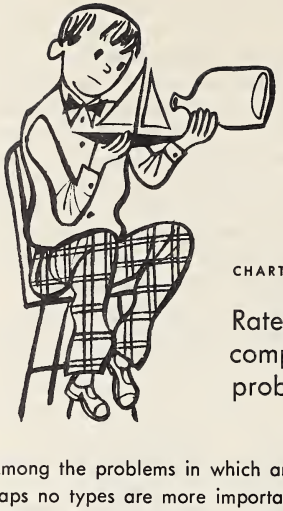
**Providing for the able pupil**

To get practice in mental addition and short division, the able pupils might like to play the game of "What's the number?" which is described in Activity 34, page 395.

**Helping the slow learner**

"Short way home" is a game that provides practice in short division with remainders. See Activity 35, described on page 395.





CHARTING THE COURSE

### Rate and comparison problem situations

Among the problems in which arithmetic is used, perhaps no types are more important than those that involve rate. For example, suppose 12 rosebushes cost \$28. At this rate, how much would 3 rosebushes cost? In this example a rate, which is 12 per 28, is given. The problem calls for finding the cost of 3 rosebushes at this known rate. The meaning of rate and problems in which the rate was to be expressed in symbols were introduced in *Seeing Through Arithmetic 5*. The meaning of rate is carefully retaught on pages 32-37 of *Seeing Through Arithmetic 6*. Pages 112-116 show pupils how to deal with situations in which the rate is known, but is to be expressed in different terms.

Traditionally, problems such as the example above have usually been taught as multiple-step problems. One common instructional procedure (the "method of unitary analysis") is to tell the pupil to proceed somewhat as follows: "First, find the cost of 1 rosebush. Divide 28 by 12. One rosebush would cost $\$2\frac{1}{3}$. To find the cost of 3 rosebushes, multiply $2\frac{1}{3}$ by 3. Then $3 \times \$2\frac{1}{3} = \$7$, which is the cost of 3 rosebushes."

Another procedure is to tell the pupil to compute as follows: "The rosebushes are priced at 12 for \$28. You want to know the cost of 3. Now, $12 \div 3 = 4$, so 3 is $\frac{1}{4}$ of 12. You must find $\frac{1}{4}$ of \$28. Then $\frac{1}{4} \times \$28 = \$7$, which is the cost of 3 rosebushes."

It should be noted that these instructional procedures are highly verbal and often consist principally of giving a set of rules. After solving many examples of this type in one of these ways, some pupils become able to deal with such problems when they arise. However, usually

pupils do not understand the basic principle involved or recognize that its generality extends far beyond problems of buying. It can hardly be overemphasized that the traditional presentation of these methods consists of a sequence of rules for short cuts. The apparently greater simplicity of such "explanations" is a consequence of their failure really to explain the procedure in a fundamental way. The truth of this statement is likely to be more apparent to pupils than to teachers who, through long experience with the short-cut explanation, have come to believe they understand it.

In the example above, and in others of the same general type, the idea of rate is basic. The rate is known and is expressed by writing a pair of numerals as a ratio. The problem requires this same rate to be expressed by another pair of numerals, and one of this pair is also known. The other member of the pair is not known, and must be found. In general, two computational "steps" are required. In one of them the process of division is used. In the other, the process of multiplication is used.

Reference again to the specific example above, and to pages 112-116 of *Seeing Through Arithmetic 6* will make this clear. The rate in the example is expressed by the ratio 12/28. In the new form, the cost term is at first unknown and may be temporarily represented by x. The number of rosebushes is 3; so the new ratio is $3/x$. Both ratios must express the same rate; so $12/28 = 3/x$. Here, as usual, several procedures may be used to find a numeral to replace $x$.

The general procedure is actually shorter than the "short cuts." Since $12/28 = 3/x$, the idea is merely to replace 12/28 by an equivalent ratio whose first term is 3. Both the first term 12 and the second term 28 must be divided by the same number. This number is found by dividing 12 by 3. When the result, 4, is used as a divisor of both 12 and 28, the equivalent ratio is 3/7, which shows that $x = 7$.

*Seeing Through Arithmetic 6* uses this general procedure and provides a full explanation of it so that children can understand. The authors believe that short cuts and verbal rules are not conducive to the development of understanding, and these have been avoided throughout the initial presentations.

Similar procedures are followed when the problem situation involves a comparison. The solution of problems of this type is explained in detail on pages 111-124 of *Seeing Through Arithmetic 6*. For example, suppose the problem is as follows: "Don and Pat have a small printing press. One day Don printed 48 cards on the press, and Pat printed 16 cards. Don printed how many times as many cards as Pat that day?" (See page 122.) In this example the ratio 48/16 expresses the comparison of the number Don printed with the number Pat printed. This ratio must be replaced by another in which the second term is 1, and the first term is unknown. This ratio may be written as $x/1$. As in the rate problems, a divisor must be found to reduce the ratio 48/16. The divisor is 16, and when both 48 and 16 are divided by 16, the new ratio is 3/1. This means that Don printed 3 cards for each 1 that Pat printed. This fact would often be stated in the form: "Don printed 3 times as many cards as Pat." The solution consists in expressing the same comparison with different numerals. In this particular example it may appear that all one needs to do is divide 48 by 16 and get 3, then state the answer as indicated above. This is a case where appearances are definitely deceiving. In the first place, the answer is found by dividing *both* 48 and 16 by 16, and not merely 48 by 16. In the second place, this is a special case in which one of the terms is a multiple of the other, and the answer is easily obtained.

It is important to note that the methods outlined above serve to unify or bring under one fundamental procedure several types of problems that usually have been taught separately. Although the details differ from example to example, the basic ideas are the same throughout. Moreover, these same basic ideas will be essential when problems requiring the multiplication or division of fractions are taught. The methods used in *Seeing Through Arithmetic 6* are chosen not because they may seem novel or different to those familiar with traditional arithmetic instruction. On the contrary they are chosen and presented solely because they are fundamental and in spite of their novelty at this level.

The introduction of these fundamental ideas at the points where they are first needed will keep arithmetic from becoming merely a collection of short cuts learned by drill methods.

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## 112-116 Exploring problems

Lesson Briefs for this lesson are on pages 120-123.

**Overview**

The concept of rate may be applied to many kinds of problems in the arithmetic of everyday life. Hence,

the writing of ratios to express rates and then using these ratios to solve problems is a very useful mathematical procedure. All too frequently pupils have been given little help in solving rate problems. Usually they have been taught to treat these problems as multiple-step problems. Rarely have they been given a procedure that applies to all rate situations.

It should be noted that in all rate problems, two ratios, both expressing the same rate, are involved. Of the four terms in these two ratios, a replacement for one term must be found by computation. Examination of pages 112-116 in the pupils' book will show how equal ratios are written to describe a problem situation and how the computation is performed to complete one of the ratios. Pupils have already had experience in writing ratios to express rates, in finding the missing numeral in a pair of equal ratios, and in writing the proper equation. (Sometimes equations of the type used in the problem situations on pages 112-116 are referred to as "proportions.")

Throughout the teaching emphasize that it does not matter which numeral becomes the first term in the first ratio. However, once the first ratio has been established, the relationship determined by the position of the terms must be maintained when the second ratio is written.

**Teaching the whole class**

**Page 112:** Before proceeding with the work on this page, you may wish to review briefly the changing of ratios to equivalent forms. Write a ratio such as 3/4 on the board. Ask the pupils to think of this ratio as expressing a rate and to give examples of such a rate (3¢ for 4 lollipops, for example). Then write the equation $3/4=6/n$ on the board. Get someone to explain what $6/n$ is [a ratio equal to the ratio 3/4, but with one numeral missing]. Let someone explain how to find the missing numeral. Then interpret the completed equation in terms of the original problem. Thus, $3/4=6/8$ can be interpreted as meaning that, at the rate of 3¢ for 4 lollipops, 6¢ would buy 8 lollipops (or the rate of 3¢ per 4 lollipops is the same rate as 6¢ per 8 lollipops).

Now direct attention to the problem and Picture A. First get the pupils to state the rate. Then have them explain what 12 and 28 stand for in the ratio 12/28. Get them to see that they must find another ratio to express this rate and that 3 is a term of this new ratio. Point out that the problem asks them to find the cost when the number of rosebushes is fewer than that expressed in the original ratio. Discuss why the cost will be less than the cost expressed in the original ratio.

Direct attention to the text opposite Picture B. Get pupils to see that in the new ratio, the numeral for the cost (or missing term) is written *below* the 3, which represents the number of rosebushes. (Pupils should understand that the rate could have been expressed by the ratio 28/12, but that then the new equal ratio would have to be $x/3$.) Emphasize that when the numeral that replaces $x$ is found, it will be a numeral that will make $3/x$ equal to 12/28.

**Page 113:** Direct attention to Pictures C and D, and let pupils take turns relating the pictures to the text. In the equation, discuss the fact that since the 3 can be found by dividing 12 by 4, the 28 must also be divided by 4. Thus the numeral to replace $x$ is 7. The pupils should interpret this 7 as $7 and realize that the rate of 12 rosebushes for $28 is the same rate as 3 rosebushes for $7. In other words, the cost of 3 rosebushes is $7.

A pupil may demonstrate the solution of this problem with play money and objects. (Use play money in the same denominations as those pictured in the pupils' book.) You can also ask a pupil to find the cost of 6 rosebushes at the rate of 12 bushes for $28. He should divide the objects into 2 groups of 6 bushes. Then he should separate the money into two equal groups (each containing 2 five-dollar bills and 4 ones) to find that 6 bushes would cost $14.

The problem in the second SEE step at the bottom of the page also involves a rate. Note that the number of bulbs is greater than that expressed in the original ratio. Have a pupil relate Picture A to the problem and to the ratio 230/25.

**Page 114:** Discuss the new ratio with the pupils. Since the numeral for the cost is written as the first term in the first ratio, a numeral for cost must *again* be used as the first term of the equivalent ratio. The 150, which represents the number of bulbs, must be written below the line.

Get the pupils to explain why the equation $230/25 = m/150$ is correct for this problem. [The two ratios express the same rate.]

Next discuss the solution of the equation in the COMPUTE step. Point out that since the problem asks for the cost of 150 bulbs, these 150 bulbs can be thought of as 6 groups of 25, since the cost of 25 bulbs is known. We know that the rate for 150 bulbs is the same as the rate for 25 bulbs. We know that 150 bulbs is 6 times as many bulbs as 25 bulbs; so the cost of 150 bulbs must be 6 times as much as the cost of 25 bulbs. We know that the cost of 25 bulbs is $2.30; so the cost of 150 bulbs must be 6 times $2.30, or $13.80.

**Page 115**: Discuss Problems A to D with the class. Stress the fact that the arrangement of numerals representing the data in the second ratio must correspond with the arrangement of numerals representing the data in the first ratio. Let pupils take turns explaining each equation and finding the missing numeral.

In Problem C, be sure pupils are not misled into supposing that they are dividing the 1785 in the first term by the 15 in the second term. Because the second term of the second ratio is 1, it follows that 15 can be divided by 15 to get 1. Then 1785 must also be divided by 15. This 15, however, does *not* stand for the 15 lb.

Assign Problems E to P as written work. (Problems I to P will be found on page 116.) Provide answers and discuss any troublesome problems.

**Page 116**: Complete the discussion of problems that gave trouble.

The exercises in the "Keeping skillful" section should be assigned as a separate lesson.

**Providing for the able pupil**

Able pupils might enjoy using a toy or gift catalog that has many items priced at different rates: 4 for so much, 6 for so much, etc. They could make up a list of items they would like to order, with prices, and then alter the number of items or the amount to be spent. (6 fish hooks for 75¢; 12 fish hooks for how much? 2 fish hooks for how much?) ($6/75=12/n$; $6/75=2/r$)

Able pupils will also enjoy playing "Spin-a-rate," Activity 20, page 384.

**Helping the slow learner**

Let slow learners use objects to work out problems. Encourage them to make up problems from their own experiences of buying, traveling, etc. Some pupils may write the ratios that express the rates on the board, and the others may discuss the ratios and give solutions. Problems should be kept simple, such as "12 pencils cost 50¢. At this rate, what should I pay for 6 pencils?" or "35¢ buys 2 boxes of cereal. At this rate, how much will 6 boxes cost?"

The game of "Ratio," Activity 15, page 380, will be helpful for slow learners.

## 117-121 Exploring problems

Lesson Briefs for this lesson are on pages 124-127.

**Overview**

In this lesson the pupils review how to make equations for problems involving comparisons and how to solve these equations. If the pupils can solve problems involving rate, they will have no trouble with the solution of problems involving comparison. You may wish to use play money and other objects to illustrate comparison situations.

**Teaching the whole class**

**Page 117**: You may wish to review the idea of comparison and its expression by a ratio. If necessary, refer to pages 38-42 of the pupils' book. Then write a ratio such as 4/8 on the board. Ask the pupils to state some comparisons, using 4 and 8. Then have them turn to the problem on page 117.

First of all the class must see why this is a comparison situation (the amounts two boys spend for bus fare are being compared). Let them then relate the coins in Picture A to the boys' fares for one day. Also have them relate the numerals in the ratio to the amounts mentioned in the problem and to the coins in Picture A. Be sure the pupils read the ratio as "50 *to* 30." They should note that the numeral representing Philip's fare is above the line, and the numeral for Paul's is below the line.

The pupils should realize that, to solve the problem, they need another ratio equal to the ratio 50/30. Get them to see that the problem gives one numeral of this new ratio. [The total amount of Philip's fare, $2.50, is given.] They should be able to explain why the numeral representing this amount can be used in the first term of the new ratio. Point out that it is convenient to think of $2.50 as 250 cents. Help the pupils see that since they do not know the total amount of Paul's fare, they can write $a$ in the second term of the new ratio. Then discuss the equation at the bottom of the page. Let a pupil explain the position of 250 and $a$ in the second ratio.

**Page 118**: Ask the pupils to read the text and look at Picture C. Then have them explain how 250 can be obtained from the 50 [by multiplying 50 by 5].

Next have the pupils recall that for every 50¢ Philip spent, Paul spent 30¢. Point out that Picture C shows 30¢ matched with each 50¢. Five groups of 30¢ are needed. When the computation is considered, the pupils can see that 50 can be multiplied by 5 to get 250. So 30 must also be multiplied by 5. Thus the numeral 150 replaces $a$. Remind the pupils that in this case 150 means 150 cents, or $1.50. They may wish to verify this computation by referring to Picture C.

If play money is available, ask a pupil to show the comparison of 50¢ to 30¢. Then tell him to put down enough additional half dollars to make $2.50. Since Paul spends 30¢ every time Philip spends 50¢, tell the pupil to match 30¢ with each half dollar. It will

then be clear that $1.50 is the total amount Paul will have spent. Have the pupils note that 5 half dollars and 5 amounts of 30¢ were put down. Ask them to multiply 50¢ by 5 and 30¢ by 5 to see that the total amounts can be obtained this way, too. Then ask someone to write the equation on the board and explain what must be done to find the numeral to replace *a*.

The new problem at the bottom of the page may be handled in much the same way. Get the pupils to see that the problem states a comparison between the number of squares Ann wove and the number May wove. Then help them understand that they are to express this same relationship with smaller numbers, one of which is not given in the problem. Discuss the ratio 16/12 and relate it to Picture A.

**Page 119:** Discuss the equal ratios and the equation as you did for the previous problem. Help the pupils see why Ann's 12 squares should be thought of as divided into groups of 3. When they know the number of groups of 3 squares, they can divide May's 16 squares into the same number of groups for matching purposes and find how many are in each group. Let someone work this out with objects representing the squares. Also discuss how to arrive at the same answer by computation.

**Page 120:** When you discuss the problem at the top of the page, be sure the pupils understand what groups are being compared, and that the second ratio expresses the same relationship as the first ratio. Point out that since the second ratio involves larger amounts than the first ratio, multiplication is suggested. You may wish to demonstrate the solution of this problem.

When you discuss Problem B and its equation, help the pupils see why they divide to find the replacement for *d*.

Treat Problems C to F as you did the preceding ones. Point out that for each problem two equations have been written to describe the situation. Remind the pupils that it does not matter which quantity is expressed in the first term of the first ratio, provided the relationship in the second ratio is the same. Have each pair of equations solved at the chalkboard and explained by the pupils.

**Page 121:** Assign Problems G to N as written work. Tell the pupils they may write their ratio equations whichever way they wish, but the data must be consistent within the equation. The same kinds of things must correspond to the first term of each ratio. Then the same kinds of things will be represented in the second term of each ratio. Afterward, in discussion, have the pupils tell why they multiplied or divided in each equation.

The "Keeping skillful" exercises should be treated as a separate lesson and assigned when there is time.

#### Providing for the able pupil

Sixth graders have had some experience in working with information obtained from diagrams. (See the lesson on page 100.) But so far they have not used information obtained from tables. The following activity will give the able pupils this type of experience, as well as practice in solving ratio equations.

Write on the board a ratio equation in which the equal ratio is indicated by a pair of letters. For example, you might write $150/30=a/n$. Beside the equation write a table, such as the following:

| $a$ | $n$ |
|---|---|
| 50 | ? |
| 300 | ? |
| 450 | ? |
| 150 | ? |

Tell the pupils to find $n$ in the equation when $a$ is replaced by each of the four values in the table. Be careful to set up values for $a$ that are either multiples or factors of the number represented by the opposite numeral in the ratio equation.

Additional equations and tables might be devised by the pupils for exchange and solution. Be sure to examine the equations and tables to see that the solutions do not require fraction numerals.

For another activity, the able pupils may make up problems involving comparisons to be solved by the slow learners. See the details below under "Helping the slow learner."

#### Helping the slow learner

Pupils in the sixth grade are usually interested in making collections, earning money, etc. Any such activities can be the basis for problems involving comparisons created by the able pupils. The problems may be used in several ways:

(1) You may type them on 3″ x 5″ cards, to be used by the slow learners in the way suggested in Activity 10, page 378. This method will enable you to remove any problems that require fraction numerals in the solution.

(2) The problems may be illustrated to show comparisons between two groups of objects. On the top half of a sheet of paper have the pupils illustrate the original ratio for each problem; on the lower half have them illustrate the equal, completed ratio. On each page the pupils should write the problem and the equation, first with a letter in the proper place and then with the

proper numeral inserted. Such drawings may be displayed by the pupils in individual notebooks or on the bulletin board.

## 122-124 Exploring problems

Lesson Briefs for this lesson are on pages 127-129.

### Overview

Using the expression "times as many as" in a problem is really a special way of speaking about a situation. The comparison between two groups is made by expressing what the relationship is when one of the groups consists of only 1 object. Traditionally, this type of problem has been solved directly by multiplication or division—a procedure that is really a short cut made possible because the second term of one of the ratios is 1. (See the discussion under "Comments" in the Lesson Briefs, pages 127-128, and "Charting the Course," pages 322-323.)

This lesson shows the pupils how to solve such problems by writing an equation of two ratios in which one term of one ratio is 1. The pupils learn that the equation is solved by exactly the same procedures as were used for solving equations in preceding lessons.

### Teaching the whole class

Sixth-grade pupils have had many opportunities to hear and to use such expressions as "two times as many as," "four times as many as," "twice as much as," and so on. Some pupils will have a clear idea of what such expressions mean. Others will not. So, before the work on page 122 is begun, you may wish to discuss these expressions. Have available two kinds of objects (black buttons and white buttons, for example). On a table place 1 white button and 2 black buttons. Ask a pupil to tell what ratio expresses the comparison of black buttons to white buttons [2 to 1]. Write this ratio on the chalkboard [2/1]. Then explain to the pupils that the statement "There are two times as many black buttons as white buttons" expresses the same comparison. Next have a pupil use buttons to show the meaning of the ratio 3 to 1. Ask him to state this same comparison, using the expression "times as many as." Get him to say, "There are three times as many black buttons as white buttons." Then ask another pupil to show "4 times as many black buttons as white buttons." Ask him to write the ratio that shows the comparison and to read it [4/1 and 4 to 1]. Point out that the comparison is always with 1. Conclude by asking the class to give examples of other things that can be expressed by using the phrase "times as many as." For example, "There are 3 times as many blue balloons as green balloons" and "There are four times as many yellow pencils as red pencils."

**Page 122:** To begin this lesson, have the pupils read Problem A. Have them note the now familiar expression "times as many as." Then go on somewhat as follows:

*Does the problem tell how many times as many cards Don printed as Pat printed? But we have seen that the expression "times as many as" is used when we make a comparison based on 1, haven't we? Jane, read the first statement below the problem. When we know how many cards Don printed for each card Pat printed, we will know how many times as many cards Don printed.*

*Look at the picture. Don printed how many cards? They have been divided into 16 equal groups, each group to match one of Pat's cards. How many cards of Don's are there to each one of Pat's? Can we say that we have a comparison of 3 to 1? Now let's see how we can solve this problem by computation. The problem tells us that Don printed 48 cards to Pat's 16. Can you state this comparison by using a ratio?* [*Discuss the suggested ratio.*]

Then have the pupils find the ratio in the book. Suggest that an equal ratio, in which one of the terms is 1, is needed. Remind the pupils that this second ratio will show how many cards Don printed for each 1 of Pat's. Discuss the fact that the numeral representing Don's cards (48) is used in the first term of the first ratio (48/16). Ask the pupils why Don's cards should be represented in the first term of the equal ratio. Since they do not know the number, they may use the letter $x$. Point out that the numeral representing Pat's cards is used in the second term of the first ratio. Since the new ratio should show a comparison with 1 of Pat's cards, the numeral 1 should be used in the second term of the new ratio. Have the pupils find this ratio at the top of Column 2.

Next have the pupils study the equation. Discuss how the 1 in the new ratio can be found [by dividing 16 by 16]. Ask why 48 should also be divided by 16. Have someone divide 48 by 16 to find the numeral 3 that replaces $x$. Then remind the pupils that there is another way to describe the situation when the ratio is 3 to 1. Have them find this statement in the book (3 times as many as).

**Page 123:** Problem B on this page differs from Problem A. Have the pupils read Problem B and note that they are told how many times as many chicks as ducklings Sue has. The pupils should know that "2

times as many as" means there is a relationship of 2 to 1. Have the pupils note in the equation for the problem that the ratio they know [2/1] is written first. Since the numeral representing Sue's chicks is used in the first term of the first ratio, a numeral representing Sue's chicks must also be used in the first term of the second ratio. Since the number of chicks Sue has is unknown, the letter $m$ is written to hold a place for the numeral representing this number. The numeral 19, representing Sue's ducklings, is written in the second term. Let the pupils look at the equation and determine the relationship between 1 and 19 in the two ratios. [1 can be multiplied by 19 to get 19.] Ask what 2 should be multiplied by [19]. 38 can then replace the letter $m$. So the ratio 38 to 19 means that Sue had 38 chicks to 19 ducklings.

Problem C can be discussed in the same way. Be sure the pupils understand why the known ratio is 5/1 and why the ratio $70/d$ is equal to 5/1. Keep emphasizing the relative position of the numerals in the terms.

Discuss the fact that before the pupils can find $d$, they must first understand that 5 can be multiplied by 14 to get 70. Since the 5 in the first ratio is multiplied by 14 to get 70, 1 is multiplied by 14 to get the replacement for $d$.

Discuss Problems D and E in much the same way as you discussed Problems A, B, and C. Since there are no pictures, the class will benefit from using objects to illustrate the problem situations.

Able pupils undoubtedly will see that the answer to Problem D can be found by directly multiplying 13 by 2. However, this short cut, although useful and traditional, does obscure the basic meaning of the comparison in the problem—that Paul has 2 pigs to each of Jack's pigs. Insist, therefore, that all pupils write an equation of two ratios for each "times as many as" problem. Point out, too, that the expression, "twice as many as" means the same as "2 times as many as." Let the pupils give other examples of situations where they can say "twice as many as." The answer to Problem E, of course, may also be found without the use of ratios, but the short cut again hinders the understanding of the comparison.

**Page 124:** Continue in the same way with the first two problems. For Problems H and I, have the pupils explain the positions of the numerals and letters in the equations they use. Point out in Problem H that the expression "twice as much as" means the same as "2 times as much as." Let the pupils give other examples of situations where they can say "twice as much as."

Assign the remaining problems as written work. Supply answers and discuss troublesome problems.

Handle the "Keeping skillful" exercises as a separate lesson.

#### Providing for the able pupil

If your able pupils made up original problems using comparison situations, as suggested in "Providing for the able pupil" in the preceding lesson, let them now make up problems using "times as many as" situations. These problems can be put on cards and included with the collection described in Activity 10, page 378, or they can be illustrated, as suggested in the preceding lesson in the section for the slow learner (page 326).

#### Helping the slow learner

Let the slow learners work with objects to demonstrate the three types of "times as many as" problem situations (as illustrated by Problems A, B, and C on pages 122-123).

For this activity have two boxes, each with a different kind of object in it. One box might have 50 corks in it, the other 65 buttons. Let pupils work in pairs. The type of situation illustrated by Problem B on page 123 of the text ($2/1=m/19$) is probably the easiest to start with. You might say:

*Jane, take 19 corks. Bob, you are to pick out 2 times as many buttons as corks. What ratio will show 2 times as many?* [*2/1*] *For each cork that Jane has, how many buttons should you draw?* [*2*]. [*Have the pupils do the drawing and matching.*] *Now, Bob, how many buttons do you have? Henry, write the equation on the board and find the numeral to replace* m.

For another situation you can say:

*Don, take 12 corks. Peggy, take 24 buttons. Peggy you have how many times as many buttons as Don has corks?*

Remind Peggy that she is to find how many buttons she has to each 1 of Don's corks. Have Don lay out his 12 corks. Then Peggy can divide her buttons equally into 12 groups to find the number in each group that matches each 1 of Don's. Also have the pupils solve this situation by use of an equation. For a third situation, try the following:

*Bill, draw 20 corks. If I say that you should have 5 times as many corks as John has buttons, how many buttons should John draw? What is the ratio that shows 5 times as many?* [*5/1*]

Tell Bill to arrange his corks so that he has 5 in a group. Then John can match 1 button to each group of 5 to find the number he can have. Let the pupils also solve the problem by an equation.

## 125 Thinking straight

Lesson Briefs for this lesson ore on page 130.

Expanded Notes are not considered necessary for this lesson.

CHARTING THE COURSE

### Per cent

A brief introduction to the idea of per cent is now given in some fifth-grade programs. In Grade 6 the thorough study of per cent should begin. There is justification for this because per cents are frequently found in various reading materials that the boys and girls use in Grade 6.

If pupils have had a good introduction to comparisons by ratios, the idea of per cent is a relatively easy extension, since per cent refers to any comparison in which the second term of the ratio is arbitrarily taken as 100. The only new thing to be learned is that using 100 as the second term is a convenient thing to do, and along with it goes the term *per cent* and the symbol %.

In *Seeing Through Arithmetic 6* the idea of per cent is introduced by a comparison situation in which ratios for a number of different comparisons are made, and these various ratios are used to study the relative success of six schools in raising money. This situation helps the pupil to grasp at the beginning the main advantage of using the same basis—that is, 100—for all these comparisons.

In most arithmetic programs three "cases," or kinds of per cent problems, are taught. In *Seeing Through Arithmetic 6,* problems of the three types are introduced and solved, but no effort is made to have the pupil learn to sort out the cases as such. All three "cases" are approached in the *same* fundamental way, which is to set up an equation of ratios, or a proportion. In every case, both terms of one ratio are known, and one term of the other ratio is also known. The second term of one of the ratios is always 100. These features serve to unify the approach by way of proportion, and reduce the differences among the cases to matters of little or no concern.

In the traditional "Case 1" (finding a per cent of a number), the second term of the known ratio is 100, and the second term of the other ratio is known. Its first term is to be found. The equation takes the general form $x/a = b/100$, where $a$ and $b$ in a specific situation hold places for numerals representing known numbers, and $x$ holds a place for the unknown numeral. The so-called "percentage" is to be found. In the traditional "Case 2" (finding what per cent one number is of another number), the equation takes the form $x/100 = a/b$. In this case the so-called "rate per cent" is to be found. In the traditional "Case 3" (finding a number when a per cent of it is known), the equation takes the form $a/x = b/100$. The so-called "base" is to be found. These terms *percentage, rate per cent,* and *base* are not needed and are not used in this book. The term *per cent,* of course, is used.

After a thorough and understandable introduction to this basic method, the pupil is taught the usual brief or "abstract" ways of formulating per cent problems. These ways include such brief sentences as "42 is 84% of ▒," "65% of 20 is ▒," and "50 is ▒% of 20." In the past, when pupils have had difficulty with per cent, it has often been because they were prematurely required to work with these shortened expressions without any real instruction in the basic ideas. Furthermore, children have been led to think of per cents as numbers instead of as pairs of numbers that involve a relation, and which express a comparison.

## 126-129 Moving forward

Lesson Briefs for this lesson are on pages 131-134.

**Overview**

In this lesson the pupils learn that per cent is a way to express a comparison by a ratio with 100 in its second term. The meaning of per cents more than 100% is also developed.

**Teaching the whole class**

**Page 126:** Have the pupils read Exercise A, and then discuss the piles of silver dollars in the picture. Be sure the pupils understand that the numerals near the

top of each pile indicate the number of silver dollars in that pile. They should understand that, in each case, the pile on the right shows how many dollars the school wanted to raise, and the pile on the left indicates how many dollars it actually did raise.

In discussing Exercise B, bring out that the Greenfield School wanted to raise $50 and that it did so. Discuss the other amounts in terms of what each school wanted to raise and how much each school actually did raise. Be sure it is clear to the pupils that for each school they are comparing the amount raised with the goal.

Then have the pupils try to decide which school did best. Bring out that if the goal for all schools had been the same, it would be easy to determine which school did best. Suggest that if the comparisons were expressed by ratios, it might be possible to find which school did best.

Next direct attention to Exercise C and have the pupils make ratios to show the comparisons for the Baker and Weston schools. After the two ratios have been made, the class can see that it is still difficult to tell which of these schools did better. Suggest, then, that if they find equal ratios with the same numeral in the second term, they perhaps can tell which school did better. Have them work through Exercise D. They will find that when 50 is chosen as the common second term, they can see that 46/50, or 46 to 50, is better than 40/50, or 40 to 50.

**Page 127:** Exercises E to G show how the common second term of 50 is used to determine whether the Burns School or the Fisher School did better. Discuss how each of the ratios in Column A is obtained and what each means. Explain how the ratios for the Burns and Fisher schools show that each of these schools raised more than its goal. Then let a pupil explain how an equal ratio for the Fisher School can be obtained with 50 in the second term. They should note that the ratio for the Burns School already is expressed with 50 in the second term. The pupils should realize that raising 74 dollars for a desired goal of 50 dollars is better than raising 63 dollars for a desired goal of 50 dollars.

By the time Exercise H is reached, pupils will begin to see the importance of identical second terms when comparisons such as these are made.

In Exercise I, tell the pupils that many years ago it became customary to use 100 as the common second term in making comparisons by ratio. Be sure they understand that each ratio in Column B is equal to the opposite one in Column A. Then discuss how the ratios 23/25 and 18/20 were changed to 92/100 and 90/100, respectively. The pupils will now be able to see that the Baker School did better than the Cook School. Bring out that the Baker School did not actually raise $92 out of a desired goal of $100, but that this equal ratio (92/100) expresses the same relationship as the comparison of 23 with 25.

As directed by Exercise J, have the pupils take turns explaining how each ratio in Column B was obtained.

**Page 128:** Discuss why the list should begin with the school that did the poorest if it ends with the Fisher School. Then you may assign Exercise K as written work. Direct attention next to Exercise L. You may wish to have the pupils take turns reading the expressions in Column C, page 127, so they will become familiar with them. Be sure they understand that these expressions state the same comparisons as the ratios in Column B. Point out that when we say that the Greenfield School achieved 100% of its goal, we mean that the school raised all the money it had set out to raise.

Exercise M gives meaning to the per cent sign. Be sure the pupils understand that the per cents in Column D are a short way of writing the per cents in Column C and that they express the same comparisons as the ratios in Column B and the equal ratios in Column A. Explain why the failure of a school to reach its goal can be expressed as a per cent less than 100% and why the achievement of a school that exceeded its goal can be expressed as a per cent greater than 100%.

Assign Exercises A to P as written work. Exercises A to L that follow may also be written and should help the pupils understand that a per cent is just one of many different equal ratios.

Exercises A to F (Exercise F is on page 129) bring out the idea that a per cent usually does not give any information as to the quantities involved. Be sure the pupils do not think that 100, when used in the second term of a ratio, means 100 items. They should get the idea that while a per cent does not tell numbers in the comparison, it does indicate a *relationship*.

**Page 129:** Discuss the table that is to be used with Exercises G and H. Have the pupils write the ratios that compare the goals and amounts saved, the equal ratios with 100 in the second term, and the per cents. Then, to stress that the per cent expressions make it easy to compare what the boys accomplished, have the pupils, as directed in Exercise H, write the names of the boys in the order that shows how well they did.

Exercises A to P will test the pupils' understanding of phrases indicating comparison.

The "Keeping skillful" exercises should be used as a separate lesson, as time permits.

**Providing for the able pupil**

Able pupils may be asked to collect, from newspapers and magazines, statements involving per cents. An effective bulletin-board display can be made from the collected statements.

**Helping the slow learner**

These pupils may profit from seeing per cent applied to some of their own activities. A bowling game can furnish opportunities for them to express the number of pins left standing as a per cent of all the pins. If equipment for a bowling game is not available, 10 other objects can be placed on a table in the usual bowling formation. The pupils can take turns rolling a small rubber ball to see how many they can knock down. For example, 4 out of 10 might be left standing. The pupil can see that this comparison can be expressed as the ratio 40/100, or as 40%.

## 130-133 Exploring problems; Keeping skillful

Lesson Briefs for these lessons are on pages 134-137.

Expanded Notes are not considered necessary for these lessons.

## 134-135 Exploring problems

Lesson Briefs for this lesson are on pages 137-138.

Expanded Notes are not considered necessary for this lesson, except for suggestions for the able pupil and the slow learner.

**Providing for the able pupil**

A graph device can be made that will enable pupils to compute answers for all types of per cent problems. See the description on page 396, Activity 36.

For Exercises A to L on page 135, the able pupils might make problems from which each of the sentences can be derived. These problems could be written or typed on cards, and the slow pupils might use them as an additional set of practice problems.

**Helping the slow learner**

The problems mentioned above may be assigned as extra practice work for the slow learners.

An activity with bundles of 10 sticks can give the slow learners a visual background for understanding the language of per cent. Place 4 bundles of sticks opposite 10 bundles of sticks. Then give directions and ask questions such as the following, and let the pupils manipulate the sticks when they need to do so.

*Make a ratio that compares the 4 bundles of sticks with the 10 bundles of sticks.* [*4/10*] *Now make an equal ratio with 100 in the second term.* [*40/100*] *Express the comparison as a per cent.* [*40%*] *4 is what per cent of 10?* [*40%*] *What is 40% of 10?* [*4*] *4 is 40% of what number?* [*10*]

CHARTING THE COURSE

### Using multiples of 10

Skill in computation depends in part upon certain short cuts that are possible when multiples of 10 are involved. These short cuts are often presented merely as "rules," which are applied with little understanding. However, when pupils are well grounded in the principles of the system of numeration, they can understand the generalization that the numeral for a product contains a zero for every factor of 10 the number has. This generalization enables the pupil to count the number of zeros in an expression such as 400 × 7000 and know at once that the product can be written as 28 followed by five zeros, or 2,800,000.

In similar fashion, pupils can quickly determine the number of zeros in the quotient of one multiple of 10 divided by another multiple of 10. This ability is useful, for example, in estimating the quotient in a long division example. These ideas are presented in *Seeing Through Arithmetic 6* on pages 136-137, and are followed by a set of problems and numerous practice exercises through which skill may be acquired.

## 136-137 Thinking straight

Lesson Briefs for this lesson are on pages 139-140.

Expanded Notes, except for suggestions for the able pupils, are not considered necessary for this lesson.

**Providing for the able pupil**

To become proficient in working with multiples of 10, the able pupil may be given a series of exercises, such as the following:

"If 240 divided by 6 equals 40, what does 240 divided by 60 equal?"

"If 24 divided by 6 equals 4, what does 240 divided by 6 equal?"

From these he should be able to observe and generalize what happens when the dividend increases (or decreases) and the divisor remains the same, or when the divisor increases (or decreases) and the dividend remains the same.

Other short cuts the pupils may try include methods of multiplying any number by 50 or by 25. A short cut for multiplying a number by 50 is to multiply it by 100 first (annex two zeros to the numeral), then divide the result by 2. To multiply a number by 25, multiply by 100 (annex two zeros to the numeral), then divide the result by 4. The pupils may try out these methods with any number. Be sure the pupils understand why these short cuts work.

Put the following example of a short cut for multiplying by 11 on a card (or cards). Let the pupil study it for a while. Then ask him to do several exercises of his own. He will soon see that when he adds 1 times the number to 10 times the number in regular multiplication, he is adding the same numbers as he uses in the short cut.

| Regular method | Short cut | |
|---|---|---|
| 49325 | 493250 | |
| 11 | 0+5=5, | write 5 |
| 49325 | 5+2=7, | write 7 |
| 493250 | 2+3=5, | write 5 |
| 542575 | 3+9=12, | write 2, carry 1 |
| | 1+9+4=14, | write 4, carry 1 |
| | 1+4=5, | write 5 |

Put on a card (or cards) an example like the one below to show a short cut to multiply any "teen" number by another "teen" number: First find the sum of either number and the ones of the other number. Multiply this sum by 10. Then to this result add the product of the ones in both numbers. For example, to find the product of 18 and 13, follow the steps shown below.

(1) $10(13+8)=210$

(2) $210+(8\times3)=210+24=234$

Pupils should be able to give a mathematical explanation for this short cut. The teacher may have the pupil do the multiplication as follows:

| | |
|---|---|
| | 13 |
| | ×18 |
| (a) $8\times3\rightarrow$ | 24 |
| (b) $8\times10$ | 80 |
| (c) $10\times3$ | 30 |
| (d) $10\times10$ | 100 |
| Add | 234 |

The pupil should see that Steps *b*, *c*, and *d* are combined $[10(8+3+10)]$ and that the result is 210. Then the result of Step *a* $[8\times3]$ is added to 210.

Pupils who are thoroughly familiar with the basic principles of our numeral system (base 10) may begin to get acquainted with other number bases (for example, base 5). The digits of a base 5 numeral system are 0, 1, 2, 3, and 4. The pupils could illustrate counting from 1 to 15 in base 5 with this table:

| Base | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 5 | 1 | 2 | 3 | 4 | 10 | 11 | 12 | 13 | 14 | 20 | 21 | 22 | 23 | 24 | 30 |

The pupils should be able to explain that the numeral 23 with base 5 is equal to the numeral 13 with base 10 because 23 represents 2 fives and 3 ones. When the pupils understand how to count with base 5, they may make up addition and multiplication tables as follows:

Addition Table

| + | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 10 |
| 2 | 2 | 3 | 4 | 10 | 11 |
| 3 | 3 | 4 | 10 | 11 | 12 |
| 4 | 4 | 10 | 11 | 12 | 13 |

Multiplication Table

| × | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 |
| 2 | 0 | 2 | 4 | 11 | 13 |
| 3 | 0 | 3 | 11 | 14 | 22 |
| 4 | 0 | 4 | 13 | 22 | 31 |

The teacher should not use this base 5 material unless she is fairly familiar with it.

## 138-139 Using arithmetic; Keeping skillful

Lesson Briefs for these lessons are on pages 141-142.

Expanded Notes are not considered necessary for these lessons, except for suggestions for the able pupil.

**Providing for the able pupil**

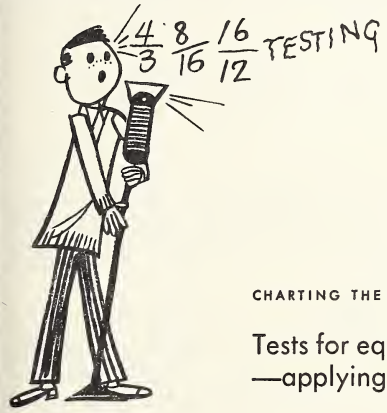
The problems in this lesson may awaken interest in certain scientific areas. Pupils who are interested should be encouraged to read science books and learn more about these subjects. Boys particularly may be interested in what happens when planes break the sonic barrier. Most of us have heard the noise, and some may have seen and felt the results (broken windows, etc.). Suggest to the able pupils that they find out what causes this and perhaps make a diagram to show what happens to the sound waves.

The word "Mach," which is used to designate the ratio of a given speed to the speed of sound, should be a familiar word to boys who read scientific magazines. They may wish to find out more about this word (how it originated, how twice the speed of sound is indicated by "Mach 2," etc.) and tell the class about it.

## 140 Thinking straight

Lesson Briefs for this lesson are on page 143.

Expanded Notes are not considered necessary for this lesson.




CHARTING THE COURSE

### Tests for equal ratios —applying the test

Statements that two ratios are equal occur frequently in mathematics, and they play a fundamental rôle. Up to this point in the *Seeing Through Arithmetic* program one ratio has been replaced by another ratio by using the same number as a multiplier (or divisor) of the numbers represented in the two terms. For example, the ratio 8/7 is replaced by the ratio 72/63 by using 9 as a multiplier. Conversely, the ratio 72/63 is replaced by the ratio 8/7 by using 9 as a divisor. The ratios 8/7 and 72/63 are equal. The basis for this, up to this point, has been quite informal. On page 141 of *Seeing Through Arithmetic 6* a formal basis for the equality of ratios is presented.

The basis is, actually, one that should be familiar to teachers if expressed verbally in the form: Two ratios are equal if (and only if) the "cross-products" are equal. Here the expression *cross-product* has a visual basis, because, in general form, if $a/b=c/d$ then $ad=bc$.

This condition—that the cross-products be equal—is admittedly formal, or "mechanical." At the same time it is the essence of the mathematical *definition* of equality for the case of two ratios. It is, therefore, of great theoretical importance, and its historical origins go back more than two thousand years to the Greek mathematician Eudoxus. At the same time, the equality of cross-products is of unquestioned practical importance. Through the application of this definition (which is called the "ratio test" in the pupils' book) it is easy to convert the statement that two *ratios* are equal to an equivalent statement that two products are equal. From the latter form one can proceed with further work in cases that are awkward to handle by the informal methods available up to this point. For example, in the proportion $5/6=n/13$, it is not easy to discover a multiplier (or divisor) to change the terms of 5/6 so the second term is 13, so that the first term would then be the correct replacement for $n$. However, an equivalent statement is $6n=5\times13$, or $6n=65$, and from this $n=10\frac{5}{6}$ can be found. When this method is used, the multiplier, which is $2\frac{1}{6}$, never has to be explicitly found or used.

An introduction to the ratio test and its use is given on pages 141-146 of *Seeing Through Arithmetic 6*. The use of this method has not been customary in Grade 6. It is introduced here not for the sake of producing something new or different, or of bringing down material from higher grades because the arithmetic program needs to be strengthened. On the contrary, it is introduced because it is fundamental to the understanding of arithmetic at this level. It does strengthen the *Seeing Through Arithmetic* program at a point where other programs are weak, but that is a consequence of the decision to teach fundamental ideas meaningfully when they are needed, rather than to follow a policy of bypassing them by giving rules that represent short cuts to more basic principles.

## 141-142 Thinking straight; Keeping skillful

Lesson Briefs for these lessons are on pages 144-145.

**Overview**
The pupils learn how to apply a test (called the "ratio test") to determine the equality of ratios. In using the ratio test, the diagonally opposite terms are multiplied and the products compared. If the products are the same, the ratios are equal.

**Teaching the whole class**
**Page 141:** Have two pupils represent the cartoon characters and read the dialogue. Point out that Picture A illustrates the method the pupils have been using to find a ratio that is equal to a given ratio. Be sure they understand that when both numbers represented by the terms of a ratio are multiplied or divided by the same number, the new ratio expresses the same relationship as the old. Pictures B to F tell how to use a test that shows whether ratios are equal or not.

To create interest and to strengthen the learning of this concept, place several pairs of ratios on the chalkboard (for example, 15/21 and 45/63; 27/18 and 135/90; 60/72 and 105/126; 124/275 and 362/550). Ask for volunteers to work at the board to determine, by means of the ratio test, if these pairs are equal. Let the class observe the work at the board, ask questions, and discuss the method. Ask other pupils to place between each pair of ratios the sign that will make a true statement. Emphasize that a true statement can be made about unequal ratios by putting the sign $\neq$, which means "is not equal to," between them.

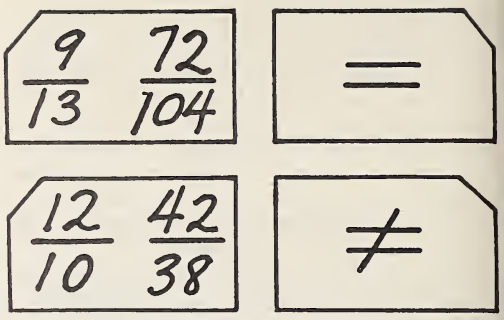
**Page 142:** After the preliminary work on page 141, the pupils should be able to follow the instructions in Exercises A to C and to explain each result with understanding.

Assign Exercises D to S as written work. To determine which pupils need special assistance, verify each pupil's computation. Some able pupil might volunteer or be asked to be a "teacher" in helping a classmate who is having difficulty.

The "Keeping skillful" blocks may be assigned as written work as time permits.

**Providing for the able pupil**
The project suggested below serves two purposes. The able pupil learns about ratios by preparing practice cards for the use of the slow learner, and the slow learner is provided with material for practice in applying the ratio test.

Have the able pupils write ratios on 3″ x 5″ cards (one pair to a card). Both equal and unequal pairs




should be prepared, and the ratios should be sufficiently difficult to make the use of the ratio test desirable.

A pair of ratios without a sign should be placed on one side of a card, and the sign that makes a true statement should be placed on the reverse side, so that the slow learners may verify their work. Have each pupil make four cards. Then have them exchange cards and verify one another's work.

**Helping the slow learner**
The cards described above may be used for practice by these pupils. Have the able pupils work with them to help them understand the procedure in using the ratio test.

## 143-145 Moving forward

Lesson Briefs for this lesson are on pages 146-148.

**Overview**
This lesson gives the pupils a way to solve ratio equations whose corresponding terms involve a relationship in which the numbers do not have common factors. There will be many applications of these ideas in later work.

**Teaching the whole class**
**Page 143:** Example A shows the different ways that an equation may be written when applying the ratio test. Too often, in arithmetic, it has been assumed that there is only one way to arrive at the correct answer. These different ways of writing an equation illustrate two kinds of commutative situations. One is that the factors of a product are commutative. The other is that the members or "sides" of an equation are commutative. Pupils do not need to learn the term "commutative," but they should understand and make use of the principles involved.

Place several pairs of equal ratios on the chalkboard (for example, 2/3=4/6; 4/5=8/10; 9/12=3/4; 10/12=5/6). Let volunteers go to the board

and see in how many ways they can write the result of applying the ratio test. The pupils who are not sure of the concept being presented will learn as they hear the equations discussed. Leave the ratio pairs on the board to be referred to later.

Example B shows a simpler way to write an equation for Example A. Have the pupils compare $3n=18\times2$ with the first equation in green in Example A. They should note that $3\times n$ has been replaced by $3n$. Explain that when a letter is used, the multiplication sign may be omitted. Remind the pupils that the multiplication sign can also be omitted when parentheses are used.

Have a pupil rewrite the four equations in green in Example A in this simpler form. Let the class decide whether the simplified form gives the same information. [$3n=18\times2$, $3n=2\times18$, $18\times2=3n$, $2\times18=3n$] Explain that writing the numeral first and then the letter is conventional in mathematics, but the reverse is also correct.

Next ask the pupils what multiplication has been done in the first equation in brown in Example B. Then go on somewhat like this:

*The equation says that 3*n *or 3 times the number that replaces* n, *must equal 36. What do you suppose 1*n *equals? Can you find the number by dividing 36 by 3? Look at the next equation in brown. Does 36 over 3 mean 36 divided by 3? Divide 36 by 3. If* n *is replaced with 12 in the equation 3*n*= 36, does it make a true statement? If 12 is put in the place of* n *in the original ratio equation in Example A, has a true statement been made? Is the equation solved?*

Example C, then, goes a step further by showing that the replacement for $n$ can also be found without actually finding the product of $18\times2$. Be sure the pupils see that 18 and 3 are divided by the same number, 3, and then crossed out, and that they understand the multiplication that follows. Then have them replace $n$ with 12 in Example A to see if a true statement results.

To illustrate the importance of the ratio test in solving equations, the original four ratio equations placed on the chalkboard can be referred to again. Use the previously taught method to determine if the statement $2/3=4/6$ is true. The pupils will probably say that both 2 and 3 have been multiplied by 2. Apply this test to the other three equations; then try to apply it to $18/3=n/2$. The pupils will see that 3 would have to be multiplied or divided by some number to get 2, but since they have not yet had the necessary work with fractions, they will be puzzled as to how to proceed. Some able pupil might see that both 3 and 18 could be multiplied by ⅔ or divided by 1½, but the class should be ready to accept the fact that the ratio test method is easier and can be used to solve any equation.

**Page 144:** In discussing Example D, some pupils may suggest other ways of dividing and crossing out. Allow them to solve the equation in other ways.

Since it has been stressed that only one number above the line should be divided, some pupil may question the fact that both 16 and 30 in Example E have been divided. It will be necessary to compare the complete solution shown in black with the work shown by the equation in red. Here, again, the pupils can be allowed to use different ways of computing. It should be stressed that any number of divisions may be done without writing out each new equation.

**Page 145:** The discussion and work that have already taken place should have prepared the pupils for the individual work at the top of this page. In the case of the slow pupils, you should be satisfied with only a few of the many ways to do Exercises A to F of Block 1. The fact that there is more than one way to arrive at a solution should encourage the slow learners to use all of their ability to achieve some success. Provide answers for both blocks of exercises so that the pupils can verify their work.

**Providing for the able pupil**

Able pupils can be given further practice in multiplying a series of factors by using different arrangements. A little guidance will enable them to make their own combinations, such as $2\times3\times4=4\times3\times2$ and $3\times7\times9\times5=9\times7\times5\times3$. Challenge them to use as many arrangements as possible, with the aim of developing skill in arranging factors in such a way that computation is simplified.

"Ratio roundup," Activity 37 on page 397, can be used by the able pupils to develop skill in recognizing equal ratios by applying the ratio test.

**Helping the slow learner**

The slow pupil needs ample time to complete his work. Give him not only opportunity to watch and listen to class presentations, but also to give some of his own. Simple pairs of ratios can be set up for the slow pupil to enable him to participate in the activity "Ratio roundup" mentioned above.

## 145-146 Using arithmetic

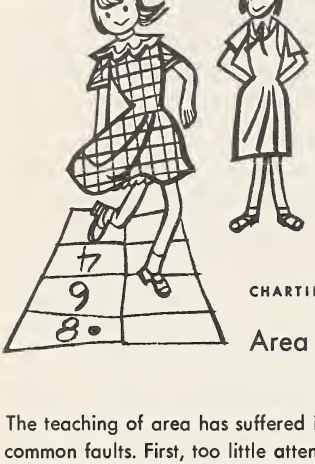
Lesson Briefs for this lesson are on pages 148-150.

Expanded Notes are not considered necessary for this lesson.

## 147 Checking up; Keeping skillful

Lesson Briefs for these lessons are on pages 150-151.

Expanded Notes are not considered necessary for these lessons.




CHARTING THE COURSE

## Area

The teaching of area has suffered in the past from two common faults. First, too little attention has been given to the concept of area. The tendency has been to proceed immediately to the computation of areas. Second, the computation of areas has depended too much on brief verbal rules that state a short-cut procedure. The fundamental reason the short-cut rules work is rarely made really clear. *Seeing Through Arithmetic 6* overcomes these weaknesses in prior instruction by providing a really new and distinctive introduction to area.

The idea or concept of area was developed in *Seeing Through Arithmetic 5* on pages 8-11. At that time the emphasis was upon the meaning, and areas were found by counting units of area and estimating. The computation of area was not introduced in these pages. Instructional materials for this important application of arithmetic were provided on pages 240-251 of Book 5. After some introductory teaching, the computation of area is retaught and extended on pages 152-160 of *Seeing Through Arithmetic 6*.

The approach to area through the rectangle and square is customary, since the area of other polygons can be found by transforming them into rectangular shapes. However, the method of computing area developed in *Seeing Through Arithmetic 6* is not the customary rule-giving one. The method used is an application of the concept of rate, which is basic in this situation. The familiar short-cut rule for the area of a rectangle, "Area equals length times width," produces the correct number, but the reason for this cannot be made clear without reference to a rate.

In the case of the rectangle, the rate is the number of units of area that can be accommodated on one of the sides of the rectangle. For example, if the length of a rectangle is 5 inches, then a row of inch squares can be accommodated on the longer side, and there will be 5 of them. The rate, then, is 5 square inches per 1 row. The other dimension (the width) determines the number of rows of inch squares that can be placed in the rectangle. If the width is 3 inches, there will be 3 rows.

The total number of square inches is at first not known, but temporarily it may be represented by $n$. Then the rate may also be represented by the ratio $n/3$, and so $5/1=n/3$. The use of the ratio test produces the equation $1n=3\times5$, from which we find the area is 15.

From this basic approach, it follows that as a short cut the area can be found by multiplying the number of square units in one row by the number of rows. So by abbreviation one arrives at the brief verbal rule: "The area of a rectangle is equal to the length times the width."

Teachers who have not themselves learned to recognize and use rates may consider the above explanation difficult to understand. The *Seeing Through Arithmetic* program developed the rate concept earlier because it is basic to a great many applications of arithmetic. The computation of area is only one of these. Although pupils may quickly satisfy themselves that the short-cut rule for computing area produces the right answer, it does not follow that they understand it. In conformity with their fundamental principles, the authors of *Seeing Through Arithmetic 6* have sought first to develop understanding and have postponed the presentation of the short-cut rule to a later grade.

## 148-149 Thinking straight

Lesson Briefs for this lesson are on pages 152-153.

**Teaching the whole class**

Before presenting this lesson, you may wish to prepare patterns, or "templates," for use in drawing

shapes on the chalkboard. Sometimes pupils have hobbies that make use of wood working tools. These pupils, with the aid of their fathers, can provide a set of these patterns. They can be made of scraps of ¼″ plywood with the following specifications: square, 8″ x 8″; rectangle, 9″ x 6″; parallelogram, 9″ x 6″; circle, 8″ in diameter; triangle, base of 9″ and altitude of 4″; another triangle, base of 8″ and altitude of 6″; trapezoid, a base of 9″ and an altitude of 6″; and a rhombus whose sides are 8″.

**Page 148:** Read and discuss Exercise A, bringing out the observation that each of the polygons has four straight sides. The pupils might be interested in knowing that the word *polygon* means "many angles." Let them give their opinions as to why these shapes, then, are called polygons. Have Exercise B read. Then let a volunteer use the patterns to place on the chalkboard drawings of the shapes that have four right angles. Another volunteer may draw the parallelogram.

Let the pupils take turns answering the questions in Exercise C, referring to the drawings on the board to verify their explanations. A review of the meaning of the word *perpendicular* may be necessary.

Have Exercise D read and let pupils discuss it. Summarize the thinking of the group to bring out that the opposite sides of the polygons are equal in length because the opposite sides are parallel. This generalization will lead the discussion into the first statement of Exercise E.

Here, again, the pupils will be interested in knowing that "gram" in the word *parallelogram* can mean "a stroke in writing" or "a line." This can open the way for discussion of why the rectangle is a special kind of parallelogram. The discussion will bring out that the opposite sides of a rectangle are parallel and its sides are perpendicular to each other, thereby forming right angles. They can conclude that Polygons A and B are both rectangles.

Let pupils take turns answering the questions in Exercise F, and guide the discussion so the pupils understand that the square is special because, although it is a parallelogram, all sides are equal and all angles right angles. Consequently Polygon B is a square.

Have Exercise G read. It may be necessary to review how to find the perimeter of a plane figure. To increase the pupils' interest in the derivation of words, they can be told that each part of the word *perimeter* has a definite meaning that should help them to remember how to find the perimeter of a figure. The "peri" means "around" and the "meter" comes from a word that means "measure." Ask questions concerning Polygons D and E to help the class observe the length of $r$ and $s$ in Polygon D, and the length of $z$ and $x$ in Polygon E. Then have pupils answer the questions and explain their answers.

In discussing Exercise H, have the pupils justify their comments.

**Page 149:** Let pupils take turns answering the questions and explaining their decisions for Exercises I and J. The words *base* and *altitude,* that are being discussed, can be placed on the chalkboard drawings so that pupils' attention can be directed to the point of discussion.

A reproduction of Polygon F and Polygon G can be placed on the chalkboard before the discussion of Exercise K. (A music staff liner guided by a ruler can make the work of reproducing these only a matter of minutes.) Have the exercise read and then ask pupils to imagine that the squares drawn on the board also represent square feet. Have the pupils work from the illustrations on the board to show that the areas of the two parallelograms are not equal and that Polygon F has the greater area. Exercise L should also be discussed and the conclusions verified by using the illustrations on the board.

Let pupils explain Exercises M and N by using the drawings on the board. They may count the number of squares in each polygon to verify the unequal areas, and they may compute the length of the perimeters to verify that they are equal. It may be difficult for pupils to realize that the slanted side of Polygon G is the same length as the side of Polygon F. This should be verified by measuring.

For Exercises O to S, place drawings of shapes A to I on the chalkboard. The size of the parallelograms can be varied by moving the patterns up, down, or across to lengthen sides. Let pupils take turns answering the questions in the exercises.

Use the drawings that were placed on the board for Exercises O to S for a quick review of this lesson. Ask a pupil to place an S in all squares, another to place a P in all parallelograms, another to place an R in all rectangles, another to place a T in all triangles, and another to place an L in all polygons. As each pupil completes his task, let the class determine if the decisions are correct. From this, the pupils should be able to make such generalizations as, "All rectangles are parallelograms, but not all parallelograms are rectangles."

Another procedure for enriching the learning of the class would be to describe certain shapes and ask pupils to find them on the board and give reasons

for their selection. Tell them that some of the descriptions will not fit the shapes; so they must be careful. Other examples may be added to the list below, if desired.

1. Find a rectangle that is not a square.
2. Find a square that is not a rectangle. (Impossible)
3. Find a parallelogram that is a square.
4. Find a parallelogram that is neither a square nor a rectangle.
5. Find a square that is not a parallelogram. (Impossible)
6. Find a four-sided polygon with no sides parallel. (Some pupil may suggest that one can be drawn.)
7. Find a parallelogram that has all sides equal.
8. Find a parallelogram that has no sides equal. (Impossible)
9. Find a four-sided polygon with one pair of parallel sides.
10. Find a parallelogram that has all sides equal, but is not a square. (Some pupil may suggest that one can be drawn.)

**Providing for the able pupil**

The able pupil can make some generalizations about bases and altitudes of triangles by cutting out a triangle and placing it in various positions. He can, then, generalize that any of the three sides may be used as a base and that the triangle has three altitudes. The same procedure may be followed with a parallelogram, in which he can find the two bases and two altitudes.

Have these pupils look up the meaning of the words *quadrilateral, pentagon, hexagon, octagon,* and *decagon.* Encourage them to report the meanings to the class and to illustrate with chalkboard drawings of these polygons.

**Helping the slow learner**

Different shaped parallelograms (cut from squared paper) having the same base length, but with different altitude lengths, can provide visual materials for pupils to compare areas and to see the relationship of the altitudes to the areas when the bases are the same.

## 150-151 Moving forward

Lesson Briefs for this lesson are on pages 153-154.

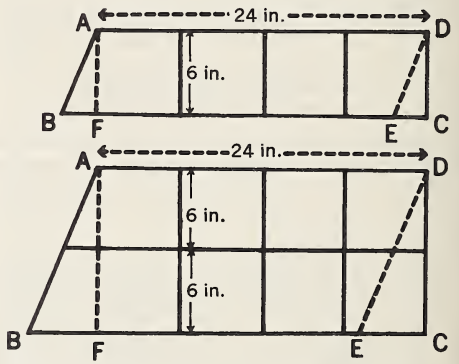
**Overview**

Pupils use what they know about areas of rectangles to explore areas of other parallelograms. Without computing the areas, the pupils count the number of square units. They use rectangles and parallelograms that have bases and altitudes the same length. They discover that all parallelograms with a base and an altitude the same length as the base and the altitude of a rectangle have the same area as the rectangle.

**Teaching the whole class**

**Page 150:** Large rectangles, similar to those in Pictures A and C, can be prepared ahead of time and marked off in square units that are large enough to be seen by the whole class. Or, if you prefer, you might make the teaching aid that is described below.

Start with trapezoids like those pictured below in which CD is perpendicular to AD, and BC is parallel to AD. Fold on DE parallel to AB and fold on AF perpendicular to AD, as the dotted lines show. Unfold DE (the corner). You now have the rectangle ADCF to begin the demonstration. When you want to use the polygon as a parallelogram, fold on DE and unfold on AF, to get Parallelogram ADEB.




**Page 151:** Have the pupils answer Exercises A, B, and C and relate them to Pictures A and B. Also show the class the large demonstration rectangle marked off in four square units and ask if anyone can suggest a way to make the rectangle into a parallelogram that is not a rectangle. Some pupil may suggest cutting off a triangular piece from one end (ABF) and moving it to the other end. Another pupil may suggest unfolding the triangular piece (DCE) at one end and unfolding the triangular piece (ABF) at the other end. The pupils can have sets of 1-inch squares and can place them over the picture in their books or on 4″ x 1″ rectangles and parallelograms drawn on paper. Sheets of 1-inch squares can easily be duplicated and each pupil can cut out a set for himself. Or, many can be made at one time by cutting a number of sheets simultaneously

on an office paper cutter. Follow the procedure described above for Exercises D, E, and F, this time using a demonstration rectangle marked off in two rows of four square units.

As the pupils read Exercise G, let them demonstrate with the large rectangle that the base line is still the same length, even though a piece has been transferred from one end of the rectangle to the other. Since the cut is on a slant starting at the end point of the base, the length of the base line is not disturbed. When the piece removed is transferred to the other end of the rectangle, the length of the base is still the same, because just the point of the triangular piece touches the end point of the base line. Pupils can also see that the altitude is still the same length in both polygons, for nothing has been cut away from the top or the bottom of the rectangle.

They can also understand that the area has not changed because the piece that was removed from one place has been put on at a different place. The pupils may demonstrate this for themselves, using their own 1-inch squares. They can cut a triangular piece off one square and put it along the side of the last square in the row. Be sure they realize that they have not changed the length of the base or the altitude and that the area is still the same. Follow this same procedure for Exercise H.

Let pupils answer Exercises I and J by counting the squares. As a supplement to this lesson, and as a help for all pupils, have drawings of various rectangles and other parallelograms duplicated on worksheets. Let the pupils find the areas of these by counting the number of 1-inch squares they can place within the outlines.

**Providing for the able pupil**
Able pupils will enjoy "Polygon puzzles," Activity 38, page 397, in which they cut up one polygon and fit the pieces together into the shape of another polygon that has the same area. Those who want a greater challenge may like to determine *how* to cut one polygon so that the pieces will fit into another one.

**Helping the slow learner**
Slow learners will also enjoy "Polygon puzzles." Limit the polygons here to familiar ones, such as rectangles and other parallelograms.

## 152-156 Exploring problems

Lesson Briefs for this lesson are on pages 155-158.

**Overview**
In this lesson the pupils learn to find the area of a parallelogram by applying what they know about rate. They learn to find area by means of an equation of two ratios. One ratio expresses the rate as square units per row along the base, that is, the number of square units per *one* row. The other ratio expresses the rate as total square units per all the rows. The pupils solve the ratio equations by applying the ratio test.

**Teaching the whole class**
**Page 152:** Begin this lesson by asking how the area of a parallelogram was found in the previous lesson [by covering the surface with 1-inch squares and counting the total number of square inches]. Then have the pupils look at Picture A and read its accompanying text. They should decide that they cannot tell the area in square inches for the diagram as shown. Discuss what can be done. [Cover the surface with 1-inch squares.] Discuss the fact that the diagrams in this lesson are scale drawings and that the squares shown are 1-inch in scale.

Then have the pupils read the text relating to Picture B. They should understand that at this stage only one row of squares has been placed on the parallelogram and that six 1-inch squares are needed for one row because the base of the parallelogram measures 6 inches. Remind them that since an inch square is the same as one square inch, the surface covered in this one row is 6 square inches. Go on somewhat like this:

*Could we find the total area by covering the entire surface of this parallelogram with inch squares? Would it be convenient to find the area of this classroom in the same way? Since this way is not convenient for large surfaces, it is to our advantage to discover a way to find the area of a parallelogram by computation.*

*Look again at Picture B. We are going to cover the parallelogram with 1-inch squares at the rate of 6 per row. We can express this rate by writing 6 over 1, as shown in the book. We will cover the whole parallelogram in rows at the same rate as for the first row. What ratio expresses the rate for 1 row?* [6/1] *Can we also express the rate by an equal ratio that will use all the rows?*

*Read the text beside the arrow pointing to Picture C. First we must find how many rows there will be. How long is the altitude of this parallelogram? The squares in each row are how large? How many rows will there be? We want to make a ratio for 4 rows that is equal to the ratio for 1 row, which is 6/1. Where is the numeral showing the number of rows placed in the first ratio?* [*Second term*] *What must the numeral in the second term*

*of the equal ratio express?* [*Rows*] *Since we don't know the number of 1-inch squares for 4 rows, we will use the letter* x. *Why do we use it as the first term of the second ratio? How can we express the new ratio?* [x/*4*] *Find this ratio in the book.*

**Page 153:** Next have the pupils look at the equation. They must understand that the rate for 4 rows is the same as the rate for 1 row; therefore the ratios are equal and can be used to write an equation.

Direct attention next to the work opposite Picture E. Ask for suggestions on how to find the numeral that replaces $x$. Remind the pupils that since 1 can be multiplied by 4 to get 4, the 6 also must be multiplied by 4. Point out in Picture E that square inches are used at a rate of 6 per row, and it takes 24 to cover the 4 rows, or the entire surface within the parallelogram. Ask the pupils to verify by counting the 1-inch squares. Review with the class what is meant by the ratio test. Ask them to write the equation they get by using the ratio test and solve it.

Direct attention to the ratio with the numeral 24 replacing $x$. Point out that this equation means that when 1-inch squares are used at a rate of 6 per 1 row, then 24 will be used for 4 rows. Be sure the pupils realize that the answer to the problem is 24 *square inches.* Relate the answer to the illustration.

The pupils may benefit by using 1-inch squares to cover a parallelogram whose base is 6″ and altitude is 4″. Have them find the number of inch squares in one row along the base of the parallelogram. Discuss the fact that they are using inch squares at a rate of 6 per 1 row. Tell them to write a ratio for this rate. Then discuss the fact that they are to find an equal ratio showing the number of 1-inch squares there will be per all the rows. Ask the pupils how they can use a few of their squares to find how many rows in all there will be. [They can lay 1-inch squares along the 4-inch altitude of the parallelogram to find that there will be 4 rows.] Have the pupils now make an equal ratio by using the letter $x$ to indicate the total number of 1-inch squares for 4 rows. Next tell the pupils to complete covering the parallelogram at the same rate of six 1-inch squares per 1 row. (They will have to cut some of their squares so that the squares cover the entire surface of the parallelogram, but do not overlap the edges. Help them see that the part of a square they cut from one end of a row can be used at the other end of the row.) Then ask the pupils to use the ratio equation to find the answer by computation.

The pupils may observe that even though the parallelograms they used for this exercise are not all the same shape (which depends on the slant of the sides), they all will need 24 of the 1-inch squares to cover them. Have a pupil explain why.

**Page 154:** The new problem is solved in exactly the same way as the first problem on page 152. Adapt the procedures used with page 152 to this page. A 24″ x 30″ (scale 6″ to 1″) reproduction of Picture G can be placed on the chalkboard so that you and the pupils may discuss each successive step that is taken in the text. Be sure the pupils understand that in a rectangle the altitude may be taken as one of the sides, while in a parallelogram this is not true. Have the pupils answer the questions in the text and complete the illustration on the board.

**Page 155:** For the work through Picture K, adapt the procedures suggested in the notes for page 153. For the parallelogram in Picture L, ask the pupils to *imagine* 1-inch squares laid along the base. Then they will know the rate of 1-inch squares per 1 row. They can also imagine 1-inch squares laid along the altitude to find the number of rows of 1-inch squares. Discuss the equation in the way you have been doing. Be sure the pupils realize that the answer means *square inches.*

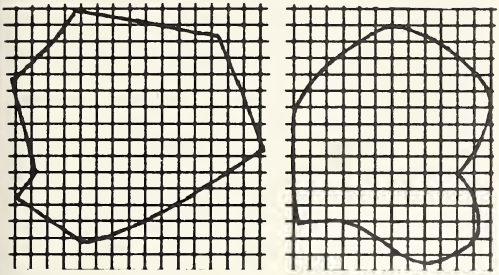
**Page 156:** Before asking the pupils to find the areas of Parallelograms A to H, discuss Parallelogram A. Have a pupil demonstrate with 1-inch squares, which he can lay out on his desk, that 14 are required to make one row. Have the pupils state the rate by writing the ratio 14/1. Also have a pupil use the 1-inch squares to show that there will be 6 rows. Then let the pupils express this rate of $n$ squares per 6 rows by writing the ratio $n/6$. Be sure they understand that the two ratios are equal. Then ask a pupil to write another equation by using the ratio test. Have the pupils solve both equations and write their answer as a number of square inches. Then instruct them to write two equations for each of Parallelograms B to H. When they have finished, supply answers and discuss the exercises that caused trouble.

Problems A to K can be used as written work. Tell the pupils to write a ratio equation for each problem, then use the ratio test and find the answer. Be sure they understand that the answer for Exercise G represents a number of *tiles.* Supply answers and discuss any problems that have caused trouble.

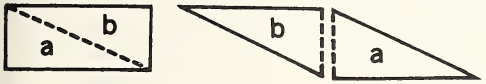
**Providing for the able pupil**

Pupils might draw irregular shapes on tissue or other transparent paper. Then, by placing these shapes over squared paper (such as graph paper) and counting the squares enclosed by the shapes, they can find the approximate areas of the shapes. (See examples shown

below. After doing this, they can appreciate more fully the short-cut method we use for computing areas of rectangles and other parallelograms.



Pupils may also convert a rectangle into a parallelogram that is not a rectangle by cutting it in half diagonally and rearranging the pieces as shown below.




Have the pupils point out the original base and altitude on the new figure and make observations concerning it. Pupils might also like to find the dimensions of familiar rectangular-shaped areas, such as the classroom, the teacher's desk, their own desks, or rooms in their own homes. The data can be written on the chalkboard, and the areas computed and verified by the pupils.

**Helping the slow learner**

Provide the slow learners with rectangles and parallelograms with bases and altitudes of various lengths (involving only whole inches). Tell them to label the bases and altitudes with their lengths and beneath each parallelogram to write the equation that can be used to find the area.

Slow learners may also like to find the areas of rectangular objects in the classroom. Be sure to select objects whose surface areas can be measured in square inches and whose dimensions do not involve fractional parts of inches.

## 157-160 Looking back

Lesson Briefs for this lesson are on pages 158-161.

**Overview**

The measurement of area is extended to include the square foot and the square yard. The square foot is introduced in terms of its area in square inches, and the square yard is introduced in terms of its area in square feet. The computational changing of units of square measure is also developed through ratio equations.

**Teaching the whole class**

So far the pupils have computed areas of parallelograms by using square inches only. In this lesson they learn to compute to find areas in square feet and square yards and to change units of square measure when they need to. To present this lesson, mark off a foot square sheet of oaktag into square inches and a yard square sheet of oaktag into square feet. It is more realistic to discuss an actual square foot and a square yard, and pupils are able to see these areas in their correct relation to each other. Keep these square units on display for ready reference.

**Page 157:** As the pupils read the text opposite Picture A, point out that the square foot is also a unit of square measure and that sometimes it is more convenient to measure area in square feet than in square inches. Show the pupils the full-scale model of the square foot, and tell them that they can use an equation to find the number of square inches in it. Have them observe the 12 one-inch squares in the row along the base. Be sure the pupils understand that the number of 1-inch squares in this row determines a rate and call attention to the ratio in the book (12/1) that expresses this rate.

Have the pupils observe that there are 12 rows of 12 1-inch squares. Help them understand that the numeral that replaces $b$ will express the number of 1-inch squares in 12 rows. Then have the pupils answer the questions opposite the equation.

Ask the pupils to explain why the ratios 12/1 and $b/12$ are equal and can be used to make an equation. Have them solve the equation by using the ratio test. When the equation is solved, point out to the pupils that they have now discovered that there are 144 square inches in one square foot. Some pupils may wish to use Picture A to verify this fact.

Have the pupils read the text opposite Picture B, and show them the full-scale model of a square yard. Explain that the square yard is also a unit of square measure and that this area can be expressed in square feet. By looking at the picture, the pupils will easily be able to count the 9 square feet in the square yard. Tell them this number can also be found by using an equation.

Let the class recall the number of feet in one yard, and point out that it takes three 1-foot squares to cover 1 row. Continue to develop the relationship between the number of square feet and 1 square yard

in much the same way that the relationship between the number of square inches and 1 square foot was developed.

**Page 158:** Direct attention to Picture C and have the pupils note its dimensions. Have someone read Problem A and write the equation ($12/1 = n/9$) that shows how to find the area in square feet. Have the class solve the equation.

Then point out that sometimes it is desirable to know how to express an area in terms of some other unit of square measure, as suggested in Problem B. (The rug in Picture C might be sold at a certain price per square yard, for example.) Tell the pupils that to change 108 square feet to square yards, they must first think of the rate of square feet per 1 square yard, or 9 per 1. When the pupils examine Picture D, help them recognize that the squares outlined in black represent a square yard, or 9 square feet. To clarify this, draw a network of lines forming squares on the chalkboard like Diagram D in the textbook. You might use a music staff liner (a device for holding five pieces of chalk) and a yardstick as a guide. Let a pupil mark off the amount representing 1 square yard. Other pupils can mark off additional square yard sections. Thus pupils can see that when areas are expressed in larger units, fewer units are needed.

Discuss how to express the rate of square feet per square yard (9 per 1) when 108 square feet are involved. Then ask some of the pupils to explain what the symbols in the equation $9/1 = 108/t$ stand for. Be sure they understand that in this equation the 1 represents 1 square yard and not 1 row. Then have a pupil explain how to find the numeral that replaces $t$ by using the ratio test. Emphasize that 108 square feet and 12 square yards are the same area by letting a pupil summarize the findings by using the chalkboard illustration as verification.

Direct attention to Picture E and Problem C. Have a pupil write and solve the equation for finding the area of the tablecloth in square yards.

To give Problem D more meaning, use the first and fourth pieces of chalk on the music staff liner to reproduce Picture E on the chalkboard. Help pupils see that information from Problem C is needed to complete Problem D. Let a pupil use the music staff liner, including the second and third pieces of chalk, to reproduce the black-lined section in Picture E, which shows there are 9 square feet in 1 square yard. Let other pupils fill in the other simulated square yards with the liner to show square feet and help the class understand that when a measure expressed in large units is expressed in smaller units, the number of units is increased. Let pupils take turns answering the questions, and make certain they know that the ratio of 9 square feet to 1 square yard will express the same relationship as $n$ square feet to 6 square yards, when the correct replacement for $n$ is found. Let a pupil solve the equation by applying the ratio test.

**Page 159:** Discuss Problems E to J with the class. Write the ratio equations on the chalkboard and have the pupils supply answers by using the ratio test. Problems H, I, and J each involve two ratio equations. Be sure the pupils know what each term in the ratios represents. When the pupils make ratio equations for changing measures from square inches to square feet or square feet to square yards, be sure they understand that the 1 in the second term of the first ratio represents 1 square foot or 1 square yard, not 1 row. Also have the pupils observe that when they change a measure from a smaller unit of measure to a larger unit, they will have a smaller number of units; but when they change from a larger unit to a smaller unit, they will have more units.

Assign Exercises A to M as written work, giving help where needed. Supply answers, letting the pupils verify their own work. Discuss any exercises that cause difficulty.

**Page 160:** In discussing Rectangle A, make sure the pupils are aware that the dimensions are expressed in different units. Following the directions in Exercise A, ask the pupils to explain how to change 4 feet to inches. Then help them make the equation for finding the area in square inches and write it on the chalkboard ($48/1 = n/36$).

Next, have the pupils find the area in the two ways described in Exercise B. They should note that the easier way is to change 36 inches to feet.

The pupils should work independently to find the areas of Parallelograms B to G. Tell the pupils that when the dimensions of a parallelogram are expressed in different units, they should change one of them to the unit that seems to make the work easier.

The "Keeping skillful" exercises may be assigned as a separate lesson or as time permits.

**Providing for the able pupil**

Able pupils might be interested in looking up information on the history of our units of measure and telling the class about it.

Let the pupils find the dimensions of rectangular objects in the classroom and record these dimensions in inches, feet, and yards. Then have them compute the areas in square inches, square feet, and square yards (do not involve fractions). They can vary the

dimensions by having one dimension in inches and the other in feet and then computing the area.

**Helping the slow learner**

Slow learners might also measure the dimensions of rectangular objects in the classroom and compute the areas. They should not be expected to do as many as the able pupils, and they may have to keep the dimensions in the same units.

## 161 Checking up; Keeping skillful

Lesson Briefs for these lessons are on pages 161-162.

Expanded Notes are not considered necessary for these lessons.

## 162-164 Side trip; Keeping skillful

Lesson Briefs for these lessons are on pages 162-164.

Expanded Notes are not considered necessary for these lessons.

CHARTING THE COURSE

### The number line

One of the most important ideas of mathematics stems from the intuitive association of numbers with the points on a line. Practical applications of this idea abound in rulers and scales of all sorts, in graphical charts, in maps, and other such familiar devices. The idea also has great theoretical importance. In the *Seeing Through Arithmetic* program such "number lines" were introduced in Grade 5 and were used to show that the same point (or the same number) can be named in many different ways, as for example, $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}$, etc.

The number line is introduced again on pages 165-168 of *Seeing Through Arithmetic 6*. Here, at the beginning, the line is used to show geometrically (or graphically) the one-to-one correspondence between the members of two different systems of numbers. One of these systems consists of the set of *natural* numbers. This is the correct mathematical name for the familiar "whole" numbers that the pupils have known from the primary grades.

A different system of numbers is found by using *pairs* of natural numbers together, as indicated by the numerals $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$, etc. These numbers are members of the set of *rational* numbers, which also includes the familiar fractions. It is customary to write $\frac{2}{1}=2$, $\frac{3}{1}=3$, etc., and to ignore the fact that the numerals $\frac{2}{1}$, $\frac{3}{1}$, etc., on one side of the "=" are from one system and the numerals 2, 3, etc., on the other side are from another system, and there is clearly no need whatever to try to teach the distinction to elementary-school children.

Of much greater importance is the idea that the same number (or the same point on a number line) has many different names. It is by changing from one name to another that we arrange to compare, to add, and to subtract fractions. This idea is already well established. We are, however, soon to introduce still another set of names, that is, the decimal fraction numerals. The number line, with its visual representation of the same number associated with different common fraction numerals, provides a useful kind of readiness experience for the study of decimal fraction numerals.

## 165-168 Moving forward

Lesson Briefs for this lesson are on pages 164-167.

**Teaching the whole class**

To supplement the illustrations and exercises in the lesson, the teacher can prepare an enlarged copy of the number line (without numerals) on a sheet of wrapping paper (7′ by 5′) or on the chalkboard as shown in the illustrations on pages 165-166 of the pupils' book. Be sure to leave plenty of space between the points on each line, and between the lines.

As the exercises in the text are discussed, have the pupils complete the number line by putting in (with chalk or crayon) the names for the points on this chart. If the chart is prepared on paper, it can be used for permanent display, and the pupils can refer to it in future lessons.

**Page 165:** Use Exercise A to reacquaint the pupils with the number line. Pupils should observe that the points on the line are the same distance apart and are named in Set A by the names of whole numbers.

Emphasize that this line extends indefinitely to the right and have pupils give names of large numbers that can be included in Set A. In discussing Exercises B to D, be sure the pupils understand that the fraction numerals in Set B are other names for the points on the line. Have them observe that all the names in Set B have "1" as the denominator.

Exercises E to I bring out the idea that a number can have many names, and each name belongs to a different set. In connection with Exercises E to H, point out that the names in Set C have a denominator of "2," and the names in Set D have a denominator of "3." Through discussion of Exercise I, have the pupils bring out that the names for 2 in Sets B, C, and D will always have a numerator equal to twice the denominator.

**Page 166:** Use Exercises J to M to emphasize that there can be more points on the line *between* the points already named. Point out that Set A actually can include *all* the points on the line and that several of these points are named in red above the line. However, Set B does not include all the points in red because they cannot be named by a pair of numerals with "1" as a denominator. However, they are included in Set C because they can be named by a pair of numerals having "2" in the denominator. Again, point out that Sets B and C are infinite, and have the pupils give other pairs of numerals for each set. Also call attention to the fact that the point named "0½" is usually called just "½" for short.

In connection with Exercises N to Q, discuss the other points that have been named in Set A, and have the pupils tell which other set (or sets) includes names for these points. Have the pupils observe that the colors of the numerals help them identify the points and their corresponding sets.

**Page 167:** Exercises R to V give the pupils practice in finding other names for various points on the line. Stress the idea that the pairs of numerals in a set have the same denominator, but that names for the same number have different denominators. Have the pupils observe that names for Point $p$ have denominators that are multiples of 2, names for Point $1\frac{2}{3}$ have denominators that are multiples of 3, etc.

Use Exercises W to Z to point out that, when adding and subtracting fractions, we make sure that their names are from the same set before computing.

Let pupils refer to the number line when making true statements for Exercises A to T. If necessary, review the meaning of the signs $>$ and $<$.

**Page 168:** Exercises A to F give pupils practice in using sets of names other than those used on the preceding pages. Some of the number lines are drawn vertically to point out that a number line need not always have a horizontal position. Have the pupils work individually on these exercises. Then verify the answers by class discussion.

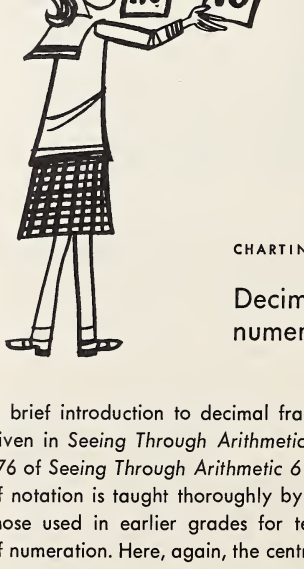
The "Keeping skillful" exercises may be assigned according to the needs and abilities of the pupils.

**Providing for the able pupil**

Able pupils may use sets of numerals having denominators of 5, 6, 7, 8, etc., on the extended number line.

**Helping the slow learner**

To emphasize the position of names on a number line and to stress the idea of different sets, the slow learners may participate in an activity in which they become "names" on a number line. Distribute among the pupils 6″ x 4″ oaktag cards that have the names of points on the number line printed on them. Then have the pupils line up in correct position for a number line. This activity can be varied by having pupils who are holding cards for *one set* to line up, or by having all the pupils holding cards with names for 2 step forward. The amount of variety in this activity will depend upon the number of pupils.




CHARTING THE COURSE

## Decimal fraction numerals

A brief introduction to decimal fraction numerals was given in *Seeing Through Arithmetic 5*. On pages 169-176 of *Seeing Through Arithmetic 6* the decimal system of notation is taught thoroughly by methods similar to those used in earlier grades for teaching the system of numeration. Here, again, the central ideas are grouping by tens and the use of "place value" in the nu-

merals, but now it is fractional parts which are tenths, and tenths of tenths (or hundredths), etc., that are represented.

Pupils who have learned that the same fraction may be represented by many different fraction numerals should have little difficulty in seeing that $\frac{1}{5}=\frac{2}{10}=\frac{20}{100}$ and that other numerals for the same fraction, namely .2 and .20, can be adopted. The great advantage that decimal fraction numerals have over common fraction numerals cannot be appreciated, however, until these numerals have been learned not as individuals, but as members of a numeral system. The numeral system in common use is a decimal system even when only the "whole" numbers are considered, and this system is merely extended when the decimal fraction numerals are introduced. In everyday language decimal fraction numerals are called "decimals," for short, and a common misconception is that they somehow constitute a quite different system of numeration, rather than a specialized system of naming familiar numbers.

In conformity with a principle followed in the *Seeing Through Arithmetic* program in earlier grades, the adjustments in the numerals that are needed to "carry" in addition and to "borrow" in subtraction are taught before the processes themselves are explained. In this way a considerable part of the difficulty that commonly arises is avoided.

## 169-172 Moving forward

Lesson Briefs for this lesson are on pages 168-170.

**Teaching the whole class**

To help teach this lesson, you can, if you wish, reproduce the picture at the top of page 169 by arranging groups of disks and fractional parts of disks. For use with the rest of the lesson, large squares 30″ x 30″ could be drawn on the chalkboard and marked off to show hundredths, thousandths, and ten-thousandths.

**Page 169:** In Picture A the pupils should see that there are 31 whole disks and $\frac{2}{10}$ of a disk in all. They already know how to write $31\frac{2}{10}$ as a mixed numeral. Have them note that the text shows how to write it in words, and point out the third way that $31\frac{2}{10}$ can be written. Point out that the word *and* is used in reading all three ways. For a decimal fraction numeral, the point is read "and." Tell the pupils that the numeral to the right of the point represents tenths. Have them relate the numerals in the spaces to the picture.

In Exercise B, have the pupils note that the "2" in the ones' space represents the 2 large red squares. They should also observe that another square is divided into 100 parts. They should see that the "7" in the hundredths' space represents the 7 small red squares. Call attention to the longer dotted line that separates the spaces. It separates the numerals for a whole number from the numerals that represent a fraction. The shorter dotted line separates the spaces in the decimal section. Have the pupils read the three ways of expressing the number.

**Page 170:** Have the pupils examine the picture and relate the numerals in the spaces to the parts that are colored red. Have them read the three ways of writing the number. For the bottom half of the page, explain that the picture represents the same number, but that now the hundredths have been regrouped into tenths. They should see that there are 4 groups of tenths and 3 hundredths. Discuss the numerals in the spaces.

**Page 171:** In discussing Exercises A and B, bring out the reason for the zeros and ask what would happen if they were omitted. Let some of the pupils write the decimal fraction numerals on the chalkboard.

To clarify the regrouping in Exercise C, explain that if a square were divided into thousandths, and 65 thousandths were regrouped into hundredths and thousandths, there would be 6 groups of hundredths and 5 thousandths. For Exercise D, explain that if a square were divided into ten-thousandths, and 411 ten-thousandths were regrouped into hundredths, thousandths, and ten-thousandths, there would be 4 groups of hundredths, 1 group of thousandths, and 1 ten-thousandth.

For Exercises E and F, help the pupils understand the regroupings and indicate the numerals for the screens and the words for the wavy lines correctly. Let them work individually on Exercises G to J. Then have some of the pupils read their answers as the others evaluate and verify theirs.

**Page 172:** Ask a pupil to read the first decimal fraction numeral by following the instructions given. Emphasize that "2" is in ten-thousandths' place. Have another pupil read the second numeral. Through questions have the pupils explain the reason for the zeros. Have the third numeral read, and emphasize that no whole number is expressed.

For Exercises A to O, have the pupils take turns reading the decimal fraction numerals. Call attention to the red numeral in each of the first five

exercises, and be sure they read the part of the numeral that represents a fraction as they would if it represented a whole number. If necessary, let the pupils arrange Exercises F to O in chart form also.

Assign Exercises A to N as individual work. Verify answers by having them written on the chalkboard.

**Providing for the able pupil**
Encourage able pupils to search through newspapers, magazines, and books to find decimal fraction numerals. A poster may then be prepared to show the importance of knowing how to read and write such numerals.

## 173-174 Moving forward

Lesson Briefs for this lesson are on pages 170-171.

**Teaching the whole class**
**Page 173:** Get the pupils to describe the first picture in Exercise A. Discussion should bring out that each small piece is a tenth of a disk. Relate the picture to the numerals in the tally spaces. Ask what is happening in the second picture. Discussion of the third picture should bring out that the regrouping results in 3 whole disks and 6 tenths of a disk. The pupils should relate the picture to the numerals in the tally spaces and write the numeral for the number, using both common and decimal fraction numerals.

Explain that regroupings like those shown on this page will be used when numbers represented in decimal fraction notation are added or subtracted. Ask if the regrouping in Exercise A is used in adding or subtracting. If necessary, help pupils see that "carrying" is involved (10 tenths are combined to form a whole) and that this type of regrouping occurs in addition.

For the first picture in Exercise B, bring out that these whole disks are also divided into tenths. Again relate the picture to the numerals in the tally spaces. It should be clearly understood that each small piece represents 1 tenth. Let a pupil write the numeral for the number, using both common and decimal fraction numerals.

Have the pupils tell what is taking place in the second picture. In the third picture they should see that 1 disk has been separated into 10 tenths and combined with the 3 tenths. They should conclude that 10 whole disks and 3 tenths of a disk have been regrouped as 9 whole disks and 13 tenths.

Ask the pupils whether Exercise B illustrates regrouping for addition or for subtraction. They should decide that separating 1 whole into 10 tenths and including them with the 3 tenths is the regrouping that is sometimes necessary in subtraction.

**Page 174:** For Exercises A to F, instruct the pupils to compare the numerals in the top row with those in the bottom row and decide what numerals can be used to replace the question marks. In Exercise A, the pupils should observe the 16 hundredths in the top row and the 6 hundredths in the bottom row. They should understand that 10 hundredths should be expressed as 1 tenth and carried to the tenths' column. They then should realize that when 1 tenth is added to 9 tenths, making 10 tenths (or 1 one), 1 can be carried to the ones' column, making 4 ones, and leaving 0 tenths.

Follow similar procedures for Exercises B to F. Let the pupils decide which exercises show the regrouping necessary in addition and which exercises show that necessary in subtraction.

The other two blocks of exercises on this page can be used orally first. Then let pupils take turns writing the numerals in Exercises A to L on the chalkboard. Let them explain their thinking. Pupils also may take turns in replacing the question marks in Exercises A to F. Discuss any exercises that are troublesome, because a pupil's success in computation involving decimal fraction numerals is based upon his understanding of regrouping.

After discussing the exercises, you may wish to assign them for written work. This will enable you to evaluate the pupils' progress, and determine which ones need further help.

**Providing for the able pupil**
Able pupils will be challenged by playing the game "Numeral spelldown," described in Activity 39 on page 397.

**Helping the slow learner**
Exercises patterned after Exercises A to F on page 174 can be written on 3″ x 5″ cards and used by these pupils. Write the numerals in the top row in one color and those in the bottom row, including the question marks, in another color. Write the numerals that replace the question marks on the backs of the cards so that pupils may verify their work.

## 175-176 Moving forward

Lesson Briefs for this lesson are on pages 172-173.

Expanded Notes other than suggestions for the able pupils and help for the slow learners are not considered necessary for this lesson.

**Providing for the able pupil**
The line made for Exercise S on page 176 shows the portion of the line between .00 and .01. To reinforce the idea that the number line has an infinite

number of points, provide a 1 ft. x 6 ft. strip of paper on which the able pupils can show the portion between .000 and .001. Allow 5 inches between points, and tell the pupils to establish names for points between .0000 and .0010, or beyond.

Able students may be interested in looking up information on Simon Stevin of Belgium, the man usually given credit for introducing decimal fraction notation.

**Helping the slow learner**

On the chalkboard reproduce the number line shown on page 175 and extend the line so it will include Points 3 and 4. (A 5-ft. line will be adequate.) Assist the pupils in writing names for the points between 2.0 and 4.0 for Sets B, C, D, and E. Pupils may take turns writing names, or each of four pupils may complete one set. An able pupil could be asked to supervise this activity.

CHARTING THE COURSE

## Addition and subtraction involving decimal fraction numerals

When the decimal system of naming numbers is well understood, the processes of addition and subtraction with decimal fraction numerals are easy. The processes are carried on exactly as they are with "whole" numbers, and the only feature that calls for special attention is a mechanical one—the numerals are usually arranged so that the numerals expressing the "ones," the "tenths," the "hundredths," etc., of the different numbers appear in the same vertical column—that is, tenths under tenths, hundredths under hundredths, etc. When this is done, the decimal points also are placed in a vertical column, and in fact, this latter feature is the one usually emphasized because it provides a visual or mechanical aid to locating the decimal point in the answer.

It should be noted, however, that this feature is not really a fundamental one. A competent computer can obtain the answer and place the decimal point correctly when the numerals are written horizontally (for example, as in $4.34+3.67=8.01$), or even if they are not written at all. The essential point is that hundredths be added to hundredths, tenths to tenths, etc. If one of the numbers is expressed to hundredths, for example, then all other numbers involved in the computation are also expressed to hundredths automatically, whether or not zeros are actually annexed to their numerals to fill the column. The result, then, is also automatically expressed to hundredths. Even in an example such as $4.33+3.67=8.00$ which "comes out even," the answer technically is not just 8, but 800 hundredths. In practical problems the retention of the zeros in 8.00 conveys information about the precision of the measurement that is concealed when the answer is written as 8.

### 177-180 Learning how; Keeping skillful

Lesson Briefs for these lessons are on pages 173-176.

Expanded Notes are not considered necessary for these lessons.

### 181-182 Using arithmetic; Checking up

Lesson Briefs for these lessons are on pages 176-178.

Expanded Notes are not considered necessary for these lessons.

CHARTING THE COURSE

## Multiplication involving common fraction numerals

In the elementary school the meaning of multiplication is introduced through concrete situations that involve the combining of equal groups. In such situations the number of objects in each group is a "natural" number, or "whole" number, and the number of groups is also a

"natural" number. The product, in such situations, is a natural number, and it expresses the total number of objects in the group formed by combining the several equal groups.

The primitive meaning of multiplication sketched above is, however, completely inadequate when fractions are involved. It is commonly pointed out that in a situation that calls for combining, for example, 5 quantities when each is represented by the fraction $\frac{3}{4}$, the total amount can be represented by $n=5\times\frac{3}{4}$. If one does not know how to "multiply a fraction by a whole number," he can in this case calculate $n$ by using $n=\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$ to obtain $n=\frac{15}{4}$. In an example such as $n=\frac{5}{6}\times\frac{3}{4}$ the addition procedure is, however, impossible. The interpretation of multiplication in terms of addition breaks down completely. In the example, any effort to interpret $\frac{5}{6}$ as indicating how many three-fourths are to be added leads to a meaningless collection of words and to utter confusion. The primitive idea of multiplication must be abandoned.

The fact is that it is virtually impossible to give a meaningful explanation of the process for multiplication of fractions by using concrete situations. Concrete situations are useful in getting children to *accept the rule*, but this acceptance should not be taken as evidence of genuine *understanding*. It does become clear that the rule "works" in the sense that it gives a sensible answer, but *why* it works remains obscure.

The basis of the rule is mathematical, rather than concrete. In mathematical terms, the use of fractions calls for a remarkable extension of the number system. One must learn to calculate in the system of rational numbers ("fractions"), and each rational number involves a set of *pairs* of natural numbers. Certain sets of pairs have all the essential properties of a natural number. For example, the rational number named by the pair 12/4 (or its equivalent pair 3/1) can be used instead of the natural number 3. Similarly, the rational number named by the pair 6/3 (or its equivalent pair 2/1) can be used instead of the natural number 2. The product of 3 and 2 is 6. Hence, the rule for multiplication of the rational numbers named by the *pairs* 12/4 and 6/3 should produce a new pair that can be used in place of 6. This rule is, of course, the familiar one that authorizes one to write

$$\frac{12}{4}\times\frac{6}{3}=\frac{12\times6}{4\times3}=\frac{72}{12},$$

and the pair 72/12 can be used instead of 6.

In general, one may say that the rule (or definition) for multiplication of rational numbers ("fractions") is formulated in such a way as to make the results consistent with those obtained when the example is, in effect, viewed as one involving only natural numbers. But many of the examples that occur cannot be viewed in this way. For example, the rational number named by the pair 5/6 does *not* have a corresponding natural (whole) number as 6/1 has. Similarly, the pair 3/4 does not have such a corresponding natural number. Nevertheless, the same rule is applied for multiplication as was used for those fractions that can be viewed as natural numbers. Thus the rule gives

$$\frac{5}{6}\times\frac{3}{4}=\frac{5\times3}{6\times4}=\frac{15}{24} \text{ or } \frac{5}{8},$$

and this result will provide a sensible answer in a concrete situation in which the numbers are applied.

A discussion in the language used above is obviously not suitable for an introduction to elementary-school pupils. It does, however, suggest a mathematically correct way of approaching the multiplication of fractions. An introduction that is *guided* by the above principles is given on pages 184-185 of *Seeing Through Arithmetic 6*. This discussion in the pupils' book is sufficiently simple that it can be understood by the pupil, and it provides a sound foundation for continued growth of understanding of fractions and, in general, of rational numbers.

Attention may be called here to the fact that some writers on the teaching of arithmetic have asserted that in an example such as $5\times\frac{3}{4}$, neither teacher nor pupil should write $\frac{5}{1}\times\frac{3}{4}$. The writing of 5 as $\frac{5}{1}$ is viewed as a "crutch." The teacher is advised to avoid it, and to rely on the meaning. However, many experienced teachers have found that it helps pupils to write the example this way. Actually, teachers should have no hesitation whatever in *teaching* pupils to write $\frac{5}{1}$ for 5 when operations with fractions are involved. Mathematically speaking, it is better to think of the calculation as done *within the system of rational numbers* ("fractions") and not in some "mixed" system in which one factor is a natural number, but the other factor and product are in the system of rational numbers. This mathematically sound way of treating the example gives good results, probably because the pupil senses the mathematical basis.

In *Seeing Through Arithmetic 6* the introduction to multiplication of fractions is followed immediately by the multiplication of numbers expressed by mixed nu-

merals. In accordance with one of the general principles by which this program is organized, a general case is presented rather than a sequence of special cases such as "whole number times fraction," "fraction times whole number," etc. (See the section called "Guiding Principles," pages 14-15.) It should be noted that in the pupils' book a problem in calculating area is used as a source of a valid multiplication example, but that the *explanation* of the process is not based on a discussion of the diagram. The pupil's attention, however, is called to the fact that the process he has been taught does produce a result that can be verified on the diagram.

On page 195 the pupil is introduced to the idea of reciprocals. This idea is basic for a sound understanding of division by a fraction. The idea is very simple, but has been avoided in the past in elementary arithmetic in favor of a mystical rule that commands the pupil to "invert the divisor and multiply." In a modern mathematics program, ideas as fundamental as those associated with reciprocals cannot be neglected. It is fortunate indeed that the concept is as simple as it is. From this point onward in his study and use of mathematics, the pupil will find this concept recurring constantly.

## 183 Thinking straight

Lesson Briefs for this lesson ore on poges 178-179.

Expanded Notes are not considered necessary for this lesson.

## 184-185 Thinking straight

Lesson Briefs for this lesson ore on pages 180-181.

Expanded Notes are not considered necessary for this lesson.

## 186-191 Learning how

Lesson Briefs for this lesson are on poges 182-185.

**Overview**

The pupils now learn how to find products when they multiply fractions expressed by proper fraction numerals and mixed numerals. In a previous lesson the pupils learned that ratios could have fraction numerals and mixed numerals in their terms. In this lesson they solve equations of such ratios by multiplication.

The pupils know that "whole numbers" can be multiplied by using improper fraction numerals as names for the "whole numbers" and then multiplying the numbers represented by numerals in the numerators and in the denominators. They can expect the same method to work for numbers expressed by mixed numerals and proper fraction numerals. The illustrations in the pupils' book show that the results achieved by this method are correct.

In the first problem situation, the pupils find the area of a rectangle with dimensions of $4\frac{1}{3}$ in. and $2\frac{1}{2}$ in. Other problems are built around rate and comparison situations. In each case, solution of the equation of ratios requires multiplication.

**Teaching the whole class**

**Page 186:** After the pupils have read the problem and looked at Diagram A, you may want to review how to find the area of a rectangle whose dimensions are expressed as whole numbers. For example, on the chalkboard draw a 4″ x 2″ rectangle. Divide the rectangle into 8 squares of 1 square inch each. The bottom row shows four squares, each representing 1 square inch. Discuss the ratio equation that can be made to find the area. Suggest that the area of a rectangle with dimensions expressed as mixed numerals can be found in the same way.

Next call attention to Diagram B. Point out that the shaded row shows $4\frac{1}{3}$ square inches. Then explain that since the length of the altitude is $2\frac{1}{2}$ inches, there will be 2 rows and part of another row. Be sure the pupils can relate the numerals in the ratio equation to the dimensions of the rectangle.

In discussing the work at the bottom of the page, the pupils should see that the multiplication is performed just as it was on pages 184-185. The only difference is that the mixed numerals are renamed as improper fraction numerals.

**Page 187:** Tell the pupils that now they can verify the answer they just found by computation. First discuss the square inches and parts of square inches marked off on Diagram C. The pupils may wonder about the $\frac{1}{6}$ square inch in the upper right corner. Help them see that this part of the area is half as large as the parts labeled $\frac{1}{3}$ square inch.

Then use the questions in the text to help the pupils find the total number of one sixths of a square inch in the rectangle. They will see that the answer they get by counting or adding agrees with the answer they got when they computed.

The pupils may use the diagram again to express the area as $10\frac{5}{6}$ square inches. They can count the 8 whole square inches, and they can count the four $\frac{1}{2}$ square inches as 2 whole square inches. Also, they can think of each $\frac{1}{3}$ square inch as $\frac{2}{6}$ square inch. Thus the total is $10\frac{5}{6}$ square inches.

Have the pupils read the next problem and look at Diagram A. Point out that each dimension of the farm is a fraction of a mile. Then have the pupils note the ratio equation and explain what the numerals in the terms stand for. Remind the pupils that this problem is like any other problem in which they are asked to find area. It is different only in that the measures of the dimensions are fractions.

**Page 188:** Discuss how the equation $t=\frac{3}{4}\times\frac{2}{3}$ is obtained from the ratio equation on the preceding page. Then discuss the multiplication of the numbers represented by the numerals in the numerators and denominators and the reduction of the resulting product. Point out that ½ here means ½ square mile.

Next turn the pupils' attention to Diagram B. Point out that the shaded part represents Mr. Jackson's farm. The text will help the pupils see why the sides of the diagram are marked off in fourths and in thirds. Have the pupils verify that the square mile is marked off into 12 equal parts. They can see that the area of Mr. Jackson's farm is 6 twelfths of a square mile, or ½ square mile. Have a pupil count the number of small shaded sections to verify this. If any pupils have trouble seeing that the area is ½ square mile, show them that for each unshaded twelfth there is a shaded twelfth, so that ½ of the square region is shaded and ½ is unshaded.

**Page 189:** After the pupils have read the problem, point out that it involves a comparison. Point out, too, that the terms of the ratios are numerals representing a whole number and a mixed number. Discuss the ratio equation and the use of the ratio test. Let pupils answer the questions in the text.

Then direct attention to the pictures and text at the bottom of the page. Point out Jane's 1¼ pages (the shaded pages). Let someone explain how the diagram shows that Ann's story was 1¾, or 2½, pages long.

**Page 190:** In the work entitled THINK, have the pupils answer the questions about the computation.

Assign Examples A to F in the TRY step as written work. Tell the pupils to cover the computation in red until they have finished each example. Then they may compare their work with that in the book to see if their computation is correct.

Point out that in Examples C and E the letter holding the place for the missing numeral appears at the right of the equals sign. Explain that the letter may be written on either side; the work is done in the same way.

**Page 191:** Problems A and B and Exercises C to W should be assigned as written work. When all pupils have finished, supply answers and verify the work. Use Exercises A to G as additional practice for those who need it.

The "Keeping skillful" exercises should be assigned as a separate lesson. The equations in Exercises D, F, I, and K in the third block of work may appear unfamiliar to some of the pupils. Point out that equations like these have been used in solving ratio equations. For example, they first solved ratio equations like $6/3=m/27$ by thinking "3 multiplied by what number equals 27?" Then to find this number, they divided 27 by 3.

**Providing for the able pupil**

The able pupils can try multiplying numbers expressed by mixed numerals without renaming them as improper fraction numerals. An example of such multiplication is shown below.

$$\begin{array}{rl} 4\frac{1}{3} & \\ 2\frac{1}{2} & \\ \hline 8 & (2\times 4) \\ 2 & (\frac{1}{2}\times 4) \\ \frac{2}{3} & (2\times\frac{1}{3}) \\ \frac{1}{6} & (\frac{1}{2}\times\frac{1}{3}) \\ \hline 10\frac{5}{6} & \end{array}$$

Explain each step of the computation, and explain how the total is found. These pupils could use Diagram C on page 187 to verify the computation above.

Some pupils could work other examples with larger numbers in this way. You also might ask them to draw diagrams illustrating the steps of the computation. For example, a diagram illustrating $53\frac{1}{2}\times 26\frac{2}{3}$ might look like this:




**Helping the slow learner**

These pupils could make tables similar to Table A and Table B shown below. Note that the number mul-

tiplied remains the same, but the multiplier becomes progressively less.

| Table A | Table B |
|---|---|
| $2\times2=4$ | $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$ |
| $1\times2=2$ | $\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$ |
| $\frac{1}{2}\times2=1$ | $\frac{1}{8}\times\frac{1}{2}=\frac{1}{16}$ |
| $\frac{1}{4}\times2=\frac{1}{2}$ | $\frac{1}{16}\times\frac{1}{2}=\frac{1}{32}$ |
| $\frac{1}{8}\times2=\frac{1}{4}$ | etc. |
| etc. | |

The pupils should observe that in Table A, as the multiplier becomes smaller, the product also becomes smaller. The same conclusion holds true for Table B, but in this case another generalization can be made: When two numbers expressed by proper fraction numerals are multiplied, the product is less than either of the factors.

## 192-193 Using arithmetic

Lesson Briefs for this lesson are on pages 186-187.

Expanded Notes are not considered necessary for this lesson.

## 194 Side trip

Lesson Briefs for this lesson are on page 188.

Expanded Notes are not considered necessary for this lesson.

## 195 Thinking straight; Checking up

Lesson Briefs for these lessons are on pages 189-190.

Expanded Notes are not considered necessary for these lessons except for suggestions for the able pupil.

**Providing for the able pupil**

Able pupils might use the idea of reciprocals with ratio equations and the ratio test. Give them some problems like the following:

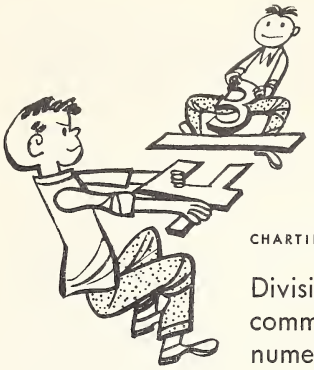
If Bob travels 1 mile in $\frac{3}{5}$ minute, how many miles does he travel in 1 minute?

Have the pupils set up the ratio equation $1/\frac{3}{5}=n/1$ and by using the ratio test, establish the equation $\frac{3}{5}\times n=1$. Of course, the pupils do not know how to divide to solve this equation; but from their knowledge of reciprocals, they should be able to tell without computing that $\frac{5}{3}$ replaces $n$.

## 196-197 Thinking straight

Lesson Briefs for this lesson are on pages 190-192.

Expanded Notes are not considered necessary for this lesson.



CHARTING THE COURSE

# Division involving common fraction numerals

One of the most difficult parts of arithmetic to explain is the procedure for dividing by a fraction. There has been much discussion among teachers and in the literature concerning methods of teaching this topic. Some authorities have despaired of making the process meaningful, and have in effect condoned the formal teaching of the familiar rule: "invert the divisor and multiply." Others have tried to obtain *acceptance* of the rule (without, however, achieving understanding) by using various particular concrete situations as a basis for the general rule. Still other authorities recommend the "common denominator method," but have never given a satisfactory explanation as to *why* this procedure produces the correct result. All of these difficulties arise because the instructional methods used do not face up to fundamental mathematical and psychological aspects of the problem. In *Seeing Through Arithmetic 6* these fundamental aspects are recognized and attacked directly.

It must be recognized, first of all, that the rule "invert the divisor and multiply" describes a short-cut procedure, but explains nothing. Where does the rule come from? Why does it work? The mathematical principle underlying the rule is that if, in a division example, both dividend and divisor are multiplied by the same number, the quotient is unchanged. This principle holds not only when the dividend and divisor are integers, or "whole" numbers, but also when they are rational numbers, or "fractions." When the divisor is expressed by a fraction numeral, the procedure consists in changing the example to an equivalent example in which the divisor is 1. This feature of the new example is secured by multiplying the divisor by its reciprocal. (See page 198

of the pupils' book.) To obtain an equivalent example, the dividend must also be multiplied by the same number, in accordance with the basic principle stated above. For example,

$\frac{\frac{1}{2}}{\frac{3}{4}}$ is changed to $\frac{\frac{1}{2}\times\frac{4}{3}}{\frac{3}{4}\times\frac{4}{3}}$, or $\frac{\frac{1}{2}\times\frac{4}{3}}{1}$, or $\frac{1}{2}\times\frac{4}{3}$.

The new division example has a new dividend that is the product of the original dividend and the reciprocal of the original divisor. Then the new divisor is 1, but division by 1 leaves the dividend unchanged. This procedure, then, has traditionally been summarized as the short-cut rule "invert the divisor and multiply."

It should be noted that the word "invert" refers to the physical or visual appearance of the numeral, and strictly speaking is not a mathematical operation at all. Multiplying the dividend by the reciprocal of the divisor is, on the contrary, a respectable mathematical operation, and is justified and motivated if the divisor also is multiplied by the reciprocal. It should also be noted that the procedure is general and applies to any example in which dividend and divisor are rational numbers (fractions). This includes "whole" numbers, "mixed" numbers, and various combinations that lead to special cases in traditional organizations of arithmetic.

In traditional teaching, concrete examples are sometimes used to get the student to accept the rule. For example, the question may be asked: "How many halves are there in 3 things?" The pupil can be shown concretely there are 6 halves, and that he may use numerals to work the example, writing $3\div\frac{1}{2}=\frac{3}{1}\times\frac{2}{1}=\frac{6}{1}=6$. Other similar examples are followed by the statement of the rule. It should be noted that the reasoning becomes awkward when the dividend is smaller than the divisor. Thus, if one asks, "How many three-fourths are there in one half?" it is difficult to find a sensible and simple concrete situation in which the question would arise, and even if this were easy, a sensible answer would be "none." Application of the rule gives $\frac{1}{2}\div\frac{3}{4}=\frac{1}{2}\times\frac{4}{3}=\frac{2}{3}$. The result, $\frac{2}{3}$, provides useful information about the *relationship* of $\frac{1}{2}$ to $\frac{3}{4}$; namely, that "one half is two-thirds of three-fourths," but the statement within quotation marks is not an answer to the original question. Thus, we can understand why this approach to the division of one fraction by another has serious shortcomings.

Recently, the "common denominator method" has become increasingly popular. When this method is used, both dividend and divisor are expressed with a "common denominator," and then the numerator of the dividend is written over the numerator of the divisor. Advocates of this method customarily say nothing whatever about what happens to the common denominator —it is simply ignored. For example, $\frac{1}{2}\div\frac{3}{4}$ is changed to $\frac{2}{4}\div\frac{3}{4}$, and the result is given as $\frac{2}{3}$. Thus a legitimate question is "What happens to the 4's?" The answer is that a division of 4 by 4 was also involved, but since the result was 1, the complete answer $\frac{2}{3}\div 1$ could immediately be expressed as $\frac{2}{3}$. However, this explanation has seldom been given.

In this connection it may be noted that the "common denominator method" is a special case of a general procedure in which the common denominator is not needed. For example, given $\frac{9}{14}\div\frac{3}{7}$, find $9\div 3=3$, and $14\div 7=2$, and express the answer as $\frac{3}{2}$. An effort to formulate this precisely as a rule leads to complex sentence structure. An informal and not precise formulation is: "Divide the numerators for a new numerator and divide the denominators for a new denominator." This rule provides a short-cut method when the divisions do not involve a "remainder," as in the example above. Also, if a common denominator is found *before* the rule is applied, the new denominator is 1, and this is the fundamental basis of the "common denominator method." In general, however, the procedure will not be a short cut, and will yield a quotient that is expressed in a form as complex as the original example.

This discussion should help make clear why one of the traditional or familiar methods of dividing by a fraction is not taught in *Seeing Through Arithmetic 6*. These methods do not lead to genuine understanding because they are short cuts and do not go to the heart of the matter mathematically. In contrast, the method that involves the idea of multiplying both dividend and divisor by the reciprocal of the divisor, thereby obtaining an equivalent example in which the divisor is 1, is not only sound and defensible from the viewpoint of the mathematician, but is also the easiest to understand, and hence defensible on psychological grounds as well.

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## 198-200 Learning how

Lesson Briefs for this lesson are on pages 192-194.

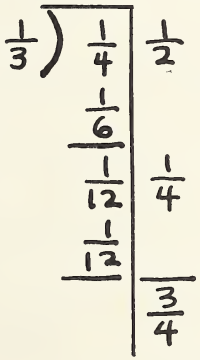
Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil.

**Providing for the able pupil**

Able pupils may study Examples A to J on pages 199-200 to see what generalizations they can make. After some observation they should discover that when the divisor is less than 1, the quotient will be greater than the dividend. When the divisor is greater than 1, the quotient will be less than the dividend, and when the divisor equals 1, the quotient equals the dividend.

Although the division of numbers expressed by fraction numerals and mixed numerals cannot be demonstrated concretely in all cases, able pupils may represent the *answer* for some cases by making diagrams. They could illustrate the problem $5\frac{1}{2} \div \frac{3}{4} = w$ in this way: Starting with a total of $5\frac{1}{2}$ disks, which could be represented by 22 quarter disks, they would ask "How many times can $\frac{3}{4}$ of one disk be taken from $5\frac{1}{2}$ disks?" Then have them take groups of fourths of a disk (each group containing 3) and arrange them in separate piles. The illustration below shows the original $5\frac{1}{2}$ disks. The pupils should see that 7 groups of 3 one fourths of a disk can be taken. The remainder, one fourth of a disk, is $\frac{1}{3}$ of a group.

Able pupils might also try to use for division of fractions the form they used with whole numbers. For example, they might divide $\frac{1}{4}$ by $\frac{1}{3}$ in this way:




## 201 Thinking straight; Keeping skillful

Lesson Briefs for these lessons ore an poge 195.

Expanded Notes are not considered necessary for these lessons.

## 202-203 Using arithmetic; Checking up

Lesson Briefs for these lessons ore on poges 196-197.

Expanded Notes are not considered necessary for these lessons.

## 204 Thinking straight

Lesson Briefs for this lesson ore on poges 197-198.

Expanded Notes are not considered necessary for this lesson.

## 205 Keeping skillful

Lesson Briefs for this lesson ore on poges 198-199.

Expanded Notes are not considered necessary for this lesson.

## 206-208 Checking up

Lesson Briefs for this lesson ore on poges 199-202.

Expanded Notes are not considered necessary for this lesson.

## 209 Thinking straight

Lessan Briefs for this lesson ore on poges 202-203.

Expanded Notes are not considered necessary for this lesson.

## 210-213 Exploring problems; Keeping skillful

Lesson Briefs for these lessons ore an poges 203-206.

Expanded Notes are not considered necessary for these lessons.

## 214-215 Exploring problems; Keeping skillful

Lesson Briefs for these lessons are on pages 206-208.

Expanded Notes are not considered necessary for these lessons.

## 216-217 Thinking straight

Lesson Briefs for this lesson are an poges 208-210.

**Overview**

In this lesson pupils apply their knowledge of decimal fraction numerals as they learn about precision in measurement. The lesson also provides opportunities for rounding off numbers.

Before taking up the exercises in the book, discuss the difference between counting and measuring. Have the pupils count the number of windows

(or any convenient objects) in the room. Then have several pupils measure the width of a desk with a ruler or a yardstick. All the pupils will agree on the number of windows in the room (if they counted correctly) because counting windows is an exact process, but they may not agree on the width of the teacher's desk. Point out that, in general, numbers resulting from counting lead to exact statements about the situation. (There are cases, such as statements about the population of a city, where the number resulting from counting gives only an approximation of the actual number.) Have the pupils give the numbers they got from measuring the width of the desk. Their answers may vary, depending upon the measuring instruments they used and how carefully they measured. Use this activity to point out that a number resulting from measurement does not describe the quantity exactly.

**Page 216:** Draw attention to the pictures at the top of the page. Explain that these are scale drawings. Have the pupils count the number of subdivisions in the foot and read Exercise A. Have a pupil explain how to write 10 tenths as a decimal fraction numeral [1.0].

Have the pupils find the points for 1.1 ft. and 1.2 ft. Then ask them to observe which point (1.1 or 1.2) is nearer to the end of the rod. Point out that the number for this point is the length of the rod to the *nearer tenth* of a foot. Have a pupil estimate the location of this point on a yardstick and give the length to the nearer foot. Then have the class find the lengths and widths of objects in the room to the nearer foot, the nearer 6 inches, the nearer inch, the nearer ½ inch, etc.

On Ruler B have the pupils distinguish the marks showing hundredths from the marks showing tenths. Have them observe that each division on Ruler B is only $\frac{1}{10}$ as long as a division on Ruler A. Ask the pupils how they would describe divisions that would be $\frac{1}{10}$ the size of those on Ruler B. Point out that it would not be practical to have a ruler graduated to $\frac{1}{1000}$ of an inch, but that there are certain precision measuring instruments, such as the micrometer, that help us measure to the thousandth or even ten-thousandth of an inch.

Be sure the pupils understand that the magnified part of the ruler shows the same units as Ruler B, but that it is drawn to a different scale for easier reading. Ask them to give the length of the rod to the nearer hundredth of a foot. They will find that they have to use the magnified part of the ruler to find this measure.

Point out that the measures of the rod under Ruler C and of the rod under Ruler A are the same to the nearer tenth of a foot. Have the pupils notice the difference when the rods are measured to the nearer hundredth of a foot (under Rulers B and D). Point out that the rulers graduated to hundredths of a foot can be read more precisely than the rulers graduated only to tenths of a foot. How precise a measure can be made will depend upon the measuring instrument and the way it is read. Have the pupils compute the difference in length of the two rods to the nearer hundredth of a foot [.06]. Point out that this is not the *exact* difference in length. It is the difference in length to the nearer hundredth of a foot. If the lengths of the rods could have been measured to the nearer thousandth of a foot, the difference in the lengths might not have been .06 ft. Ask the pupils if they could ever be sure that the *exact* difference in length had been found [no, for measurement is never exact]. Point out that we can usually find a close enough measure for practical purposes.

**Page 217:** Point out that rounding a measure to the *nearer* unit means the reading on the closer subdivision. This will sometimes involve rounding up and sometimes rounding down. When the measure is in the middle, it may be rounded either up or down, although rounding up is a common practice. Have the pupils discuss the difference in precision between "1 ft." and "1.0 ft." Get them to observe that the zero after the decimal point tells them that the measure is given to the nearer tenth of a foot, whereas "1 ft." may indicate that the measure was rounded to the nearer foot.

Have the pupils observe that to find a measure to the nearer tenth of a foot, they had to round off a measure that was given in hundredths of a foot. Get them to see that they must have a measure given to thousandths of a foot before they can round it to the nearer hundredth. Give them several measures, such as 2.165 ft., and ask them to round these measures to the nearer hundredth, tenth, and foot. After doing several of these, the pupils should observe that the nearer hundredth, tenth, etc., can be determined by the value of the numeral following the place to which the measure is to be rounded. For example, 2.165 ft. to the nearer foot is 2 ft. because the numeral following 2 represents .1 ft., and .1 ft. is less than ½ of a foot. Therefore, the measure is closer, or nearer, to 2 ft. than to 3 ft. To the nearer tenth of a foot, the measure would be 2.2 ft. because the numeral following the tenths' place represents $\frac{6}{100}$ of a foot. This is more than ½ of a tenth of a foot; there-

fore, the measure to the nearer tenth of a foot is 2.2 ft. rather than 2.1 ft. To the nearer hundredth of a foot the measure may be 2.16 ft. or 2.17 ft. because the numeral (5) following the hundredths' place represents 5 thousandths or $\frac{5}{10}$ of one hundredth. This is half of one hundredth; so the measure may be rounded either up or down.

Exercises N and O focus the pupils' attention on determining what measures could originally have been included in a rounded-off measure. The preceding exercises should have given the pupils the background to observe that a rounded-off measure could have included any measure from ½ unit below to ½ unit above the given measure. This ½ unit is the *tolerance* when a measure is said to be precise to the *nearer* unit. However, the tolerance of a measure may be more than or less than ½ unit, and in such cases is usually indicated; that is, a measure may be given as 6 ft. plus or minus .2 ft. ($6' \pm .2'$). This means that the tolerance, or the possible error, of the measure is .2 ft., and the measure may be anywhere from 5.8′ to 6.2′. Sometimes a tolerance is understood without being indicated. When we say that a person is 6 ft. tall, we generally mean that his height is within an inch or two of 6 ft. We do not necessarily mean $6' \pm \frac{1}{2}'$.

**Providing for the able pupil**

Able pupils may be interested in comparing measures of various levels of precision as described above. Objects in the room can be used.

Pupils may also be given a set of statements like those below. Have them tell whether or not the statements given are precise and why.

1. Mr. Jones bought *6* gallons of gasoline at *31* cents per gallon.
2. The diameter of the sun is *109* times the diameter of the earth.
3. There are *33* desks in the classroom.
4. The convoy included *258* trucks, each weighing *2375* pounds.
5. On the first of January, Washington, D.C., had a *10*-inch snowfall.
6. Bob is *65* in. tall.
7. Nancy bought *2*½ lb. of cookies.
8. The trip took *7* hours.

**Helping the slow learner**

Slow learners should get as much experience as possible in actually measuring objects to the nearer foot, the nearer inch, etc. Give them an opportunity to measure objects in the classroom to various degrees of precision, using different measuring instruments. Then let them compare their results.

CHARTING THE COURSE

## Multiplication involving decimal fraction numerals

When the factors of a product involve fractions that are expressed in decimal notation, the only new feature of the process is the decision as to where the decimal point is to be placed in the answer. In the past this decision has usually been reached by using a rule. Traditionally, pupils have been taught to count the number of "decimal places" in the multiplier and the multiplicand, and to "point off" the total number of these places in the answer. This formal rule works, provided the pupil remembers in which direction to count in "pointing off" his answer. If, however, the pupil is to understand why the rule works, he must think of the meaning of the symbols. Moreover, he must understand that the rule is based upon the procedure for multiplying any two rational numbers, or "fractions." When decimal notation is used, the numbers expressed by the denominators are all multiples of 10, but otherwise the situation is the same as it is when "common" fraction numerals are used. In *Seeing Through Arithmetic 6*, these ideas are carefully explained to the pupils.

When numbers expressed in decimal notation are to be added or subtracted, it is customary to arrange the numerals so that the decimal points occur in a vertical column. This provides a visual and mechanical way of placing the point in the answer—it is merely put in the same column as the others. Pupils can learn to place the point with little or no attention to the meanings of the symbols that justify the mechanical device.

In multiplication examples this simple device cannot be used. Teachers frequently do not know that it is not only possible, but also quite sensible to write the numerals for partial products and the final product so that



**Expanded notes 216-217**

the decimal points are in a vertical column, as in the example shown here. This procedure does not work automatically, however, and thought must be given to the placement of the numerals in the correct columns. In the example, the multiplication indicated by $.7 \times 3.42$ is also indicated by $\frac{7}{10} \times \frac{342}{100}$, and this product may be written in the form $\frac{2394}{1000}$ or 2.394. The result shows that the numeral for the first partial product must show three "places" to the right of the decimal point to express the thousandths in the answer.

$$\begin{array}{r} 3.42 \\ 2.7 \\ \hline 2.394 \\ 6.84 \\ \hline 9.234 \end{array}$$

The difficult thinking involved in the procedure just outlined has prevented arithmetic programs from trying to teach it. Instead, it is customary to ignore the decimal points and to multiply the "whole" numbers thus implicitly expressed. The traditional arrangement of the numerals then disturbs the vertical column of decimal points, and thus it is necessary to have some other means of locating the decimal point in the answer.

$$\begin{array}{r} 3.42 \\ 2.7 \\ \hline 2.394 \\ 6840 \\ \hline 9.234 \end{array}$$

In the long run, the safest way to be sure the point is correctly located is to understand what the symbols represent. Thus in the example above, the pupil should know that $2.7 = \frac{27}{10}$, and that $3.42 = \frac{342}{100}$. Then from his knowledge of the procedure for multiplying rational numbers, or "fractions," he finds that the product can be expressed as $27 \times 342$ "thousandths." This enables him to place the point in the answer by using his knowledge of the decimal system of numeration.

In *Seeing Through Arithmetic 6*, many examples are worked out in the above manner and explained to the pupils. If some pupils discover through this study that there is a short-cut rule for locating the point in the answer, no harm is done, provided they understand the basis for the rule. This is an entirely different situation from the traditional one in which the rule is presented with no effort to help the pupils understand it. In such situations, if mastery is acquired at all, it is achieved only through prolonged drill. Such instructional methods are characteristic of a traditional but not of a modern approach to arithmetic. In a modern approach, achievement is built upon understanding.

## 218-221 Learning how; Keeping skillful

Lesson Briefs for these lessons are on pages 210-213.

Expanded Notes are not considered necessary for these lessons.

## 222-223 Using arithmetic; Checking up

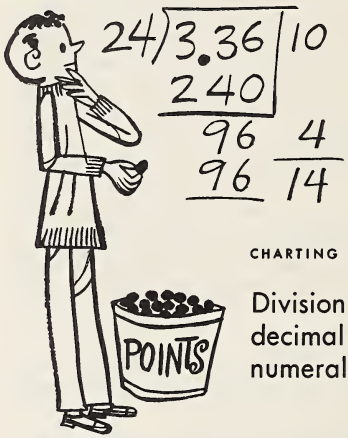
Lesson Briefs for these lessons are on pages 213-215.

Expanded Notes are not considered necessary for these lessons.

## 224 Thinking straight

Lesson Briefs for this lesson are on pages 215-216.

Expanded Notes are not considered necessary for this lesson.




CHARTING THE COURSE

### Division involving decimal fraction numerals

When a division example involves fractions expressed in decimal notation, the placing of the decimal point in the answer becomes an important part of the procedure. As in the case of multiplication, a rule governing the placement of the point is often learned without understanding. In a modern program, however, pupils should be helped to understand the principles underlying the method they are expected to learn. In *Seeing Through Arithmetic 6*, pupils are taught a slight modification of the method most widely used in this country today, and are, of course, helped to understand it. In this method the number of "decimal places" to be provided in the answer is decided at the outset, rather than in the midst of the process. Otherwise, the process is substantially the same as it is when one whole number is divided by another.

In this country today it is customary to replace a division example in which the divisor includes a fraction (expressed in decimal notation) by an equivalent example in which the divisor is a whole number. This is

done by multiplying both dividend and divisor by a multiple of ten. Thus in the example 24.48÷1.7, both 24.48 and 1.7 are multiplied by 10, and the equivalent example 244.8÷17 is thereby obtained. This is the basis of the so-called "caret" method, whether or not the caret is actually used, and even if the principle involved is never explained. In *Seeing Through Arithmetic 6* this part of the procedure is explained at the beginning. (See page 225 of the pupils' book.)

Next, one observes that the new example 244.8÷17 may be expressed in the form $\frac{2448}{10} \div \frac{17}{1}$, or $\frac{2448}{10} \times \frac{1}{17}$, and thus the division of 2448 by 17 is appropriate. The result, which is 144, must still be divided by 10, but familiarity with the decimal system enables one to write the answer as 14.4 immediately. When the usual method of writing the work for computation is used, the decimal point in 244.8 may be temporarily ignored, as the work at the right shows, and after the division by 17 has been completed, the final step of dividing 144 by 10, or expressing the answer as a whole number plus a fraction in decimal notation, is done mentally and the decimal point is written in the answer. Similarly, in the example 63.75÷5, the division of 6375 by 5 is carried through first and then, since 63.75 involves hundredths, the result (1275) is divided by 100 to express the answer as 12.75. Although explained here by the use of particular examples, the method is general.

```
17 ) 244.8  | 100
     170 0  |
      74 8  |  40
      68 0  |
       6 8  |   4
       6 8  |
            |  144
```

In the method widely taught today, the decimal point in the answer is placed, by a more or less mechanical procedure, *before* the division process itself is begun. In the method outlined above, the decimal point is placed by using the meaning of the symbols *after* the division process is completed.

The division process does not always "come out even," and often may be continued indefinitely, as in the simple example 2÷3. For this reason, it is wise to decide at the beginning upon the necessary or desired precision of the answer. The precision is indicated by the number of "places" explicitly shown in the dividend. For example, 126.3÷26 would yield an answer precise to the nearer tenth. If an answer precise to the nearer hundredth is wanted, the example may be written as 126.30÷26, and this will yield the desired precision. If an answer precise to the nearer thousandth is needed, the example can be written as 126.300÷26, and this will again yield the desired precision.

It should be clear, then, that it is possible to place the decimal point by meaningful thinking that applies these methods in any long division example. The way the work is written is only a minor aspect of the procedure, and various feasible ways of writing the work may be used. In *Seeing Through Arithmetic* the long division process is taught in a way that keeps understanding of the various "steps" always in focus. One feature of the method is that entire partial quotients are recorded, instead of only one digit at a time. When the process is used with numbers that involve fractions expressed in decimal notation, this feature is maintained by deciding at the outset how many "decimal places" are to be shown in the answer.

Although the use of the caret is not needed when the procedure taught in this book is used, the use of the caret is shown (pages 234-235). The caret is needed only when the example that is to be worked is written ready for the computer without the necessary changes in the divisor and dividend. When the computer himself writes the example in the form of computing, he naturally writes it with the changes already made.

## 225 Thinking straight

Lesson Briefs for this lesson are on pages 216-217.

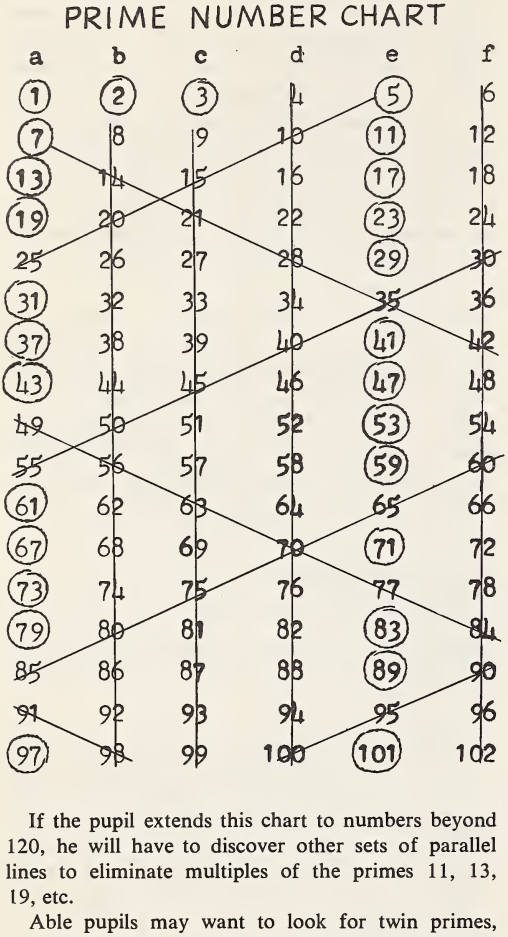
Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil and the slow learner.

**Providing for the able pupil**

Able pupils will benefit from a study of prime numbers. Point out, or have the pupils recall, that a prime number is a number that is divisible only by itself and 1. The pupils should be able to give such examples as 1, 2, 3, 5, 7, 11, etc. Have them make a chart as described below to help them find the prime numbers to 100, to 200, or as far as they wish to extend the chart.

Write the numerals from 1 to 102 (or more) in horizontal rows of six. Since prime numbers are not divisible by any numbers except themselves and 1, any multiple of a prime number has to be eliminated. Therefore, the numbers in Columns b, d, and f (except 2, which is divisible only by itself and 1) are crossed out because they are all divisible by 2. Those under "3" in Column c are crossed out because they are all divisible by 3. Multiples of 5 can be eliminated by a set of diagonal parallel lines from upper right to lower left, as shown. Multiples of 7 can be eliminated by a set of diagonal parallel lines from upper

left to lower right, as shown. The remaining numerals circled on the diagram are names of primes.




If the pupil extends this chart to numbers beyond 120, he will have to discover other sets of parallel lines to eliminate multiples of the primes 11, 13, 19, etc.

Able pupils may want to look for twin primes, such as 11 and 13, 17 and 19, etc. Mathematicians do not know how many twin primes there are.

**Helping the slow learner**

The slow learner should be given as much practice as possible in changing dividends and divisors that are decimal fractions to whole numbers by multiplying by powers of ten. Examples of the type in this lesson should be written on cards. For each example, the pupil should multiply by 10, 100, or 1000, as directed. A series of such experiences should help build up the slow pupils' skills and readiness for division of decimals.



## 226-229 Learning how; Keeping skillful

Lesson Briefs for these lessons are on pages 217-220.

**Overview**

Finding the quotient of two numbers expressed by decimal fraction numerals involves three steps: (1) changing the divisor to a whole number and changing the dividend accordingly (so the quotient will be the same), (2) computing as with whole numbers, and (3) determining the position of the decimal point in the quotient.

The pupils understand the first two steps and, therefore, the primary purpose of this lesson is to teach them how to determine the position of the decimal point in the quotient. The positioning of the decimal point is based upon the pupils' knowledge of division of numbers expressed by common fraction numerals.

**Teaching the whole class**

Before beginning this lesson, pupils should have a clear understanding of the material presented in the two previous lessons (pages 224-225). A brief review of dividing numbers expressed by common fraction numerals will also be helpful.

**Page 226:** Have pupils observe the equation for Example A. Tell them that to divide, it is convenient to express the divisor as a whole number. Have the pupils observe the second equation (having a whole number for the divisor) and note the change in the dividend. Have the pupils recall that the quotient remains the same if the divisor and the dividend are multiplied by the same number. Get the pupils to explain why 10 is the most convenient number by which to multiply 1.7 to get a whole number.

Have the pupils write the numerals in the form used for common fractions and compute. Point out that the first step in this computation is to divide the numbers 2448 and 17 by 17. This gives us the numeral in the numerator of the quotient. Have the pupils explain *why*. The numeral in the denominator of the quotient is the same as the numeral in the denominator of the dividend.

Call attention to the computation using decimal fraction numerals. Point out that the number divided is treated as if it were a whole number. The computation is the same as that used with the whole numbers in the computation with common fractions. The computation determines the numerator of a fraction. The denominator is then expressed by using the decimal point.

For Example B, get the pupils to explain that a second equation need not be written, since the divi-

sor is already a whole number. Have them notice that in the computation with common fraction numerals the whole number 6025 and the whole number 5 are each divided by 5. The answer for $6025 \div 5$ (multiplied by 1) gives us the numerator of the quotient. Ask why the numeral in the denominator is 100.

Again get the pupils to observe that the dividend is treated as a whole number when computing. Have them explain the position of the decimal point in the quotient. Point out that both the dividend and the quotient are expressed as hundredths.

**Page 227:** Discuss the equation and point out that before 2 can be divided by 80, we need another name for 2. Tell them that if 2.000 is used, the answer will "come out even." Write the new equation $(2.000 \div 80 = p)$ on the chalkboard.

Follow the procedures for the previous examples in discussing the computation with common fraction numerals and decimal fraction numerals. Have pupils explain why a zero is placed to the left of 25 in the quotient [to express thousandths]. Use the statements following Example C to guide the pupils in forming the generalization that the quotient is expressed with a decimal fraction numeral to the same number of "places" as the dividend.

Have pupils discuss the questions in the THINK examples. Have them observe that no decimal point is required in the quotient for Example A since the dividend is a whole number.

**Page 228:** In Example C have a pupil explain why a zero is placed to the left of the 2 in the quotient.

Assign the TRY examples as individual work. For each example instruct the pupils to write a second equation that will have the same quotient but with a whole number for the divisor. Have them verify their work with the computation in the text, and help them with any questions that arise.

**Page 229:** After the pupils have completed the TRY examples, assign the DO exercises as individual work. Instruct the pupils to write a second equation for each exercise as they did for the TRY examples. Supply answers so they may verify their work.

The "Keeping skillful" exercises should be assigned as separate lessons, according to pupils' individual needs and abilities.

**Providing for the able pupil**

The able pupils might profit from an investigation of the relationship demonstrated in the examples below.

| | |
|---|---|
| $23 \times .01 = .23$ | $23 \div .01 = 2300$ |
| $23 \times \frac{1}{100} = .23$ | $23 \div \frac{1}{100} = 2300$ |
| $23 \div 100 = .23$ | $23 \times 100 = 2300$ |
| $4.10 \times .1 = .410$ | $4.10 \div .1 = 41.0$ |
| $4.10 \times \frac{1}{10} = .41$ or $.410$ | $4.10 \div \frac{1}{10} = 41.00$ or $41.0$ |
| $4.10 \div 10 = .41$ or $.410$ | $4.10 \times 10 = 41.00$ or $41.0$ |

The pupils should note that *multiplying* a number by .01 (or $\frac{1}{100}$) produces the same result as *dividing* that number by 100, while *dividing* a number by .01 (or $\frac{1}{100}$) produces the same result as *multiplying* that number by 100. The same, of course, is true for tenths and 10, for thousandths and 1000, etc.

The able pupils could work a few examples of this kind to convince themselves that this relationship always holds.

## 230-231 Thinking straight

Lesson Briefs for this lesson are on pages 220-222.

Expanded Notes are not considered necessary for this lesson.

## 232-233 Using arithmetic; Checking up

Lesson Briefs for these lessons are on pages 222-224.

Expanded Notes are not considered necessary for these lessons.

## 234 Side trip; Keeping skillful

Lesson Briefs for these lessons are on pages 224-225.

Expanded Notes are not considered necessary for these lessons.

## 235 Side trip

Lesson Briefs for this lesson are on pages 225-226.

Expanded Notes are not considered necessary for this lesson.

## 236-238 Exploring problems; Keeping skillful

Lesson Briefs for these lessons are on pages 226-228.

Expanded Notes other than suggestions for the able pupil are not considered necessary for these lessons.

**Providing for the able pupil**

Pupils of this age, particularly boys, often have a favorite Big League player or team. Suggest that they look up statistics on their favorite player or team and make up problems similar to those in the lesson. The problems could be used as a follow-up lesson after the teacher has screened them for difficulty. Or the pupils may gather information about a local team and keep a record as a class project.

## 239-240 Exploring problems; Keeping skillful

Lesson Briefs for these lessons are on pages 229-230.

**Overview**

Pupils who have used the *Seeing Through Arithmetic* program have a method for solving comparison problems by writing equations of ratios (in addition to comparing by subtracting). They have no need to translate the conditions of a problem into a sentence, such as "57 is .95 of what number?" or "What number is .875 of 76?" before solving it. However, they will encounter statements like these in many standardized tests, and they should know that such a statement can be expressed by an equation of ratios. This lesson, involving decimal fraction numerals, is presented to give pupils the needed understanding. A similar lesson, involving per cents, is on pages 134-135.

**Teaching the whole class**

**Page 239:** Have Problem A read and discussed. Point out that Carol's weight (68 lb.) is compared with Jim's weight (76 lb.). Ask what ratio expresses this comparison [68/76]. Remind the pupils that, to find what fraction Carol's weight is of Jim's weight, they must find how much Carol weighs for each pound Jim weighs. Since Carol's total weight is less than Jim's total weight, she weighs less than 1 pound for each pound Jim weighs. In the equation, $n$ represents a fraction of pound, and the problem requires that this fraction be expressed as a decimal fraction numeral in hundredths.

Discuss the equation of ratios. Have the pupils explain that the first term of each ratio represents Carol's weight, and the second terms represent Jim's corresponding weight. Have a pupil write the equation resulting from the ratio test [$76\times n=68\times 1$] and do the computation necessary to find the replacement for $n$. Then replace $n$ in the original equation with the correct decimal fraction numeral.

Have the pupils observe the completed equation of ratios. Point out that the ratio 68/76 has been expressed as another ratio [.89/1] that has a decimal fraction numeral as its first term. If we wanted to express this ratio with a fraction numeral as its first term, we would still use the equation resulting from the ratio test [$76\times n=68\times 1$] and would find $n=$ $^{68}\!/_{76}$ (or $^{17}\!/_{19}$). Replacing $n$ in the ratio equation, we would have:

$$\frac{68}{76}=\frac{\frac{68}{76}}{1}$$

ratio ... ratio ... fraction numeral

It is important to realize that originally 68/76 represented a *ratio,* but in the solution, $^{68}\!/_{76}$ is a *fraction numeral* that is the first term of the ratio $^{68}\!/_{76}/1$.

Refer again to Problem A and point out that the problem can be restated as "68 is what fraction of 76?" Be sure the pupils understand that this is another way of stating the conditions or data in the problem.

Have Problem B read and have the pupils explain each ratio in the equation. Have them observe what the ratio .875/1 tells us [that a year ago Jim weighed .875 of a pound for every pound he weighs now]. $k/76$ represents the comparison of his *total* weight a year ago with his *total* weight now.

A pupil may give the equation that results from using the ratio test and do the computation on the chalkboard. Through discussion the pupils should arrive at the sentence.

In a similar manner discuss Problem C. Have the pupils point out what the first ratio tells us [that for every 1 inch of Cal's height now, his height was only .95 of an inch a year ago]. The second ratio compares his *total* height a year ago to his *total* height now. Through discussion arrive at the sentence.

Have the pupils read Problem D, give the equation of ratios, and explain each ratio. Then have them do the computation and choose the correct sentence to express the problem. They should see that Sentence 2 is the correct sentence for this problem.

**Page 240:** Apply the procedures suggested for Problem D to Problems E and F.

Assign Exercises A to J as individual work. Tell the pupils to make an equation of ratios for each exercise and then to solve the equation. Guard *against* the practice of merely trying to replace "is" by "=" and "of" by "×." This is a mechanical short cut which contributes nothing to meaning and understanding. Emphasize that the statements compare one number with another and, therefore, can be expressed by equations of ratios. The equation used for computation results from the ratio test, *not* from a mechanical replacement of words by other symbols.

When verifying the pupils' work, have them discuss each equation. Also have them evaluate each solution in terms of the sentence.

The "Keeping skillful" exercises give pupils additional practice with statements involving per cents and with the multiplication and division of numbers expressed by decimal fraction numerals. These exercises should be assigned as a separate lesson or lessons, as class needs demand. The examples are few in number, but considerable time is required to solve them.

**Providing for the able pupil**

Provide a list of statements, such as ".89 of 66 is ▒," etc. Have the able pupils make up a problem for each of the statements. For example, the statement ".89 of 66 is ▒" may express this problem: Alice's weight is .89 of George's weight. George weighs 66 lb. What does Alice weigh?

Also it would be valuable to provide a set of equations of ratios and have the pupils write a statement for each of these equations.

Practice in relating the statement to the problem situation, or an equation of ratios to the statement, will help the able pupil to see the relation between these statements and the problems they express.

## 241-242 Thinking straight

Lesson Briefs for this lesson ore on poges 231-232.

Expanded Notes for this lesson are not considered necessary except for suggestions for the able pupil.

**Providing for the able pupil**

Able pupils may like to try to find a common fraction numeral that is not a "repeater" when written as a decimal fraction numeral. Suggest that they use fraction numerals that have denominators between 1 and 15. Point out that some fractions, like $\frac{1}{7}$ and $\frac{1}{13}$, will have to be expressed to several decimal places before they will repeat. They will discover that every fraction will eventually repeat the same sequence of digits when expressed by a decimal fraction numeral. Those fractions that "come out even" repeat zeros.

It may also be of interest to the able pupils to see that, once they know the decimal fraction numeral for $\frac{1}{2}$, they can use a short cut involving division by 2 to express the family of common fraction numerals $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc., as decimal fraction numerals. The process is as follows:

$\frac{1}{2}$=.5000 Then $\frac{1}{4}$=.5000÷2 or .2500
$\frac{1}{8}$=.2500÷2 or .1250
$\frac{1}{16}$=.1250÷2 or .0625
etc.

The same procedure works for $\frac{1}{3}$ and $\frac{1}{5}$:

$\frac{1}{3}$=.3333 $\frac{1}{5}$=.2000
$\frac{1}{6}$=.3333÷2 or .1667 $\frac{1}{10}$=.2000÷2 or .1000
$\frac{1}{12}$=.1667÷2 or .0833 $\frac{1}{20}$=.1000÷2 or .0500
etc. etc.

## 243 Thinking straight

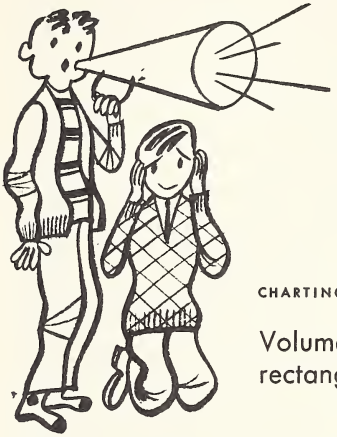
Lesson Briefs for this lesson are on page 233.

Expanded Notes are not considered necessary for this lesson.

## 244-246 Exploring problems

Lesson Briefs for this lesson ore on poges 234-236.

Expanded Notes are not considered necessary for this lesson.



CHARTING THE COURSE

### Volume of the rectangular prism

Pupils were introduced to the concept of volume on pages 13 to 16 of *Seeing Through Arithmetic 6*. At that time the emphasis was entirely upon the meaning associated with the word *volume*. For example, pupils were expected to find the volume of a box by the direct method of counting (or estimating) the number of cubic units that would "fill" it. The computation of volume by more economical procedures is taught on pages 248-258 by methods that are similar to those used to teach the computation of area on pages 152-161.

To an adult, and especially to a teacher, the rule for the computation of the volume of a rectangular prism may seem very simple and obvious. One merely has to multiply the length by the width, and then this product by the height. Long familiarity with the rule tends to obscure the reasons underlying it. Actually, the rule serves as an indirect way of finding the volume by using three measures of length and a computation to arrive at the number of cubic units the prism contains.

In the *Seeing Through Arithmetic* program, pupils are helped to see that the fundamental idea underlying the computation of volume is the concept of rate. Since this concept is also used in many other situations, it is a basic unifying concept. The "rate" approach to the computation of volume thus serves to bring this problem also within the scope of the more fundamental idea.

The computation of volume requires a double use of the rate concept. First, the number of cubic units that can be accommodated on the base can be found by the method used to compute area. This result determines another rate, namely, the number of cubic units per 1 "layer." For example, suppose the base of a rectangular prism is 5″ x 4″, and the height is 3″ (see page 248 of the pupils' book). In this example, the number of units in the length, 5, determines a rate of 5 cubic inches per row. By using the number of units in the width, 4, the total number of cubic units on the bottom layer may be written as $5\times4$. This number of cubic inches on the bottom of the box determines another rate that is indicated by the ratio $(5\times4)/1$, or 20 cubic inches per 1 layer. However, it is easier to see the basis for the rule (or formula) that will eventually emerge if, instead of computing the value 20, the ratio is left in the form $(5\times4)/1$.

Next, each unit of height determines a corresponding layer of cubic units. In the example mentioned above, the height of 3″ shows there will be 3 layers (each of 20 cubic inches) contained in the prism. The second rate, then, can also be expressed by another ratio, $w/3$, in which the first term, $w$, represents the total number of cubic inches, and the second term, 3, is determined by the number of layers.

Now, since both ratios express the same rate, the equation $5\times4/1=w/3$ may be written. Finally, the use of the ratio test in this example produces the familiar equation $w=5\times4\times3$.

This development can, of course, be generalized and will lead to the familiar rule that applies to each rectangular prism, whatever its dimension numbers may be. One who has never before seen such a development of the rule (or formula) can hardly fail to recognize that he has up to that point been accepting and using the rule on faith or on the basis of its success in practical problems, rather than on the basis of his understanding of the mathematical reasoning underlying it.

The traditional mathematics programs were content to state computational rules and then to drill on their use. The demand for sounder modern programs stems in part from the recognition that the development of genuine mathematical competence requires more than this. Pupils must be helped to understand the fundamental basis for the rules. When they do, the rules themselves are hardly needed. In *Seeing Through Arithmetic* 6, the rule is not formally stated, and its presentation as a formula is deferred to a later grade.



## 247 Thinking straight

Lesson Briefs for this lesson are on pages 236-237.

Expanded Notes are not considered necessary for this lesson except for suggestions for the able pupil and the slow learner.

**Providing for the able pupil**

Diagram B on page 247 may provide motivation for able pupils to make models of similar prisms. The completed models could be displayed with labels showing the surface areas.

Able pupils might also be interested in building the polyhedrons described in Activity 40, page 399.

**Helping the slow learner**

To help these pupils understand the concept of surface area, you might have them make diagrams of the type shown in Diagram B. Give the pupils dimensions for a rectangular prism, such as 8″ x 3″ x 5″, and ask them to make a diagram like the one shown in Diagram B. The diagrams need not be drawn to exact scale, but the dimensions should be labeled.

## 248-252 Exploring problems

Lesson Briefs for this lesson are on pages 237-240.

**Overview**

In this lesson the pupils learn how to compute the volume of a rectangular prism in cubic inches by using an equation of ratios. The first ratio represents the number of cubic inches per one layer, and the second ratio represents the total number of cubic inches per all the layers. Pupils should observe that in each case the second term of one ratio is 1; therefore they can use the short cut shown on page 249 for solving equations of ratios. The resulting equation is similar to the equation expressing the traditional rule, "Volume is equal to measure of the length times measure of the width times measure of the height." But the pupils have arrived at this short cut through an understanding of what is taking place, not by blindly following a formula.

**Teaching the whole class**

**Page 248:** In presenting the problem on this page, it will be helpful for the teacher to construct a rectangular prism of the given dimensions and to have available 1-inch cubes to demonstrate the action taking place for each step of the solution.

Have the pupils observe the dimensions of the box and notice how many 1-inch cubes can be placed in one layer on the bottom. Ask the pupils how many square inches are the area of the base of the prism. Have them observe that the number of square inches

is the same as the number of 1-inch cubes that can be placed in one layer on the base of the prism. Get them to explain that these two numbers are equal because the area of the base of each cube is 1 sq. in.

Point out that the ratio expressing the number of cubes per one layer (20/1) can also be written with the first term expressing the dimensions of the base: $(5\times4)/1$.

Get the pupils to explain that the box will hold three layers of 1-inch cubes, because the height of the box is 3 in. and each layer of cubes is 1 in. high. Have them explain the ratio $w/3$.

**Page 249:** Be sure the pupils understand that the rate for three layers of cubes is the same as the rate for one layer, and, therefore, an equation of ratios can be written. Have the pupils observe that the first term of each ratio represents the number of cubic inches and the second term represents the number of layers. Ask the pupils to apply the ratio test to the equation. Have them observe that the second term of the first ratio is 1; therefore they can find $w$ by multiplying $(5\times4)$ by 3. The product (60) is the number of cubic inches in the box.

**Page 250:** Have the pupils observe the dimensions of the piece of cheese. Ask them to study Picture H and imagine how the prism would look if it were 1 in. high. Point out that it would then have 1 layer containing $(3\times2)$ one-inch cubes; therefore the rate of cubic inches per layer is $(3\times2)$. Have the pupils give the ratio that expresses this rate.

Point out that to find the volume of the prism means to find the number of cubic inches in *all* the layers. The rate expressed in terms of the number of cubic inches per *all* the layers is the number of cubic inches per ¾ layer. Have the pupils give the ratio for this rate and explain its terms. Be sure they understand that the rate for one layer is the same as the rate for ¾ layer and that they can write the equation of ratios $[3\times2/1=d/¾]$. Again have them observe that since the second term of one ratio is 1, they can find $d$ by multiplying $(3\times2)\times¾$. Point out that the parentheses indicate which numerals are in the first ratio but that the equation can be written without them $[d=3\times2\times¾]$. Have the pupils recall the associative property of multiplication, which they studied on page 31, and let them give different ways of obtaining the product.

In the equation of ratios for this problem, the first terms of both ratios represent number of cubic inches, and the second terms represent number of layers. When the dimension measure is a fraction, a new unit is involved. For example, if the height is ¾″, we can think of a new unit of volume in which the base is a 1-inch square, and the height is ¼″. This, of course, is not a *standard* unit, but it is nevertheless a useful unit. There will be 3 layers of these units. It takes 4 of these units to make 1 cubic inch. When we compute, we multiply by the 3 of the ¾. This tells us how many of these new units there are in all. We must group these into groups of 4 to find how many cubic inches there are. Thus when we divide by the 4, we find by computation how many cubic inches there are in 3 layers of the new units, or in one "layer" that is only ¾″ in height.

**Page 251:** Have the pupils relate the pictures to the problems and write an equation of ratios for each problem. Be sure the pupils understand that the height of each prism tells the number of layers of volume units the prism holds. They should also realize that the number of square inches in the base of the prism is the same as the number of units in the volume of one layer. Have them apply the ratio test to each equation. Call attention to the short cut that can be used when the second term of one ratio is 1. By this time some of the pupils will probably observe that multiplying the three dimensions will give the number of cubic inches. However, it is important that the pupils understand the rates involved.

**Page 252:** Have the pupils write equations of ratios to find the volumes of the prisms shown at the top of the page. When the pupils have finished the exercises, supply answers so that they may verify their work. Discuss any exercises that have caused difficulty.

When assigning Problems A to F, remind the pupils that they may have to use information from preceding problems. Also point out that some of the answers are to be rounded off.

**Providing for the able pupil**

Able pupils may compute the volumes of the prisms that they constructed in the previous lesson. Display these prisms with only the dimensions given. Have the pupils fill in information about the prisms on individual charts like the following:

| Prism | Surface Area | Volume |
|---|---|---|
| John | | |
| Paul | | |

After the pupils have verified each other's work, the information can be displayed with the prisms.

**Helping the slow learner**
Slow learners will benefit from verifying their computation by discovering for themselves the number of 1-inch cubes that will fill various rectangular prisms.

## 253-256 Moving forward

Lesson Briefs for this lesson are on pages 241-243.

**Overview**
The concept of volume is extended to include the cubic foot and the cubic yard. The cubic foot is introduced in terms of its volume in cubic inches, and the cubic yard is introduced in terms of its volume in cubic feet. The computational reduction of cubic measure is developed.

**Teaching the whole class**
So far the pupils have computed the volumes of rectangular prisms in cubic inches only. In this lesson, they learn to compute to find volumes in cubic feet and cubic yards and to change units of cubic measure when necessary. When presenting this lesson, you may want to have on hand a 1-foot cube and a 1-yard cube. Then pupils can see the volumes in correct relationship.

**Page 253:** Let the pupils observe the picture at the top of the page and/or the full-size model. Point out that the cubic foot is a unit of cubic measure, just as is the cubic inch, and that sometimes it is more convenient to measure with cubic feet than with cubic inches. Get the pupils to think of each edge of the cube as 12 in. Have them explain why the first layer will contain 144 cu. in. Have this expressed by a ratio [144/1] with the first term representing the number of cubes and the second term representing the number of layers.

Ask the pupils how many such layers the prism will hold. Get them to explain that there will be 12 layers, because each layer is 1 in. high and the prism is 12 in. high. Point out that the rate in terms of cubes per 12 layers can be expressed as $z/12$, with $z$ representing number of cubes and 12 representing number of layers.

In connection with Exercise C, get the pupils to explain that since each layer contains the same number of cubes, cubic inches will be used at the same rate for 12 layers as for 1 layer. Therefore, an equation of ratios $(144/1=z/12)$ can be written. Since the second term of one ratio is 1, the replacement for $z$ can be found by multiplying 144 by 12. Have the pupils explain why the equation $z=12\times12\times12$ can be used. Point out that now the number of cubic inches in 1 cu. ft. has been determined.

**Page 254:** Treat Exercises D to F as you did Exercises A to C on the preceding page. When the exercises have been discussed, point out that we now know the number of cubic feet in 1 cu. yd.

Discuss Exercise G and observe the accompanying diagram. Have the pupils write an equation of ratios and verify that the volume of the packing case is 12 cu. yd. Point out that Exercise H asks that this volume be expressed in cubic feet. Get the pupils to observe, by studying Diagram H, that there will be 27 cu. ft. per 1 cu. yd., and that this rate will be the same for 12 cu. yd. Be sure they interpret the ratio equation in this way. Have the pupils compute to find the replacement for $e$ and verify the answer. Some pupils may suggest verifying the answer by expressing each dimension in feet and writing an equation of ratios for this situation $[(9\times6)/1=e/6]$.

**Page 255:** The pupils should note that Exercise I requires that cubic feet of volume be expressed in terms of cubic inches. Follow the same procedures as you did for Exercise H. Get the pupils to observe that the rate of cubic inches per cubic foot is 1728 per 1 (1728/1) and that this rate is used in this exercise. Have the pupils explain each term of the equation of ratios and compute to find the replacement for $a$.

For Exercise J, have the pupils verify the statement that the room contains 1344 cu. ft. Point out that Exercise K asks that this volume be expressed in cubic yards and have the pupils explain the equation of ratios needed. Let them compute to verify the answer.

Point out that Exercise L requires the volume of the chest of drawers to be expressed in cubic feet. Since the dimensions are given in inches, the volume can first be found in cubic inches.

**Page 256:** Discuss the dimensions of Diagram A and point out that some of the dimensions will have to be expressed differently to compute the volume in cubic inches. Have the pupils compute the volume in cubic inches. Point out the two ways in which the volume can be found in cubic feet and have the pupils write an equation of ratios for each way suggested in Exercise N.

Use Exercises O to Q as individual written work. Before assigning the exercises, remind the pupils to observe the dimensions for the prisms in Exercise O and tell them to use whichever dimensions seem simpler. Point out that using fraction numerals to write dimensions may make computation simpler.

**Providing for the able pupil**
Able pupils can measure orange crates, tool chests, or other types of rectangular prisms and then compute

the volumes. Measurements may be given in different units, or pupils might find the equivalent volumes in other units of measure.

**Helping the slow learner**
Slow learners will also benefit from finding volumes of various rectangular prisms in different units of measure. Keep the dimensions simple.

### 257 Using arithmetic

Lesson Briefs for this lesson ore on poge 244.

Expanded Notes are not considered necessary for this lesson.

### 258 Checking up; Keeping skillful

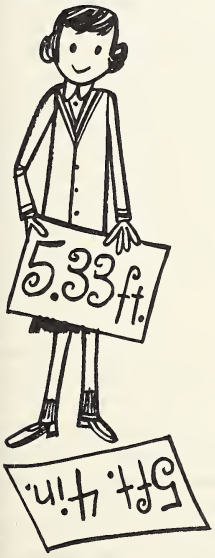
Lesson Briefs for these lessons ore on poges 245-246.

Expanded Notes are not considered necessary for these lessons.

### 259-261 Exploring problems

Lesson Briefs for this lesson are on poges 246-249.

Expanded Notes are not considered necessary for this lesson.




CHARTING THE COURSE

## Mixed measures

In the arithmetic programs of a generation ago much attention was given to computations involving measurements expressed in two different units, traditionally termed "compound denominate numbers." This topic has gradually been diminishing in importance in practical applications, and a modern arithmetic program must put much less emphasis on it than the traditional programs do. The gradual acceptance and increasing use of the decimal system has contributed to this trend. Thus a measure that formerly would have been reported as 5 ft. 4 in. will today often be given as 5.33 ft. The increasing use of the metric system of measures is only one instance of a general trend toward the use of simpler and more efficient measures.

In *Seeing Through Arithmetic 6*, pupils are introduced to a few simple computational problems involving compound denominate numbers. This is justified not only because such problems occur in standard tests, but also because they do still arise outside the classroom.

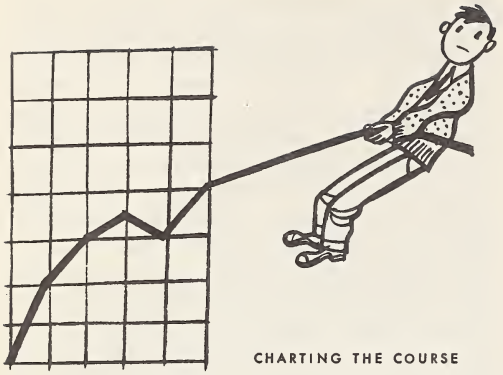
The essential idea that underlies operations with expressions involving different units is that careful attention must be given to the units. The central problem is not so much arithmetical as it is physical and practical. A measurement reported as 4 ft.+8 in. can be combined with one of 3 ft.+6 in. to produce a total length of 7 ft.+14 in. The arithmetic is very simple, requiring merely two addition facts, $4+3=7$ and $8+6=14$. Since in this case both units are associated with length, the result is meaningful. One would not, however, add 7 and 14, because of difficulty in the interpretation of the result. However, 14 in. is equivalent to 1 ft. + 2 in., so that 7 ft. + 14 in. becomes 7 ft. + 1 ft. + 2 in. or 8 ft. + 2 in. In such examples it has been customary to omit the plus sign, just as it is when $3+\frac{1}{2}$ is written as $3\frac{1}{2}$.

The above procedure is analogous to the regrouping that is done when "carrying" in an addition example, and when "borrowing" in a subtraction example. In the *Seeing Through Arithmetic* program, the underlying principles have been taught in lessons on the base ten numeral system. A little work on measurements expressed in two different units would doubtless be justified merely because of the extra appreciation it brings of the underlying principles common to the various concrete measurement systems and the Hindu-Arabic numeral system.

### 262-264 Side trip; Keeping skillful

Lesson Briefs for these lessons ore on poges 249-252.

Expanded Notes are not considered necessary for these lessons.



CHARTING THE COURSE

# Graphs

Nowadays the graphical presentation of statistical data is so common that even young children can profit from a simple introduction to the interpretation of graphs. They see graphs in newspapers, schoolbooks, pamphlets, and even in cartoons. To an adult, the interpretation of elementary graphs seems quite obvious. The problem of interpretation is not so simple for the pupils in elementary schools. They need some instruction on it.

*Seeing Through Arithmetic 6* provides materials on pages 265-272 to help pupils acquire some of the basic concepts and skills of interpretation. The primitive aspects of negative numbers are introduced. Pupils are shown how line graphs can be used to express rates. Simple frequency tables are introduced, and several ways of depicting them graphically are shown and interpreted in an elementary way.

No doubt more and more attention will be given to topics of this kind in the future. They are comparative newcomers to the mathematical and social scene.

Traditional courses of study and pressure to achieve computational skill have delayed the day when the interpretation of tables and graphs will be accepted as an important part of the arithmetic program.

## 265 Thinking straight

Lesson Briefs for this lesson are on pages 252-253.

Expanded Notes are not considered necessary for this lesson.

## 266-269 Moving forward; Keeping skillful

Lesson Briefs for these lessons are on pages 254-257.

**Overview**

In "Moving forward" the pupils see how familiar number pairs that represent rates can be made to correspond to points in a two-dimensional graph. Emphasis should be placed on determining number pairs and naming points in a graph, to prepare pupils for drawing graphs in the lesson that follows.

**Teaching the whole class**

**Page 266:** You may wish to use page 266 as a basis for discussion. Ask a pupil to state the information given in the poster in Picture A. Proceed somewhat as follows:

*The poster tells us that balloons cost 3¢ apiece. Let's make a table of that information [draw a form for a table as in Picture B]. If we buy 1 balloon, we will have to pay 3 cents. So let's put the numeral pair (1, 3) in the table. What other numeral pairs can be put in the table?*

Point out that as soon as the number of balloons is known, the cost can be found. The table has two sets of numerals, one set showing the number of balloons and the other set showing the cost of the balloons. The members of one set are paired with the members of the other set to make the numeral pairs. Then continue in the following manner:

*Now let's locate the points corresponding to these numeral pairs in a graph. Here are two lines for our graph. [It is not necessary here to discuss why the lines are at right angles. The important thing to be brought out is that each numeral pair in the table can be represented by a point in the graph.] On one line we'll put numerals to show the number of balloons; and on the other line we'll put numerals to show the cost of the balloons. What numerals shall we use?*

Have the pupils supply the numerals from the table. Be sure they notice that only numerals for multiples of 3 are used on the line representing cost, and only numerals for whole numbers are used for the number of balloons. You may ask why this is so; or you may ask the pupils to imagine that the balloons cost 5 cents apiece, and ask what the numerals on the cost scale would be in this case.

Then locate in the graph the point for the numeral pair (1,3). Have the pupils observe where it is located (count over to 1 and up to 3). Tell the pupils that the name for this point is "one, three" and is written (1,3). Write the name beside the point. Let the pupils come to the board and locate points for the other

pairs in the table. Also beside each point have them write the name of the point.

To emphasize the ideas introduced thus far, ask the pupils how the cost of balloons at 5 cents apiece could be represented in a graph. Let the pupils supply numeral pairs for a table. Draw two lines at right angles and ask the pupils to tell what numerals belong on each line. Have them locate and name the numeral pairs for this graph.

**Page 267:** When you discuss Exercise A, point out that the ratio test can be used to find the cost of any number of balloons. You may wish to have the pupils check the numeral pairs on page 266 by using the ratio test. As the pupils answer Exercise C, have them explain how they found the numeral pairs. Any of the pairs can be found by using the ratio test. However, the pupils may have used other ways, such as noticing that the number represented by the second numeral of each pair is 3 times the first; the first is ⅓ of the second; or the second in each pair is 3 more than in the preceding pair.

Be sure the pupils understand that fractions are not used in this situation because it is not possible to buy a part of a balloon or spend fractional parts of a cent. Also discuss why numerals for 1 cent, 2 cents, 4 cents, etc., are not needed in the graph.

The form for writing numeral pairs discussed in Exercise G is one the pupils will use in later grades. Have the pupils observe what each numeral stands for. To stress the importance of how the numeral pair is written, have the pupils locate points in a graph as (2,6) and (6,2) or as (3,9) and (9,3).

As the pupils give some of the numeral pairs they have chosen, have them explain how they determined the pairs and locate the pairs in the graph on the chalkboard. (If necessary, extend the scales.) Have the pupils observe that this set of numeral pairs gives a series of unconnected points lying in a straight line. Be sure they understand that points between those in the graph would express fractions.

**Page 268:** Since the pupils have had experience with numeral pairs on the two preceding pages, they should have little or no difficulty in writing the answers for Exercises A to D. When they have finished, discuss each exercise. Have the pupils explain how they found the missing numerals for Exercise C. Emphasize that fraction numerals can be used for this situation, and, therefore, the number of pairs is unlimited.

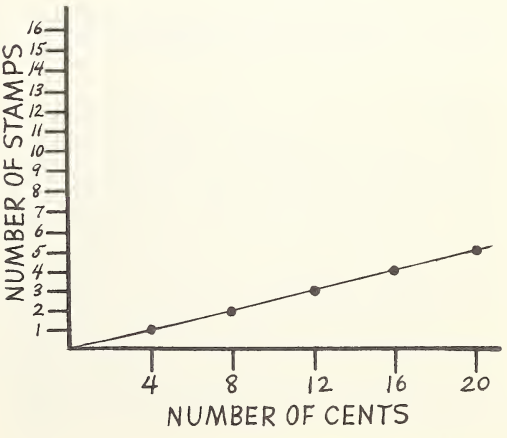
You may put a large graph on the chalkboard like the one on page 268 and refer to it while discussing Exercise E. Have the pupils put the numerals mentioned in Exercise E in the scale and then locate the numeral pairs they found for Exercises C and D. Help the pupils understand that since the number of pairs is unlimited, the graph will be a line.

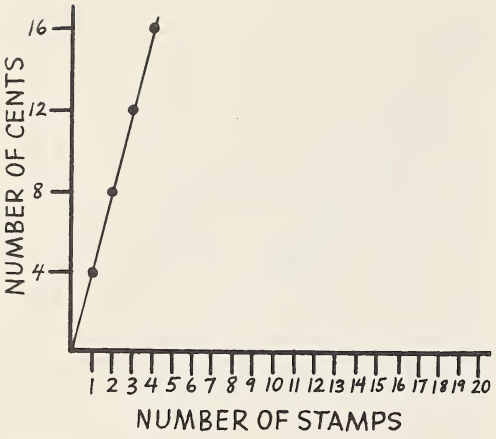
**Page 269:** Discuss Exercises H to K. Be sure the pupils understand that each numeral pair in the table is included in the line. Label any point A in the line on the chalkboard graph. Have the pupils try to count the number of points at the left of A to help them understand that the number of points to the left of A is unlimited. Then refer to the graph on page 266. Look at any point in this graph. Have the pupils count the number of points at the left of this point. See if the pupils can explain the difference in the two graphs. For Exercise I, call attention to the fact that the name for any point in the line can be put in the table because the numeral pairs for each two points satisfy the ratio test, but it would be impractical to list all of the points, since the number is unlimited.

After the pupils have completed Exercises L and M, discuss each exercise. Have them explain how they found the numeral pairs for Exercise L, and why they could use fraction numerals in Exercises 3 and 5. The pupils may have a variety of answers for Exercises N to Q. Discuss the answers to make sure they are sensible.

**Providing for the able pupil**

Able pupils may wish to draw graphs for Exercise L. Be sure they observe which graphs will have isolated points and which will have lines. The pupils may also wish to investigate what happens when they interchange the scales for any graph. For example, the graph of the cost of postage stamps at 4¢ each may be either of the following:






The pupils should observe that the graphs for Exercise 3 will be the same, even though the scales are interchanged.

Pupils may also bring to class graphs they have found in newspapers and magazines. Have them make tables of the information given by the graphs and select numeral pairs in these graphs.

**Helping the slow learner**

Slow learners should get as much experience as possible in determining numeral pairs, locating them in a graph, and naming points in a graph. Provide other rates like those in Exercise L, and have the pupils find numeral pairs for them. They might also locate some of these points in a large graph on the chalkboard. Or, draw a large graph on the chalkboard and have the pupils name various points in it.

## 270-272 Moving forward; Keeping skillful

Lesson Briefs for these lessons ore on poges 257-259.

**Overview**

Statistical data in the form of test scores are used to teach the use of frequency tables and frequency graphs. Three ways of representing the numeral pairs in the table are shown: points, bars, and "broken lines." The pupils also learn how to use information from the frequency table to find the average.

**Teaching the whole class**

Throughout the pupils' study of graphs, they should look for graphs in newspapers, magazines, and books. Have these graphs put on the bulletin board.

**Page 270:** There are times when we want to know how the class as a whole is doing, without paying special attention to any one pupil. Test scores or grades can be listed, and statistical tables then compiled. The work on this page is built around such a statistical compilation.

To begin the lesson, call attention to the arithmetic scores listed at the left side of the page. Be sure the pupils realize that each score stands for the number correct out of the entire test. Have the pupils read Exercise A, and then let them answer the questions. Also call on pupils to answer the first two questions in Exercise B. Then discuss the meaning of the word *frequency* as applied to the test scores. Discuss the remaining questions in Exercise B, and lead the class to see that it is hard to get a clear picture of comparisons among the scores.

Next call attention to the table at the top of the page. Point out that here the scores with their frequencies are tabulated. For Exercise C, have the class verify each of the frequencies by referring to the scores in the typewritten list.

In discussing Exercise D, ask if test scores from 0 to 6 could have been given in the table, and what frequency would be listed after each score. [0]

Next discuss the meaning of the numerals in the column headed "Total Points." Point out that when three people got a score of 14, they were responsible for $3 \times 14$, or 42 points. Have the pupils complete Exercise E.

**Page 271:** Before considering Exercise F on this page, ask the pupils to look back at the list of test scores on the left side of page 270. Ask them to find the average score. Discuss the difficulty of adding such a long column. Tell the class that another way to find averages is to use a frequency table. Then turn attention to Exercise F on page 271. Point out that when many scores are involved, this method of finding the total of all the scores is easier, since each column has only nine numerals. Then discuss why the total of the frequency column gives the total number of pupils. [The frequency column gives the number of pupils that obtained each score.] Have the pupils find the average. Discuss the advantages of using the frequency table method.

Next direct attention to Graph A. Ask why, when there are actually 30 pupils in the class, only 9 numerals are listed at the left side of the graph. [The highest frequency was 8; so 9 is more than enough.] Then point out the break in the horizontal scale for scores. Tell pupils the break indicates that the part of the scale shown on the graph does not start at 0. You might ask the better students how the graph would change in appearance if the numbers 0 to 6 had been represented on this horizontal line. [The

horizontal line would be longer, and the vertical and horizontal lines would meet at a point whose name is (0,0).]

Have the pupils answer the questions about Graph A in Exercise G. Bring out that the score with the highest frequency is easily identified because it is at the highest point shown in the graph. Then have pupils verify the position of each of the points by referring to the frequency table on page 270.

Next have the pupils look at Graph B. Discuss the meaning of the numerals on the horizontal and vertical lines. Bring out that the tops of the bars correspond with the points in Graph A. Have the pupils verify the pairs of numerals involved in each case by referring to the point graph and to the frequency table on page 270. Then have the questions in Exercises H, I, and J answered. The pupils should agree that Graph B gives a better picture of the frequencies of the scores than Graph A, in that it makes comparison easier.

In discussing Graph C and Exercises K and L, bring out that the points in Graph A are now shown connected by short lines. Explain that the points on the lines between the points do not represent actual number pairs in the problem, since there are no scores or frequencies other than those shown in the original table. Have the pupils verify this fact by referring to the frequency table. Then refer them to the graph on page 268, in which each point of a line does have a meaning. Tell them that, in considering any line graph, they need to determine whether the line connects significant points only to give a better outline or picture of changes, or whether the line represents points corresponding to number pairs that are meaningful in the problem.

**Page 272:** Have the pupils look over the arithmetic test scores for the Friday test, then follow the directions in Exercise M and complete the frequency table on their papers.

The pupils should have no difficulty in finding the class average by means of the frequency table. You might ask them to find the average by using the original list, too, to verify the answer.

Then ask the pupils to make the three graphs called for in Exercise O. Have pupils display their graphs afterward.

As a follow-up activity, you could tell the pupils that they will use the scores they obtain in Exercises A to T in the "Keeping skillful" section to make a class frequency table and graph. Keep this work for another day, if necessary. After the pupils have scored their own "Keeping skillful" answers, ask them to turn in their papers to you. You can then assign identifying letters to each final score and record them on the board in that way, so that no one in the class knows who attained each of the scores. The frequency table and graph can be made at the chalkboard, with the entire class contributing.

**Providing for the able pupil**

Frequency tables and graphs that use statistics other than class scores can be made. For example, a group of able pupils might want to collect data about the kinds of pets owned by the pupils' families, to discover, among other things, which kind of pet is the most popular. Or a listing of the musical instruments played by the pupils can be made. After the tables and graphs have been made, they should be displayed and used for discussion.

The newspaper and magazine graphs that have been on display on the bulletin board may be used for class discussion. You should stress the fact that most graphs represent sets of number pairs. Have the pupils point out some of the pairs in each graph.

**Helping the slow learner**

Slow learners should also make their own frequency tables and graphs. For them, however, use a smaller number of items. If the compilation concerns data about classmates, limit it to facts about a small group of pupils rather than the whole class. Or suggest activities that involve a small range—for example, the number of members in class families.

## 273 Side trip

Lesson Briefs for this lesson ore on poge 260.

Expanded Notes are not considered necessary for this lesson.

## 274-278 Looking back

Lesson Briefs for these lessons ore on poges 261-265.

Expanded Notes are not considered necessary for these lessons.

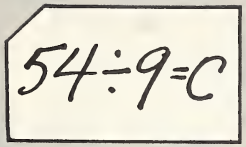
## 279-282 Checking up

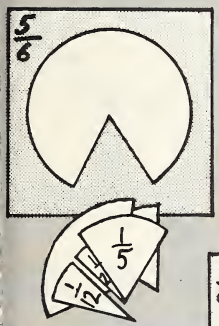
Lesson Briefs for these lessons ore on poges 266-269.

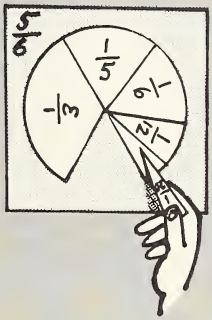
Expanded Notes are not considered necessary for these lessons.

Your notes

# Activities



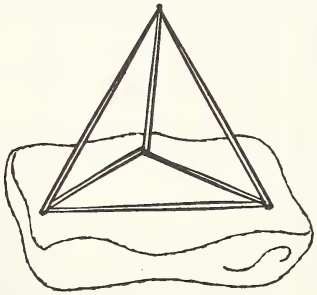



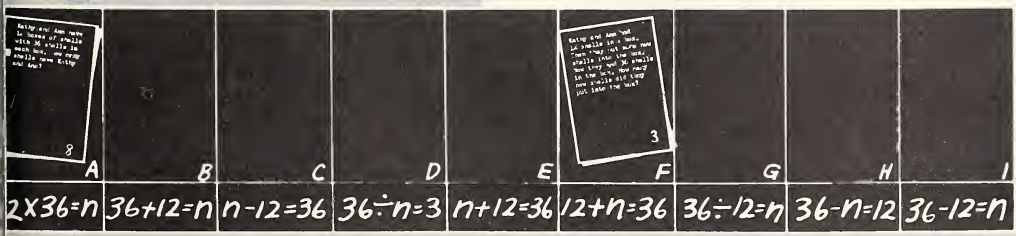

This section of the *Teaching Guide* contains suggested activities specially designed to extend the child's concepts and generalizations about arithmetic. Specific references are made to these activities in the Expanded Notes.

The activities are arranged in the order of their use in the Expanded Notes and are numbered to make reference to them easier.

You will undoubtedly discover new ways of using these activities and ways of adapting them to other topics.

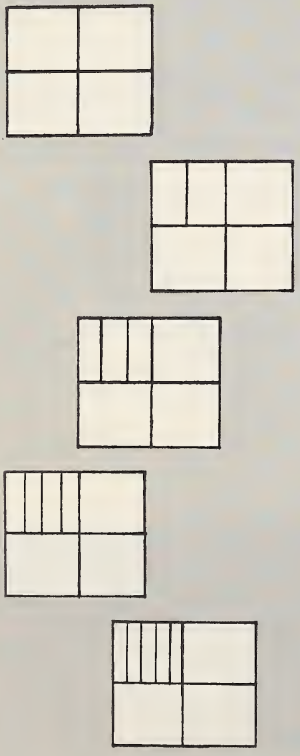
You will also be sure to think up other activities as good as or better than the ones described here.





# Activities

**Useful arithmetic activities**
are those that lead
to good thinking
and generalizations.
They should not require
time-consuming
and mechanical work.
They should be
directly arithmetical.



**Activity 1**

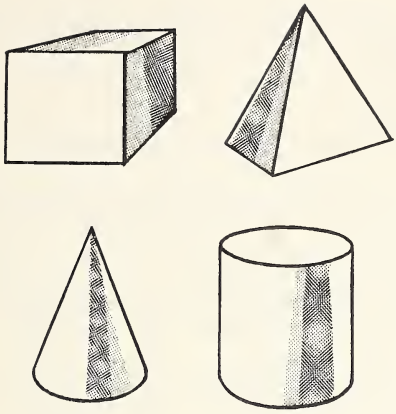
## Models of three-dimensional geometric shapes

Models of three-dimensional geometric shapes have many uses in the classroom. A committee of able pupils might be assigned the project of making a permanent collection from plaster of Paris. The following suggestions may be helpful to you.*

Before the models can be cast, molds of the desired shapes must be obtained. These can be made of oaktag or cardboard that is fastened together by painters' masking tape. Or perhaps the pupils can find school and household items to serve as molds. For example, a cardboard mailing tube may be used as a mold for a cylinder, a paper cup for a cone, and any small pasteboard box for a rectangular prism. Molds for pyramids and multi-sided geometric shapes will have to be constructed. A hollow rubber ball cut in half can be used as a mold for a half sphere. For a sphere, two half spheres can be made and the models fastened together with plaster of Paris.

When the molds have been prepared, they should be supported in an upright position and filled with a mixture of two parts of plaster of Paris and one part of water. About one hour should be allowed for the plaster to set. Then the molds can be peeled off and the surfaces of the plaster of Paris models sandpapered to remove imperfections. If the models are

*Based on an article by Wallace L. Hainlin, which appeared in the May, 1955, issue of *The Mathematics Teacher*.



to be painted, at least one full week should be allowed for drying before the painting is attempted.

Labels should be made to identify the models.

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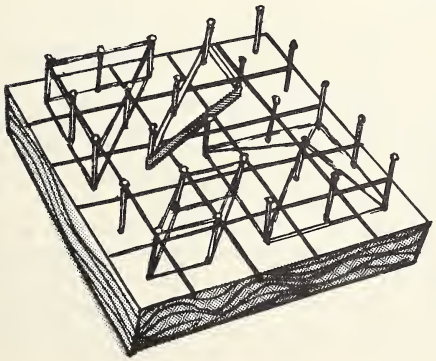
**Activity 2**

## Making polygons and prisms

With a peg board and some rubber bands of different colors, the pupils can get practice in making polygons and prisms.

A board about 7″ x 7″ and ¾″ thick may be marked off into 25 squares (or any other number of squares you wish to make). Then a 1½″ finishing nail may be partly driven into the center of each square.

Any polygon can be shown by stretching a rubber band around the nails. Prisms can be formed by using two rubber bands. (See the illustration below.)



For any prism, push one band down to the board and place the other one near the heads of the nails.

Pupils may work separately or in pairs. They can make polygons and prisms as directed or as they want to. To check polygons and prisms that have been made, ask the pupils to give the name of each, and refer to it by the color of the rubber band used.

---

**Activity 3**

## Experiments with polygons

The experiments below will help to develop certain generalizations about polygons. For example, the work with four-sided polygons will show that the square is not the only four-sided polygon that has equal sides. And the work with three-, four-, and five-sided polygons will make the pupils aware of angles other than right angles.

Each experiment can be typed on a 4″ x 6″ card. Pupils should work independently. When they have found the answers through experimenting, they should write the answers, and the teacher should inspect their work. Note that answers are given.

1 Can you construct with 3 toothpicks of equal length a triangle having a right angle if 1 toothpick is used for the base and 1 toothpick is used for the altitude?

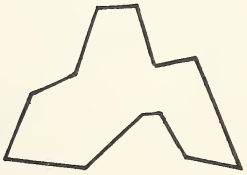
2 Using toothpicks of equal length, construct a triangle whose sides are equal. What else will be equal for this triangle?

3 Using the full length of 6 toothpicks of equal length, construct *only* 4 triangles. Each triangle must have equal sides.

4 How many different four-sided polygons can you make with 4 toothpicks of equal length, using the full length of the toothpicks? What differences can you see among these polygons?

5 How many different five-sided polygons can be made with 5 toothpicks of equal length? If someone asked different people to draw a five-sided polygon with equal sides, what would they all need to know to produce the same polygon?

6 Is it possible to describe the polygon below so that someone can duplicate it?

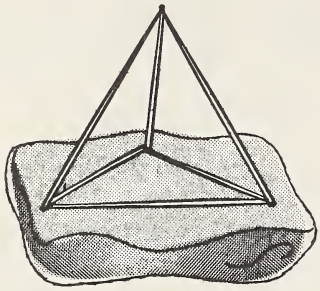


Answers to experiments

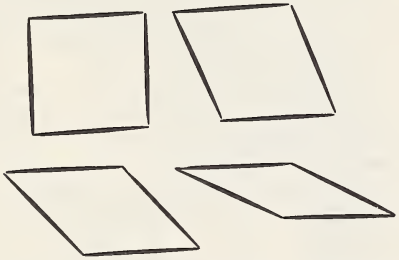
1 No. The altitude must be perpendicular to the bases, and 1 toothpick is not long enough to join the other two sides.

2 The angles of an equilateral triangle are equal.

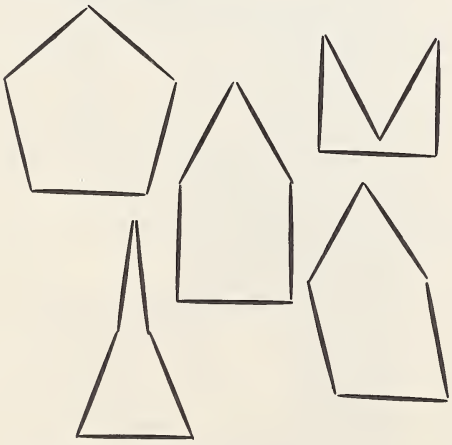
3



4 An unlimited number; Sizes of angles differ. Some pupils will notice that the areas are different.



5 A great many; They would need to know how big to make the angles.



6 This polygon is very difficult to duplicate from description unless line lengths and sizes of angles are given.

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**Activity 4**

## Working with number series

For each number series the pupils are to find the pattern used in making the series and to write numerals for enough additional numbers to complete at least one more sequence of the series. Sample number series of suitable types are indicated below for use with various pages of the pupils' book.

For page 18:

37, 35, 33, 31, 32, 30, 28, 26, 27, . . . (−2, −2, −2, +1; −2, −2, −2, +1)

201, 197, 193, 189, . . . (−4 each time)

6, 30, 120, 240, 1200, 4800, 9600, . . . (×5, ×4, ×2; ×5, ×4, ×2)

53, 55, 110, 112, 224, . . . (+2, ×2; +2, ×2)

25, 150, 50, 300, 100, . . . (×6, ÷3; ×6, ÷3)

For page 24:

172, 86, 43, 44, 22, 11, 12, . . . (÷2, ÷2, +1; ÷2, ÷2, +1)

52, 208, 104, 416, 208, . . . (×4, ÷2; ×4, ÷2)

For pages 99 and 102:

⅙, ½, ⅚, 1⅙, . . . (+⅓ each time)

17/24, 7/12, 11/24, . . . (−⅛ each time)

1, 1¼, 1 1/12, 1⅙, ⅚, . . . (+¼, −⅓; +¼, −⅓)

For page 182:

32.3, 32.7, 33.1, 33.5, 33.9, . . . (+.4 each time)

40.0, 39.3, 38.6, 37.9, 37.2, . . . (−.7 each time)

26.1, 26.6, 26.4, 26.9, 26.7, 27.2, 27.0, . . . (+.5, −.2; +.5, −.2)

---

**Activity 5**

## Other methods of multiplication

The procedures used in the methods of multiplication shown below may be written on the chalkboard or copied on cards for the pupils. Then the pupils should study the methods and try to "think out" the procedures by themselves.

Multiplication has not always been done as we do it now. The method described below was used many years ago. The exercise $43 \times 52 = n$ is used as an example. As shown on page 375, 52 is written first, then a vertical line is drawn, and the numeral 1 is written at the right. This means that $1 \times 52 = 52$.

Next, the number in each column is doubled, and the products are written under the 52 and the 1 ($2\times1=2$; $2\times52=104$). Thus, each time the number in the second column is doubled, the number in the first column is doubled. This doubling procedure is continued until the multiplier found by this procedure approaches the multiplier in the exercise. For this exercise, when the multiplier has reached 32 we do not double it again, because 2 times 32 is greater than 43 (the multiplier in the exercise).

| | |
|---|---|
| 52 | 1 |
| 104 | 2 |
| 208 | 4 |
| 416 | 8 |
| 832 | 16 |
| 1664 | 32 |

| | |
|---|---|
| 1664 | 32 |
| 416 | 8 |
| 104 | 2 |
| 52 | 1 |
| 2236 | 43 |

When $32\times52$ has been reached, $11\times52$ is still needed to equal $43\times52$. At this point, then, we look among the multipliers for numbers that, when added together, equal 11. Since 8, 2, and 1 equal 11, the products for these numbers (416, 104, and 52) are added to the product of 32 and 52 (1664). The answer is then found to be 2236.

To verify the answer to the exercise just finished, our usual method of multiplication is shown at the right.

$$\begin{array}{r} 52 \\ \times\ 43 \\ \hline 156 \\ 2080 \\ \hline 2236 \end{array}$$

By using the very old method just described, it is possible to find the answer by two methods. Look at the work for $15\times545=n$ shown below. For this particular exercise, the second method is the easier.

Method 1

| | |
|---|---|
| 545 | 1 |
| 1090 | 2 |
| 2180 | 4 |
| 4360 | 8 |

| | |
|---|---|
| 4360 | 8 |
| 2180 | 4 |
| 1090 | 2 |
| 545 | 1 |
| 8175 | 15 |

Method 2

| | |
|---|---|
| 545 | 1 |
| 1090 | 2 |
| 2180 | 4 |
| 4360 | 8 |
| 8720 | 16 |

| | |
|---|---|
| 8720 | 16 |
| 545 | 1 |
| 8175 | 15 |

Instead of stopping with $8\times545=4360$, the 8 and 4360 are doubled ($16\times545=8720$). Then, since the exercise is $15\times545$, which is $1\times545$ less, 545 can be subtracted from 8720 to get the answer. In an exercise such as this, in which the number we want to multiply by is one less than the next multiplier, it is easier to subtract 545 than it is to add the products, as is done in Method 1, and there is less chance of making an error.

Here is another ancient method. The exercise $52\times43=n$ is worked by this method as shown below.

The work is written in three columns labeled A, B, and C. First the numerals representing the numbers to be multiplied are written, one in each of the first two columns. The 52 in Column A is *divided* by 2 (remainders are ignored), and the 43 in Column B is *mutiplied* by 2. (It does not matter, however, which number is divided and which is multiplied. 43 can be divided and 52 multiplied.) This process of halving and doubling is continued in each column until only the numeral 1 remains in Column A.

Then the odd numbers represented in Column A are located. The numerals at their right in Column B are written in Column C. The numbers represented in Column C are then added to get the answer to the exercise.

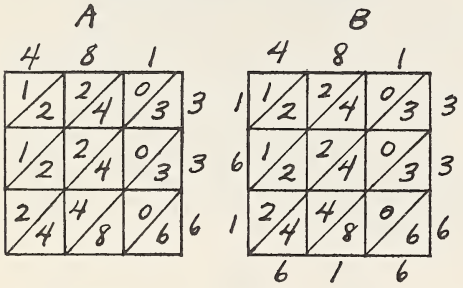
| A | B | C |
|---|---|---|
| 52 | 43 | |
| 26 | 86 | |
| 13 | 172 | 172 |
| 6 | 344 | |
| 3 | 688 | 688 |
| 1 | 1376 | 1376 |
| | | 2236 |

A third multiplication method of long ago is shown below. The exercise $336\times481=n$ is used to illustrate this method. Since the multiplication involves two three-figure numerals, nine spaces are marked off in three rows. 481 is written above the top row, and 336 is written to the right of the three rows, as shown.

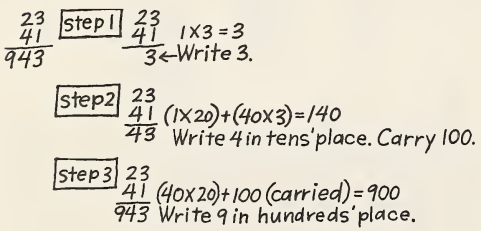
Now diagonal lines are drawn in each space. The space below the diagonal line is for ones, and the space above the diagonal line is for tens. Then the 4, 8, and 1 in 481 are multiplied in turn by the 3, 3, and 6 in 336, starting in the upper-right corner as follows: $3\times1=3$; write 3 for ones below the diagonal; since there are no tens, write 0 above the diagonal. $3\times8=24$; write 2 for tens and 4 for ones in the next space. $3\times4=12$; write 1 for tens and 2

for ones. The 4, 8, and 1 in 481 are each then multiplied by the second 3 in 336, and by the 6 in 336.

In the next step, beginning in the lower-right corner, the numbers expressed by numerals in the *diagonal* rows are added. The first row has only one numeral, 6; so 6 is written below the space. The next diagonal row contains 8, 0, and 3, or 11; so 1 is written below the next space, and 1 is remembered, or carried. The next diagonal row, when added, is 16, including the 1 that is carried; so 6 is written and 1 remembered. The next diagonal row is 11, including the 1 that is carried; so 1 is written to the left of the bottom row. The addition is then continued for the last two rows, and the answer is read, starting at the top-left corner. Multiplying 481 by 336 in the usual way, we find that the answer 161,616 is correct.




The following "short multiplication" method can be used as an exercise in mental calculation.

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**Activity 6**

## Basic fact lists

To give pupils a review of the multiplication and division basic facts in a different way, have them arrange the facts according to number groups. For this activity, instruct the pupils to write the numerals 40, 42, 45, 48, 49, 54, 56, etc., in rows across the top of their papers. Show them how to leave sufficient space between and below numerals so that they can write the complete equations in a column under each number. (See the sample below.)

| 40 | 42 | 45 |
|---|---|---|
| 5×8 = 40 | 6×7=42 | 5×9 = 45 |
| 8×5 = 40 | 7×6=42 | 9×5 = 45 |

This activity may be used for the division basic facts also. Instruct the pupils to put the dividends for the division groups across the top of their papers in the way explained above for multiplication. Then tell them to show in equation form all the basic facts that belong in each group. For the 40, 42, and 45 groups, their work would look like this:

| 40 | 42 | 45 |
|---|---|---|
| 40÷5 = 8 | 42÷6 = 7 | 45÷5 = 9 |
| 40÷8 = 5 | 42÷7 = 6 | 45÷9 = 5 |

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**Activity 7**

## Target toss

The game of "Target toss" can furnish practice in column addition.

Materials needed are a large square board (the end of an orange crate will do), five drapery angle screws or hooks, five or more jar rubbers, and a set of large cards, each with a different numeral on it (one-figure, two-figure, or three-figure). Punch a hole in the top center of each card. Make the target by screwing the drapery hooks into the board, one hook in the center and two rows of two hooks each. Then hang a card on each hook.

The difficulty of the game may be controlled by the size of the numbers whose numerals you choose to hang up and by the number of rings that are thrown. Be sure to keep the difficulty within the range of the players.

Decide before the game begins how many "ringers" a player may have at each turn at play. For example, the game can be limited to two ringers at each turn, with a maximum of five jar rings to throw. When a player has made two ringers, he is through for the turn. He may throw any number of jar rings up to

five to get two ringers. The game can be made more difficult by requiring three ringers in five throws. If the pupils find it hard to get two (or three) ringers, provide jar rings for more throws.

Let the players take turns throwing jar rings, one at a time, at the hooks from a distance of six feet. One pupil may be designated as scorekeeper to write the ringed numerals on the board under a player's name. Then let each player add up his own total score. The player with the highest total score wins. Ties may be broken in a toss-off. Abler pupils can try adding the numbers mentally, but the slower pupils should use paper and pencil.

**Activity 8**

## Multiplication and division chart

A chart for the multiplication and division basic facts may be prepared by the pupils.

Have the pupils mark off a 4½″ square into half-inch spaces, 9 across and 9 down. Beginning with the second from the left space at the top, have them write the numerals from 2 to 9 in the spaces going *across*. Then have them write the numerals from 2 to 9 in the spaces going *down* the left side, starting with the second from the top space. Tell the pupils to multiply the 2 at the top by the 2 at the left, and write the answer directly under the top 2 and opposite the side 2. Then tell them to multiply the 3 at the top by the 2 at the left, and write the answer beneath the 3 and opposite the 2. Have them continue multiplying by 2 the numbers represented in the top row and writing in the answers until the second horizontal row is filled. Then have them multiply the numbers represented in the top row by the 3 at the left side and write the answers in the proper columns. Have them continue in this way until all multiplication basic facts have been filled in. Once the chart has been completed correctly, the pupils can use it to verify any multiplication basic fact they are unsure of.

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 |

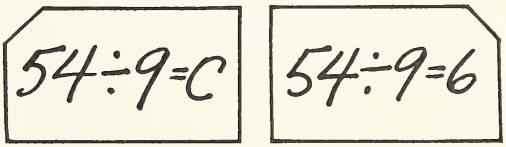
This chart, of course, can also be used to review the division basic facts. When the pupils want to check division facts, they use the numerals in the first column at the left as divisors. The numerals in the rows to the right show numbers that are to be divided, and the numerals in the top row express the answers.

**Activity 9**

## Trouble spots

Individual practice cards that contain only the basic facts the pupil finds difficult provide a useful activity. Let each pupil put the facts he finds troublesome on small pieces of drawing paper. Have him write the fact in equation form (without the answer) on one side of the card and the same fact in equation form with the answer on the other side. Lay the cards so that all sides without answers are face up and cut off a small part of the upper-left corners. The cut corners will help the pupil arrange the cards so that all the sides with answers or all the sides without answers are facing the same way.

Practice cards may be made for addition, subtraction, multiplication, and division basic facts.




Commercially prepared cards, for both individual and class use, are available (in computational form only).*

**Arithmetic Flash Cards for Individual Use* (2″ x 3½″ in envelope). Set 5: Addition and Subtraction (200 cards); Set 6: Multiplication and Division (190 cards). Scott, Foresman and Company.

*Arithmetic Flash Cards for Class Use* (4″ x 7″, boxed). Set 1: Addition (100 cards); Set 2: Subtraction (100 cards); Set 3: Multiplication (100 cards); Set 4: Division (90 cards). Scott, Foresman and Company.

**Activity 10**

## Problem cards

Prepare 3″ x 5″ cards (lettered A, B, C, etc.) and write or paste one verbal problem on each card. You may make up these problems or cut them from old textbooks. Be sure the problems are within the range of the work done thus far.

Let each pupil pick out three cards, five cards, or as many as you specify. On his paper, opposite the letter of the card, he is to write the equation for the problem, with the letter *n* where the answer is to be inserted. Then he is to do the computation and rewrite the equation with the answer in place of *n*. If a problem requires computation on paper, the pupil should show his computation.

Another version of this activity that is interesting to pupils is to have committees prepare word problems from "real life" experiences. These committees should include pupils of all levels of ability. The slower pupils can acquire data by referring to various sources such as mail order catalogues. The abler pupils can construct problems by using data collected by other pupils. All the pupils on each committee should participate in the solution of the problems prepared.

Data acquired in this manner should be subjected to questions concerning its reliability. Perhaps mistakes were made in the collection of the data or the sources of the data were not reliable. In other words, the pupils should begin to develop a critical attitude.

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**Activity 11**

## Make up a problem

Have the pupils work in groups and take turns being "It," as described below. Let one pupil do the timing. Have him experiment with periods of 1 minute to 3 minutes to determine how much time should be allowed. (It will be helpful if you can provide the group with a three-minute egg timer, a stop watch, or a watch with a second hand.) The pupil who is "It" writes on the board an equation with the letter *n* to hold a place for the answer. Then he points to someone in the group and asks him to make up a verbal problem to fit the equation. If the chosen pupil can state a problem and give the answer before the time is up, he becomes "It." If he cannot think of a problem, the pupil who is "It" asks someone else to make up a problem.

Everyone in the group should verify the answers. Supervise the activity to make sure that the pupils do not go beyond the range of the processes studied thus far. Permit discussion of all disputed problems.

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**Activity 12**

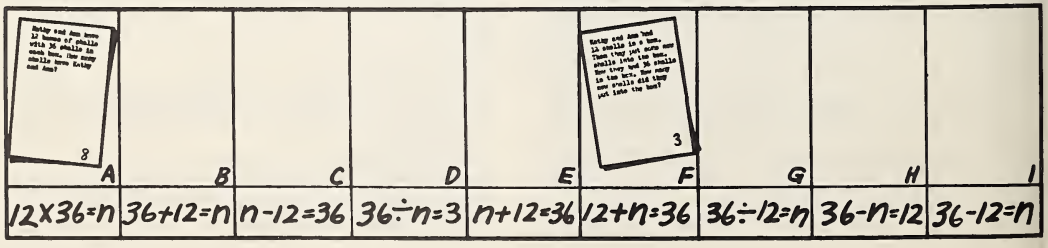
## Solve it

The following activity will give the pupils practice in recognizing the proper equations for problem situations. The activity can be used for problems that can be solved in one step. This is an individual activity, not a group activity.

The pupil will need a board and some problem cards. To make the board, mark off a piece of oaktag or cardboard 22½″ x 4½″ into spaces 2½″ x 4½″. Then draw a line about an inch above the lower edge of the board to make small spaces below the larger spaces. Letter each large space A to I, as shown in the illustration, and write an equation in each of the smaller spaces.

To carry out the activity, the pupil is given a board and a set of problem cards. (As explained below, he should have more than 9 problem cards.) He selects a problem card and looks for an equation that fits the problem. He then places the problem card in the space above the equation. Then he solves the equation and writes on a piece of paper the letter of the space, the number of the problem, and the answer. For the first equation shown in the illustration below, he would write "A, 8, 432."

Since each board will show 9 equations, you should prepare more than 9 problem cards for each board.



The problems should be written on 2″ x 3″ cards and should be numbered. A sample set of problems and equations is given below. Extra problems are made so that the pupil will always have to do some thinking in deciding which equation fits a given problem. Be sure to warn the pupils that there is more than one card for some of the spaces on the board. The problems should differ only in wording. Objects, names of characters, and numbers used should be the same.

Several boards may be made with different equations and different sets of problems. The pupils can either verify their own answers or exchange boards and verify one another's work.

Here is a sample set of problems and equations:

1 Kathy had 36 shells in a box. Ann put 12 more shells into the same box. How many shells were in the box then? $(36+12=n)$

2 Kathy had some shells in a box. Ann put 12 more shells into the same box. Then Kathy had 36 shells in the box. How many shells were in the box to begin with? $(n+12=36)$

3 Kathy and Ann had 12 shells in a box. Then they put some new shells into the box. Now they had 36 shells in the box. How many new shells did they put into the box? $(12+n=36)$

4 Kathy had 36 shells in a box. She gave some of them to Ann. Then she had 12 shells left. How many shells did she give to Ann? $(36-n=12)$

5 Kathy had 36 shells in a box. Ann took 12 of them out of the box. How many shells were left in the box? $(36-12=n)$

6 Kathy had some shells in a box. She took out 12 of them. Then 36 shells were left in the box. How many shells did she have in the box to begin with? $(n-12=36)$

7 Kathy had 36 shells in a box. Ann helped her put them into 3 smaller boxes, with the same number in each box. How many shells did they put into each of the smaller boxes? $(36\div n=3)$

8 Kathy and Ann had 12 boxes of shells with 36 shells in each box. How many shells did Kathy and Ann have? $(12\times 36=n)$

9 Kathy had 36 shells in a box. Ann helped her put 12 shells into each of several small boxes. Into how many small boxes did Kathy and Ann put the shells? $(36\div 12=n)$

10 Kathy had 36 shells in a box, and Ann had 12 shells in another box. The girls put the shells into a larger box. How many shells were in the larger box? $(36+12=n)$

11 Ann had 12 shells in a box. Kathy put some more shells into the box. Then there were 36 shells in it. How many shells did Kathy put into the box? $(12+n=36)$

12 Kathy put 11 shells in one box, 12 shells in another box, and 13 in a third box. What was the average number of shells per box? $(36\div n=3)$

---

**Activity 13**

## Problem relay

This game can be played by the whole class or by only the slower pupils, if supervised. The game involves group solving of problems. As described below, the game gives practice in solving multiple-step problems, but it can be adapted to give practice in solving one-step problems.

You will need to provide some easy multiple-step problems. You might get them from old textbooks or make them up yourself. Ditto a set of them so that each pupil may have a copy.

Separate the players into two teams. Appoint one pupil as captain for Team A and another pupil as captain for Team B. Captain A selects a problem from the list to start the game. He reads it aloud and then consults with the members of his team to determine the proper equation for its solution. He writes this equation on the board and explains it. If Team B accepts this equation as correct, Team A gets 1 point. One pupil may keep track of points earned by writing the number of points on the board or on paper under the team names.

When it has been determined that the equation is correct, Captain A calls on the next person in line on his team to find the number that the expression in parentheses in the equation stands for. If computation is necessary, this pupil should compute on the board. When he has finished, Team B decides whether or not to accept his work. If Team B accepts it, Team A earns another point and proceeds to do any other work indicated by parentheses. Finally, after all the preliminary work has been done, Team A finishes the equation by finding the numeral that replaces *n*, or the answer to the problem. If Team B does not challenge the answer, Team A earns another point. It is thus possible to earn a total of 3 points for a two-step problem and 4 points for a three-step problem.

After Team A has successfully solved the first problem, Team B selects a problem and proceeds to solve it step by step, just as Team A did. At any step along the way, the opposing team may challenge a solution. If it can be shown that the thinking or

computation is incorrect, that team gets a chance to take over and make points.

If you wish, you can use this game with one-step problems. Only 2 points may be scored for the solution of each of these problems—1 point for the correct equation and 1 point for the correct answer.

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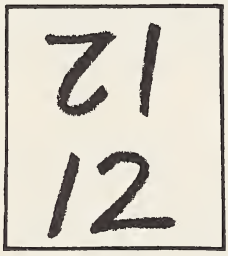
**Activity 14**

## Call the average

This game provides practice in estimating and computing averages. It also helps the pupils make generalizations about how the average is affected by zero or by one or more numbers that are much larger or much smaller than the other numbers.

Prepare in advance sets of cards with numerals on them. (3″ x 5″ cards may be cut in half to make 3″ x 2½″ cards.) Write a numeral on each card so that it can be read from two positions. (See the drawing below.) You can control the difficulty of the game by the numerals used on the sets of cards. For example, if you are preparing a set of cards for the slower pupils, make 3 cards each of the numerals 0 to 9 and about 25 cards more, each with a two-figure numeral.




For the able pupils, prepare a set of cards made up of two-figure and three-figure numerals and a few zeros and one-figure numerals. For example, you can prepare cards with any of the numerals from 20 to 50, five cards showing numerals for larger numbers like 95, 105, 115, five cards for the numerals 1 to 5, and two or three cards with zero.

Five or more pupils may play the game at one time. Place a set of cards face down in the middle of a table. Each player then draws one card from the pack and places it face up in front of him. All the players look at the exposed cards, and each player estimates the average. Someone makes a list of the estimates. Then all the players compute to find the average. Remainders should be disregarded, both in estimating and in computing. The winner is the player whose estimate is nearest to the computed average. In case of a tie after a predetermined number of rounds have been played, subsequent rounds can be played to determine who came closest to calling the average the greatest number of times. Used cards should be placed to one side until the pack has been used up. Then the whole pack may be reshuffled, and play can continue as before.

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**Activity 15**

## Ratio

"Ratio" is a game designed to give pupils practice in recognizing ratios that express rates illustrated by pictures.

Picture cards and ratio cards will have to be made in advance.

First, select six rates and make two picture cards (5″ x 5″) for each rate (12 cards altogether). The picture cards may be made with X's and O's, or two different kinds of objects may be drawn, such as apples and pennies. This work may be done by pupils who draw fairly well. The following guide is suggested for drawing objects on the cards:

4 groups of 3 per 5
3 groups of 2 per 4
2 groups of 6 per 5
3 groups of 2 per 5
3 groups of 4 per 6
2 groups of 5 per 10

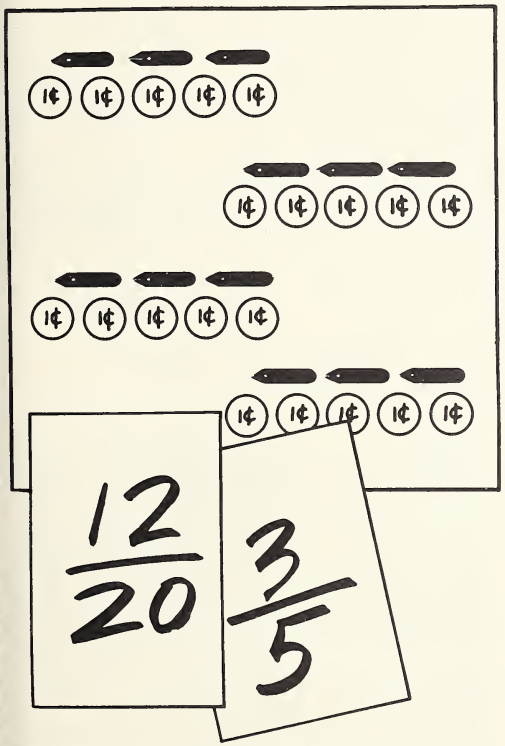
Then make 32 ratio cards (2″ x 3″). On each card write a ratio that expresses a rate illustrated by one of the picture cards. Although any number of ratios can be written for the six rates illustrated on the picture cards, use only 32 ratios, and make one card for each. For the set of picture cards described above, the following ratios may be used on the ratio cards: 3/5, 5/3, 6/10, 10/6, 12/20, 20/12, 2/4, 4/2, 4/8, 8/4, 6/12, 12/6, 6/5, 5/6, 12/10, 10/12, 2/5, 5/2, 4/10, 10/4, 6/15, 15/6, 4/6, 6/4, 8/12, 12/8, 12/18, 18/12, 5/10, 10/5, 10/20, 20/10

For a second set of picture cards (two for each rate), the following rates may be used as a guide.

3 groups of 3 per 4
2 groups of 2 per 10
2 groups of 5 per 8
4 groups of 2 per 3
3 groups of 4 per 5
2 groups of 8 per 15

Ratio cards for this second set may include these ratios:
3/4, 4/3, 6/8, 8/6, 9/12, 12/9, 2/10, 10/2, 4/20, 20/4, 5/8, 8/5, 10/16, 16/10, 2/3, 3/2, 4/6, 6/4, 6/9, 9/6, 8/12, 12/8, 4/5, 5/4, 8/10, 10/8, 12/15, 15/12, 8/15, 15/8, 16/30, 30/16

Four pupils may play the game at one time. One pupil deals and gives each player 3 picture cards and 8 ratio cards. Each player lays his picture cards face up in front of him and holds the ratio cards in his hand. Each player then examines his ratio cards to find ratios that are illustrated on his picture cards. Each ratio card that matches a picture card should be placed beside that picture card. (See the illustration below.)




After the players have laid down all the ratio cards they can, the player to the right of the dealer draws one ratio card from those that the player to his right is still holding in his hand. If the ratio card drawn illustrates one of his picture cards, he lays it beside the correct picture card. If it does not match any of his picture cards, he puts it in his hand with the other ratio cards he is holding. The next player draws a card from the player on his right, and the game continues.

The object of the game is to place all the ratio cards beside picture cards. The player who first gets rid of all the ratio cards in his hand is the winner.

This game can also be adapted to give the pupils experience in using ratios expressing comparisons. New cards should be made with pictures that show the comparison of groups.

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**Activity 16**

## Flannel board

A sturdy felt board or flannel board is a very helpful device, since it can be used many times throughout the year. It is a useful device for providing experiences with fractions.

Three fourths of a yard of 36″ flannel or felt, preferably in a dark color, will be sufficient to cover a 24″ x 32″ piece of heavy cardboard or easel-board. Cutouts to be used with this board may be made of oaktag, felt, or colored construction paper. Small pieces of colored felt are sold in most hobby shops.

If you use oaktag or construction paper for cutouts, paste a small strip of coarse sandpaper on the back of each cutout. The cutout will then stick to the board when it is given a slight downward pressure.

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**Activity 17**

## Fraction kit

The making of a fraction kit can be a valuable learning experience for pupils, and the kit itself is useful for making many discoveries about fractions.

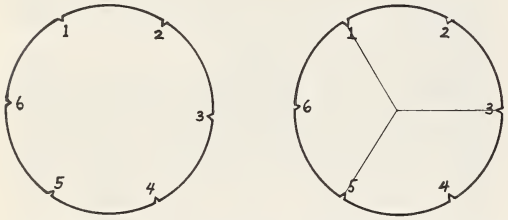
Construction paper of different colors may be used for the sets of fractional parts. (For example, green can be used for whole disks, red for halves, blue for fourths, yellow for eighths, lavender for thirds, and pink for sixths.) One sheet of 12″ x 18″ construction paper will provide six disks that are each six inches in diameter. Oaktag patterns can be given to the pupils to use in making the disks.

Have the pupils first cut out the whole disks and put them aside. Then ask them to cut out other disks and fold each disk to show halves. Then tell the pupils to open the disks and label each half. Next have them cut along the line of fold. It is suggested that one disk be left uncut.

For making fourths, instruct the pupils to follow the procedure used for halves, but to fold the folded half again in half. Have them label each fourth and then cut the disks apart. Have one disk left uncut.

When the pupils make eighths (which require one more fold than fourths), again have them label the eighths and leave one disk uncut.

For making sixths, provide the pupils with oaktag patterns that have been notched at six points along the edge. These six points are found by marking off the circle with compasses into sixths (by using the radius). Have the pupils number the points consecutively around the circle. (See the drawing below.) Tell them to use the notches and numerals as guides to fold three diameters, 1-4, 2-5, and 3-6. Remind them to label the sixths. Again one disk should be left uncut.




The pupils may use the notched oaktag patterns to make thirds. Tell them to number the points consecutively around the circle. This time they will not do any folding. Ask them to line up Points 1 and 4 with a ruler and draw a light pencil line from Point 1 to the center of the disk. Then have them line up Points 2 and 5 and draw another line from Point 5 to the center. Next have them line up Points 3 and 6 and draw a third line from Point 3 to the center. Again the thirds should be labeled, and one disk left uncut.

A room kit that contains more fractional parts can be used more extensively, but such a kit should be made by the teacher so that it may be as accurate as possible.

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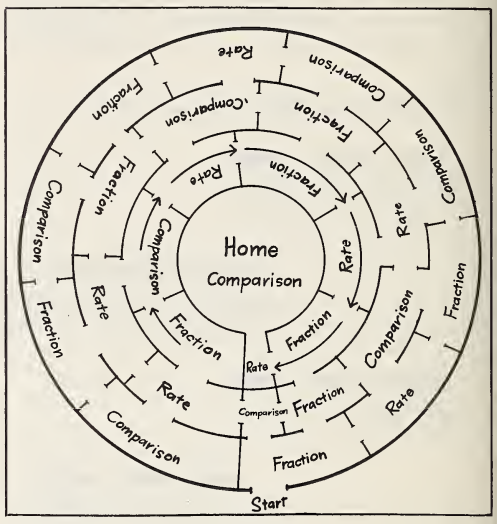
**Activity 18**

## Rate, comparison, or fraction?

This game will provide practice in distinguishing among fraction, rate, and comparison situations. Each group participating should not exceed four—one to read the exercises and three to play. Each of the three players will need a game board and a marker. Three versions of the game board are needed. The basic pattern of each board is the same, but the words "rate," "comparison," and "fraction" should be placed in the spaces in a different order on each board. Be sure the space opposite "Start" is labeled with a different word on the three boards and that the "Home" space contains a different word. The board illustrated below begins with "Fraction" and has "Comparison" in the "Home" space. Copies of the three game boards may be reproduced from master duplicating stencils.




Prepare at least 36 exercises that involve a rate, a comparison, or a fraction situation (12 for each situation) and write each exercise on a 3″ x 5″ card. (See page 46 of the pupils' book for sample exercises.) Label each card with a numeral. Also prepare a master card that shows a classification for each exercise to use in case of dispute. (This card should be a simple listing of each exercise label followed by its classification. For Exercise 1, for example, the list could read, "1—rate"; for Exercise 2, "2—fraction"; and so on.) The individual cards are placed face down in front of the pupil who is to do the reading.

Be sure each player has a different version of the game board. To start the game, each of the three players places a marker over the word "Start" on his game board. The reader reads the first exercise, and the players classify the situation by saying "rate," "comparison," or "fraction." When the classification is agreed upon, the player whose board begins with

that classification must move his marker into the first space opposite "Start." For example, if the exercise expresses a fraction idea, the player whose board begins with the word "Fraction" moves his marker into the first space marked "Fraction" (see the picture of the game board). If the first exercise expresses a comparison situation, the player whose board starts with "Comparison" must move his marker into that space.

Another exercise is now read and classified, and any player who can move his marker does so. A move is not always possible, since every move must be made through a "door" leading into an adjoining space having the required classification. This space *must* be adjoining, and it must open into the space formerly occupied by the marker of the player. If no such opening exists, the player may not move. A player may sometimes have a choice of direction, either right or left. Although he will try to keep moving in the direction of "Home," he may sometimes be forced to move backward, because he *must* move every time it is possible. When he reaches the inner circle, he must move in the direction of the arrows in order to get "Home." The pupil who reaches "Home" first is the winner.

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**Activity 19**

## Problem tag

A flannel board for mounting the tags used in this activity and verbal problems written on cards can form the basis for an activity that will help improve the pupils' ability to make equations to symbolize problems. (See Activity 16, page 381, for instructions on how to make a flannel board.)

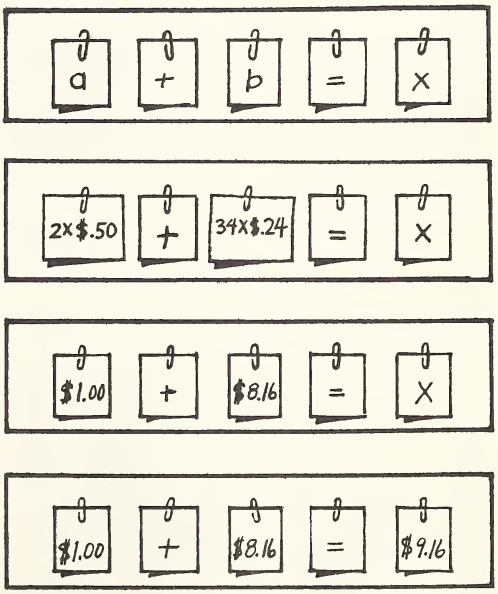
The pupils can make their own problems and write or type them on 3″ x 5″ cards, or they can get them from old arithmetic books. If you have some old books that may be cut up, the pupils can cut out problems and mount them on cards. See that the cards prepared by the pupils contain only problems of types that they have studied.

Besides a flannel board and problems on cards, you will need small pieces of sandpaper for tags, on which the pupils can write symbols, such as numerals, operational symbols ($+$, $-$, $\times$, $\div$), equals signs, expressions within parentheses, and letters. Other place-holder symbols, such as asterisks and question marks, may also be used. Be sure the flannel board is large enough to accommodate the equations you want to make.

A pupil reads a problem, then writes the symbols he needs on the tags to form an equation for the problem. He then arranges the tags on the flannel board. Or, he may put each tag on the flannel board as he prepares it.

This activity will be especially good in getting across the idea of replacing letters and other place-holder symbols with numerals and renaming numerals. For example, for Problem J on page 51 of the pupils' book, the equation would first appear on the flannel board as $a+b=x$. Then the first tag, which is marked $a$, would be removed and changed to $(2\times\$.50)$; the second tag, marked $+$, would remain; the third tag, marked $b$, would be changed to $(34\times\$.24)$; the equals sign would remain; and $x$ would remain. Then the preliminary computation would be done and the numerals inserted in place of the expressions in parentheses. Finally, $x$ would be replaced with the answer.

The drawing below shows how each progressive change in the equation would be symbolized in solving Problem J.




Instead of a flannel board, a large sheet of cardboard with slits for the insertion of paper clips may be used. The slits should be in a horizontal row and spaced far enough apart to permit the tags to hang from the clips.

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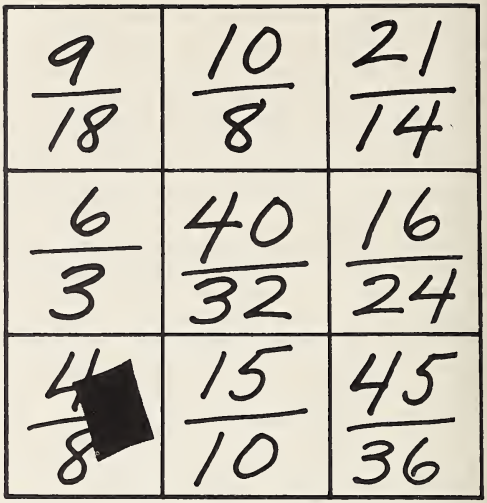
Activity 20

## Spin-a-rate

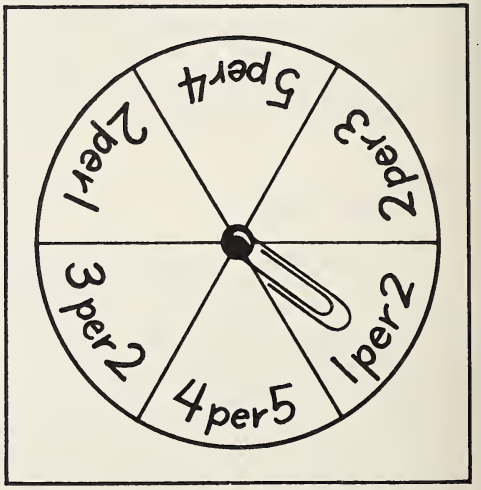
"Spin-a-rate" will give the pupils practice in recognizing different ratios that can be written for one rate.

Make Spin-a-rate cards 4½″ square. Mark off each card into nine 1½″ squares. In each square write a ratio that expresses one of the six rates you choose to use for the game. Vary the ratios on the cards so that no two cards are exactly alike and no card contains the same ratios for all six rates. This will assure occasional spins on a rate for which there is no ratio on a pupil's card. Some of the ratios, of course, may be repeated on different cards. A sample card is shown at the right. Make as many cards as you need (one for each player). The game may be played either by a few pupils or by a large group.




On a spinner, like the one pictured at the right, write the six rates you have chosen and for which ratios appear on the Spin-a-rate cards. Write the rates in the form "2 per 1," "3 per 2," etc.

The spinner may be made of a piece of oaktag or cardboard about 4″ square, a paper clip, and a brass brad. Draw a circle about 3″ in diameter and divide it into six sections for the six rates you are going to use. Then fasten the paper clip to the center of the card by putting the double flanges of the brad through the clip and then through the center of the circle. (It is important to fasten the brad loosely, so that the clip can spin easily.)




You will also need some markers for the pupils to place on the Spin-a-rate cards. (Corn, buttons, or small pieces of paper may be used.) Each player will need nine or more markers.

When the game is played by a small group (two to six), have the players take turns spinning the spinner and playing. The player uses the rate given in the section where the spinner stops. If a ratio on his card is equal to it, the player puts a marker on that ratio. Only one ratio space can be filled at each spin. The player who fills his card first is the winner.

When the game is played by a large group, one pupil may serve as caller. He spins the spinner and calls out the rate indicated. Each time he spins and calls a rate, any pupil who has the correct ratio on his card places a marker on that ratio. The winner is the player who covers all the ratios on his card first. The winner becomes the caller for the next game.

Besides the set of equal ratios shown in the illustration of the Spin-a-rate card, eight more sets of ratios are listed below for the rates shown on the spinner. Each set contains enough ratios for one card. If you want sets of ratios for more cards, they can be made up by rearranging the same ratios, or you can substitute others of your own choosing.

Set 1—3/6, 5/10, 16/8, 10/15, 18/12, 14/21, 12/15, 28/35, 24/30

Set 2—6/12, 10/5, 8/12, 9/6, 8/10, 16/20, 32/40, 12/8, 10/15

Set 3—7/14, 8/4, 4/6, 12/18, 15/10, 8/12, 21/14, 15/12, 25/20

Set 4—8/16, 10/20, 12/6, 6/9, 18/27, 20/25, 36/45, 20/16, 35/28

Set 5—9/18, 6/3, 14/7, 24/16, 6/4, 27/18, 16/20, 30/24, 35/28

Set 6—4/8, 18/9, 20/10, 16/24, 18/27, 8/10, 24/30, 36/45, 40/32

Set 7—16/8, 8/4, 6/9, 12/18, 15/10, 20/25, 12/15, 45/36, 10/8

Set 8—3/6, 10/20, 4/6, 14/21, 27/18, 28/35, 15/12, 25/20, 30/24

This activity may be adapted to the work with comparisons by writing comparisons on the spinner in the form "3 to 2," "2 to 1," etc.

When the pupils have completed the game, ask them to tell how ratios that are equal to other ratios can be found. The pupils should be able to generalize by replying that if both terms of a ratio are multiplied or divided by the same number, the new ratio will be equal to the original ratio.

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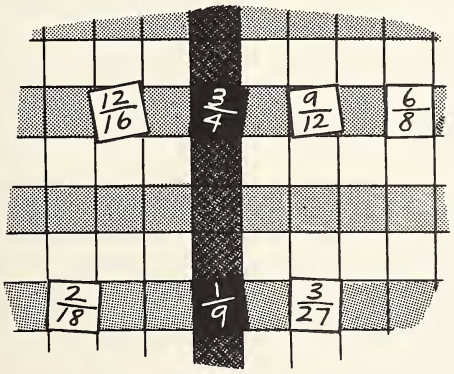
**Activity 21**

## Fraction row

First make a playing board. Mark off a sheet of 8½″ x 14″ paper (oaktag or cardboard) into one-half inch squares. If 14″ paper is not available, one and one-half sheets of 8½″ x 11″ paper can be pasted or stapled together. You will need 27 horizontal rows of squares and 17 vertical rows of squares. The middle vertical row of squares should be colored red, and the top horizontal row and every other horizontal row should be colored green. You will have 14 green rows.




Cut from colored cardboard or paper 14 one-half inch squares. On each, write the first fraction numeral (½, ⅓, etc.) from each of the following horizontal rows. These are the master cards. Cut from white cardboard or paper 54 more squares. On each, write one of the other fraction numerals (⅔, ⅜, etc.). These are the equal fraction numeral cards.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ½ | ⅔ | ⅜ | ⅝ | ⅖ | ⅗ | ⅞ | ⅛ | ⅑ | ⅒ | ⅛ |
| ⅓ | ⅙ | ⅓ | ⅘ | ⅖ | ⅗ | ⅞ | | | | |
| ⅔ | ⅙ | ⅙ | ⅔ | ⅒ | ⅗ | ⅛ | | | | |
| ¼ | ⅛ | ⅜ | ⅘ | ⅖ | | | | | | |
| ¾ | ⅝ | ⅑ | ⅛ | ⅗ | | | | | | |
| ⅕ | ⅖ | ⅗ | ⅘ | | | | | | | |
| ⅖ | ⅘ | ⅗ | ⅔ | | | | | | | |
| ⅗ | ⅗ | ⅗ | ⅛ | | | | | | | |
| ⅘ | ⅔ | ⅛ | ⅛ | | | | | | | |
| ⅙ | ⅖ | ⅜ | ⅘ | | | | | | | |
| ⅚ | ⅒ | ⅗ | ⅖ | | | | | | | |
| ⅐ | ⅖ | ⅜ | | | | | | | | |
| ⅛ | ⅖ | ⅜ | | | | | | | | |
| ⅑ | ⅖ | ⅜ | | | | | | | | |

To play the game, first shuffle all 68 cards and place them face down in a pile before the players (two, three, or four pupils may play.) Each player in turn draws five cards and lays them face up in front of him. If he finds he has drawn a master card, he should place it on any red square in a green row on the playing board.

After a master card has been put on the playing board, other cards having equal fraction numerals on them can be put on green squares to the left or right of the red square. For example, after someone has played ⅗, then ⅘, ⅗, and ⅘ may be played beside it in either direction and in any order. However, no equal numeral card can be played until the master card has been put on the playing board.

At each turn, the player may place on the board as many cards (master or equal numeral) as he can. At the end of each turn, the player records his count (see below), and then draws as many new cards as he has just played. Thus, if he lays four cards on the board, he draws four new cards. If he cannot play, he may still draw one new card. The player may draw as long as any cards are left.

The count for each play is 5 points for each master card played and 1 point for each equal numeral card played. The player with the largest score at the end of the game wins. A game is over when no cards remain to be drawn and when no plays can be made.

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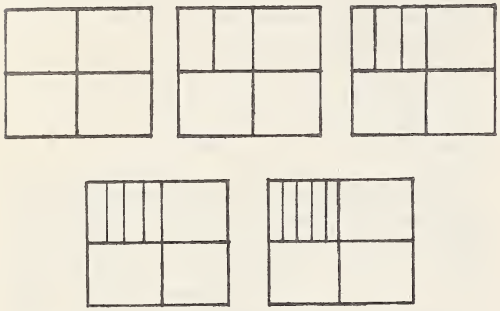
**Activity 22**

## Fraction papers

Sets of fraction papers can be used to help the pupils discover fraction equivalents and common

denominators. For each set, mark off five sheets of transparent paper.

To make a set showing fourths, eighths, twelfths, sixteenths, and twentieths, first mark off five sheets into fourths. Keep one sheet to show fourths. Subdivide one section (the same section on each sheet) of each of the other four sheets so that a different fraction of the whole paper is illustrated on each. (See the illustration below.)



Use five transparent papers to show some equivalents of one half in the same way. Mark off all five sheets into halves. Keep one sheet to show halves and subdivide one section in each of the other four sheets to show a different fraction in each—one to show fourths, one to show sixths, one to show eighths, and one to show tenths.

A set may also be made for one third. Divide five sheets into thirds. Then, keep one sheet to show thirds and subdivide one section of the other four sheets to show sixths, ninths, twelfths, and fifteenths.

Show the pupils how to superimpose one paper over another, and get them to see "$\frac{1}{4}=\frac{3}{8}$," "$\frac{1}{4}=\frac{4}{16}$," and "$\frac{3}{8}=\frac{6}{16}$." In finding a common denominator for $\frac{1}{2}$ and $\frac{2}{3}$, for example, the pupils superimpose a sheet showing thirds over other sheets until they find that the lines coincide with those on a sheet showing sixths. They then determine whether $\frac{1}{2}$ is equal to a certain number of sixths. They find that it is, and then they know that 6 is a common denominator for halves and thirds.

You may find that one set of fraction papers is sufficient, but additional sets could be made by the able pupils for class use.

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**Activity 23**

## Ratio bingo

This game (for any number of players) will give practice in finding the missing term in a pair of equal ratios.

Materials consist of a large master card made of oaktag, one or two smaller cards per player (since able pupils might play two cards at a time), scratch paper, and markers such as beans, corn, shell macaroni, small buttons, etc. About a dozen markers will

$\frac{56}{8}=\frac{7}{w}$ [1] $\frac{6}{15}=\frac{n}{45}$ [18] $\frac{11}{17}=\frac{22}{n}$ [34] $\frac{8}{9}=\frac{t}{72}$ [64]

$\frac{18}{24}=\frac{m}{4}$ [3] $\frac{38}{26}=\frac{w}{13}$ [19] $\frac{44}{19}=\frac{88}{k}$ [38] $\frac{5}{18}=\frac{20}{h}$ [72]

$\frac{9}{15}=\frac{3}{g}$ [5] $\frac{5}{7}=\frac{15}{g}$ [21] $\frac{22}{15}=\frac{n}{30}$ [44] $\frac{25}{1}=\frac{s}{3}$ [75]

$\frac{49}{14}=\frac{m}{2}$ [7] $\frac{46}{84}=\frac{s}{42}$ [23] $\frac{9}{5}=\frac{h}{25}$ [45] $\frac{7}{9}=\frac{63}{c}$ [81]

$\frac{50}{5}=\frac{t}{1}$ [10] $\frac{12}{50}=\frac{6}{h}$ [25] $\frac{8}{12}=\frac{32}{a}$ [48] $\frac{22}{11}=\frac{f}{44}$ [88]

$\frac{48}{72}=\frac{8}{e}$ [12] $\frac{9}{84}=\frac{3}{s}$ [28] $\frac{4}{13}=\frac{16}{d}$ [52] $\frac{8}{31}=\frac{24}{k}$ [93]

$\frac{4}{28}=\frac{2}{w}$ [14] $\frac{62}{8}=\frac{n}{4}$ [31] $\frac{3}{2}=\frac{r}{36}$ [54] $\frac{19}{10}=\frac{w}{50}$ [95]

$\frac{2}{6}=\frac{s}{48}$ [16] $\frac{8}{1}=\frac{w}{4}$ [32] $\frac{19}{27}=\frac{h}{81}$ [57] $\frac{6}{25}=\frac{24}{f}$ [100]

be needed for each card played, in addition to 30 or 40 markers for the use of the caller. A section of the master card and two sample players' cards are shown below. Able pupils can construct these materials.

| | | | |
|---|---|---|---|
| $\frac{56}{8}=\frac{7}{w}$ [1] | $\frac{6}{15}=\frac{n}{45}$ [18] | $\frac{11}{17}=\frac{22}{n}$ [34] | $\frac{8}{9}=\frac{t}{72}$ [64] |
| $\frac{18}{24}=\frac{m}{4}$ [3] | $\frac{38}{26}=\frac{w}{13}$ [19] | $\frac{44}{79}=\frac{88}{k}$ [38] | $\frac{5}{18}=\frac{20}{h}$ [72] |
| $\frac{9}{15}=\frac{3}{\square}$ [5] | $\frac{5}{\square}=\frac{15}{9}$ [27] | $\frac{22}{\square}=\frac{n}{\square}$ [44] | $25=\frac{5}{\square}$ [75] |

| | | | |
|---|---|---|---|
| 1 | 5 | 7 | 14 |
| 18 | 19 | 28 | 31 |
| 44 | 45 | 54 | 57 |
| 64 | 75 | 88 | 95 |

| | | | |
|---|---|---|---|
| 3 | 10 | 12 | 16 |
| 21 | 23 | 25 | 32 |
| 34 | 38 | 48 | 54 |
| 72 | 81 | 93 | 100 |

The master card will show 32 ratio equations, each having one missing term, which will appear in brackets after the equation. The equations on page 386 should be written on the master card in four columns, in the order shown. Note that the bracketed answers go down each column in numerical order. This arrangement makes it easy for the caller to check the answers the other players get by computation.

Make individual cards for the players 4″ x 4″, ruled off in 1″ squares. Each card will contain 16 numerals, and each of these numerals will be an answer to an equation on the master card. Twelve sets of 16 numerals are shown at the right. It is important to use the exact arrangement shown for each set, since the combinations have been worked out so that no two rows in any direction are alike, and the numerals are in sequence for easy reference.

To play the game, choose one pupil as the "caller." Give him a supply of markers and the master card. (He should place it in such a way that only he can see it.) Each of the remaining players should be supplied with pencil and scratch paper, markers, and one or two playing cards apiece, depending on his ability. A pupil who is playing two cards will need a double allotment of markers. The caller will call out ratio equations, choosing them at random from among those on the master card. He should use this method of calling the equation: "56 *over* 8 equals 7 *over w*," since there is no attempt here to distinguish between a rate and a comparison. As he calls each

| | | |
|---|---|---|
| 1 5 7 14 | 3 10 12 16 | 7 10 12 14 |
| 18 19 28 31 | 21 23 25 32 | 18 19 21 31 |
| 44 45 54 57 | 34 38 48 54 | 44 45 57 64 |
| 64 75 88 95 | 72 81 93 100 | 72 75 88 95 |
| 1 3 5 16 | 5 7 14 16 | 1 3 7 12 |
| 21 23 25 31 | 21 25 31 32 | 18 19 23 28 |
| 34 44 48 52 | 44 52 57 64 | 34 38 45 54 |
| 54 81 93 100 | 75 81 93 100 | 57 72 88 95 |
| 1 7 14 18 | 3 10 14 18 | 3 5 10 12 |
| 21 23 31 32 | 19 23 25 31 | 18 19 25 28 |
| 38 44 45 57 | 38 44 54 57 | 34 38 48 52 |
| 72 75 93 100 | 75 88 93 95 | 64 75 81 95 |
| 1 3 10 12 | 1 5 14 16 | 5 7 10 12 |
| 16 25 28 32 | 19 28 32 34 | 16 21 23 28 |
| 34 45 48 52 | 38 45 48 52 | 32 48 52 54 |
| 64 72 88 100 | 81 88 95 100 | 64 72 81 93 |

ratio equation, he must put a marker on the master card to show which equation he has called. He should space his calls so that pupils have a reasonable time to write each equation and compute to find the missing term. As soon as a player discovers what numeral replaces the missing term, he should try to find a duplicate of this numeral on his card, and put a marker on it. The object is to fill an entire row with markers (down or across, or diagonally). Filling each of the 4 corners may also be considered as a "row." A player's claim to having a complete row should be checked by the caller against the bracketed answers of the equations he has marked on the master card. The first player to fill a row wins the game and becomes the caller for the next game.

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**Activity 24**

## Fraction numeral set-up

The three versions of this game suggested below provide both the slow and the able pupils with

experiences in assembling sets of equal fraction numerals.

**For slow learners**

Cut 3″ x 5″ cards in thirds to make 80 small cards, each $1\frac{2}{3}$″ x 3″. On each of these cards write one of the 80 fraction numerals in Rows A to H in the chart on page 66 of the pupils' book. Write the first fraction numeral in each row, which is in lowest terms, in red. These cards with the red numerals are the "key" cards. Write the other numerals in black.

Ten cards are distributed to each of two, three, or four players. The remaining cards are kept in a pile, face down, on the table and drawn, one at a time, as the game proceeds. The players first examine their cards to see if they have any key cards. For each key card a player has in his hand, he is to find two or more cards with fraction numerals that are equal, so that sets of equal fraction numerals may be formed. The fraction numerals in a set need not be in sequence.

To play the game, the first player draws a card from the top of the center pile and lays down any sets (including a key card) of three or more equal fraction numerals. He then discards a card from his hand and places it beside the center pile of cards. The other players then take turns drawing a card from either the center or the discard pile and laying down sets if they can. A player may build onto his own sets, one card at a time, as he gets fraction numerals that are equal. He may also lay down cards that belong to sets other players have laid down. (Each player places such cards on the table with his own sets so that he can easily count his points later.)

The game continues until someone has placed all his cards on the table. He may or may not discard on the last play. The winner of the game is the one who has the most points. Key cards count as two points; other cards, one point. Any cards left in the hand count the same number of points against the player. If the point count for the cards remaining in a player's hand is greater than the point count for the cards he has laid down, his score is 0.

Perhaps the slow learners will need to be reminded that, to find sets, the fraction numerals must be reduced to lowest terms. If the resulting fraction numeral matches a fraction numeral on a key card, it is a member of that set.

**For able pupils**

For able pupils a simple adaptation can be made in the game.

Use four of the higher equal fraction numerals on key cards, and use the numerals on the original key cards on "other" cards. For example, $\frac{27}{45}$, $\frac{8}{48}$, $\frac{45}{72}$, and $\frac{18}{60}$ can be written in red and used as key cards, and $\frac{3}{5}$, $\frac{1}{6}$, $\frac{5}{8}$, and $\frac{3}{10}$ can be used as "other" cards.

The game is played in exactly the same way as described for the slow learners. The only difference in the game is that occasionally the players will work from key cards with fraction numerals that are not in lowest terms and consequently, once a set is laid down, a player may build onto it at either end.

In another version of the game, the players may collect sets of equal fraction numerals in sequence. The playing cards described in the first version of the game may be used. The key cards have no significance in this game, since a sequence may be started anywhere. For example, a pupil may lay down a sequence such as $\frac{66}{72}$, $\frac{77}{84}$, and $\frac{88}{96}$. Equal fraction numerals may be laid down at either end of this set.

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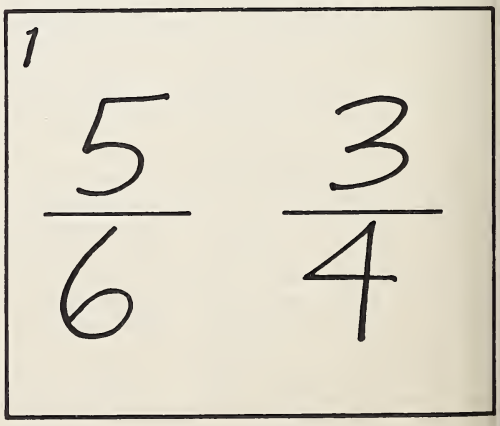
**Activity 25**

## Finding the common denominator

This activity provides practice in finding a common denominator for fraction numerals that have unlike denominators.

The activity requires some small cards on which pairs of fraction numerals with unlike denominators ($\frac{3}{4}$ and $\frac{2}{3}$, $\frac{1}{5}$ and $\frac{1}{6}$, etc.) have been written. Label each card with a numeral. Prepare 25 to 35 of these cards, depending upon the number of pupils participating and upon the number of cards you want for each pair of players. A typical card is shown below.




Give each pair of pupils from five to ten cards. One of the two pupils turns a card face up. Both

pupils write the identifying number of the card and the fraction numerals as they appear on the card. Then they rewrite them with a common denominator. If you wish, you may also tell them to circle on their papers the fraction numeral that represents the larger fraction. The next card is then turned up, and the activity continues in the same way. To vary the activity, the pupils can be instructed to circle the fraction numeral that represents the smaller fraction.

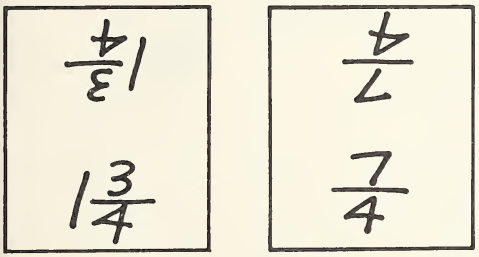
---

**Activity 26**

## Fraction match

The game of "Fraction match" will provide practice in changing improper fraction numerals to mixed numerals.

Provide a set of 36 cards (2½" x 3") on which improper fraction numerals with denominators through ninths have been written. (See the chart on page 307 of this *Teaching Guide* for a listing of the 36 improper fraction numerals.) Also provide one set of 36 cards of another color (2½" x 3") with mixed numerals on them. For each improper fraction numeral on a fraction numeral card there should be an equivalent mixed numeral on a mixed numeral card. The numerals should be written twice on each card so they can be read from either side. (See the illustration below.)




Three to six pupils can play the game. One pupil should distribute the set of 36 fraction numeral cards, 6 cards to each player. The remaining cards in this set, if any, are put aside. The set of mixed numeral cards is placed in a central position, face down, where all can reach it.

The object of the game is to see who can match all his fraction numeral cards with corresponding mixed numeral cards. (You will find that some pupils will have to compute on paper to change the improper fraction numerals to mixed numerals.)

The players take turns drawing a mixed numeral card from the pile. If the numeral is equal to the one indicated by a fraction numeral card in the player's hand, he can lay both cards down in front of him. If a player cannot match any cards, he must put the mixed numeral card in a discard pile. The pupil next to him continues the play.

If a player lays down two cards that do not match, he should be challenged by another player. If possible, the challenger should match the mixed numeral card that was incorrectly laid down with a fraction numeral card in his hand. If the mixed numeral card that was incorrectly laid down does not match any fraction numeral card of the challenger, it should simply be discarded. The player who made the error must take back the fraction numeral card he tried to play and wait for his next turn before he plays again. Consequently, all players will find that it pays to watch each time another player lays down a pair of mixed numeral and fraction numeral cards.

When the set of mixed numeral cards has been used up, the discard pile should be reshuffled and play continued until one player has matched all the fraction numeral cards in his hand with mixed numeral cards. He is the winner.

---

**Activity 27**

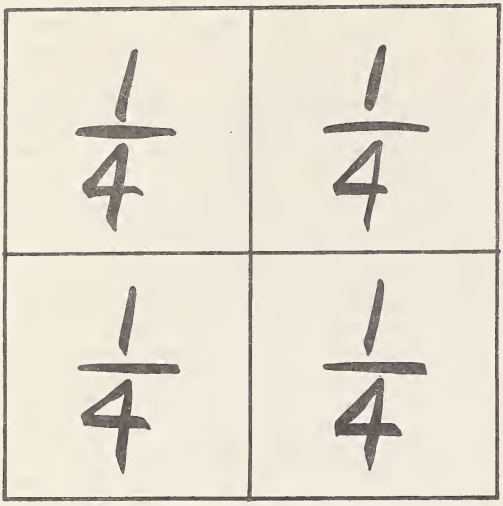
## Make 1

The game of "Make 1" helps pupils discover how fractional parts can be used to make a whole.

This game requires a set of fraction numeral cards. To make these cards, divide 11 equal-sized square pieces of stiff paper or cardboard into fractional parts that show halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, twelfths, fourteenths, and sixteenths. One card is cut into halves, one is cut into thirds, etc. Label each fractional part. See in the picture at the top of page 390 how the square is marked off into fourths and labeled before it is cut apart.

Put all the cards into a container, and let the pupils take turns drawing one card at a time from the box. The cards drawn by a pupil are to be rearranged into a square so that the sum of the fractions represented on the reassembled cards equals 1. The purpose of the game is to see which player can complete his square first.

Any card not immediately usable may be kept, or it may be traded for a card another player does not want. Each player has the privilege of making such a trade instead of drawing a new card from the con-




tainer. Any time a player trades a card, he must give one of his own unused cards in exchange. The winner is the player who is first to complete a square. For most pupils the game has the greatest value if all fraction cards are the same color.

The game works out best when it is played by groups of two, three, or four pupils.

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**Activity 28**

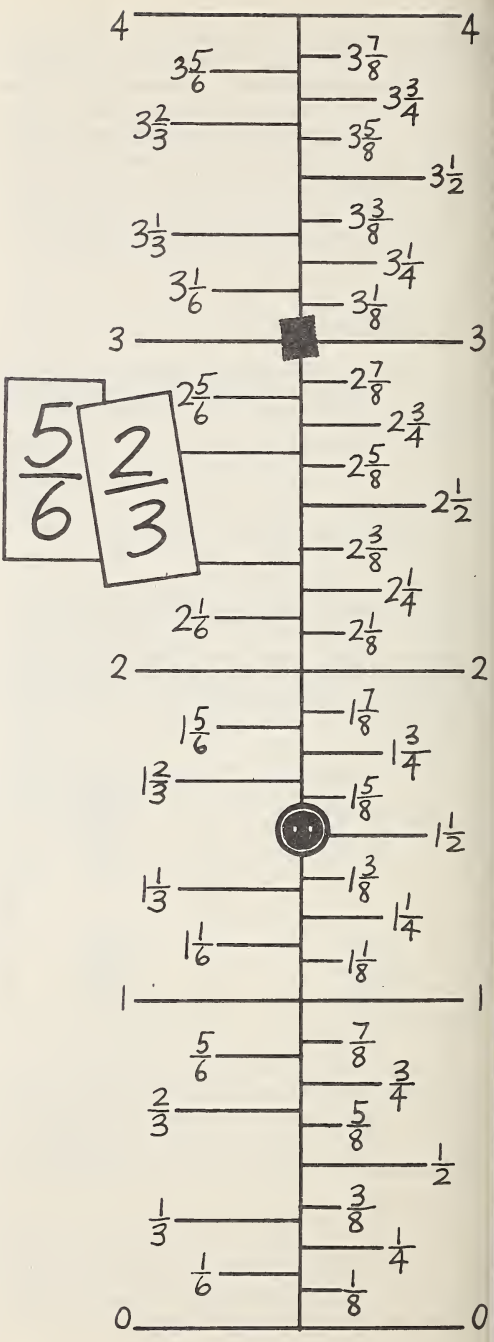
## Fraction race

This game gives practice in adding numbers expressed as mixed numerals and fraction numerals.

You will need to prepare a game board made of oaktag 8″ x 28″. (See the picture in the next column.) On the game board draw a vertical line 24″ long and divide it into four equal parts. Then draw a horizontal line at each end of the line and also at each division you have made and label these lines 0 to 4. Between these numerals (0-4) make and label subdivisions for halves, fourths, and eighths on the right side of the line and for thirds and sixths on the left side.

Also make a set of 82 fraction numeral cards (1½″ x 3″) using the fraction numerals listed below. Make two cards for each numeral.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1/16 | 2/16 | 3/16 | 4/16 | 5/16 | 6/16 | 7/16 | 8/16 |
| 9/16 | 10/16 | 11/16 | 12/16 | 13/16 | 14/16 | 1/8 | 2/8 |
| 3/8 | 4/8 | 5/8 | 6/8 | 7/8 | 1/6 | 2/6 | 3/6 |
| 4/6 | 5/6 | 1/3 | 2/3 | 1/4 | 2/4 | 3/4 | 1/12 |
| 2/12 | 3/12 | 4/12 | 5/12 | 6/12 | 7/12 | 8/12 | 9/12 |
| 10/12 | | | | | | | |



Have a marker available for each player. Each marker should be of a different kind.

Two, three, or four players can play this game. The object is to see who can move his marker from 0 to 4 first. To begin the game, the pack of cards should be shuffled, and seven cards should be dealt to each player. The remaining cards are put face down in a pile in the center of the table. The first player to the right of the dealer then draws a card from the top of the pile, puts it with those in his hand, and examines the cards in his hand to see if he has two cards showing names of fractions whose sum is one of the numbers named on the board. He should look for a combination of cards that will give him the largest possible mixed number and thus enable him to move his marker the farthest. He moves his marker to that place and lays the two cards he used face up in front of him. Scratch paper and pencils should be available for those who need them to compute. Anyone who makes an error loses the play and cannot move his marker that round. Whether the player has moved or not, he then discards one card from his hand and puts it face down beside the pile in the center.

On the first play the player must add the fractions named on two cards. On the following plays he draws one card, puts it in his hand, selects a fraction named on one of the cards in his hand to add to the fraction indicated by his marker on the board, advances to another place on the board, and discards one card. A round ends when a player reaches 4. To reach 4, he must use a fraction that, when added to the number indicated by his marker on the board, makes exactly 4. A round also ends when a player has laid down all his cards. This will happen more often. However, this does not mean that the player who lays down all his cards first wins, because another player, who is still holding cards, may have advanced farther than he has. The winner is the one who has advanced to the highest number, and that number is his score. Only the score of the winner is recorded for each round, and five rounds make a game. The winner of the game is the pupil with the highest total score in the five rounds.

This game can be adapted for subtraction of numbers expressed by mixed numerals by having the children start at 4 and, by subtracting the fraction named on one card at a time, move down to zero. Complete details for this adaptation are given in "Providing for the able pupil" in connection with the lesson on subtraction of numbers expressed by mixed numerals. See pages 315-316 of this *Teaching Guide*.

**Activity 29**

## Fraction bingo

Two forms of the game "Fraction bingo" are suggested here so that pupils may get practice in replacing mixed numerals with improper fraction numerals and with mixed numerals like $2\frac{3}{5}$. Form A, replacing mixed numerals with improper fraction numerals, is described below first.

Use $7\frac{1}{2}$" squares of oaktag for the bingo cards. Divide each of these cards into 16 spaces. Each player will need one of these bingo cards.

Make a list of 33 mixed numerals and their improper fraction numeral equivalents ($2\frac{1}{2}=\frac{5}{2}$, for example), and from this list select 16 mixed numerals and their equivalents to fill the spaces on each bingo card. Make this selection in such a way that no two bingo cards are alike. When writing the numerals in the spaces on the cards, omit the numerators of the improper fraction numerals. (A bingo card, with two numerator cards inserted, is shown below.)

| Form A | | | |
|---|---|---|---|
| $2\frac{1}{2}=\frac{\boxed{5}}{2}$ | $6\frac{3}{8}=\frac{}{8}$ | $4\frac{2}{5}=\frac{}{5}$ | $2\frac{1}{7}=\frac{}{7}$ |
| $2\frac{2}{9}=\frac{}{9}$ | $2\frac{5}{12}=\frac{}{12}$ | $3\frac{3}{4}=\frac{}{4}$ | $2\frac{3}{16}=\frac{}{16}$ |
| $1\frac{3}{12}=\frac{\boxed{15}}{12}$ | $5\frac{1}{8}=\frac{}{8}$ | $1\frac{5}{16}=\frac{}{16}$ | $3\frac{4}{10}=\frac{}{10}$ |
| $4\frac{5}{8}=\frac{}{8}$ | $6\frac{5}{8}=\frac{}{8}$ | $5\frac{3}{4}=\frac{}{4}$ | $4\frac{1}{3}=\frac{}{3}$ |

Since a wide variety of mixed numerals and equivalent improper fraction numerals may be used for this game, choose those that you feel will be most helpful to your class. Whatever your selection, your list should never exceed 33, because if too many numerals are involved, the possibility of filling a bingo card and thus of winning is lessened.

Next, make two sets of numerator cards. These should be small cards, much smaller than the spaces on the bingo cards. On each card write a numerator that belongs with one of the 33 improper fraction numerals used on the bingo cards.

The game may be played by a few pupils or by the entire class. When only a few pupils play, put the two sets of numerator cards in a container and have

the players take turns drawing the cards. Whenever a numerator card completes a fraction numeral on a pupil's bingo card, he places it in the proper space on his bingo card. If he cannot use the numerator card, he puts it back in the container.

When the whole class plays the game, give a bingo card and some markers (corn, buttons, small pieces of paper, etc.) to each pupil and choose a caller. Ask the pupils to compute on scratch paper before the start of the game to find the missing numerators on their bingo cards. Then they can quickly refer to that list when the numerals are called or drawn. The caller, who should be one of the abler pupils, draws one numerator card at a time and reads the numeral on it to the class. He then lays it face up on the table so that he can check the winner's bingo card at the end of the game. Each time the caller reads a numeral, each pupil decides if he can use the numeral in one of the spaces on his bingo card. If he can, he places a marker in that space. A pupil is to fill only one space on his bingo card at a time.

When a pupil has filled four spaces in a vertical, a horizontal, or a diagonal row, he calls "Bingo!" The caller checks the spaces that have been filled on the player's bingo card to make sure that he has won.

For Form B, make a list of mixed numerals (the list for Form A may be used) and their equivalents in the form in which the equivalent is made up of a numeral for a whole number and an improper fraction numeral ($3\frac{3}{4}$). From this list fill in another set of bingo cards. Omit the numerators of the improper fraction numerals. (A card, with two numerator cards inserted, is shown below.)

| Form B | | | |
|---|---|---|---|
| $2\frac{1}{2}=1\frac{}{2}$ | $6\frac{3}{8}=5\frac{11}{8}$ | $4\frac{2}{5}=3\frac{}{5}$ | $2\frac{1}{7}=1\frac{}{7}$ |
| $2\frac{2}{9}=1\frac{}{9}$ | $2\frac{5}{12}=1\frac{}{12}$ | $3\frac{3}{4}=2\frac{7}{4}$ | $2\frac{3}{16}=1\frac{}{16}$ |
| $1\frac{3}{12}=\frac{}{12}$ | $5\frac{1}{8}=4\frac{}{8}$ | $1\frac{5}{16}=\frac{}{16}$ | $3\frac{4}{10}=2\frac{}{10}$ |
| $4\frac{5}{8}=3\frac{}{8}$ | $6\frac{5}{6}=5\frac{}{6}$ | $5\frac{3}{4}=4\frac{}{4}$ | $4\frac{1}{3}=3\frac{}{3}$ |

Next write on small cards the numerators that are to be filled in on the bingo cards. Make two sets of these numerator cards. The game is then played in the same way as Form A.



**Activity 30**

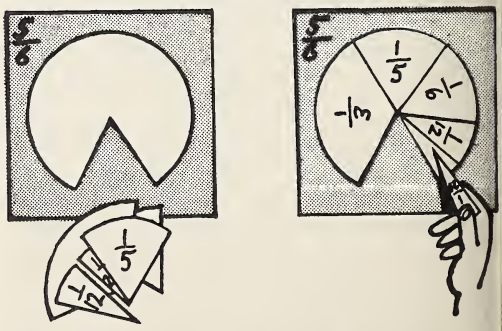
## Fraction draw

This game will give the pupil experience in judging the fractional parts of a whole.

Use cardboard to make half a dozen seven-inch square playing boards. In the upper-left corner of each playing board write one of the following fraction numerals: ⅔, ¾, ⅘, ⅚, ⅞, and 9/10. Now set your compasses to measure 2¾" between the arms and use this radius to draw on each playing board whatever fractional part of a disk matches the fraction numeral previously written in the corner of the board. (See sample board shown at the left in the illustration below.) Using the same radius, make 6 complete disks on oaktag. The following wedge-shaped fractional pieces may be cut from them:

| | |
|---|---|
| 2 one thirds | 3 one twelfths |
| 3 one fourths | 3 one fifteenths |
| 4 one fifths | 4 one sixteenths |
| 5 one sixths | 4 one twentieths |
| 6 one eighths | 8 one twenty-fourths |
| 8 one tenths | 5 one thirtieths |

Label each wedge with the numeral for the fraction it represents and put all the wedges into a box. The box should be roomy enough to enable the wedges to be tossed about when the box is shaken. Having been cut from disks of the same size as those on the playing boards, the wedges will fit into the disks when the boards are in play. The game requires each player to select appropriate wedges to cover that part of a disk shown on his particular playing board. For example, a board showing five sixths of a disk might be covered with wedges marked, respectively, ⅓, ⅙, ⅙, 1/12, and 1/20, since $\frac{1}{3}+\frac{1}{5}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}=\frac{5}{6}$. (See board in play at the right in the illustration.)




The game is played in this manner. Seat the players about a small table. Pile books on the table to a

height just above the players' eye level to serve as a stand for the box of wedges. Choose one pupil as the dealer. Have the dealer give each player, including himself, a playing board. The dealer then shakes the box of wedges, removes the lid, and replaces the box on the pile of books. Without looking, he reaches into the box and picks up wedges, distributing them among the players, including himself. He should give each player five wedges.

Each player then tries to fit as many of his five wedges as possible into the space illustrated by the incomplete disk on his playing board. (The *missing* part of the disk is not a factor in this game and should be ignored.) It is unlikely that a player will be able to cover the space illustrated with fraction wedges and thus go out this early in the game. In the event that someone can do so, his claim to winning the game should be checked by the other players. They should add the fractions represented by the numerals on the wedges used to cover the space on the "winning" playing board, to be sure that their sum can be expressed by the fraction numeral at the top left of the board.

If no one can go out by filling his space completely, the play continues clockwise from the dealer. Each player in turn reaches into the box, takes a new wedge, and discards one from his hand. Discards are placed on top of one another on the table. After the first such play, the subsequent players have the option of taking a wedge from the box or from the discard pile. A player who selects from the discard pile may take only the top wedge, or he may take the entire pile, *providing he can play the top wedge on his board.* Should he take the whole pile, he must look through it to select other fraction wedges for immediate play. Then he must play these pieces and put back into the discard pile *all* the unplayable pieces, instead of his usual discard of one. If he wishes, he may rearrange his board, discarding pieces already played, whenever it is his turn to discard. Play continues until one player goes out, as described above. This player then becomes the dealer and a new game is started.

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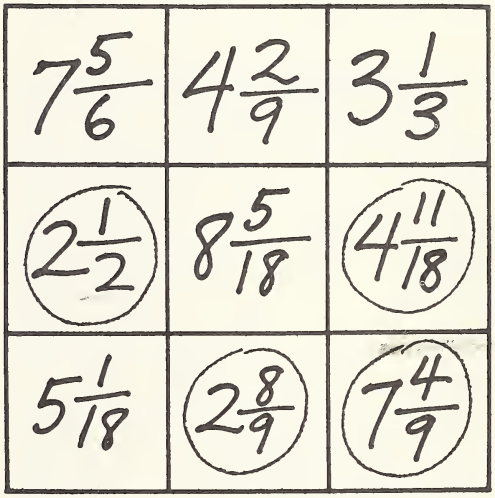
**Activity 31**

## Magic squares with mixed numerals

In this activity the pupils add and subtract numbers expressed by mixed numerals.

Rule 4″ x 6″ cards into three rows of three rectangles each. Then arrange mixed numerals, one in each rectangle, in such a way that the sum of the numbers expressed in each column and each row is the same. These are the teacher's master cards. Note that on the sample master card shown below two mixed numerals are circled in each of two rows. These spaces are left blank on the pupils' cards. The object of the activity is to find the mixed numerals that are missing and thus complete the magic square.

In making the pupils' cards, then, two mixed numerals should be omitted in each of two rows or columns. It is necessary to show numerals in all the rectangles in one row or column because that is the only way in which the pupils can find out what the sum of the numbers expressed in each row or column should be. For the card shown in the illustration, the sum is $15\frac{7}{18}$.




Each pupil may be given a completed card with mixed numerals in all except four rectangles, or the card may be reproduced on the chalkboard, so that the pupil can make his own set of nine rectangles on paper and write the mixed numerals in the proper spaces. If necessary, give some hints on how to proceed, but do not give help unless the pupil asks for it. Hints may consist of discussion and questions to help him see that he finds the desired sum from the full row and that the other mixed numerals he needs to get the sum can also be found by using mixed numerals already on the card. In other words, this is the type of problem situation in which it is necessary to find how much is to be added.

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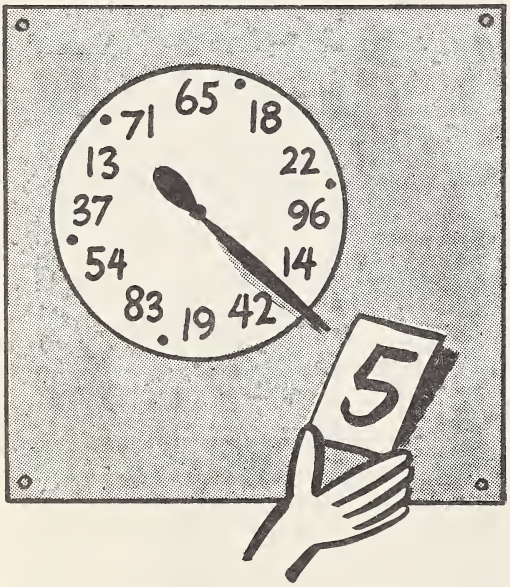
Activity 32

## Mental calculations at the flannel board

A flannel board, an oaktag disk with 12 two-figure numerals, and a pack of small cards with a one-figure numeral on each of them can form the basis for activities that give the pupils practice in multiplying mentally.

Make the oaktag disk 10 inches in diameter, and write on it 12 two-figure numerals, equally spaced, as shown in the illustration below. Make an oaktag arrow, or spinner, 6 inches long and ½ inch wide. Punch a small hole through the center of the disk and toward one end of the spinner. Fasten the disk and spinner to the flannel board with a tack. Use about 6 thumbtacks to hold the disk firmly on the flannel board.




Cut eight 3″ x 5″ cards in half to make 3″ x 2½″ cards. Write two sets of the numerals 2 to 9 on these cards, one numeral to a card.

For an activity based on multiplying without paper and pencil, let all the pupils participate, if they wish. The pack of cards is placed face down in a central position. The pupils take turns spinning the arrow on the disk. When a pupil spins the arrow, he notes the numeral at which it stops. (If it stops between two numerals, he may use the numeral for the larger number.) He then draws the top card from the pack and reads the numeral on it aloud. Then, calculating mentally, he must find the product of the two numbers named by the numeral on the disk and the numeral on the card. He should explain his thinking aloud. For example, suppose the arrow points to "42," and the pupil draws the card with the numeral 5 on it. He could tell the group that he thinks of 42 as 40 and 2, and that he multiplies 40 by 5 to get 200, and 2 by 5 to get 10, making 210 in all.

Continue the lesson by permitting the pupil who participated last to call upon the next person to spin the arrow and do the mental calculation for a different set of numerals.

All pupils benefit by recognizing that a correct solution may be obtained by different means. The length of time devoted to such an activity would depend upon the interest and participation of class members, but the time spent probably should not exceed fifteen minutes in one lesson period.

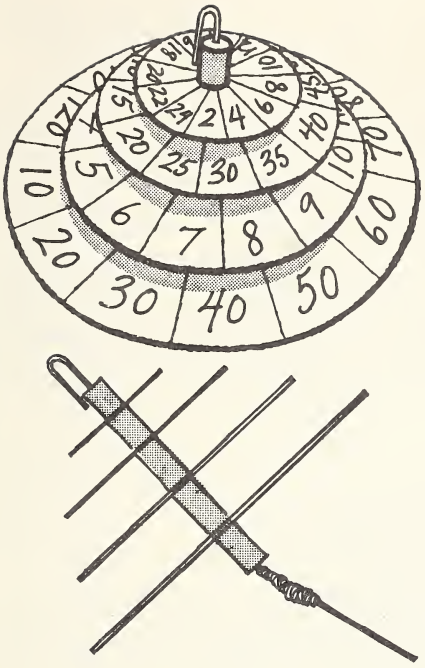
Activity 33

## Adding without pencil and paper

To give the pupils practice in adding as many as four addends mentally, prepare a spinning device made of four disks cut out of oaktag. Some pupils may make their own disks to use at their desks or to take home. The following directions can be used.

Cut disks with diameters of 5″, 4″, 3″, and 2″. Use compasses to make these disks. Make a hole in the center of each disk and divide the disk into 12 sections. Draw in the complete diameters of the smallest disk. Only the outer portions of the diameters need to be drawn in the other three disks. In each section of the largest disk write one of the decade numerals from 10 to 120 in order. In the sections of the 4″ disk, write the numerals from 1 to 12. In the sections of the 3″ disk write the numerals from 5 to 60 by fives. And in the sections of the smallest disk write the numerals from 2 to 24 by twos.

Thread the disks on a large paper clip that has been straightened except for one bend, and use wooden beads as dividers. Arrange the disks according to size so that the smallest disk is at the top and the largest disk at the bottom. A bead should be placed below the largest disk and a rubber band wound around the straightened part of the clip to hold the disks in position. (See the drawing at the top of the next page.)




Now the device is ready to use. A pupil holds it by the projecting part of the paper clip and spins the largest disk. When the largest disk is spun, the other disks will move also. When the disks stop spinning, the pupil is to note where the bent part of the paper clip is pointing and mentally add the four numbers expressed by one numeral from each disk. If the clip points to a line on any of the disks, the numeral on either side may be used.

This activity may be used by small groups. A pupil spins the disks and adds his four numbers, explaining his thinking as he does so. Time spent will vary according to the interest of the pupils.

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**Activity 34**

## What's the number?

This game provides practice in both addition and short division.

Prepare two sets of cards, 36 cards in one set and 16 cards in the other. (3″ x 5″ cards may be cut in half to make 2½″ x 3″ cards.) Use a different color for each set.

On the set of 36 cards write any two- and three-figure numerals. On the set of 16, write the numerals 2 to 9 (two cards for each numeral).

The two packs of cards are placed face down in a central spot, and a group of six (or fewer) players begins the activity. Each player, except the one designated as the checker, selects a card from the pack of two- and three-figure numerals and turns it face up in front of him. Everyone except the checker mentally adds the numbers named by the numerals on the cards of all the players. (There can be as many as five addends if there are that many players besides the checker.) Each player should record the total on scratch paper. The checker, meanwhile, adds in the usual way with paper and pencil. When the correct total has been determined (the checker may be challenged), each player who has the correct total gets that total as his score. The checker keeps a record of individual scores.

Next, a pupil draws a one-figure numeral card and places it where all can see it. Now the players work a short division exercise, using the correct sum for the addition exercise they have just done as the dividend, and the one-figure number as divisor. Those who get the correct answer add the number in the quotient to their scores. After several rounds of drawing cards from both packs, the scores are added, and the pupil with the highest total wins.

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**Activity 35**

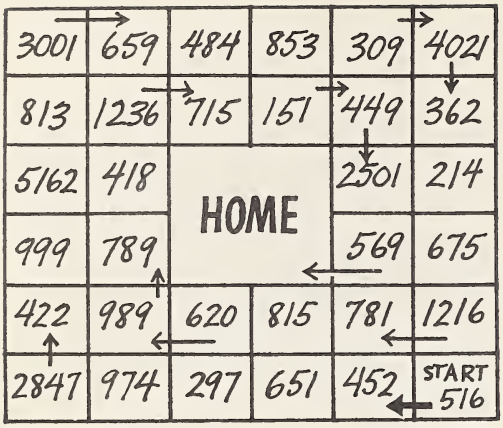
## Short way home

This game provides practice in short division with remainders.

Use a 5″ x 6″ piece of oaktag to make a playing board. Divide it into spaces as shown on page 396. Erase the lines that form the four center spaces to make the space labeled "Home." In each of the 32 small spaces, write a three- or a four-figure numeral. Write the word "Start" in the space in the lower right corner. Draw arrows to direct the way "Home." Entrance to "Home" is at the lower right corner of the large space.

On 24 small cards write the numerals 2 to 9 (each numeral will be used three times).

Two or three pupils can play the game. Each pupil should have an identifying marker. To begin the game, a player puts his marker on the space labeled "Start." He turns over the top card of the pack of one-figure numerals. Using short division, he is to divide the number named by the numeral under his marker by the one-figure number named by the nu-




meral on the card he has turned up. (He may use scratch paper.) If there is a remainder, he can move his marker as many spaces as the remainder indicates. Otherwise, his marker must remain where it is for that round. Thus, in the first play, if the numeral on the card is 7, he divides 516 (the first number named on the sample playing board) by 7 and gets 73 and 5 remainder. He advances his marker 5 squares to the numeral 2847. The other players check his work. Then the next player takes his turn. The first player to reach "Home" is the winner. On the last play, any remainder that is not short of the required number is acceptable.

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**Activity 36**

## Device for solving per cent problems

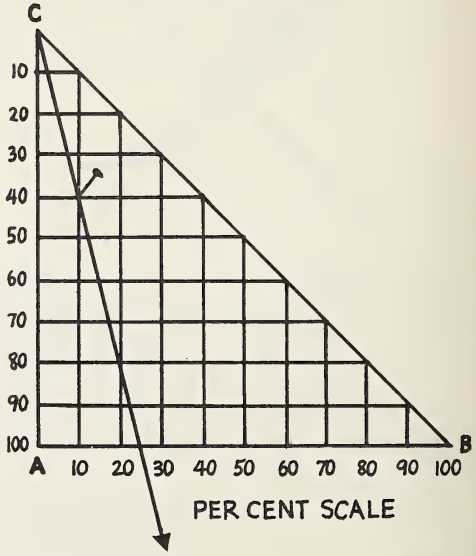
The able pupils may construct a device for finding answers for all types of per cent problems. The materials needed are a piece of string (or heavy black thread), a sheet of graph paper (10 squares to the inch is convenient), a piece of stiff cardboard larger than the graph paper, and a pin.

Mount the graph paper on the cardboard. Draw a horizontal line *AB* (see the picture in the next column) 100 units (squares) long near the bottom of the graph paper. This line will represent a per cent scale and should be labeled at 10-unit intervals as indicated in the drawing. At *A*, draw a line perpendicular to *AB* and also 100 units long (Line *AC* in the drawing). Then connect Points *B* and *C* with a line.

Now draw lines parallel to *AB* at 10-unit intervals from *C*. Label these lines at the left according to their length. Also draw lines perpendicular to *AB* at 10-unit intervals from *A*. Punch a hole at *C* and attach a string slightly longer than the diagonal *BC*.

To find what per cent 10 is of 40, find the horizontal line between *AC* and *BC* that is 40 units long. Locate the point that is 10 units to the right of *AC* on this line and place a pin at this point. Hold the string (represented by the line with the arrow) taut to form the straight line that passes through *C* and the pin point. The point where the string crosses the per cent scale indicates the per cent (25%).

The other two types of per cent problems can also be solved. For example, to find 60% of 30, hold the string taut to form the straight line that passes through *C* and 60 on the per cent scale. The point at which the string crosses the horizontal line that is 30 units long is 60% of 30 (18).




To find the number of which 10 is 20%, stretch the string from *C* to 20 on the per cent scale. The result will be at the point where the string crosses the vertical line labeled 10 (50). In a case where the result will be more than 100 (i.e., 80 is 40% of ▒) the number scale at the left will have to be extended below the per cent scale to include numbers greater than 100.

To find per cents greater than 100%, the per cent scale will have to be extended to the right to include these per cents.

---

Activity 37

## Ratio roundup

To prepare for this game, write ratios on 30 cards (3″ x 5″). On each card write a pair of equal ratios in such a way that when the card is cut in two, a ratio will be centered in each 3″ x 2½″ section. Suggested pairs are:

| | |
|---|---|
| 7/42 and 12/72 | 4/9 and 48/108 |
| 5/7 and 10/14 | 28/40 and 49/70 |
| 4/20 and 6/30 | 7/15 and 14/30 |
| 7/8 and 35/40 | 4/38 and 8/76 |
| 16/24 and 10/15 | 10/28 and 15/42 |
| 16/20 and 28/35 | 3/7 and 18/42 |
| 5/50 and 8/80 | 8/16 and 25/50 |
| 7/12 and 21/36 | 3/8 and 12/32 |
| 7/18 and 14/36 | 3/9 and 10/30 |
| 3/13 and 18/78 | 21/35 and 12/20 |
| 4/14 and 10/35 | 30/54 and 5/9 |
| 3/4 and 15/20 | 9/10 and 18/20 |
| 3/24 and 8/64 | 16/34 and 32/68 |
| 8/36 and 4/18 | 6/40 and 12/80 |
| 9/36 and 11/44 | 9/16 and 18/32 |

The object of the game is to match as many pairs of equal ratios as possible. A pupil may use the previously taught method to identify some of the equal ratios and then verify the choice by the ratio test.

Two to six players can participate. Each player draws five cards. The remaining cards are placed face down in the center of the table. To start the game, one player draws a card from the pile on the table and examines his cards to see if he has a pair of equal ratios. One pair, only, may be put down at a single play. If the player cannot make a pair of equal ratios, he discards the card drawn or a card from his hand. Discards are placed face up in a separate pile. If a player can make a pair of equal ratios, the two cards are placed on the table for all players to see and to verify the equality. Then the player draws a card so that he has five cards in his hand at all times. The game proceeds in this manner, with each player making a pair of equal ratios or discarding. When a player draws a card, he may draw from those face down or from the discard pile.

When all the cards from the original center pile have been drawn, the discard pile is shuffled and placed face down in the center. Discards are again placed alongside, face up. When all cards have been used from both piles, each player must discard one from his hand at the end of his turn. This will keep a card on the table and diminish those held in the hand so that the game can come to a conclusion.

Each correct pair of equal ratios counts one point. If a player challenges another player, he scores a point if he is correct, and the player challenged loses a point. If the challenger is incorrect, the reverse is true. When a player has finally used all his cards, the remaining players deduct from their score one point for each card held; then the combined score of deducted points is added to the score of the player who has finished first. The winner is the one who has the most points, although he may or may not be the player who finishes first.

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Activity 38

## Polygon puzzles

Many pupils will be interested to learn that each of Squares A, B, C, D, shown on page 398, can be cut into smaller polygons, as indicated by the dotted lines, and the pieces then fitted together into the shape of another polygon or polygons.

Provide the pupils with duplicated cutouts of these squares. Have them cut on the dotted lines and fit the pieces together so that they form the polygons shown at the right. The labels and directions, as well as the drawings showing the polygons the pupils are to form, are for the teacher's use only. Each of Squares A, B, C, and D may have any length of side so long as the dotted lines are drawn as shown.

Ask the pupils to use the pieces indicated in Square A to form a rectangle that is not square; to form a parallelogram that is not a rectangle of the pieces indicated in Square B; to form an equilateral triangle of the pieces indicated in Square C; and to form two squares of the pieces indicated in Square D.

The pupils should be encouraged to think of other polygons that can be cut in pieces and put together to make different polygons.

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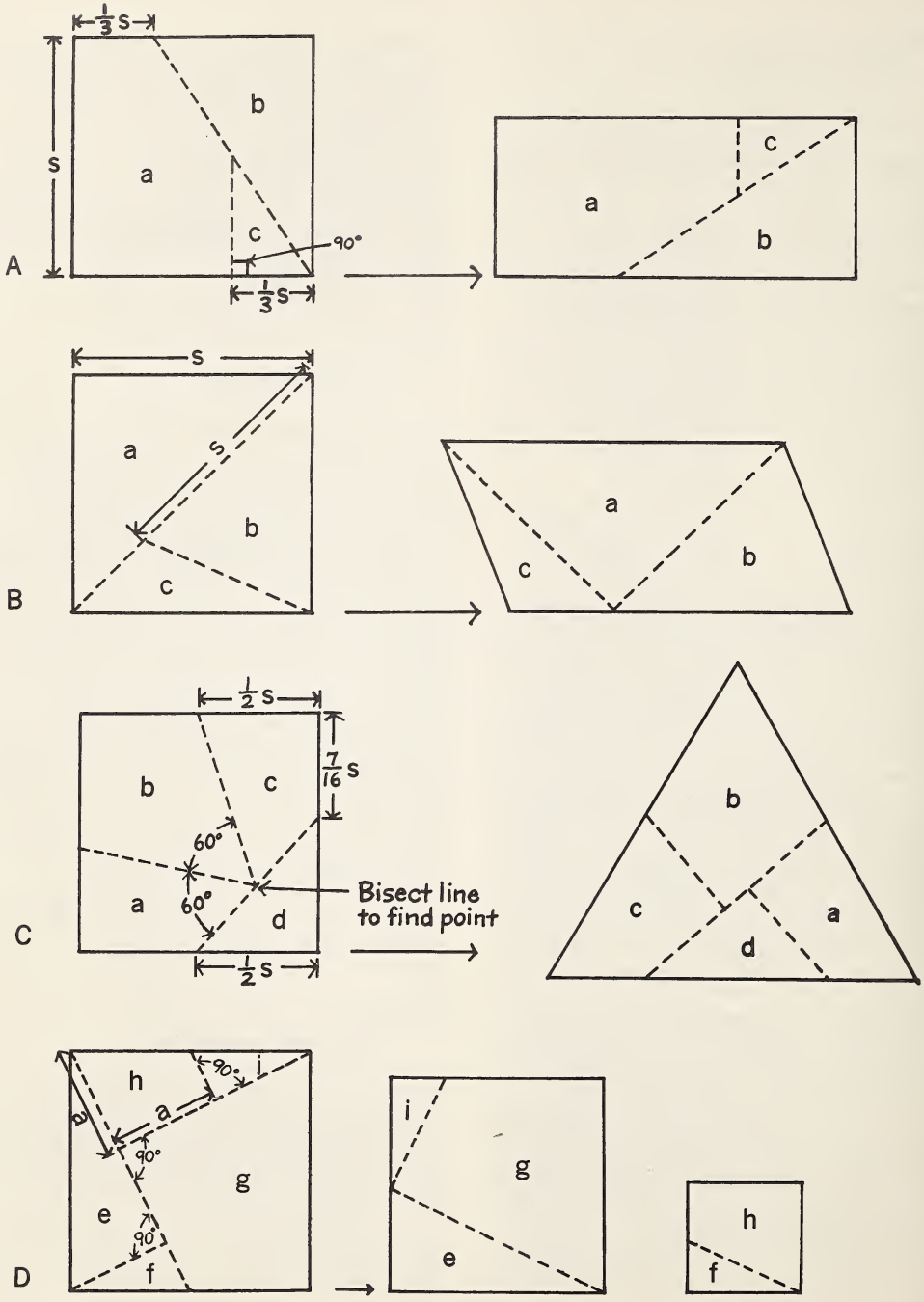
Activity 39

## Numeral "spelldown"

To give pupils practice in regrouping (ten-thousandths to thousandths, thousandths to hundredths, hundredths to tenths, tenths to ones), let them have a spelldown as described below. The pupils should first "number off" with "evens" on one team and "odds" on the other.

To start the game, the leader of one team asks the leader of the other team to change 3 tenths 14 hundredths 8 thousandths, for example, to the simpler


A
B
C
D
s
a
b
c
90°
60°
Bisect line
to find point
d
e
f
g
h
i

form (.448). If the opposing member does so correctly, it is his turn to make up an expression containing tenths, hundredths, thousandths (or ones, tenths, hundredths, etc.) and ask the second person on the first team to express it in the simpler form. When these numerals are changed by the team members, they should be written as well as spoken so that the teams can verify each answer. The pupils should understand that one of the groups in the numeral must be more than 9. When a player misses, he must sit down and give the next member on the opposing team a chance to answer. The pupil who remains standing at the end is the winner.

Another way to play the game is to give a numeral —for example, .134—and ask that it be changed to tenths, hundredths, and thousandths in such a way that one of the groups will contain more than 9. (For example, .134 may be thought of as 13 tenths and 4 thousandths or as 1 tenth 2 hundredths 14 thousandths.)

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**Activity 40**

## Building polyhedrons

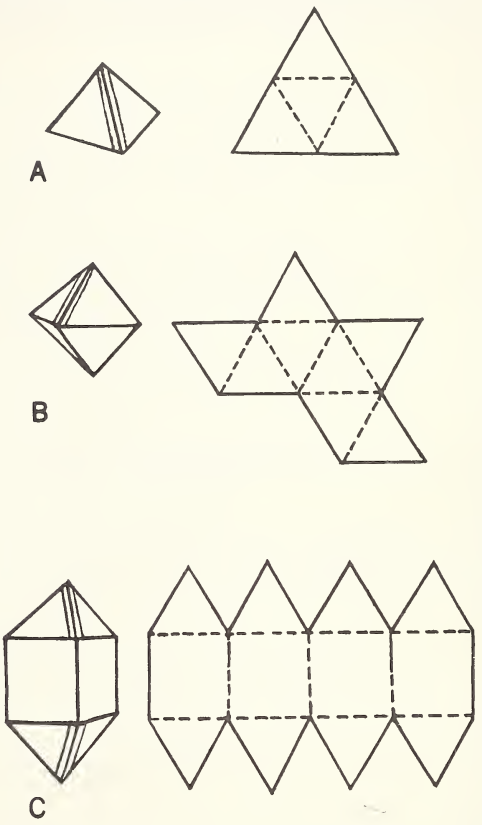
Pupils gain insight and information about three-dimensional shapes by building geometric shapes, or "solids," having various kinds of faces. The picture in the next column shows, at the left, drawings of several such shapes, and their patterns are shown at the right. These patterns should be dittoed so that each pupil has a model from which he can construct the "solid" of tag-board or heavy paper. Each "solid" is formed by cutting out the pattern, folding it along the dotted lines, and then fastening the edges with cellulose tape or stamp hinges.

Experiences of this kind will help the pupil form generalizations about the great number of "solids" and their construction from plane faces. The pupils should be encouraged to form other "solids" by making patterns of their own and putting them together.

To form a pattern for Polyhedron A, draw an equilateral triangle with any convenient length for the side. Bisect each side and connect these mid-points by dotted lines.

The pattern for Polyhedron B is constructed from congruent equilateral triangles placed as shown in the pattern below.

The pattern for Polyhedron C may be formed by drawing a rectangle of any convenient size. Use dotted lines for the two long sides. Then divide this rectangle into fourths as shown below. One fourth of the length of the rectangle then becomes the base of each of eight congruent isosceles triangles, placed as shown on the diagram. Any desirable altitude may be used for these eight congruent triangles.



Your notes

# Reference materials

This section of the *Teaching Guide* provides you with charts and tables that will enable you to understand better both the entire program and the materials for this grade.

**Bibliography**

A list of books and articles that should add interest to and help in the teaching of arithmetic

**Chart of problem types**

An analysis of the types of verbal problems that the child learns to solve

**Time schedule**

The authors' estimate of the rate of progress you may expect to make during the year

**Grade placement chart**

A graphic means of discovering the grade placement of the important topics in arithmetic for Grades 1 to 6 in this program

**Word list**

A complete alphabetical list of all words used in the pupils' book

**Mathematical content**

A detailed topical list of the mathematical content of the book

**Index**

Entries are for the teaching aspects of arithmetic

# Reference materials

# Bibliography

## Arithmetic content and method

This list includes books on method and on mathematical content of the arithmetic curriculum that have been published during the last ten years. No attempt has been made by the authors of *Seeing Through Arithmetic 6* to evaluate these books.


Adler, Irving, and Adler, Ruth. *The New Mathematics.* New York: John Day and Company, 1958.

Brueckner, Leo J. *Improving the Arithmetic Program.* New York: Appleton-Century-Crofts, Inc., 1957.

Clark, John R., and Eads, Laura K. *Guiding Arithmetic Learning.* Yonkers, New York: World Book Company, 1954.

Hickerson, J. Allen. *Guiding Children's Arithmetic Experiences.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1952.

Hollister, George E. *Teaching Arithmetic in Grades I and II.* Boston: D. C. Heath and Company, 1954.

Hunnicutt, C. W. *Research in the Three R's.* New York: Harper and Brothers, Publishers, 1958.

McIntyre, J. V. *Teaching of Arithmetic.* London, England: J. M. Dent and Sons, Ltd., 1958.

McSwain, E. T. *Understanding and Teaching Arithmetic in the Elementary School.* New York: Henry Holt and Company, 1958.

Maria, May Hickey. *The Structure of Arithmetic and Algebra.* New York: John Wiley and Sons, Inc., 1958.

Marks, John L., Purdy, C. Richard, and Kinney, Lucien B. *Teaching Arithmetic for Understanding.* New York: McGraw-Hill Book Company, 1958.

Morton, Robert L. *Teaching Children Arithmetic: Primary, Intermediate, Upper Grades.* New York: Silver Burdette Company, 1953.

Mueller, Francis Joseph. *Arithmetic: Its Structure and Concepts.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1956.

Risden, Gladys. *How Big? How Many? Arithmetic for Home and School.* Boston: Christopher Publishing House, 1951.

Rosenquist, Lucy Lynde. *Young Children Learn to Use Arithmetic.* Boston: Ginn and Company, 1949.

Spencer, Peter L., and Brydegaard, Marguerite. *Building Mathematical Concepts in the Elementary School.* New York: Henry Holt and Company, 1952.

Spitzer, Herbert F. *The Teaching of Arithmetic.* 2d ed. Boston: Houghton Mifflin Company, 1954.

Stern, Catherine. *Children Discover Arithmetic: An Introduction to Structural Arithmetic.* New York: Harper and Brothers, Publishers, 1949.

Stokes, C. Newton. *Teaching the Meanings of Arithmetic.* New York: Appleton-Century-Crofts, Inc., 1951.

Swain, Robert L. *Understanding Arithmetic.* New York: Rinehart and Company, 1957.

Wheat, Harry Grove. *How to Teach Arithmetic.* Evanston, Illinois: Row, Peterson and Co., 1956.

Williams, Catherine. *Teaching Arithmetic in the Elementary School.* New York: Hinds, Hayden, and Eldredge, 1946.

Wilson, Guy M., and Others. *Teaching the New Arithmetic.* 2d ed. New York: McGraw-Hill Book Company, 1951.


## Modern learning theory

Many of the changes that are being made in the methods of teaching arithmetic arise from the application of modern principles of teaching to arithmetic. These principles are discussed in the following books.


Cole, Lawrence E., and Bruce, William F. *Educational Psychology.* Yonkers, New York: World Book Co., 1950. Ch. 11-14.

Commins, W. D., and Fagin, Barry. *Principles of Educational Psychology.* 2d ed. New York: The Ronald Press, 1954. Ch. 12-16.

Douglas, O. B., and Holland, B. F. *Fundamentals of Educational Psychology.* New York: Macmillan Company, 1938. Ch. XIV.

Gates, Arthur I., and Others. *Educational Psychology.* 3d ed. New York: Macmillan Company, 1948. Ch. IX-XIV.

Werner, Heinz. *Comparative Psychology of Mental Development.* New York: Harper and Brothers, Publishers, 1940. Ch. IX.

Wertheimer, Max. *Productive Thinking.* New York: Harper and Brothers, Publishers, 1945. Ch. I.

Woodruff, A. D. *The Psychology of Teaching.* 3d ed. New York: Longmans, Green and Company, 1951. Ch. XIV.

## Selected research studies

Anderson, G. Lester. "Quantitative Thinking As Developed Under Connectionist and Field Theories of Learning," *Learning Theory in School Situations,* pp. 40-73. University of Minnesota Studies in Education, No. 2. Minneapolis: University of Minnesota Press, 1949.

Brownell, William A. "Arithmetical Readiness As a Practical Classroom Concept," *Elementary School Journal,* LII (September, 1951), 15-22.

Brownell, W. A., and Carper, Doris V. *Learning the Multiplication Combinations.* Duke University Research Studies in Education, No. 7. Durham: Duke University Press, 1943.

Brownell, W. A., and Moser, Harold E. *Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction.* Duke University Research Studies in Education, No. 8. Durham: Duke University Press, 1949.

Brownell, W. A., and Others. *Arithmetic in Grades I and II, A Critical Summary of New and Previously Reported Research.* Duke University Research Studies in Education, No. 6. Durham: Duke University Press, 1941.

Brownell, W. A., and Others. *Learning As Reorganization, An Experimental Study in Third Grade Arithmetic.* Duke University Research Studies in Education, No. 3. Durham: Duke University Press, 1939.

Buswell, Guy T. (editor). *Improving the Program in Arithmetic.* Proceedings of the Conference on Arithmetic, 1946. Chicago: The University of Chicago Press, 1947.

Dutton, Wilbur H. "Attitudes of Prospective Teachers Toward Arithmetic," *Elementary School Journal,* LII (October, 1951), 84-90.

Ilg, Frances, and Ames, Louise B. "Developmental Trends in Arithmetic," *Pedagogical Seminary and Journal of Genetic Psychology,* LXXIX (September, 1951).

Koenker, Robert H. "Arithmetic Readiness at the Kindergarten Level," *Journal of Educational Research,* XLII (November, 1948), 218-223.

McLaughlin, Katherine L. *A Study of Number Ability in Children of Ages Three to Six.* Ph.D. Dissertation, Chicago: The University of Chicago Press (private edition), 1935.

Martin, William E. "Quantitative Expression in Young Children," *Genetic Psychology Monographs,* XLIV (November, 1951), 147-219.

Osburn, W. J. "Division by Dichotomy As Applied to the Estimation of Quotient Figures," *Elementary School Journal,* L (February, 1950), 326-330.

Passehl, George. "Teaching Arithmetic Through Activity Units," *Peabody Journal of Education,* XXVII (November, 1949), 148-152.

Plank, Emma N. "Observations on Attitudes of Young Children Toward Mathematics," *The Mathematics Teacher,* XLIII (October, 1950), 252-263.

Swenson, Esther J. "Organization and Generalization As Factors in Learning, Transfer, and Retroactive Inhibition," *Learning Theory in School Situations,* pp. 9-39. University of Minnesota Studies in Education, No. 2. Minneapolis: University of Minnesota Press, 1949.

Thiele, C. L. *The Contribution of Generalization to the Learning of the Addition Facts.* Contributions to Education, No. 763. New York: Bureau of Publications, *Teachers College Record,* Columbia, 1938.

Van Engen, Henry, and Gibb, E. Glenadine. *Some General Mental Functions Associated with Division.* Educational Services Studies, No. 2. Cedar Falls, Iowa: Iowa State Teachers College, 1956.

## Selected articles, monographs, and yearbooks

Amsden, Constance E. "Seeing the Meaning of Plus and Minus," *Educational Screen,* XXX (March, 1951), 100-102.

Brownell, William A. "The Revolution in Arithmetic," *The Arithmetic Teacher,* I (April, 1954), 1-5.

Burch, Robert L. "Skills Instruction in Arithmetic," *National Elementary Principal,* XXIX (Dec., 1949).

Buswell, Guy T. (editor). *Arithmetic 1947.* Supplementary Educational Monographs, No. 63. Chicago: The University of Chicago Press (Oct., 1947).

Buswell, Guy T. (editor). *Arithmetic 1948.* Supplementary Educational Monographs, No. 66. Chicago: The University of Chicago Press (Oct., 1948).

Buswell, Guy T., and Hartung, Maurice L. (editors). *Arithmetic 1949.* Supplementary Educational Monographs, No. 70. Chicago: The University of Chicago Press (November, 1949).

Clark, John R. "Issues in Teaching Arithmetic," *Teachers College Record,* LII (Jan., 1951), 205-212.

Deans, Edwina. "The Contribution of Grouping to Number Development," *Childhood Education,* XVII (March, 1941), 307-310.

Gibb, E. Glenadine. "Take-Away Is Not Enough!" *The Arithmetic Teacher,* I (April, 1954), 7-10.

Grossnickle, Foster E. "How to Find the Position of the Decimal Point in the Quotient," *Elementary School Journal,* LII (April, 1952), 452-457.

Hartung, Maurice L. "Estimating the Quotient in Division," *The Arithmetic Teacher,* IV (April, 1957), 100-111.

Hartung, Maurice L. "Forward Look at Evaluation," *The Mathematics Teacher,* VLII (Jan., 1949), 29-33.

Hartung, Maurice L. "Fractions and Related Symbolism in Elementary-School Instruction," *The Elementary School Journal,* LVIII (April, 1958), 377-384.

Hartung, Maurice L. "Major Instructional Problems in Arithmetic in the Middle Grades," *Elementary School Journal,* L (October, 1949), 86-91.

Hartung, Maurice L. "Mathematics in the Total School Program," *The Mathematics Teacher,* LI, No. 5 (May, 1958), 336-343.

Hartung, Maurice L. "The Order of Operations in Elementary Mathematics," *School Science and Mathematics,* XLVI (November, 1946), 752-754.

Judd, Charles Hubbard. *Psychological Analysis of the Fundamentals of Arithmetic.* Supplementary Educational Monographs, No. 32. Chicago: The University of Chicago Press, 1927. Ch. V, VI.

Moser, Harold E. "Can We Teach Pupils to Distinguish the Measurement and Partition Ideas in Division?" *The Mathematics Teacher,* XLV (February, 1952), 94-97+.

Moser, Harold E. "Two Procedures for Estimating Quotient Figures When Dividing by Two-Place Numbers," *Elementary School Journal,* XLIX (May, 1949), 516-522.

National Council of Teachers of Mathematics Tenth Yearbook: *The Teaching of Arithmetic.* Washington, D.C.: National Council, 1935.

National Council of Teachers of Mathematics Sixteenth Yearbook: *Arithmetic in General Education.* Washington, D.C.: National Council of Teachers of Mathematics, 1941. See especially Chapter XI: "Recent Trends in Learning Theory: Their Application to the Psychology of Arithmetic," by T. R. McConnell.

National Council of Teachers of Mathematics Twenty-First Yearbook: *The Learning of Mathematics.* Washington, D.C.: National Council of Teachers of Mathematics, 1953. See especially Chapter II: "Motivation for Education in Mathematics," by Maurice L. Hartung.

National Society for the Study of Education Forty-First Yearbook, Part II: *The Psychology of Learning.* Distributed by The University of Chicago Press, 1942. See especially Chapter I: "Conditioning: A Theory of Learning in Terms of Stimulus, Response and Association," by E. R. Guthrie; Chapter VI: "Field Theory and Learning," by Kurt Lewin; and Chapter VII: "Reconciliation of Learning Theories," by T. R. McConnell.

National Society for the Study of Education Forty-Fifth Yearbook, Part I: *The Measurement of Understanding.* Distributed by The University of Chicago Press, 1946.

National Society for the Study of Education Forty-Ninth Yearbook, Part I: *Learning and Instruction.* Distributed by The University of Chicago Press, 1950. See especially Chapter I: "The General Nature of Learning," by G. Lester Anderson and Arthur I. Gates; Chapter IV: "How Children Learn Information, Concepts, and Generalizations," by William A. Brownell and Gordon Henrickson; Chapter X: "Applications of Learning Principles to the Improvement of Teaching in the Early Elementary Grades," by Esther J. Swenson; and Chapter XI: "Implications for Improving Instruction in the Upper Elementary Grades," by G. Max Wingo.

National Society for the Study of Education Fiftieth Yearbook, Part II: *The Teaching of Arithmetic.* Distributed by The University of Chicago Press, 1951.

Roudebush, Elizabeth J. "An Arithmetic Bulletin for Parents," *The Mathematics Teacher,* XLIV (May, 1951), 289-292.

Smith, Rolland R. "Meaningful Division," *The Mathematics Teacher,* XLIII (January, 1950), 12-18.

Trimble, H. C. "Fractions Are Ratios Too," *Elementary School Journal,* XLIX (Jan. 1949), 285-291.

Van Engen, Henry. "An Analysis of Meaning in Arithmetic," *Elementary School Journal,* XLIX (February-March, 1949), 321-329, 395-400.

Van Engen, Henry. "Some More Thoughts on Placing the Decimal Points in Quotients," *The Mathematics Teacher,* XXXVIII (Oct., 1945), 243-245.

Van Engen, Henry. "Teaching the Mathematical Skills in the Elementary School," *Progressive Education,* XXIX (October, 1951), 17-20.

Van Engen, Henry. "Which Way Arithmetic?" *The Arithmetic Teacher,* II (Dec., 1955), 131-140.

Weaver, J. Fred. "A Crucial Aspect of Meaningful Arithmetic Instruction," *The Mathematics Teacher,* XLIII (March, 1950), 112-117.

Weaver, J. Fred. "Misconceptions about Rationalization in Arithmetic," *The Mathematics Teacher,* XLIV (October, 1951), 377-381.

Weaver, J. Fred. "Some Areas of Misunderstanding about Meaning in Arithmetic," *Elementary School Journal,* LI (September, 1950), 35-41.

Williams, Catharine M. "Arithmetic Learning in an Experience Curriculum," *Educational Research Bulletin,* XXVIII (September, 1949), 154-162+.

## Useful bibliographies

Burch, R. L., and Moser, H. E. "Teaching of Mathematics in Grades I Through VIII," *Review of Educational Research,* XXI (October, 1951), 290-304.

Johnson, Harry C. "Problem-Solving in Arithmetic: A Review of the Literature," *Elementary School Journal,* XLIV (March-April, 1944), 396-403, 476-482.

National Council of Teachers of Mathematics Sixteenth Yearbook, Ch. XIV and XV.

Van Engen, Henry. "A Summary of Research and Investigations and Their Implications for the Organization and Learning of Arithmetic," *The Mathematics Teacher,* XLI (Oct., 1948), 260-265.

# Mathematical content

### Numerals and the number system

- Rounding off numbers (reteaching)
- Common fraction numerals (reteaching)
- Decimal fraction numerals (reteaching and extension to ten-thousandths)
- Positive and negative numbers
- Regrouping as required in carrying and borrowing (involving decimal fraction numerals)
- Number line—naming points; sets of names

### Division of whole numbers

- Division by two-figure and three-figure divisors (reteaching)
- Estimating partial quotients
- Traditional method of division
- Mental computation (one-figure divisors)

### Fractions (involving common fraction numerals)

- Concept of fractions (reteaching and extension)
- Reduction
- Comparison of fractions
- Addition of proper fractions and mixed numbers
- Subtraction of proper fractions and mixed numbers
- Multiplication of proper fractions and mixed numbers
- Division of proper fractions and mixed numbers
- Using the remainder in division to make a fraction numeral in the quotient

### Fractions (involving decimal fraction numerals)

- Extension of the number system to include decimal fraction numerals to ten-thousandths
- Addition
- Subtraction
- Multiplication
- Division
- Expressing the remainder in division
- Reduction of common fraction numerals to decimal fraction numerals

### Per cent

(See Ratios)

### Ratios

(Also see Problem solving)

- Ratios as expressions of rate (reteaching)
- Ratios as expressions of comparison (reteaching)
- Per cents as ratios expressing comparison
- Reduction of ratios (reteaching)

### Measurement

- Measures of area—square inch, square foot, square yard, square rod, acre (reteaching)
- Measures of volume—cubic inch, cubic foot, cubic yard
- Reduction of measures (reteaching and extension)
- Precision in measurement
- Metric measures—gram, meter, liter, kilogram, kilometer, centimeter

### Graphs

- Graphs representing rates
- Statistical graphs

### Geometry

- Parallel lines and right angles (reteaching and extension)
- Recognition of polygons and circle (reteaching)
- Recognition of cylinder, various prisms, cone, pyramid, sphere, half sphere
- Concept of area (reteaching and extension)
- Concept of volume
- Measurement of area—rectangles and other parallelograms (reteaching and extension)
- Measurement of volume—rectangular prisms
- Perimeters
- Surface area of rectangular prisms

### Problem solving

- Review and reteaching of previously taught multiple-step problems
- Using a variety of symbols as place holders in equations for multiple-step problems
- Using ratios to solve problems involving rate
- Using ratios to solve problems involving comparison
- Using ratios to solve problems involving comparison expressed in terms of "times as many as"
- Using ratios to solve comparison problems involving per cent
- Using the ratio test to solve equations for rate and comparison problems
- Using fraction numerals (common and decimal) as terms of ratios in equations for rate and comparison problems
- Making abstract statements for comparison problems involving common fraction numerals, decimal fraction numerals, and per cents
- Comparison problems solved by addition and subtraction

# Problem types

## additive and subtractive situations

| type no. | problem description | sample problem | equation | compu-tation | situation-process description | grad |
|---|---|---|---|---|---|---|
| AS-1 | Finding the resulting total when the original number and the number added are known | John had 9 apples. Tom gave him 3 apples. How many apples did John have then? | $9 + 3 = n$ | $9 + 3$ | additive-addition | 3 |
| AS-2 | Finding how many were added or how many more are needed when the original number and the resulting total or desired total are known | John had 9 apples. After Tom had given him some more apples, he had 12 apples. How many apples did Tom give him? | $9 + n = 12$ | $12 - 9$ | additive-subtraction | 3 |
| AS-3 | Finding how many there were to begin with when the number added and the resulting total are known | John had some apples. Tom gave him 3 apples. Then he had 12 apples. How many apples did John have to begin with? | $n + 3 = 12$ | $12 - 3$ | additive-subtraction | 3 |
| AS-4 | Finding the number left when the original number and the number gone are known | John had 12 apples. He lost 3 of them. How many apples did he have then? | $12 - 3 = n$ | $12 - 3$ | subtractive-subtraction | 3 |
| AS-5 | Finding the number gone when the original number and the number left are known | John had 12 apples. Then he lost some. He had 9 apples left. How many apples did he lose? | $12 - n = 9$ | $12 - 9$ | subtractive-subtraction | 4 |
| AS-6 | Finding the original number when the number gone and the number left are known | John had some apples. He lost 3 of them. Then he had 9 apples left. How many apples did he have to begin with? | $n - 3 = 9$ | $9 + 3$ | subtractive-addition | 4 |
| | **rate situations** | | | | | |
| R-1 | Finding the total number when the number in each group and the number of equal groups are known | John had 4 boxes of 3 apples each. How many apples did he have? | $4 \times 3 = n$<br>$\frac{3}{1} = \frac{n}{4}$ | $4 \times 3$ | multiplicative-multiplication<br>rate-multiplication | 3<br>5 |
| R-2 | Finding the number in each of the equal groups when the resulting total number and the number of groups are known | John put 4 equal groups of apples into a box. Then he saw that he had 12 apples in the box. How many apples were in each group that he put into the box? | $4 \times n = 12$<br>$\frac{n}{1} = \frac{12}{4}$ | $12 \div 4$ | multiplicative-division<br>rate-division | 5<br>5 |
| R-3 | Finding the number of equal groups when the resulting total number and the number in each of the equal groups are known | John put several groups of 3 apples each into a box. Then he saw that he had 12 apples in the box. How many groups of apples did he put into the box? | $n \times 3 = 12$<br>$\frac{3}{1} = \frac{12}{n}$ | $12 \div 3$ | multiplicative-division<br>rate-division | 3<br>4 |
| R-4 | Finding the number of equal groups when the original total and the number in each of the equal groups are known | John had 12 apples. He put a group of 3 apples into each of several boxes. How many boxes did he use? | $12 \div 3 = n$<br>$\frac{3}{1} = \frac{12}{n}$ | $12 \div 3$ | quotitive-division<br>rate-division | 3<br>4 |
| R-5 | Finding the number in each of the equal groups when the original total and the number of equal groups are known | John had 12 apples. He put equal groups of these apples into each of 4 boxes. How many apples did he put into each box? | $12 \div n = 4$<br>$\frac{n}{1} = \frac{12}{4}$ | $12 \div 4$ | partitive-division<br>rate-division | 4<br>5 |
| R-6 | Finding the original number when the number in each of the equal groups and the number of equal groups are known | John had some apples to put into 4 boxes. He put 3 apples into each box. How many apples did he have to put into the boxes? | $n \div 3 = 4$<br>$\frac{3}{1} = \frac{n}{4}$ | $4 \times 3$ | divisive-multiplication<br>rate-multiplication | 4<br>5 |

**comparison situations**

| ype no. | problem description | sample problem | equation | compu-tation | situation-process description | grade |
|---|---|---|---|---|---|---|
| C-1 | Finding how {many more / much more} or how {many fewer / much less} one quantity is thon another | John has 12 apples and 9 oranges. How many {more apples / fewer oranges} than {oranges / apples} has he? | $12 - 9 = n$ | $12 - 9$ | comparison-subtraction | 3 |
| C-2 | Finding the smaller quantity when the larger quantity and the excess of the larger quantity or the deficiency of the smaller quantity are known | John has 12 apples. He has 3 {more apples than oranges. / fewer oranges than apples.} How many oranges has he? | $12 - n = 3$ | $12 - 3$ | comparison-subtraction | 6 |
| C-3 | Finding the larger quantity when the smaller quantity and the deficiency of the smaller quantity or the excess of the larger quantity are known | John has 9 apples. He has 3 {more oranges than apples. / fewer apples than oranges.} How many oranges has he? | $n - 9 = 3$ | $9 + 3$ | comparison-addition | 6 |
| C-4 | Finding the ratio of one quantity to another (situations involving "times as many as," "fraction as many as") | John has 12 apples and 4 oranges. He has how many times as many apples as oranges? He has what fraction as many oranges as apples? | $\frac{n}{1} = \frac{12}{4}$ $\left(\frac{1}{n} = \frac{12}{4}\right)$ | $12 \div 4$ $(4 \div 12)$ | comparison-division | 5 6 |
| C-5 | Finding a quantity when the other quantity and the ratio ("number times as many as" or "fraction as many as") of the first quantity to it are known | John has 12 apples. He has {3 times as many / ⅓ as many} apples as oranges. How many oranges has he? | $\frac{3}{1} = \frac{12}{n}$ $\left(\frac{⅓}{1} = \frac{12}{n}\right)$ | $12 \div 3$ $(12 \div ⅓)$ | comparison-division | 5 6 |
| C-6 | Finding a quantity when the other quantity and its ratio ("number times as many as" or "fraction as many as") to the first quantity are known | John has 12 oranges. He has {3 times as many / ⅓ as many} apples as oranges. How many apples has he? | $\frac{3}{1} = \frac{n}{12}$ $\left(\frac{⅓}{1} = \frac{n}{12}\right)$ | $3 \times 12$ $(⅓ \times 12)$ | comparison-multiplication | 5 6 |

## Time schedule

The authors do not believe it possible to provide a rigid schedule for teachers to follow in using *Seeing Through Arithmetic 6*. Differences in the backgrounds and abilities of children make a uniform schedule impractical. Nevertheless, many teachers and schools feel a need for some kind of schedule that will serve as a general guide in covering the year's work. The schedule at the right, given in six-week periods, is based upon the authors' judgment and experience with the materials and upon the experience of teachers now using Book 6. No attempt should be made to force a class to maintain the pace indicated or to retard a class to that pace.

| period | pages of book 6 | content |
|---|---|---|
| 1 | 3-65 | solids; concept of volume; expressing rates, comparison, and fractions |
| 2 | 66-124 | addition and subtraction with common fraction numerals; inequalities; rate and comparison problems |
| 3 | 125-164 | per cent; ratio test; area of parallelogram; traditional method of division (optional) |
| 4 | 165-208 | number line; common fraction numerals (multiplication and division) |
| 5 | 209-246 | fraction numerals in ratios; multiplication and division with decimal fraction numerals |
| 6 | 247-282 | volume of rectangular prism; subtractive comparison problems; integers; graphs; review |

| topic | grade 1 | grade 2 | grade 3 |
|---|---|---|---|
| groups and number systems | recognizing, combining, separating, and rearranging through 10 | recognizing, combining, separating, and rearranging through 18 | recognition through 36 |
| decimal system of enumeration | through 99; place value | through 999; place value | through 9999; place value; preparation for carrying and borrowing |
| basic facts | | through sums and minuends of 10; informally through products and dividends of 10 | through sums and minuends of through products and dividends of 36 |
| money | coins through dime; related values; counting | coins through quarter; related values; counting | equivalents of $1; writing; making change; addition and subtraction |
| measures | idea of standard unit; development of judgment and physical competence | idea of standard unit; concept of inch, foot, pint, quart; development of judgment | equivalents; dozen; calendar; time; weight; reduction |
| processing addition and subtraction of whole numbers | | | 3-figure numbers; carrying and borrowing |
| processing multiplication of whole numbers | | | |
| processing division of whole numbers | | | |
| meaning of fractions | | judgment in selecting halves and fourths | unit and non-unit through eig reading and writing |
| processing addition and subtraction of fractions | | | |
| processing multiplication of fractions | | | |
| processing division of fractions | | | |
| ratio (see per cent) | | | preparation for reduction |
| graphs and scale drawing | | | |
| geometry | | | identification of circle, square, triangle |
| decimal fraction numerals | | | |
| per cent | | | |
| problem solving (see chart on pages 408-409) | | pictorial representations; development of visualizations; introduction of action ideas | types AS-1, AS-2, AS-3, AS-4, R-1, R-4, R-5, C-1 |
| meaning of the processes | | associating combining and separating actions with problems | order of addends and fact inverse nature of process |
| topic | grade 1 | grade 2 | grade 3 |

## Grade placement of topics

Seeing through arithmetic

Scott, Foresman

| grade 4 | grade 5 | grade 6 | topic |
|---|---|---|---|
| | | number line for system of integers and for system of fractions | groups and number systems |
| through 999,999,999; place value | reading and understanding decimal numerals through hundredths | development of decimal notation as a system | decimal system of enumeration |
| through products and dividends of 81 | used in processes | extension to decimal fraction numerals (as a system) | basic facts |
| multiplication and division | | | money |
| scale; weight; liquid (fl. oz.); dry; mile; abbreviations | square measures; acre; | cubic measures; meter; gram; liter; kilogram; kiloliter | measures |
| retaught; extended to larger numbers | extended to larger numbers | maintained; associative law for addition | processing addition and subtraction of whole numbers |
| 1-figure and 2-figure multiplier; carrying | extended to larger numbers | maintained; associative law; distributive law | processing multiplication of whole numbers |
| -figure and 2-figure divisors re divisors optional); remainders | 2-figure divisors; remainders | maintained; distributive law | processing division of whole numbers |
| quivalent; improper fraction merals and mixed numerals; use in measurement | reduction; comparing | maintained | meaning of fractions |
| | proper fraction numerals and mixed numerals | maintained | processing addition and subtraction of fractions |
| | | proper fraction numerals and mixed numerals | processing multiplication of fractions |
| | | proper fraction numerals and mixed numerals | processing division of fractions |
| | using ratios to express rate and comparison; reduction | more complex applications of rate and comparison; use of the ratio test | ratio (see per cent) |
| meaning of linear scale on map | reading scales and scale drawing; interpreting bar graphs and pictographs | making graphs of sets of ordered pairs | graphs and scale drawing |
| parallel lines; right angle; rectangle | concept of area; perimeters; area of rectangle | area of general parallelogram; identification of solids; concept of volume; volume of rectangular prism | geometry |
| | reading and understanding through hundredths | addition; subtraction; multiplication; division | decimal fraction numerals |
| | introduction (as ratio) | complete development | per cent |
| s AS-5, AS-6, C-4 (limited to "how many times as many as") | types R-1, R-4, R-5 (extended to ratios); R-2, R-3, R-6 (complete); C-4, C-5, C-6 (fractions excluded) | types C-4, C-5, C-6 (extended to fractions); C-2, C-3 (complete) | problem solving (see chart on pages 408-409) |
| | extended to include common fraction numerals | extended to include decimal fraction numerals | meaning of the processes |
| grade 4 | grade 5 | grade 6 | topic |

# Word list

This list contains the 1176 different words that appear in *Seeing Through Arithmetic 6*. Of these 1176 words, 841 were used in the preceding book, *Seeing Through Arithmetic 5*. These 841 words are in italics. Children who have completed The New Basic Reading Program of the Curriculum Foundation Series, through the new *More Days and Deeds* ($5^2$), will be familiar with 210 of the 335 words not used in *Seeing Through Arithmetic 5*. Words in this list that are used in The New Basic Reading Program, through the new *More Days and Deeds* ($5^2$), are indicated by asterisks (*). For children who have used The New Basic Reading Program, only 125 words will be new in this book. Of these 125 words, 38 are used in the new *People and Progress* ($6^1$) and in *More People and Progress* ($6^2$). Children using The New Basic Reading Program will have to cope with only 88 additional words.

The numeral at the right of each word indicates the page of first occurrence. For a list of words introduced in each lesson in the pupils' book, see the Lesson Briefs for the lesson in this *Teaching Guide* (pages 22 to 269).

The following forms of known words are not counted as new: inflected forms made by adding or dropping the endings *s, es, ed, ing, n, en,* and *er, est* of comparison (including those forms made by changing *y* to *i* or *f* to *v*, by dropping the final *e*, or by doubling the final consonant in the root word); possessives, derivatives formed by adding the prefixes *a-, be-, dis-, fore-, il-, im-, in-, ir-, out-, over-, re-, un-,* or *under-*, and the suffixes *-able, -al, -ally, -ation, -en, -er, -ful, -ion, -ish, -less, -ly, -ment, -ness, -or, -ous, -ship, -ward,* or *-y*, and *-teen, -th,* or *-ty* of numerals; compounds made up of known words; contractions, nonsense words or syllables that represent sounds. Also, homographs are not counted as separate words; for example, if *pen* meaning "a writing instrument" has been introduced, *pen* meaning "an enclosure" is not counted as new.

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